Internal structure of static-light states and pionic couplings of the B meson

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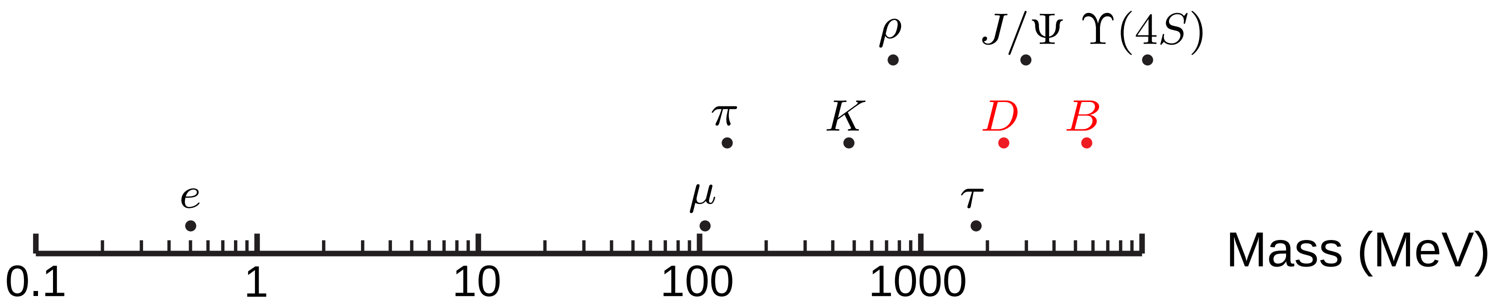
- B, D mesons phenomenology, regularization of the heavy quark
- Static-light meson density distributions to extract pionic couplings
- Multihadron states
- Outlook

[B. B., J. Bulava, M. Donnellan and A. Gérardin, PRD**87**, 9, 094518 (2013)]
[B. B. and A. Gérardin, PRD**94**, 7, 074504 (2016)]

B, D mesons phenomenology, regularization of the heavy quark

Heavy-light sector

Non exhaustive spectrum of observed charged leptons and mesons:

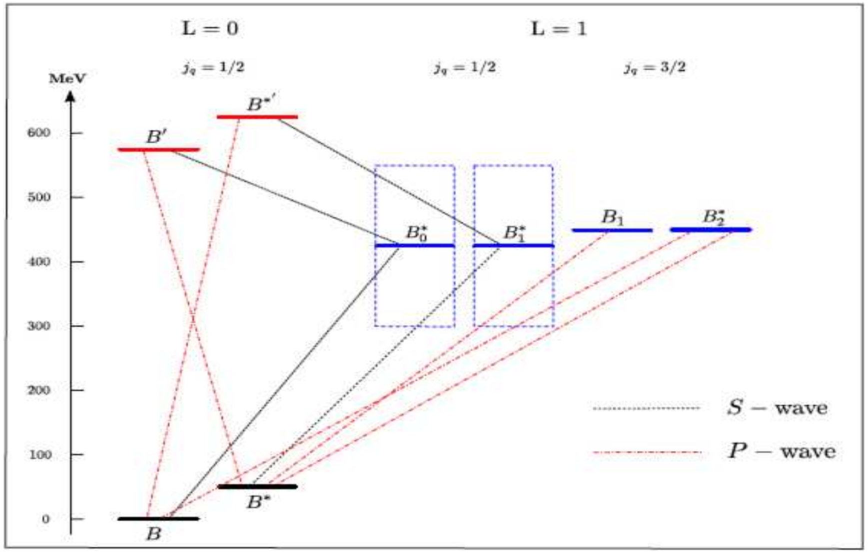


Rich B and D mesons phenomenology thanks to their high number of decay channels

Angular momentum: $J = \frac{1}{2} \oplus j_l$ Heavy-light mesons put together in doublets

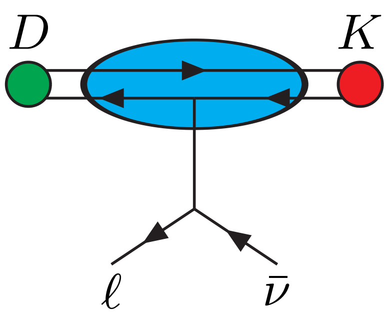
$H = B, D$:

j_l^P	J^P	orbital excitation	radial excitation
$\frac{1}{2}^-$	0^- 1^-	H H^*	H' $H^{*\prime}$
$\frac{1}{2}^+$	0^+ 1^+	H_0^* H_1^*	
$\frac{3}{2}^+$	1^+ 2^+	H_1 H_2^*	



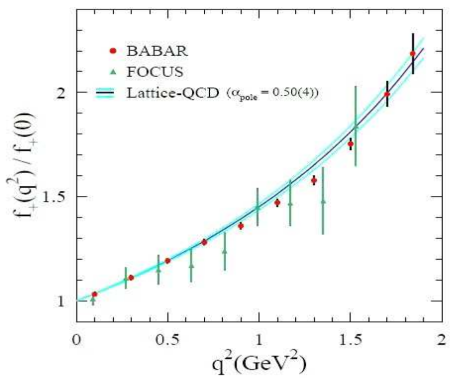
Interplay between experiment, lattice QCD and analytical methods

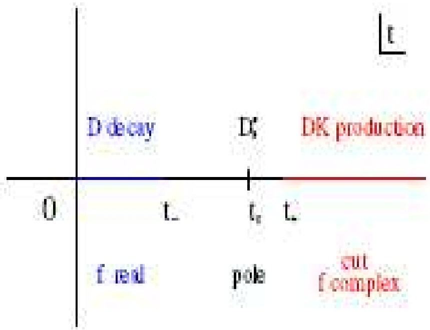
Extract form factors in particular regions of the phase space: better constrain models describing them at every q^2. Illustration on $D \to K$ semileptonic decay:



$$\langle K(p')|\bar{s}\gamma_\mu c|D(p)\rangle = \left(p_\mu + p'_\mu - q_\mu \frac{m_D^2 - m_K^2}{q^2}\right) f_+(q^2) + q_\mu \frac{m_D^2 - m_K^2}{q^2} f_0(q^2)$$

$$q = p - p' \qquad t = q^2$$





Cuts along the real axis:

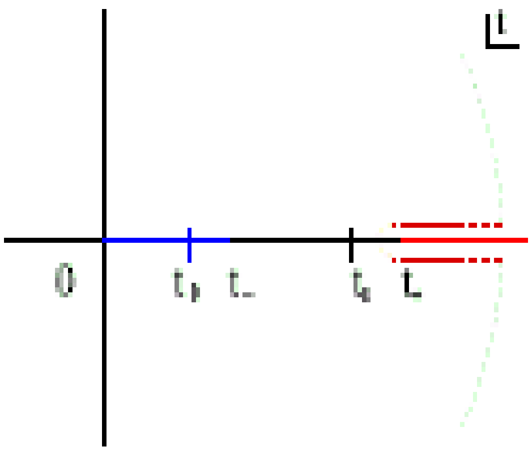
$$t_- = (m_D - m_K)^2 \qquad 1.87 \text{ GeV}^2$$
$$t_s = m_{D_s^*}^2 \qquad 4.46 \text{ GeV}^2$$
$$t_+ = (m_D + m_K)^2 \qquad 5.56 \text{ GeV}^2$$

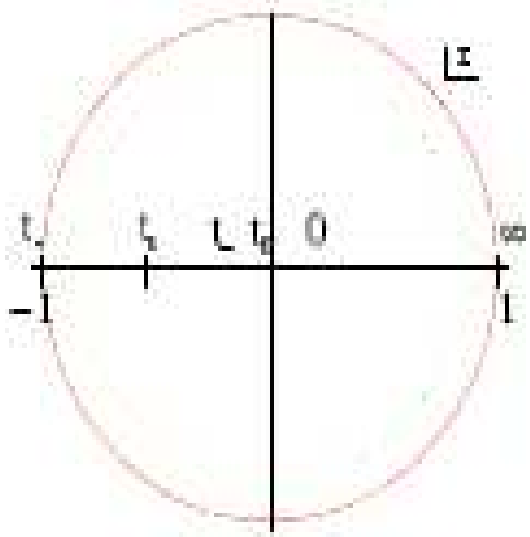
$$f_+(t) = \frac{\text{Res}(f_+)}{m_{D_s^*}^2 - t} + \frac{1}{\pi}\int_{t_+}^{\infty} dt' \frac{\text{Im}\, f_+(t')}{t' - t}, \qquad V_{cs} \text{ extracted at } f_+(0).$$

Parametrisations of f_+:

– Polynomial: $f_+(t) = \frac{f_+(0)}{1 - \frac{t}{m_{D_s^*}^2}}\left[1 + c_1 \frac{t}{m_{D_s^*}^2} + c_2 \frac{t^2}{m_{D_s^*}^4}\right]$

– Bećirević-Kaidalov: $f_+(t) = \frac{f_+(0)}{\left[1 - \frac{t}{m_{D_s^*}^2}\right]\left[1 - \alpha \frac{t}{m_{D_s^*}^2}\right]}$





conformal mapping of the variable t:

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

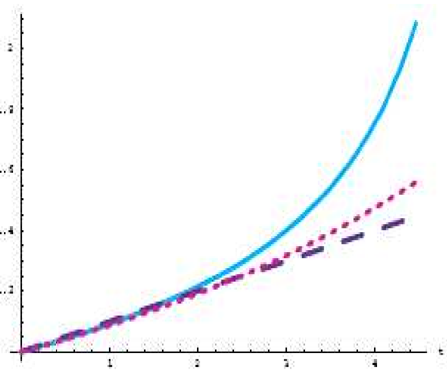
$$f_+(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k z^k$$

$$P(t) = z(t, m_{D_s^*}^2)$$

ϕ contains perturbative factors

Unitarity constraint: $\sum_{k=0}^{\infty} a_k^2 = \mathcal{O}(1)$

$\left(1 - \frac{t}{m_{D_s^*}^2}\right) \frac{f_+(t)}{f_+(0)}$ extrapolated up to D_s^* pole for various parametrisations [S. Descotes-Genon *et al*, '08]:



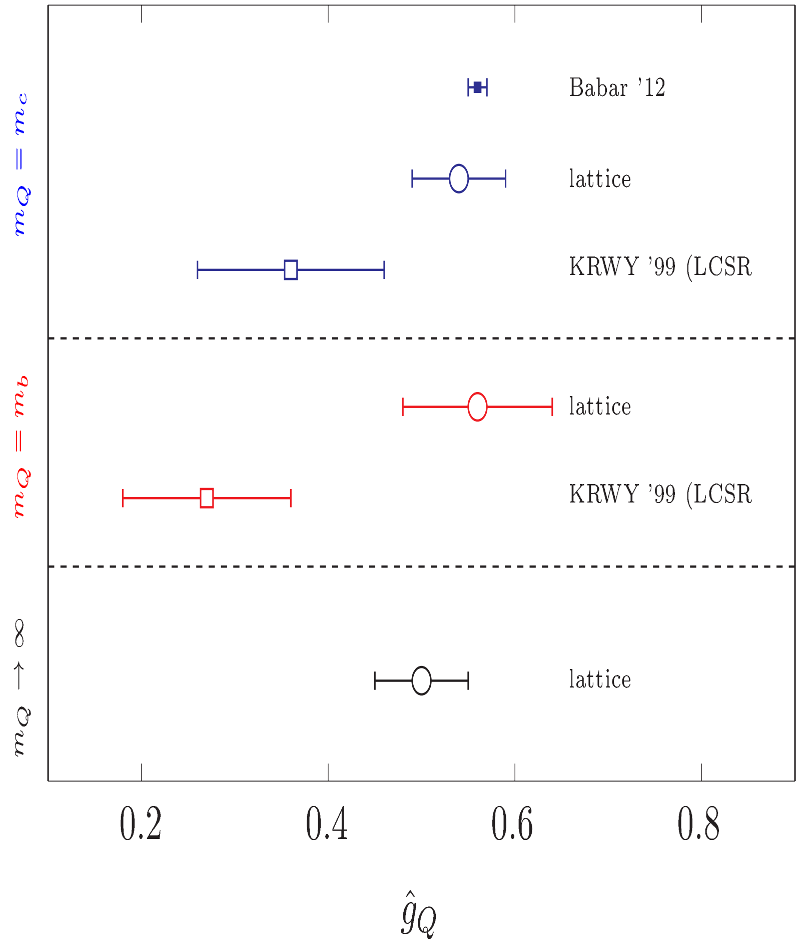
light blue: z expansion [I. Caprini *et al*, '98; C. Bourrely *et al*, '09]
dark blue: linear
purple: BK

z expansion not convergent at all outside the physical region. Spurious pole introduced in ϕ at the DK threshold.

z expansion; residue at the D_s^* pole definitely too high ($\hat{g} = 1.02!$).

$D^* \to D\pi$: an ideal process to test analytical computations based on the soft pion theorem:

$$\langle D(p')\pi(q)|D^*(p,\epsilon_\lambda)\rangle = g_{D^*D\pi}\, q\cdot\epsilon_\lambda, \qquad g_{H^*H\pi} \equiv \frac{2\sqrt{m_H m_{H^*}}\,\hat{g}_Q}{f_\pi}$$



Claim: a negative radial excitation contribution to the hadronic side of LCSR might explain the discrepancy between $g^{\rm exp}_{D^*D\pi}$ and $g^{\rm LCSR}_{D^*D\pi}$ [D. Becirevic *et al*, '03].

Without any radial excitation:

$$g_{D^*D\pi} = \frac{f(M^2)}{f_D f_{D^*}}$$

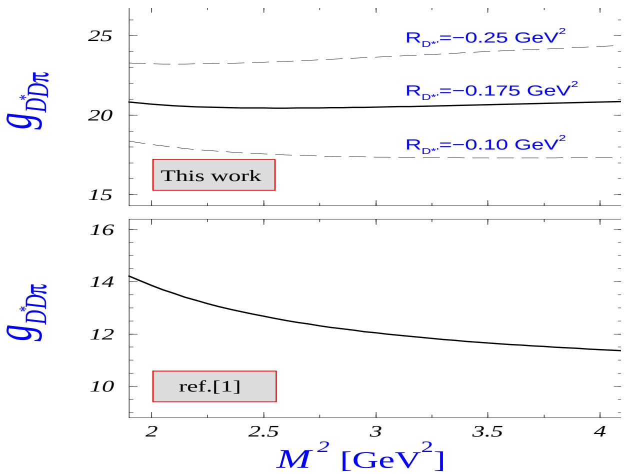
$$f(M^2) = \frac{m_c^2}{m_D^2 m_{D^*}}\ f_\pi \phi_\pi(1/2)\ M^2 \exp\left(\frac{m_D^2 + m_{D^*}^2}{2M^2}\right)\left[e^{-m_c^2/M^2} - e^{-s_0/M^2}\right] + \dots$$

With radial excitation:

$$g_{D^*D\pi} = \frac{1}{f_D f_{D^*}}\left[f(M^2) - R_{D'} \exp\left(-\frac{m_{D'}^2 - m_D^2}{2M^2}\right) - R_{D^{*\prime}} \exp\left(-\frac{m_{D^{*\prime}}^2 - m_{D^*}^2}{2M^2}\right)\right]$$

$$R_{D'} = \left(\frac{m_{D'}}{m_D}\right)^2 f_{D'} f_{D^*} g_{D^*D'\pi} \qquad R_{D^{*\prime}} = \frac{m_{D^{*\prime}}}{m_{D^*}} f_D f_{D^{*\prime}} g_{D^{*\prime}D\pi}$$

Assuming $m_{D'} = m_{D^{*\prime}}$, $f_{D'} = f_{D^{*\prime}}$ and $g_{D^*D'\pi} = g_{D^{*\prime}D\pi} = g'$: $\frac{R_{D'}}{R_{D^{*\prime}}} = \frac{m_{D^{*\prime}} m_{D^*}}{m_D^2} \frac{f_{D^*}}{f_D}$



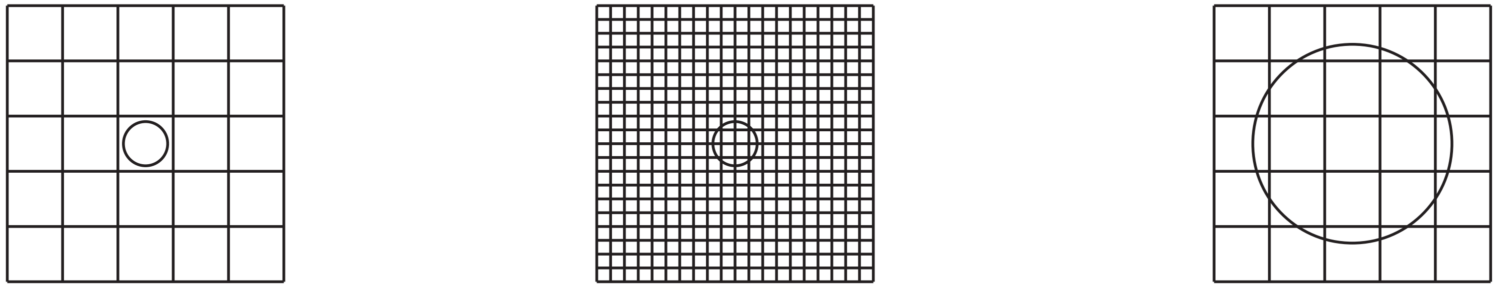
$$-0.25\ \text{GeV}^2 < R_{D^{*\prime}} < -0.1\ \text{GeV}^2$$
$$\Longrightarrow 17\ <\ g_{D^*D\pi}\ <\ 25$$

Much better stability in Borel window

Our proposal: check on the lattice that statement in the heavy quark limit.

Heavy quark on the lattice

Systematics coming from potentially large discretisation effects ($\Lambda_{\mathrm{Compt}} \sim 1/m_Q$).



Cut-off Effects cut-off effects cut-off effects

Several strategies are proposed in the literature to deal with those cut-off effects:

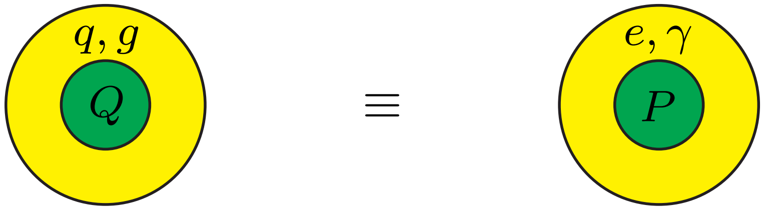
- Use NRQCD to describe the heavy quark [P. Lepage and B. Thacker, '91]; though, no continuum limit when the theory is regularised on the lattice
- Define an action with counterterms that are tuned to get $\mathcal{O}(a)$, $\mathcal{O}(am_Q)$ and $\mathcal{O}(\alpha_s am_Q)$ improvements [A El Khadra *et al*, '96; N. Christ *et al*, '06]
- Computation within Heavy Quark Effective Theory: approach chosen by the ALPHA Collaboration [J. Heitger and R. Sommer, '03]
- Computation within QCD: use of the HQET scaling laws to interpolate easily a quantity between m_c and the (exactly known) $m_Q \to \infty$ limit [B. B. *et al*, '09]

Heavy Quark Effective Theory

Effective theory "derived" by expanding in $\frac{\Lambda_{QCD}}{m_Q}$ the Lagrangian and currents of QCD.

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(iv\cdot D)h_v + \mathcal{O}(1/m_Q) \equiv \mathcal{L}^{\rm stat}_{\rm HQET} + \mathcal{O}(\Lambda_{\rm QCD}/m_Q) \quad p_Q = m_Q\, v + k$$

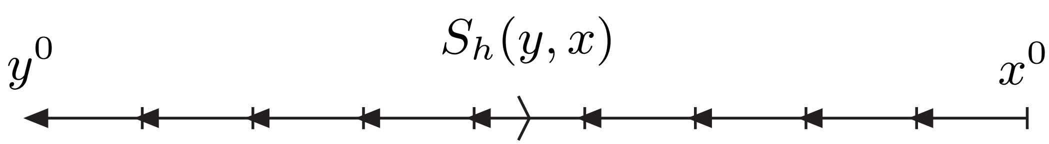
Symmetry SU(2$\mathrm{N_h}$) for $\mathcal{L}^{\rm stat}_{\rm HQET}$: flavor $\times$ spin



Heavy-light meson   Atom of hydrogen

$L^{\rm stat,\, lat}_{\rm HQET}$: $S_h = a^3 \sum_x \bar{h}(x)\left[h(x) - \mathcal{U}_0^\dagger(x-\hat{0})h(x-\hat{0})\right]$

Static propagator: $\mathcal{S}_h(y,x) = \delta_{\vec{x}\vec{y}}\frac{1+\gamma^0}{2}P_{\vec{x}}(y^0,x^0), \quad P_{\vec{x}}(y^0,x^0) = \prod_{x^0-\hat{0}}^{y^0} \mathcal{U}_0^\dagger(\vec{x},t)$



Static-light meson density distributions to extract pionic couplings

Transition amplitude under interest, with $q = p' - p$, $\mathcal{A}^\mu = \bar{d}\gamma^\mu\gamma_5 u$,
$T^{mn\,\mu} = \langle B_m(p)|\mathcal{A}^\mu|B_n^*(p',\lambda)\rangle$ and $\epsilon_\perp^\mu(p',\lambda) = \epsilon(p',\lambda)^\mu - \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu$:

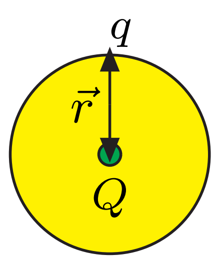
$$T^{mn\,\mu} = 2m_{B_n^*} A_0^{mn}(q^2)\frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu + (m_{B_m} + m_{B_n^*}) A_1^{mn}(q^2)\epsilon_\perp^\mu(p',\lambda)$$
$$+ A_2^{mn}(q^2)\frac{\epsilon(p',\lambda)\cdot q}{m_{B_m} + m_{B_n^*}}\left[(p+p')^\mu + \frac{m_{B_m}^2 - m_{B_n^*}^2}{q^2} q^\mu\right]$$

With $\langle B_m(p)|q_\mu\mathcal{A}^\mu|B_n^*(p',\lambda)\rangle = 2\,m_{B_n^*}\,A_0^{mn}(q^2)\;q\cdot\epsilon(p',\lambda)$, PCAC relation, LSZ reduction formula and $\sum_\lambda \epsilon_\mu(k,\lambda)\,\epsilon_\nu^*(k,\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$:

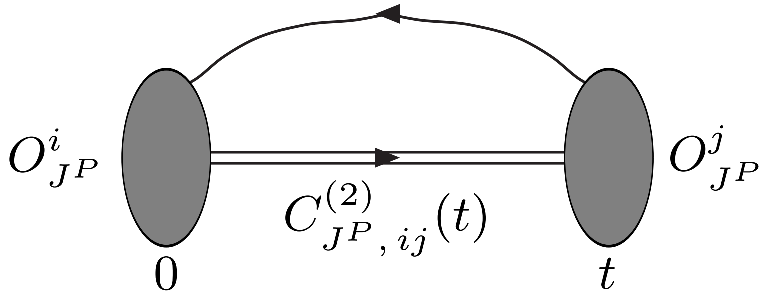
$$g_{H_n^* H_m \pi} = \frac{2\,m_{H_n^*} A_0^{mn}(0)}{f_\pi}\,,\; A_0^{mn}(q^2) = -\sum_\lambda \frac{\langle H_m(p)|q_\mu\mathcal{A}^\mu|H_n^*(p',\lambda)\rangle}{2m_{H_n^*}\,q_i}\,\epsilon_i^*(p',\lambda)$$

Back to the x space: $A_0^{mn}(q^2=0) = -\frac{q_0}{q_i}\int d^3r\, f_{\gamma_0\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}} + \int d^3r\, f_{\gamma_i\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}$

Axial density distributions $f_{\gamma_\mu\gamma_5}^{mn}(r)$ defined
in terms of 2-pt and 3-pt HQET correlation functions



Variational method: define an operator $O^n_{J^P}$ weakly coupled to other states than $|n\rangle$
[C. Michael, '85] [M. Lüscher and U. Wolff, '90] [B. B. *et al*, '09]



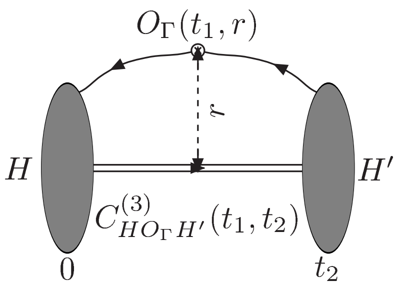
– Compute an $N \times N$ matrix of correlators

$$C^{(2)}_{P(V),\,ij}(t) = \sum_{\vec{x},\vec{y}} \langle \Omega | \mathcal{T}[O^i_{P(V)}(\vec{x},t) O^j_{P(V)}(\vec{y},0)] | \Omega \rangle$$

$$O^i_{P(V)}(\vec{x},t) = \sum_{\vec{z}} \bar{q}(\vec{x},t)[\Gamma \times \Phi(|\vec{x}-\vec{z}|)]^i_{P(V)} q(\vec{z},t)$$

– Solve the generalised eigenvalue problem

$$C^{(2)}_{P(V)}(t)\, v_{P(V),\,n}(t,t_0) = \lambda_{P(V),\,n}(t,t_0)\, C^{(2)}_{P(V)}(t_0)\, v_{P(V),\,n}(t,t_0), \;\; \lambda_n(t,t_0) \sim e^{-E_n\,(t-t_0)}$$



Two ratio methods, GEVP and sGEVP [J. Bulava *et al*, '11], to extract $f^{mn}_{\gamma_\mu\gamma_5}(r)$:

$$\mathcal{R}^{\mathrm{GEVP}}_{mn}(t,t_1;r) = \left(v_m(t_2,t_0), C^{(3)}_{\gamma_\mu\gamma_5}(t_1+t_2,t_1;r)\, w_n(t_1,t_0)\right) G_n(t_1) G'_m(t_2)$$

$$= f^{(mn)}_{\gamma_\mu\gamma_5}(r) + \mathcal{O}\left(e^{-\Delta_{N+1,m} t_2}, e^{-\Delta_{N+1,n} t_1}\right), \Delta_{nm} = E_n - E_m$$

$$G_n(t) = \frac{\widetilde{\lambda}_n(t_0+1,t_0)^{-t/2}}{\left(w_n(t_1,t_0), C^{(2)}_V(t_1) w_n(t_1,t_0)\right)^{1/2}},\; G'_n(t) = \frac{\lambda_n(t_0+1,t_0)^{-t/2}}{\left(v_n(t_1,t_0), C^{(2)}_P(t_1) v_n(t_1,t_0)\right)^{1/2}}$$

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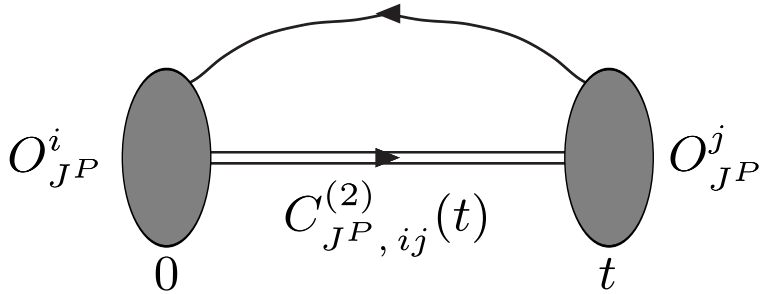

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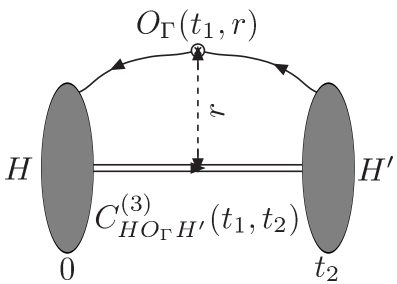
$$O^i_{P(V)}(\vec{x},t) = \sum_{\vec{z}} \bar{q}(\vec{x},t) [\Gamma \times \Phi(|\vec{x}-\vec{z}|)]^i_{P(V)} q(\vec{z},t)$$

– Solve the generalised eigenvalue problem

$$C^{(2)}_{P(V)}(t)\, v_{P(V),\,n}(t,t_0) = \lambda_{P(V),\,n}(t,t_0)\, C^{(2)}_{P(V)}(t_0)\, v_{P(V),\,n}(t,t_0), \;\; \lambda_n(t,t_0) \sim e^{-E_n\,(t-t_0)}$$



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$$\mathcal{R}^{\mathrm{sGEVP}}_{mn}(t,t_0;r) = -\partial_t \left(\frac{|(v_m(t,t_0), K'(t,t_0;r) w_n(t,t_0))|\, e^{\Sigma_{mn}(t_0,t_0)t_0/2}}{\sqrt{[v_m(t,t_0), C^{(2)}_P(t_0) v_m(t,t_0)][w_n(t,t_0), C^{(2)}_V(t_0) w_n(t,t_0)]}} \right)$$

$$\mathcal{R}^{\mathrm{sGEVP}}_{mn}(t,t_0;r) = \underbrace{f^{(mn)}_{\gamma_\mu\gamma_5}(r) + \mathcal{O}\left(t\, e^{-\Delta_{N+1,n} t}\right)}_{n>m}, \;\; \underbrace{f^{(mn)}_{\gamma_\mu\gamma_5}(r) + \mathcal{O}\left(e^{-\Delta_{N+1,m} t}\right)}_{n<m}$$

$$\Sigma_{mn}(t,t_0) = E_n(t,t_0) - E_m(t,t_0),\; K'(t,t_0,r) = K(t,t_0;r)/\widetilde{\lambda}_n(t,t_0) - K(t_0,t_0;r),$$

$$K(t,t_0;r) = \sum_{t_1} e^{-(t-t_1)\Sigma_{mn}(t,t_0)} C^{(3)}_{\gamma_\mu\gamma_5}(t,t_1;r)$$

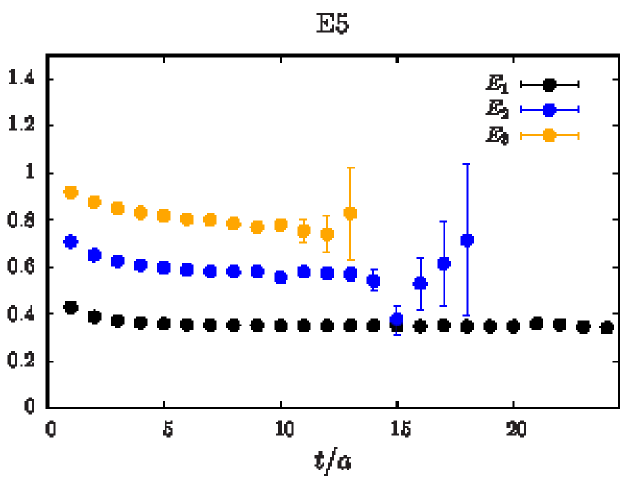
Results

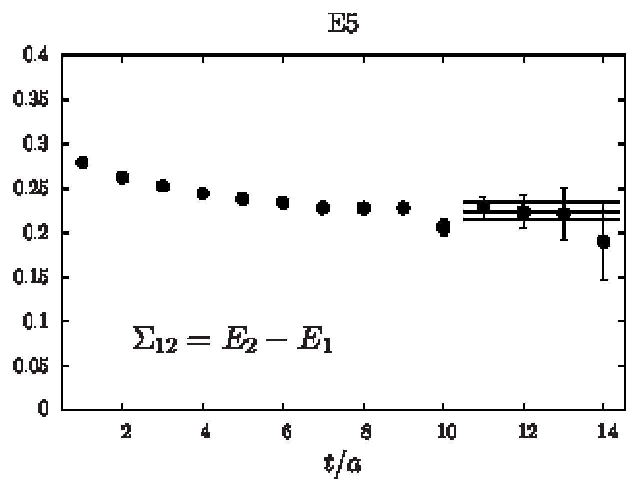
Lattice set-up: $\mathcal{O}(a)$ improved Wilson-Clover (light quark), HYP2 (static quark)

CLS based

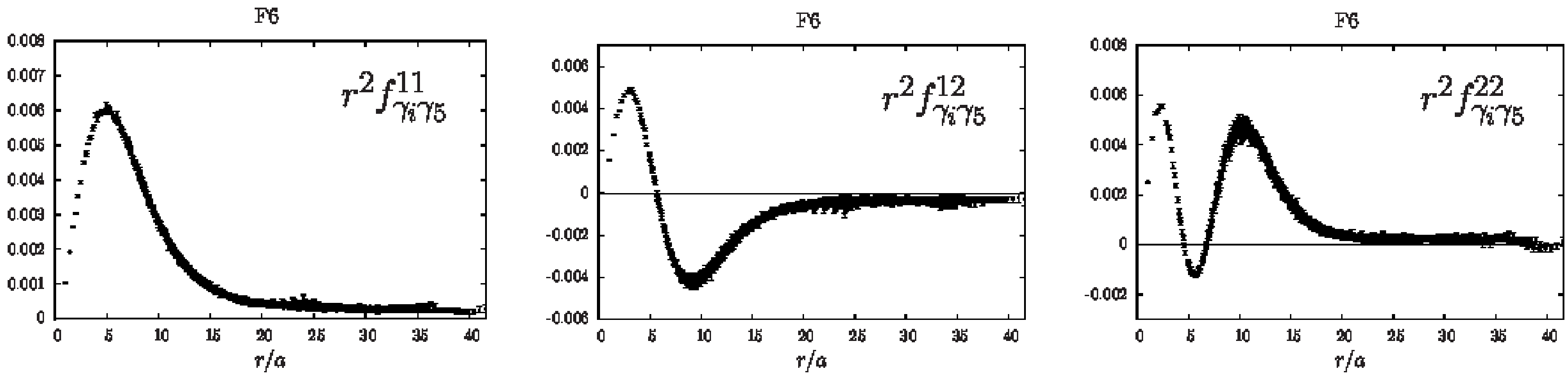
lattice	β	$L^3 \times T$	a[fm]	m_π[MeV]	Lm_π
A5	5.2	$32^3 \times 64$	0.075	330	4
B6		$48^3 \times 96$		280	5.2
D5	5.3	$24^3 \times 48$	0.065	450	3.6
E5		$32^3 \times 64$		440	4.7
F6		$48^3 \times 96$		310	5
N6	5.5	$48^3 \times 96$	0.048	340	4
Q1	6.2885	$24^3 \times 48$	0.06	-	-
Q2	6.2885	$32^3 \times 64$	0.06	-	-

Basis of interpolating fields (4×4 matrix of correlators) large enough to well isolate the ground state and the first excited state.





Spatial component of the axial density distributions: data can be exploited

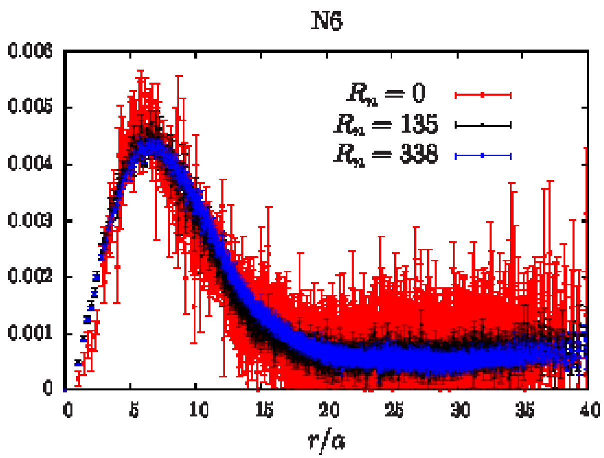


$f^{11}_{\gamma_i\gamma_5}(r)$: positive everywhere; $f^{12}_{\gamma_i\gamma_5}(r)$: there is a node; $f^{22}_{\gamma_i\gamma_5}(r)$: almost positive, negative part interpreted by relativistic effects

Issues: densities do not vanish at large r, curves are not smooth in r

Need to investigate several sources of systematics

- Contamination from excited states, test different interpolating fields

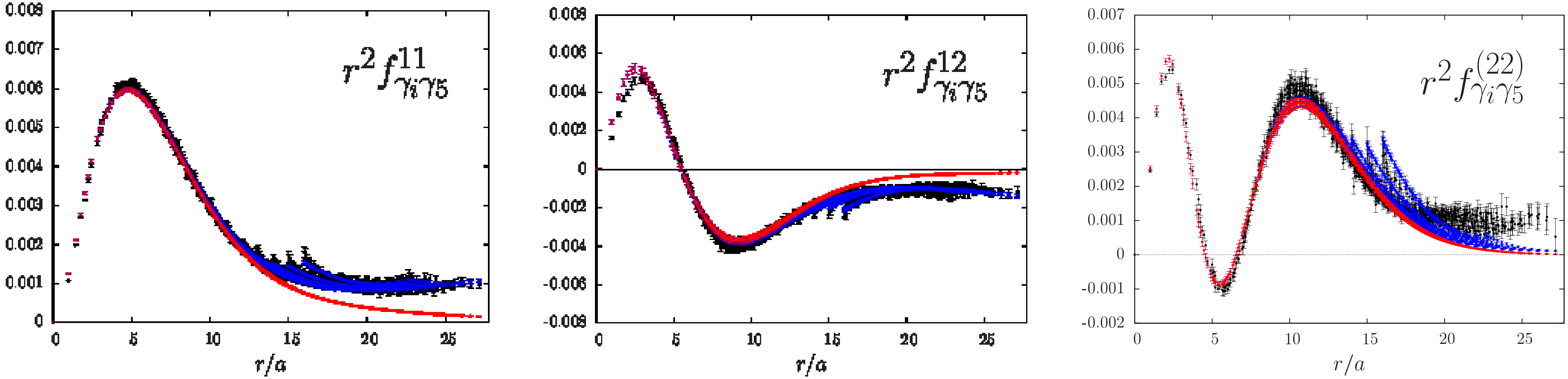


- Cut-off effects

Smoothen the fishbone structure [C. Roiesnel and F. de Soto, ’07; B. B. *et al*, ’11].
Different lattice points (r_1, r_2, r_3) can have the same r^2 but different $r^{[4]} \equiv \sum_{i=1}^{3} r_i^4$.
Preferred fit form: $a^3 f_\alpha(r^2, r^{[4]}, r^{[6]}) = a^3 f_\alpha(r^2, 0, 0) + A \times \frac{a^2 r^{[4]}}{r^6} + B \times \frac{a^2 r^{[6]}}{r^8}$, $f_\alpha(r^2, 0, 0)$,
A and B are fit parameters. Small impact of cut-off effects at large r.

- Finite volume effects

With periodic boundary conditions in space, $a^3 f^{\mathrm{lat}}_{\gamma_i\gamma_5}(\vec{r}) = \sum_{\vec{n}} a^3 \widetilde{f}_{\gamma_i\gamma_5}(\vec{r} + \vec{n}L)$, $n_i \in \mathbb{Z}$.
$\widetilde{f}_{\gamma_i\gamma_5}(\vec{r})$ can still differ from $f_{\gamma_i\gamma_5}(\vec{r})$ due to interactions among periodic images.
Assumptions: no interaction among images, $n_i = 0, -1$ taken into account.
Fit form: $f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = P_{mn}(r)\,\exp\left(-r/r_0\right)$, P_{mn} is a polynomial.



Distributions vanish at large r; good fit, nothing relevant if terms like $\exp(-(r/r_1)^\beta)$ added

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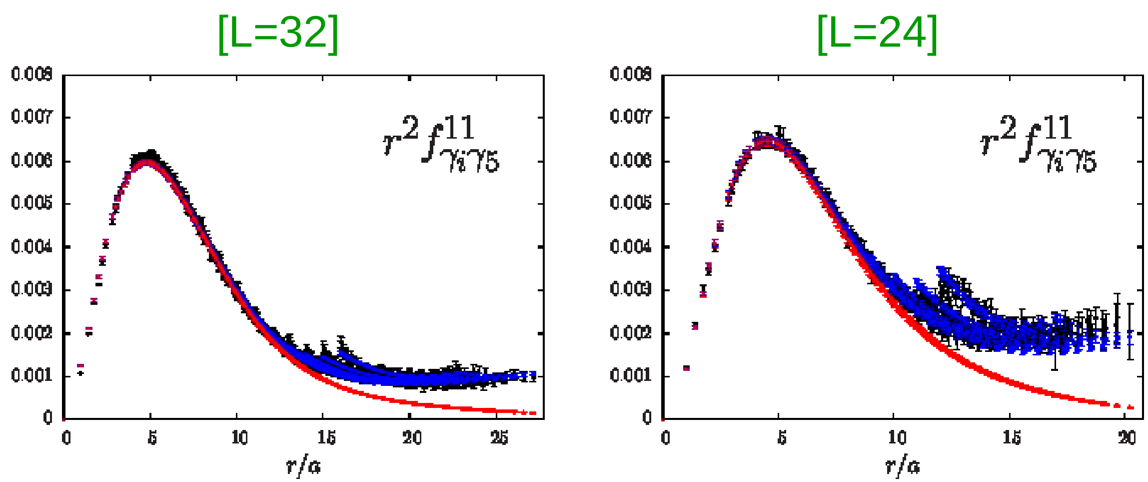
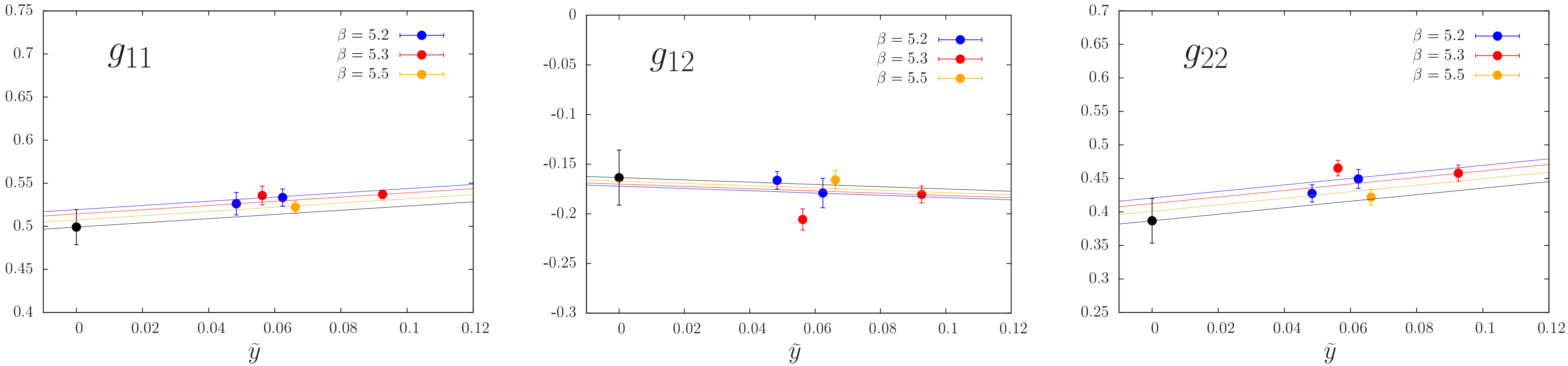
With periodic boundary conditions in space, $a^3 f^{\text{lat}}_{\gamma_i\gamma_5}(\vec{r}) = \sum_{\vec{n}} a^3 \widetilde{f}_{\gamma_i\gamma_5}(\vec{r} + \vec{n}L)$, $n_i \in \mathbb{Z}$.
$\widetilde{f}_{\gamma_i\gamma_5}(\vec{r})$ can still differ from $f_{\gamma_i\gamma_5}(\vec{r})$ due to interactions among periodic images.
Assumptions: no interaction among images, $n_i = 0, -1$ taken into account.
Fit form: $f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = P_{mn}(r)\ \exp\left(-r/r_0\right)$, P_{mn} is a polynomial.

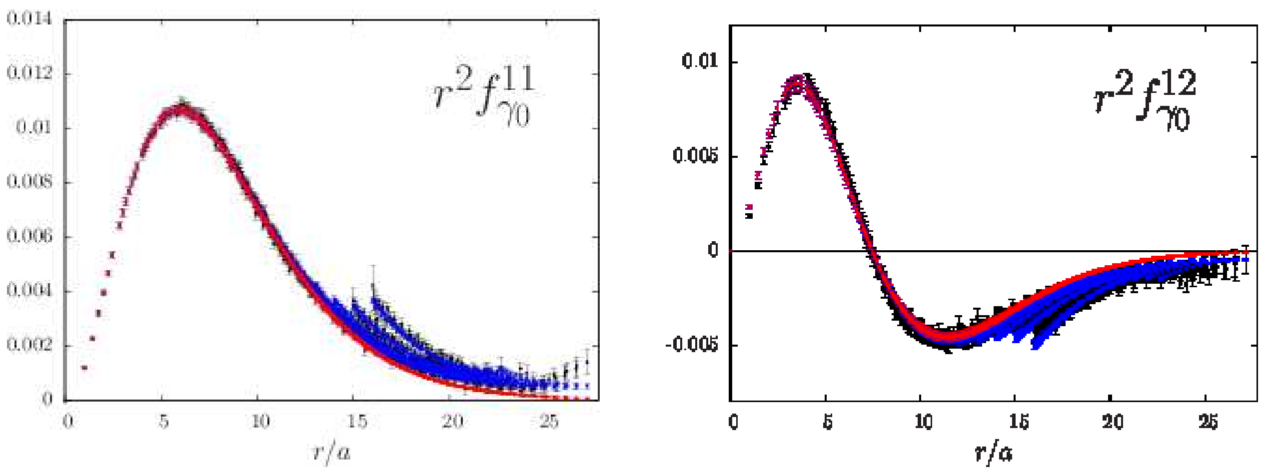


Fit parameters of data ($L = 32$) describe also data ($L = 24$).

Summation over r of $f^{mn}_{\gamma_i\gamma_5}(r)$ to get g_{mn}. After renormalisation, a continuum and chiral extrapolation is possible: $\overline{g}_{nm}(a, m_\pi) = \overline{g}_{nm} + C_1\,a^2 + C_2\,m_\pi^2/\left(8\pi f_\pi^2\right)$

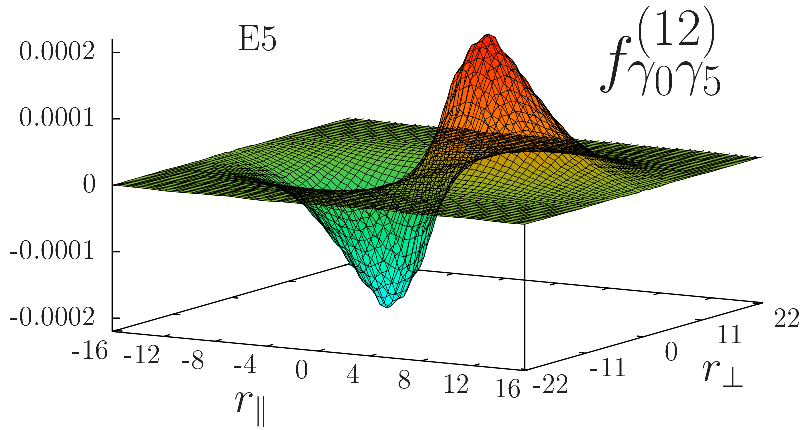


Technique employed also for the charge density distribution $f^{mn}_{\gamma_0}(r)$



Including Z_V, $\int dr\, r^2\, f^{11}_{\gamma_0}(r)$ compatible with 1. $\int dr\, r^2 f^{12}_{\gamma_0}(r)$ compatible with 0.

Time component of the axial density distribution: systematics more tricky to estimate

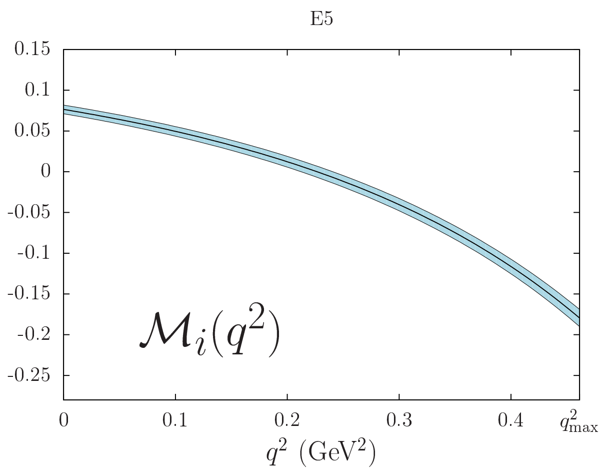


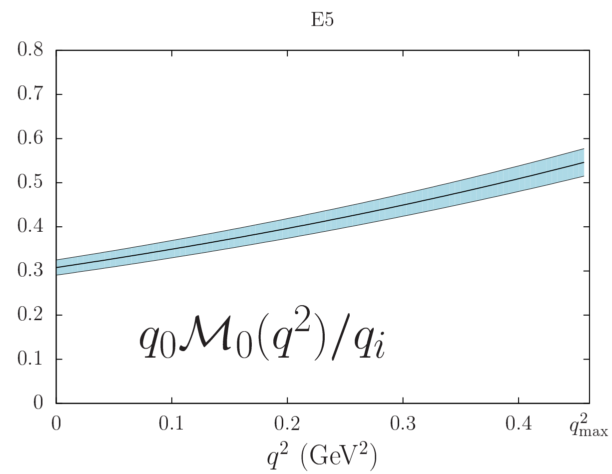
Distribution odd in $r_\parallel$, along the vector meson polarisation

Matrix elements obtained at q after a Fourier transform of the distributions to get $g_{B^{*\prime} B\pi}$

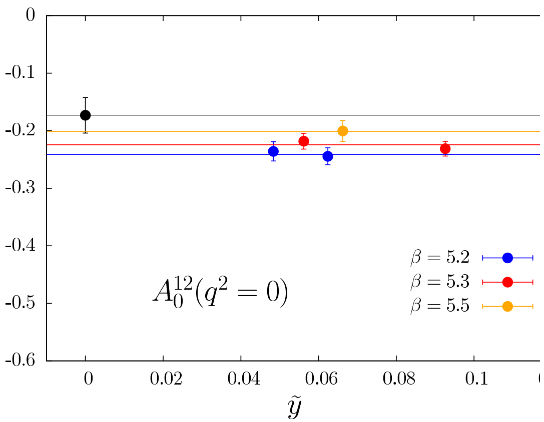
$$\mathcal{M}_i(q^2_{\rm max} - \vec{q}^{\,2}) = 4\pi \int_0^\infty \mathrm{d}r\, r^2\, \frac{\sin(|\vec{q}|r)}{|\vec{q}|r} f^{(12)}_{\gamma_i\gamma_5}(\vec{r})$$

$$\frac{q_0}{q_i}\mathcal{M}_0(q^2_{\rm max} - \vec{q}^{\,2}) = -q_0 4i\pi \int_0^\infty \mathrm{d}r_\parallel \int_0^\infty \mathrm{d}r_\perp\, r_\perp\, f^{(12)}_{\gamma_0\gamma_5}(r_\parallel, r_\perp)\, \frac{\sin(|\vec{q}|\, r_\parallel)}{|\vec{q}|}$$





Lattice results and comparison with quark models (*à la* Bakamjian-Thomas/Godfrey-Isgur, Dirac) [A. Le Yaouanc, orivate communication]



Extrapolation of $A_0^{12}(q^2=0)$ to the physical point:
$A_0^{12}(0, m_\pi^2) = D_0 + D_1 a^2 + D_2 m_\pi^2/(8\pi f_\pi^2)$

Normally, $\mathcal{O}(a)$ effects at $q^2 \neq q^2_{\mathrm{max}}$, not visible in our data.

$$A_0^{12}(0) = -0.173(31)_{\mathrm{stat}}(16)_{\mathrm{syst}}, \quad g_{B^{*\prime}B\pi} = -15.9(2.8)_{\mathrm{stat}}(1.4)_{\mathrm{syst}}$$

Quenched result ($m_q = m_s$): $A_0^{12}(0) = -0.143(14)$

	Latt		BT		D	
q^2	q^2_{max}	0	q^2_{max}	0	q^2_{max}	0
$q_0\mathcal{M}_0(q^2)/q_i$	$0.402(54)(27)$	$0.237(27)(28)$	0.252	0.173	0.219	0.164
$\mathcal{M}_i(q^2)$	$-0.172(16)(6)$	$0.064(9)(13)$	-0.103	0.05	-0.223	-0.056

Lattice: $q_0 = 0.701(65)$ GeV
Bakamjian-Thomas with Godfrey-Isgur potential: $q_0 = 0.538$ GeV
Dirac: $q_0 = 0.576$ GeV
global sign of hadronic matrix elements fixed with conventions $f_B > 0$ and $f_{B^{*\prime}} > 0$

Qualitative agreement between lattice and quark models: $q_0\mathcal{M}_0/q_i$ dominates in $A_0^{12}(q^2)$ and explains why $A_0^{12}(q^2=0) < 0$.

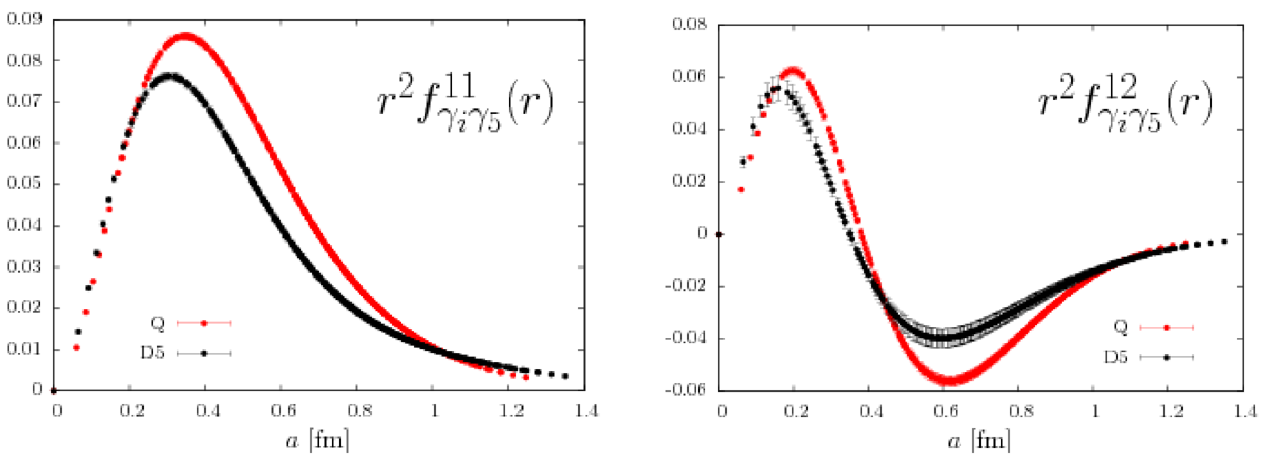
Multihadron states

A possible unpleasant systematics of our results is an uncontrolled mixing between radial excitations ($B^{*'}$) and multihadron states ($B_1^*\pi$ in S wave) close to threshold.

$$\delta = m_{B_1^*} - m_B$$

lattice	$a\Sigma_{12}$	$a\delta + am_\pi$
A5	$0.253(7)$	$0.281(4)$
B6	$0.235(8)$	$0.248(4)$
E5	$0.225(10)$	$0.278(6)$
F6	$0.213(11)$	$0.233(3)$
N6	$0.166(9)$	$0.176(3)$

Comparison with quenched data: behaviour of $f^{11}_{\gamma_i\gamma_5}$ and $f^{12}_{\gamma_i\gamma_5}$ similar



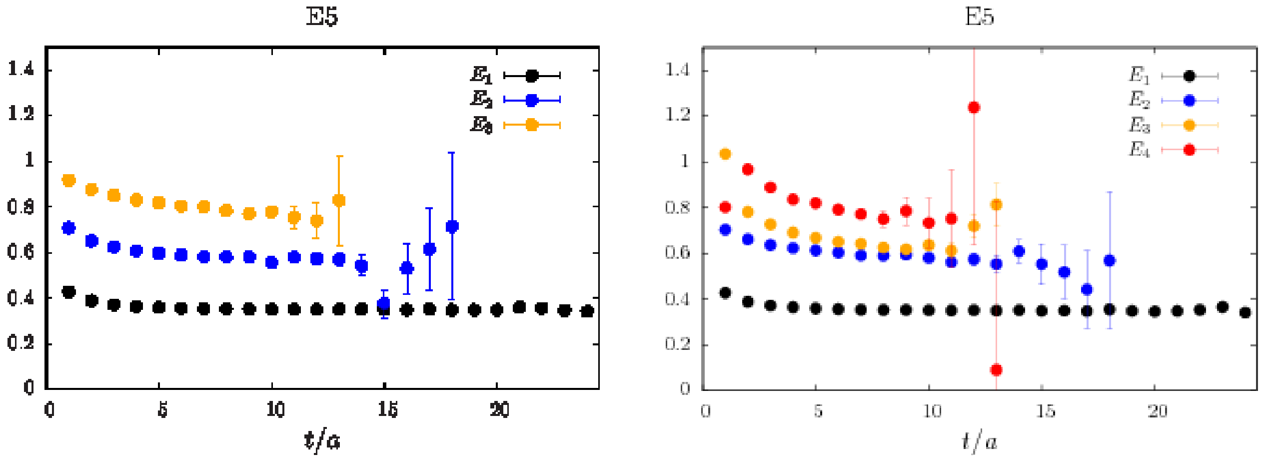
At $\mathrm{N_f} = 2$, position of the node of $f^{12}_{\gamma_i\gamma_5}$ weakly dependent of m_π in the range we have considered

lattice	m_π [MeV]	r_n^{12} [fm]
A5	330	$0.369(13)$
B6	280	$0.374(12)$
E5	440	$0.369(11)$
F6	310	$0.379(20)$
N6	340	$0.365(12)$

Change observed when $\bar{q}\nabla_k h$ is included in addition to $\bar{q}\gamma_k h$ to couple to $B^{*'}$

$\bar{q}\gamma_k h$ only

with $\bar{q}\nabla_k h$



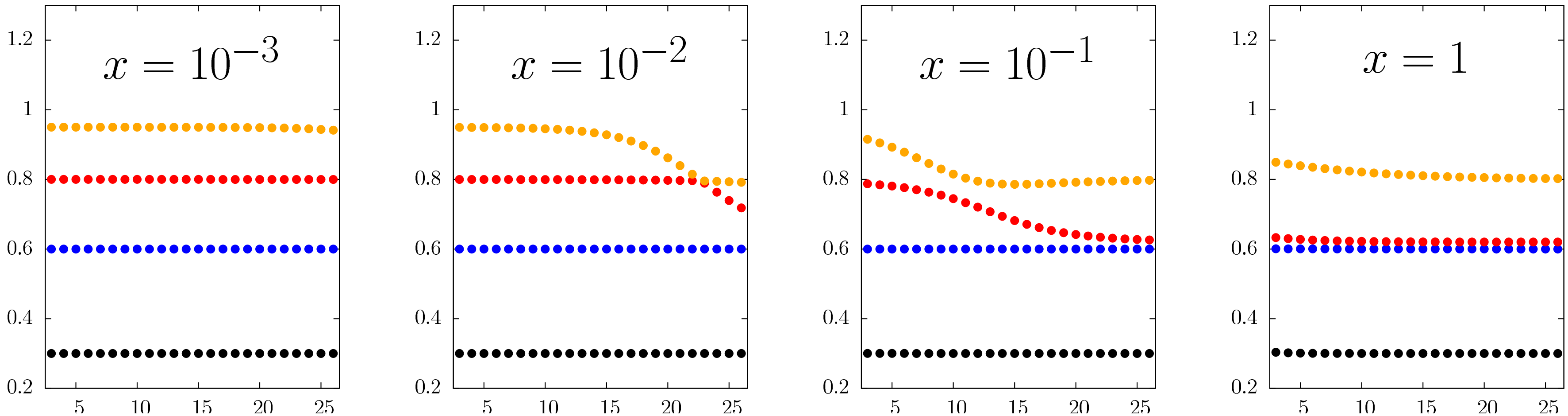
A new state, not seen before, is present in the spectrum close to the first excited state.

A toy model with 5 states in the spectrum to understand this fact:

spectrum
0.3
0.6
0.63
0.8
0.95

Matrix of couplings

$$\begin{bmatrix} 0.6 & 0.25 & x \times 0.4 & 0.1 & 0.5 \\ 0.61 & 0.27 & x \times 0.39 & 0.11 & 0.51 \\ 0.58 & 0.24 & x \times 0.42 & 0.12 & 0.52 \\ 0.57 & 0.25 & x \times 0.41 & 0.1 & 0.49 \\ 0.56 & 0.26 & x \times 0.36 & 0.08 & 0.48 \end{bmatrix}$$



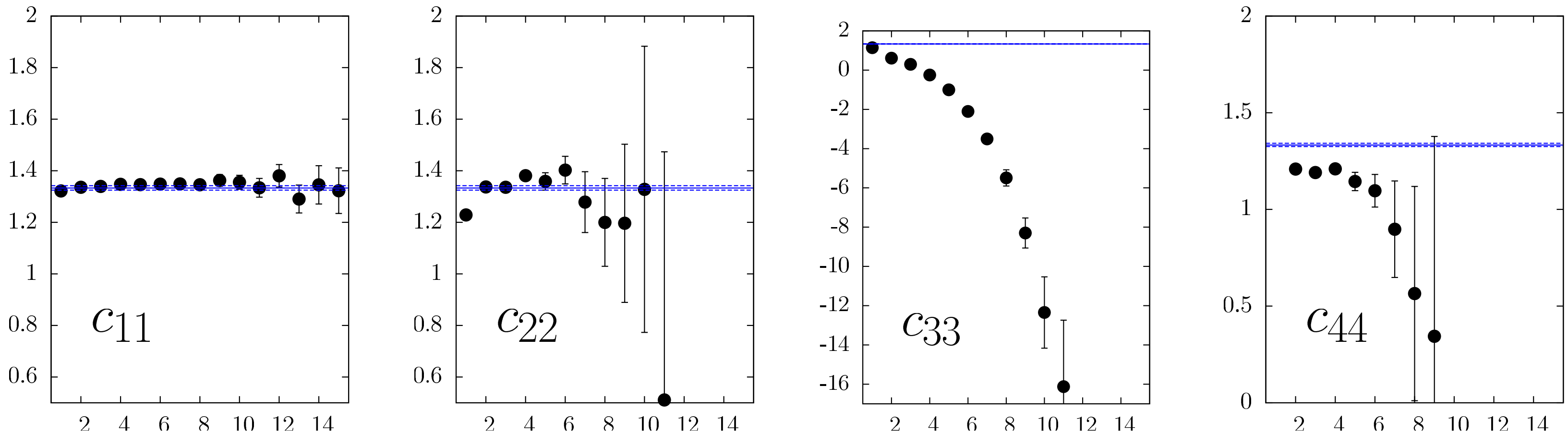
$x \ll 1$: GEVP isolates states 1, 2, 4 and 5; $x \to 1$, GEVP isolates states 1, 2, 3 and 4

A GEVP can "miss" an intermediate state of the spectrum if, by accident, the coupling of the interpolating fields to that state is suppressed.

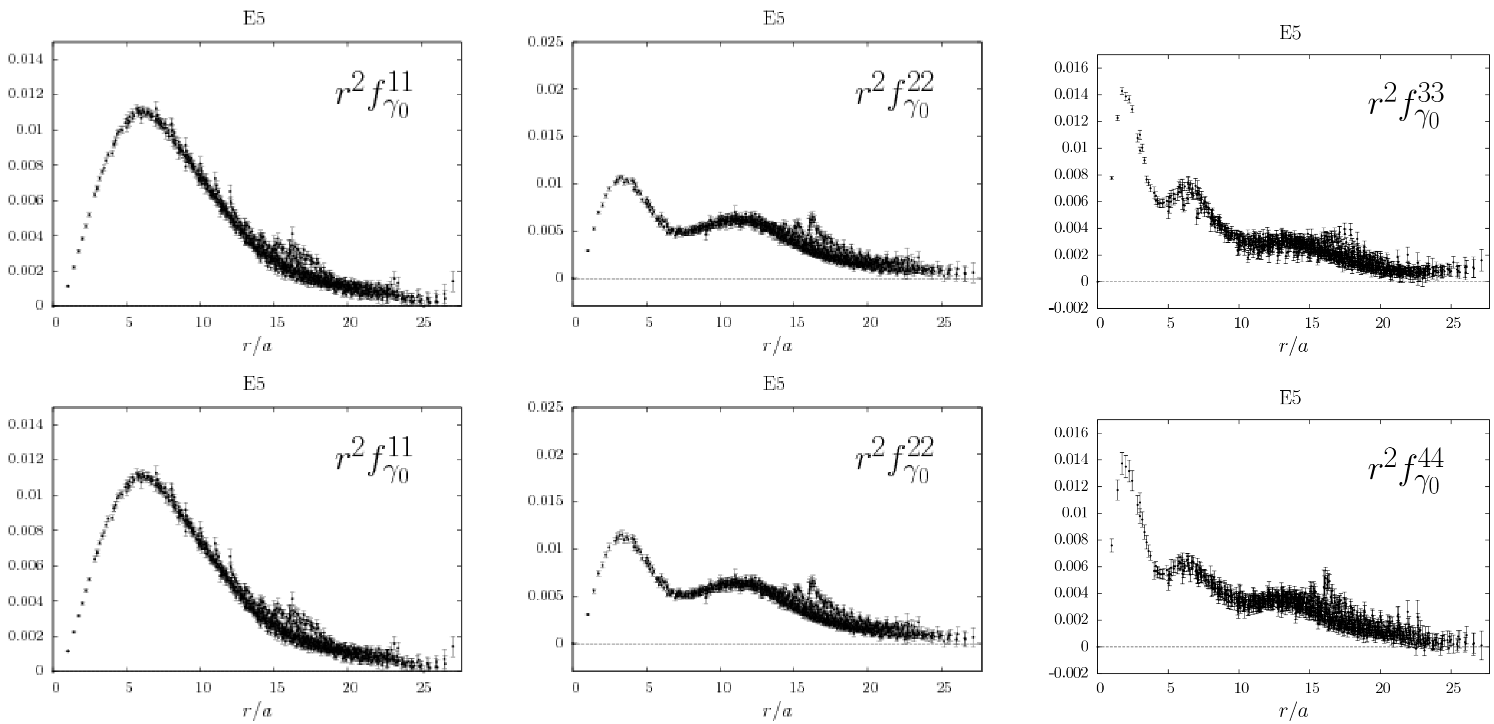
Our claim: using interpolating fields $\bar{q}\gamma_k h$, no chance to couple to multi-hadron states while inserting an operator $\bar{q}\nabla_k h$ may isolate the $B_1^*\pi$ two-particle state.

Clues come from density distributions obtained with that interpolating field.

Conservation of vector charge: not verified in the case of second excited state if the basis of interpolating fields incorporates $\bar{q}\nabla_k h$.



Including or not $\bar{q}\nabla_k h$ does not change the profile of $f^{11}_{\gamma_0}$ nor $f^{22}_{\gamma_0}$: it does in the case of $f^{33}_{\gamma_0}$.



Outlook

- Excited meson states are massively produced in experiments. To exploit fruitfully the numerous data at Super Belle and LHCb, theorists do have to put an important effort in confronting their models predictions with measurements, by proposing to experimentalists unambiguous observables to look at.

- Extract density distributions of the B meson is beneficial to get the form factors at $q^2 = 0$ associated to pionic couplings. Lattice computations allow a detailed comparison with quark models.

- Density distributions may be a check of the absence of any unwanted coupling between a given interpolating field of the B meson and a multihadron state $B\pi$.

- Other possible applications: determine whether exotic X, Y, Z are molecules or tetraquarks, estimate of the QED correction to f_B and f_D.