Gauge theories, quantum magnets and 't Hooft anomaly matching

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Based on:

M. Anber, E. Poppitz, TS,
PRD 2015, 1501.06773

TS, H. Shao, A.W. Sandvik, M. Ünsal
PRL 2017, 1608.09011

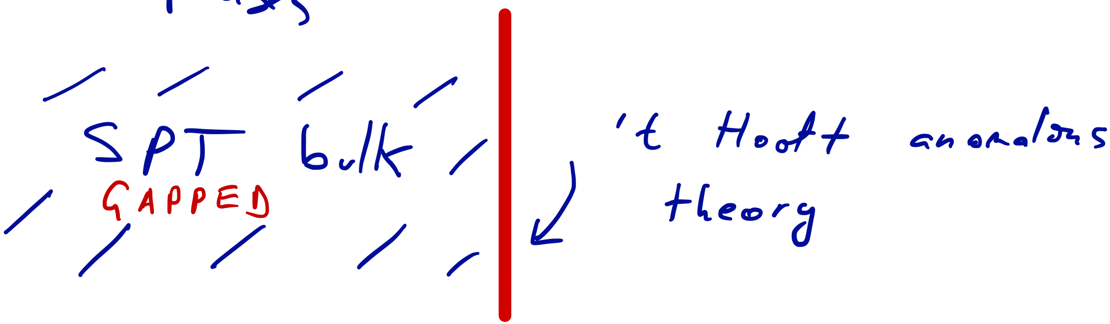
Komargodski, TS, Ünsal
PRB 2018, 1706.05731

TS, Y. Tanizaki
PRD 2018, 1802.02153

Y. Tanizaki, TS
1805.11423

't Hooft anomaly inflow

- 't Hooft anomaly is a property whereby ~~good~~ global symmetries cannot be promoted to gauge symmetries unless non-local terms are introduced into the theory.
- Such systems can be seen as boundary theories of a gapped phase called the SPT phases



When gauging the D-dimensional boundary theory we must introduce the gauge fields A. If however we have that the gauge variation of the action does not vanish, i.e.

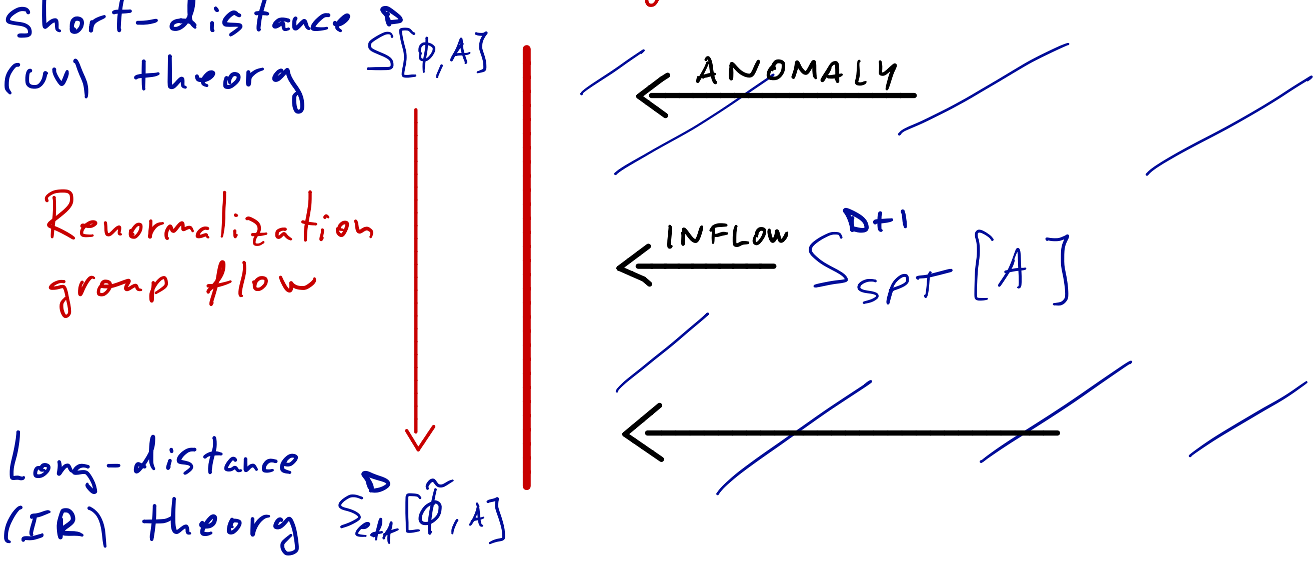
$$\delta S^D = S^D(\phi, A+d\lambda) - S^D(\phi, A) \neq 0$$

we say that there is a 't Hooft anomaly.

In order to gauge the theory we must extend the system to an extra dimension and define an SPT action $S^{D+1}_{SPT}(A)$ such that

$$S^D(\phi, A) + S^{D+1}_{SPT}(A) \text{ - gauge inv.}$$

Anomaly Inflow



SPT is gapped, so it is <u>not affected</u> by the RG flow at the boundary, so that the effective boundary theory in the IR must match the gauge variation from the bulk. This is called the 't Hooft anomaly matching. ('t Hooft 1979)

Anomaly matching in QCD with adjoint matter

$$\mathcal{L}_{ferm} = \sum_{I=1}^{n_f} \bar{\lambda}_I \not{D} \lambda^I$$ – classical $U(1)$ symmetry

$$\lambda \to e^{i\alpha} \lambda$$

However Jackiw-Adler-Bell tell us that the action changes by

$$\Delta S = i\alpha (2N)\, Q_{top}$$ so that $\mathbb{Z}_{2N} \in U(1)$ is still a symmetry.

If we gauge the center $\mathbb{Z}_N$ symmetry, it turns out that Q_{top} is no longer quantized in integer units (Gaiotto, Kapustin, Komargodski, Seiberg)

+ refs therein

Gauging the center symmetry

$$A \in SU(N) \longrightarrow A \in U(1)$$

$$F \longrightarrow F - \frac{\operatorname{tr} F}{N}$$

Notice that the theory has a $U(1)$ 1-form g.i.:

$$A \longrightarrow A + \partial \mathbb{I} \qquad (*)$$

But the theory is not the $SU(N)$ gauge th. because the fund. Wilson loops are no longer gauge inv.

$$\mathcal{P}\, e^{i\int A} \quad \text{- not inv under } (*)$$

$$\mathcal{P}\, e^{i\int \left(A - \frac{\operatorname{tr} A}{N}\right)} \quad \text{- not inv under } U(1) \text{ g.t.}$$

We must define wilson loops as

$$e^{i\int_{\partial\Sigma} A} \quad e^{i\int_{\Sigma} B}, \qquad \boxed{B = \frac{d(\mathrm{tr} A)}{N}}$$

$B \to B + d\lambda$ under the 1-form g.t.

So $\oint_\Sigma B$ is only well defined mod 2π

$e^{i\oint_\Sigma B} \in \mathbb{Z}_N$ — B measures a 't Hooft flux through surface Σ

The theory we have is $U(N)/U(1) = \dfrac{SU(N)}{\mathbb{Z}_N}$ gauge theory.

Chiral U(1) rotation : $\lambda \to e^{i\alpha}\lambda$ induces a change in the action (via fermion measure)

$$\Delta S = i\frac{2N\alpha}{8\pi^2}\int \mathrm{tr}\, F\wedge F = i(2N\alpha)\, Q_{top}$$

So if $e^{i\alpha} \in \mathbb{Z}_{2N}$ is non-anomalous

But now gauge center $F \to \underset{\nwarrow\, U(N) \text{ field str.}}{F} - \overset{\to\, \mathbb{Z}_N \text{ center g.f.}}{B}$

$$\frac{i}{8\pi^2}\int \mathrm{tr}(F\wedge F) \xrightarrow{\text{gauging center}} \frac{i}{8\pi^2}\int \Big(\underbrace{\mathrm{tr}\, F\wedge F - \mathrm{tr}\, F\wedge \mathrm{tr}\, F}_{2^{nd}\text{ Chern Class}}\Big) + i\frac{N(N-1)}{8\pi^2}\int B\wedge B$$

$$e^{i\frac{N(N-1)}{8\pi^2}\int B\wedge B} \in \mathbb{Z}_N \quad \text{so} \quad \mathbb{Z}_{2N} \longrightarrow \mathbb{Z}_2$$

$$\boxed{T^4 \to \mathbb{R}^4}$$

Gauge field for $\mathbb{Z}_N$-center are

$\longrightarrow$

equivalent to center-symmetry twists $\Leftrightarrow$ 't Hooft fluxes

$$\boxed{T^4 \to \mathbb{R}^4 \quad \text{charged}}$$

$$\cancel{\frac{\mathbb{Z}_{2N}}{\mathbb{Z}_2}}$$

$SU(N)$ + adjoint

- $\mathbb{Z}_N$-center
- $\mathbb{Z}_{2N}$-chiral

> Vacuum is charged under chiral symmetry in the presence of twists at infinity

$\Downarrow$

Vacuum cannot be trivial!!!!

So assuming center unbroken & vac gapped

$\Rightarrow$ vacuum not invariant under chiral sym.

So $\mathbb{Z}_{2N}$-chiral is explicitly broken down to $\mathbb{Z}_2$ in the presence of the center symmetry gauge fields. This implies a 't Hooft anomaly between the $\mathbb{Z}_N$-center and $\mathbb{Z}_{2N}/\mathbb{Z}_2$ - chiral symmetry.

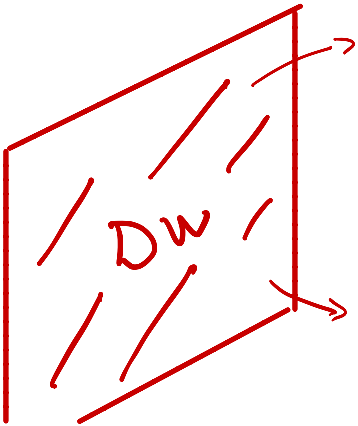
This predicts that:

* $\mathbb{Z}_{2N}/\mathbb{Z}_2$ - chiral is spont. broken if the theory is gapped & confining
* $T_\chi \geq T_{deconf}$ (known from lattice)
* χ-Domain Walls are deconfining (Komargodski, TS, Ünsal)

Domain wall deconfinement

* $\mathcal{N}=1$ and $\mathcal{N}=2$ Super Yang-Mills ($\mathbb{Z}_{2N}$ axial sym)
* QCD with adjoint matter ($\mathbb{Z}_{2N}$ axial sym)
* Pure Yang-Mills (CP symmetry)

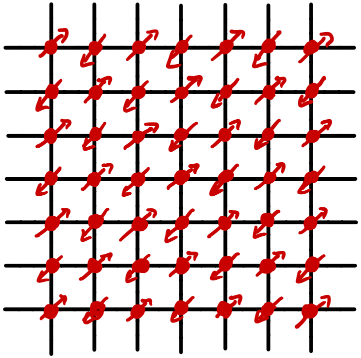
Such theories support stable domain walls



Quantum magnets.

Quantum anti-ferromagnets

The Néel phase

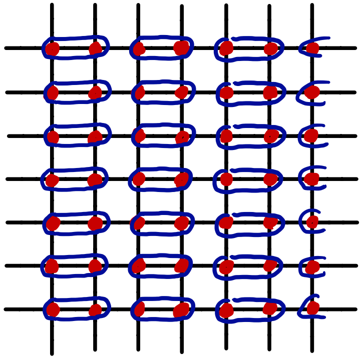


$\nearrow$ – $|\uparrow\rangle$

$\swarrow$ – $|\downarrow\rangle$

Breaks SO(3) spin symmetry & the translational symmetry

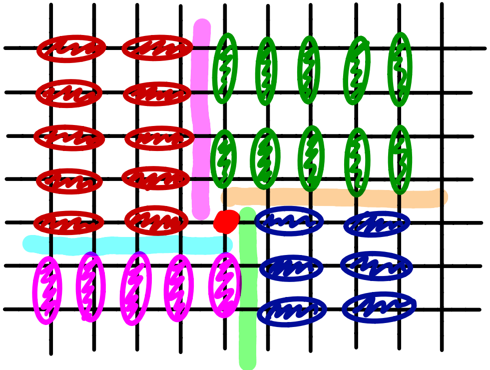
The Valence Bond Solid phase

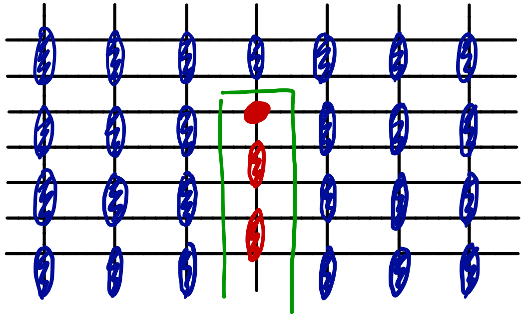


$$\underbrace{(\bullet\!-\!\bullet)_{1\,2}}_{\text{singlet}} - |\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle$$

Breaks lattice rotational symmetry and translational symmetry

Spin-$\frac{1}{2}$ confined in VBS phase





The parallels are striking between VBS and QCD-like theories in certain regimes [Anber, Poppitz, TS (2015); TS, Shao, Sandvik, Ünsal (2016); Komargodski, TS, Ünsal 2017]

CLAIM: The VBS phase is described by an Abelian-Higgs action

$$\frac{1}{4e^2}\tilde{F}^2 + |D_\mu w|^2 + M^2|w|^2 + V(|w|^2) + \boxed{\text{even monopoles}}$$

(Haldene '88, Senthil, Sachdev ~2004)

Related to the $\mathbb{Z}_2$ lattice symmetry

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \text{ - } SU(2) \text{ doublet}$$

Symmetries:

- $u \to Uu$, $U \in SU(2)$

  but $u \to -u$ is a gauge symm. so global symmetry is $\frac{SU(2)}{\mathbb{Z}_2} = SO(3)$

- $j = \frac{1}{2\pi} \star F$ $U(1)^T$-topologic.

  $$U(1)^T \xrightarrow{\text{2-mon.}} \mathbb{Z}_2^T$$

- $u \to u^*$, $A \to -A$ } C-Symmetry

Related to lattice symmetries

Gauge $U(1)$ - topological

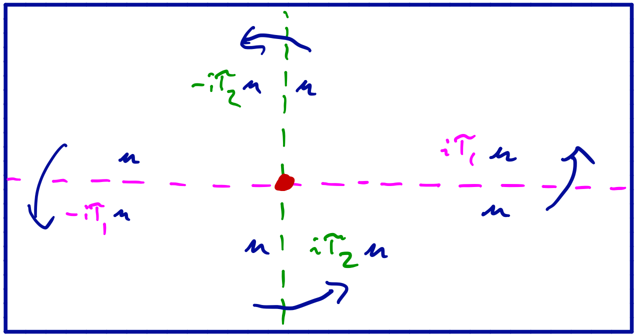
insert $\quad e^{i\int A^T \wedge * j} = e^{\frac{i}{2\pi}\int A^T \wedge F}$

$A^T \longrightarrow A^T + d\varphi$ is a gauge symmetry

because $\int F = 0 \mod 2\pi$

But now let us gauge the $SO(3)$ symmetry. In fact let us just gauge a $\mathbb{Z}_2 \times \mathbb{Z}_2$ subgroup, generated by $R_x(\pi)$, $R_y(\pi)$ - $SO(3)$ rotation matrices around the x & y axis.

Gauging $\mathbb{Z}_2 \times \mathbb{Z}_2 \in SO(3)$



T^2

--- transition function is $R_x(\pi)$

--- transition func. $R_y(\pi)$

at $\bullet$ $\quad -i\tau_1 \; -i\tau_2 \; i\tau_1 \; i\tau_2 = -1$ — center of $SU(2)$

So as we go around $\bullet$ we must also have

$$\int F = \pi \mod 2\pi$$

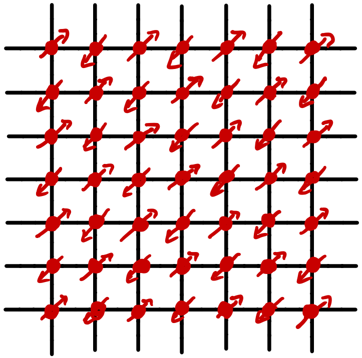
$\uparrow$ $U(1)$ dynamical gauge field.

In fact we have $\int F = \int B \mod 2\pi$

~~SO(3)~~ ~~Lattice Sym~~

The Néel phase

The Valence Bond Solid phase



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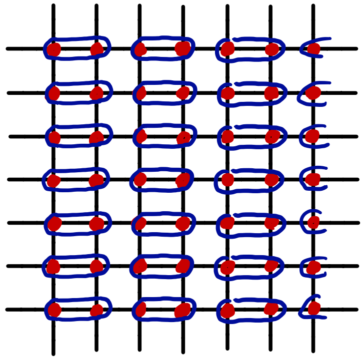


 – $|\uparrow\rangle$

 – $|\downarrow\rangle$

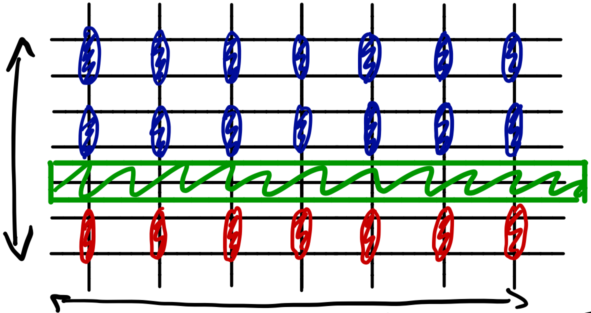
Breaks SO(3) spin symmetry & the translational symmetry

$\underbrace{1 \quad 2}_{\text{singlet}}$ – $|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle$

Breaks lattice rotational symmetry and translational symmetry

Domain walls as spin-chains

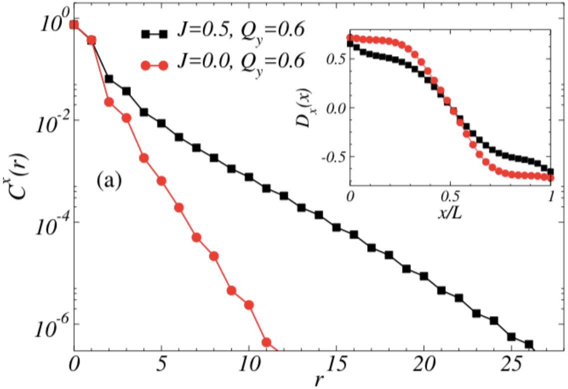
TS, Shao, Sandvik, Ünsal PRL (2017)

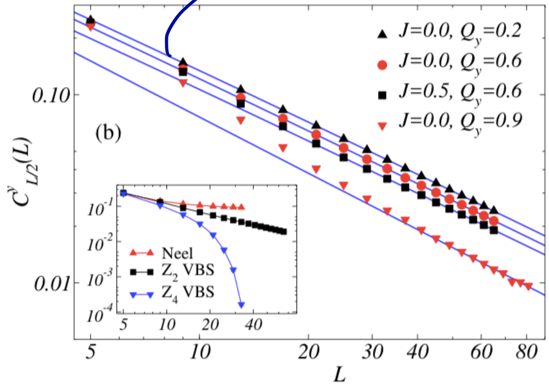


The J-Q model

$$H = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j + \underbrace{\sum_{\langle ijkl\rangle}\left(Q_x \, {}^{k}_{i}\ {}^{l}_{j} + Q_y \, {}^{k}_{i}\ {}^{l}_{j}\right)}_{\text{DRIVING VBS ORD.}}$$

fit to a spin-$\frac{1}{2}$ chain





transvers correlations gapped

longitudinal correlations critical

$SU(N)/U(1)^{N-1}$ non-lienear sigma models in 1+1D

Tanizaki, TS 2018

* The $N=2$ example is the $O(3)$ model
* It allows one θ-angle
* $\theta=\pi$ is the effective theory of a spin-half chain and the effective theory of the 2+1D VBS domain wall (Komargodski, TS, Ünsal 2017)
* Has interesting space-time symmetry anomalies (TS, Tanizaki 2018)

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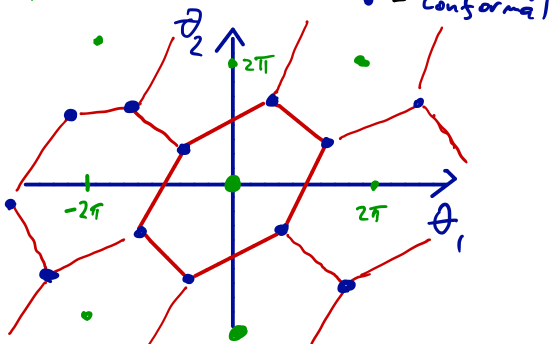
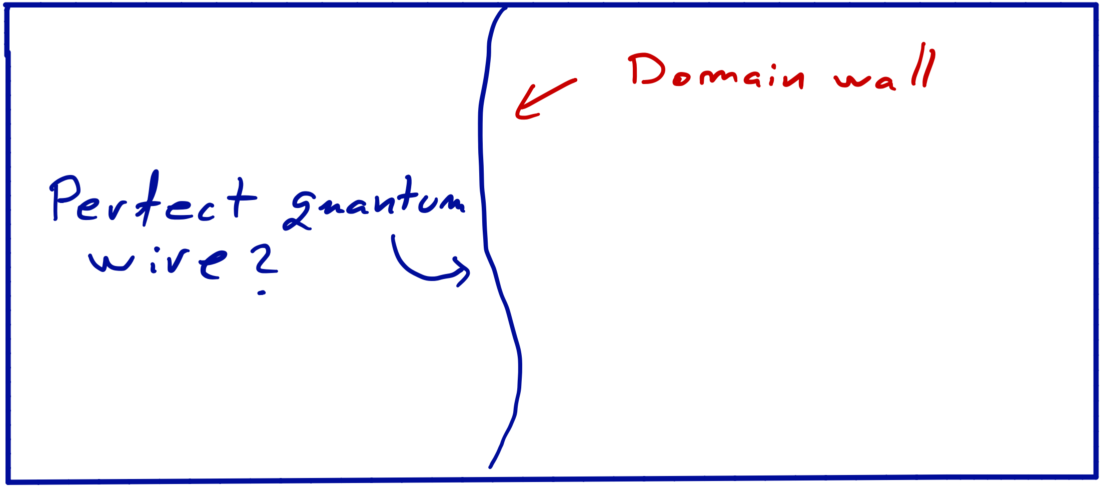
* The $N=3$ is the effective theory of an SU(3) spin chain (Lajko et al. 2017 & Bykov 2011)
* Has two θ-angles
* Is an effective theory of an 2+1D SU(3) anti-ferromagnet?
* Phase diag. highly constrained by anomaly-like arguments

Tanizaki, TS 2018



Domain walls as quantum wires



Are its properties protected from impurities?

Conclusions

* Anomaly matching provides powerful constraints on QFTs

* Domain Wall physics is particularly interesting and shows parallels between QCD & Quantum mag.

* Numerical confirmation in the J-Q model

* No QCD-like system allows simulations it would be nice to find one

* 1+1D theories which descend from domain-walls are also interesting with rich phase structure

* Possibility of simulating field theories directly avoiding the sign problem (work in progress)

Thank you!