KIAS, 26 March, 2018
Cquest, Sogang u., 29 March, 2018
MIT, CTP, 4 Apr, 2018
MPI, AEI, 13 Apr, 2018
HET group, Osaka, 30 May, 2018
DLAP2018 workshop, Osaka, 1 June, 2018
Paris QCD workshop, 11 June, 2018
Yau Center, China, 15 June, 2018

Deep Learning and AdS/CFT

Koji Hashimoto (Osaka u)

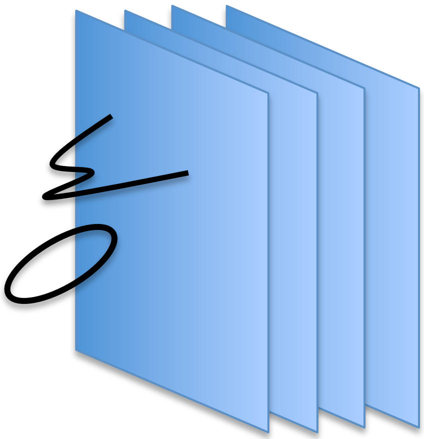
ArXiv:1802.08313 w/ S. Sugishita (Osaka),
A. Tanaka (RIKEN AIP),
A. Tomiya (CCNU)

Watch how machine learns AdS black hole

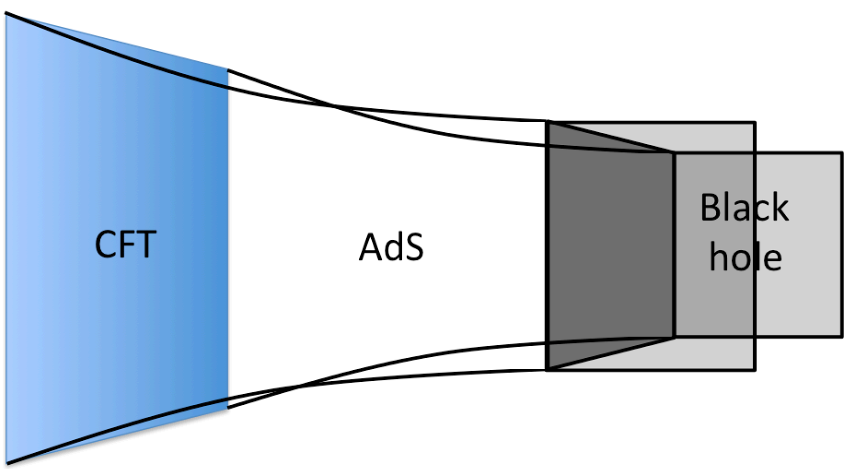
0-epochs

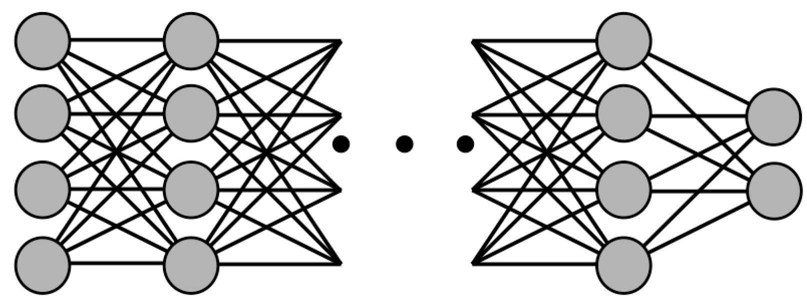
AdS radial direction

Brane
(Superstring theory)

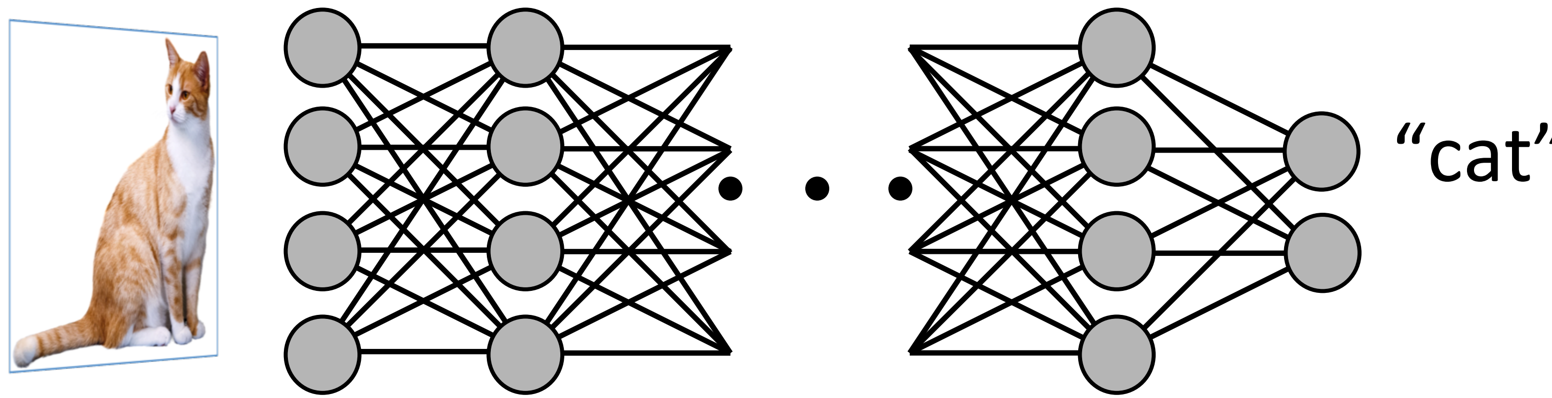


Brain
(Neuroscience)



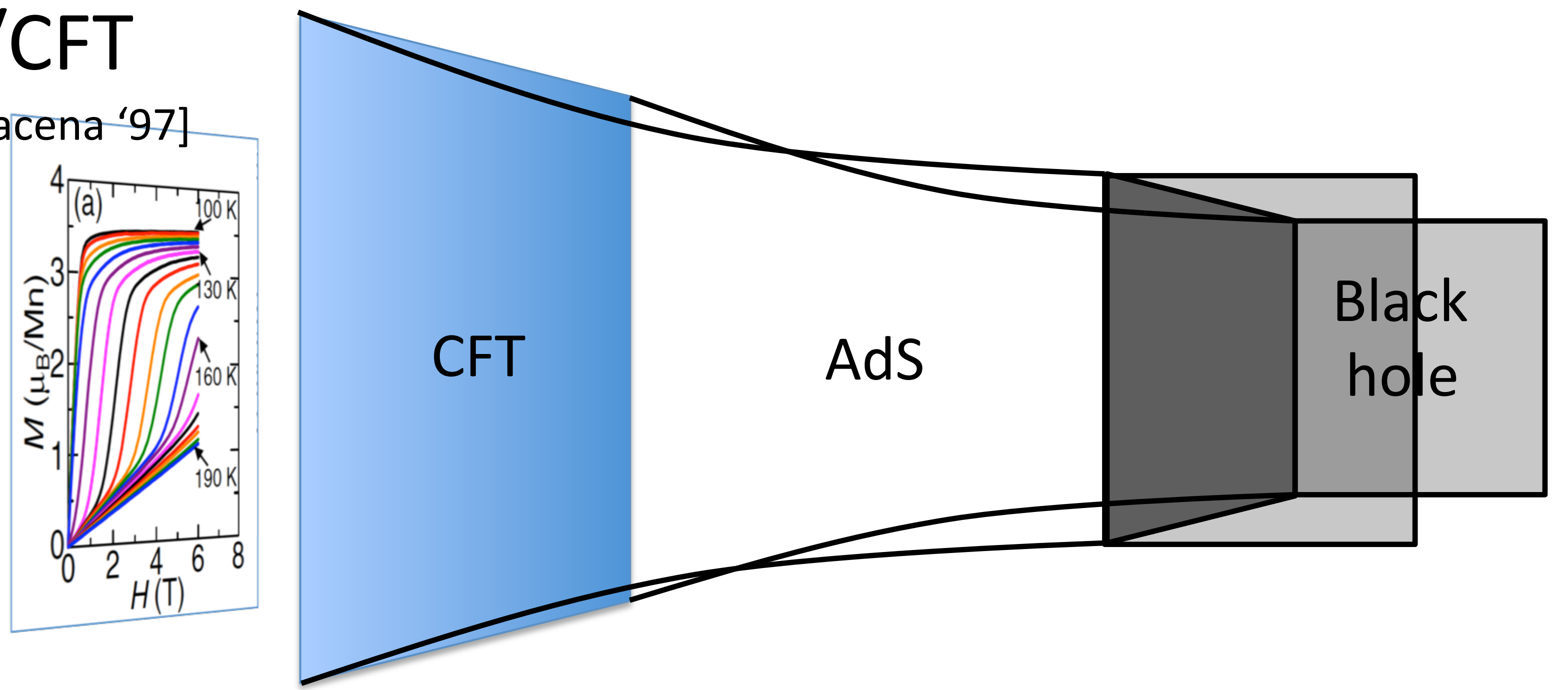


Deep Learning



AdS/CFT

[Maldacena '97]



1. Formulation of AdS/DL correspondence

2. Implementation of AdS/DL and emerging space

1. Formulation of AdS/DL correspondence

1-1 **Solving inverse problem**

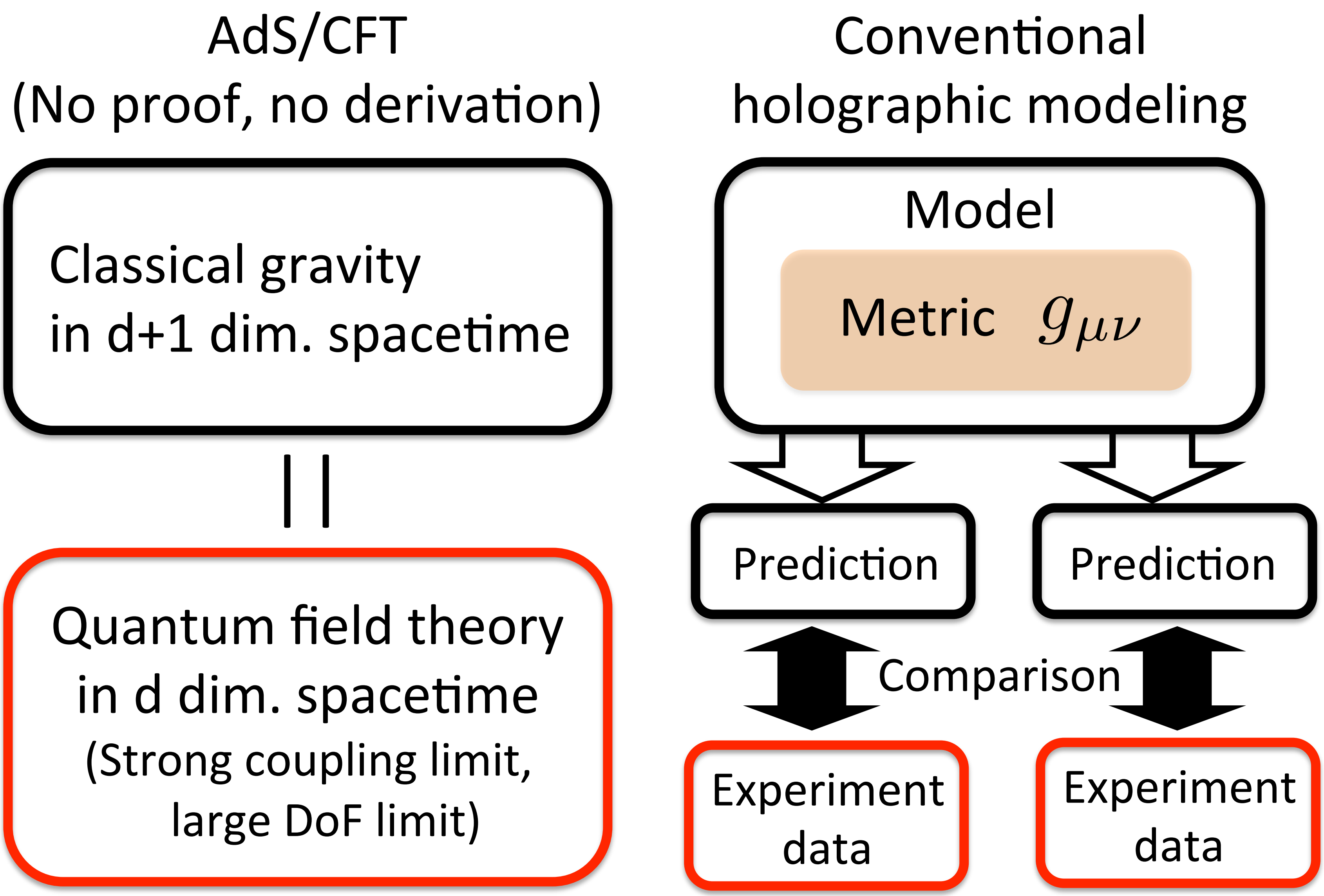
review **AdS/CFT: quantum response from geometry**

review **Deep learning: optimized sequential map**

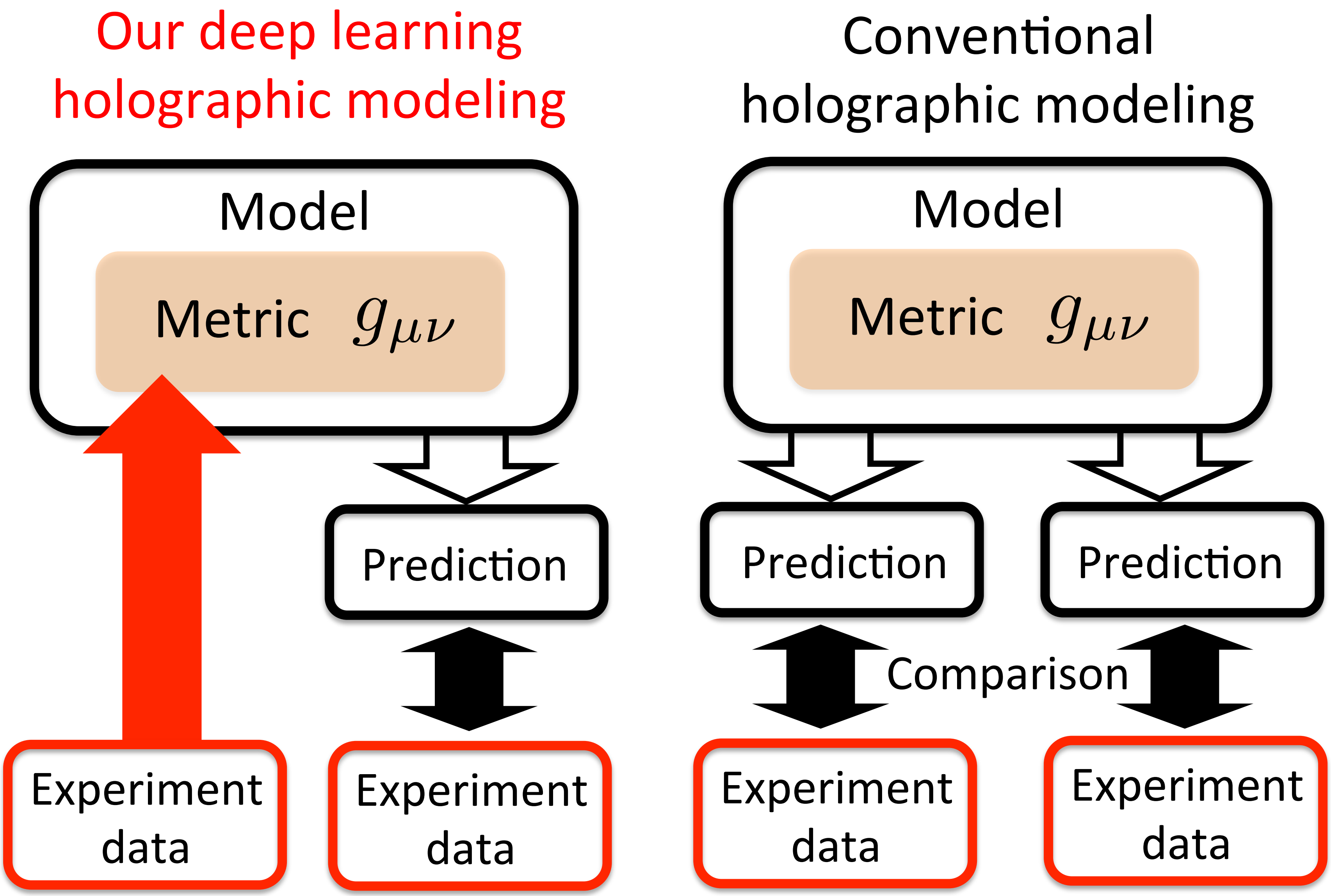
1-2 **From AdS to DL**

1-3 **Dictionary of AdS/DL correspondence**

1-1 Solving inverse problem



1-1 Solving inverse problem



review

AdS/CFT: quantum response from geometry

[Klebanov, Witten]

Classical scalar field theory in (d+1) dim. geometry

$$S = \int d^{d+1}x \sqrt{-\det g}\,\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2)$$

- AdS boundary ($\eta \sim \infty$) : $f \sim g \sim \exp[2\eta/L]$
- Black hole horizon ($\eta \sim 0$) : $f \sim \eta^2,\ g \sim \text{const.}$

Solve EoM, get response $\langle \mathcal{O} \rangle_J$. Boundary conditions:

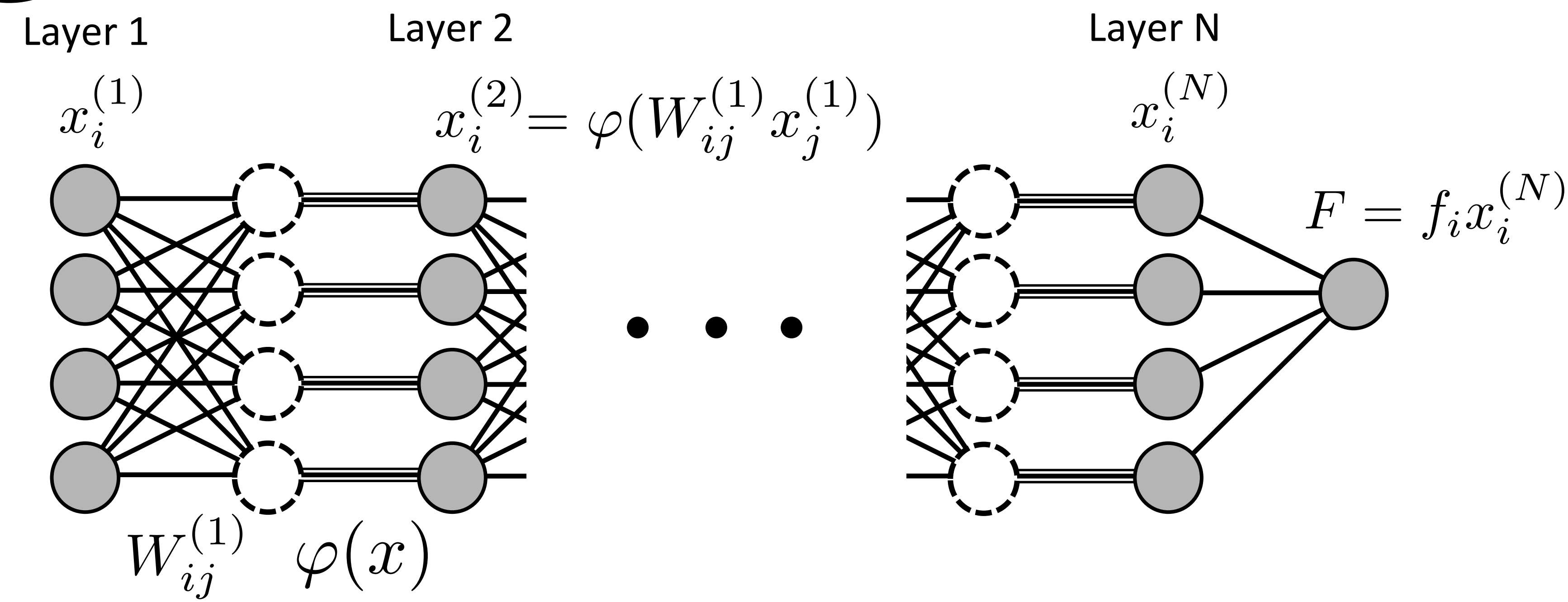
- AdS boundary ($\eta \sim \infty$) :

$$\phi = J e^{-\Delta_- \eta} + \frac{1}{\Delta_+ - \Delta_-}\langle \mathcal{O} \rangle e^{-\Delta_+ \eta}$$

- Black hole horizon ($\eta \sim 0$) : $\partial_\eta \phi \,\Big|_{\eta=0} = 0$

review

Deep learning : optimized sequential map



"Activation function" (fixed nonlinear fn.)

"Weights" (variable linear map)

1) Prepare many sets $\{x_i^{(1)}, F\}$: input + output
2) Train the network (adjust W_{ij}) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f_i(\varphi(W_{ij}^{(N-1)} \varphi(\cdots \varphi(W_{lm}^{(1)} x_m^{(1)})))) - F \right|$

1-2 From AdS to DL

Bulk EoM

$$\partial_\eta^2 \phi + \underline{\underline{h(\eta)}}\partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$$

metric $h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

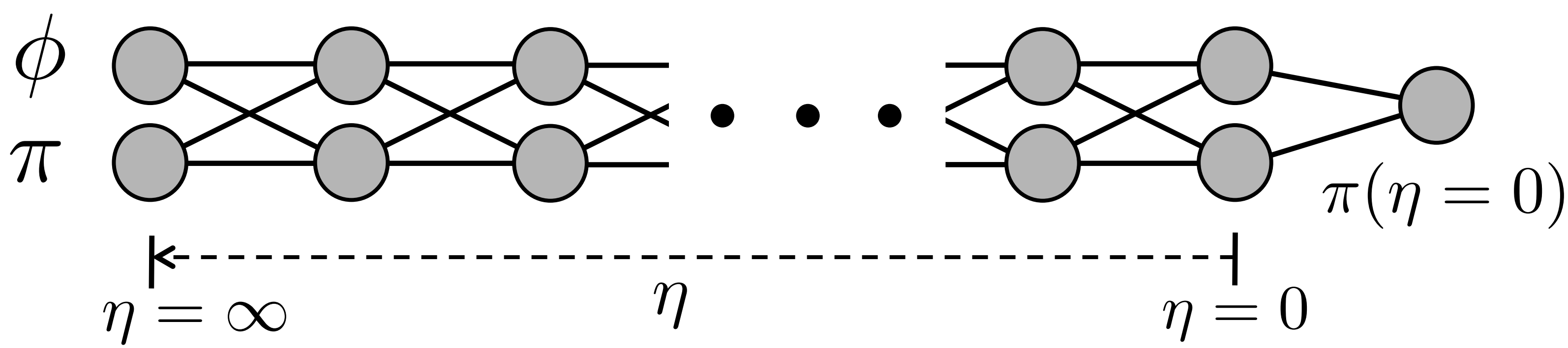
⇩

Discretization, Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$

⇩

Neural-Network representation



1-2 From AdS to DL

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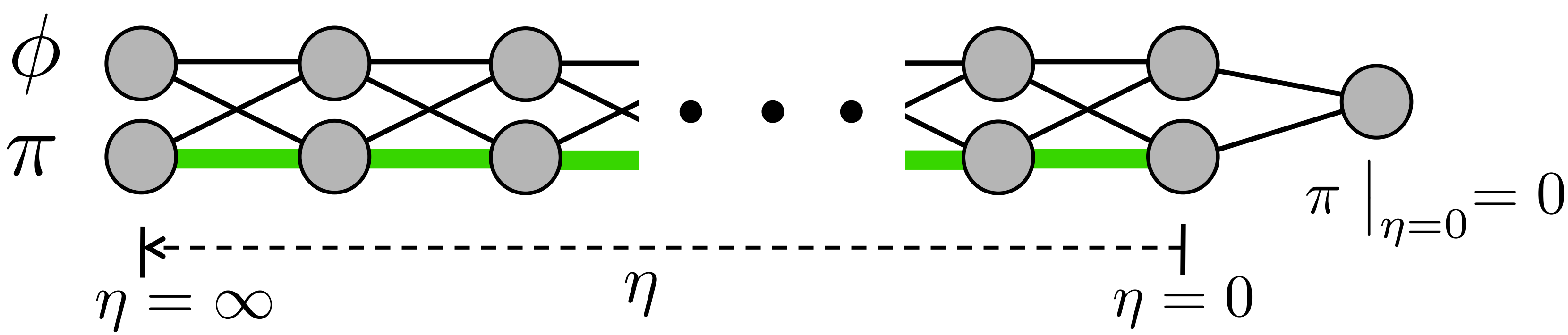

1-3 Dictionary of AdS/DL correspondence

AdS/CFT	Deep learning
Emergent space $\infty > \eta \geq 0$	Depth of layers $i = 1, 2, \cdots, N$
Bulk gravity metric $h(\eta)$	Network weights $W_{ij}^{(a)}$
Nonlinear response $\langle \mathcal{O} \rangle_J$	Input data $x_i^{(1)}$
Horizon condition $\partial_\eta \phi \big\|_{\eta=0} = 0$	Output data F
Interaction $V(\phi)$	Activation function $\varphi(x)$

1. Formulation of AdS/DL correspondence

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1. Formulation of AdS/DL correspondence

2. Implementation of AdS/DL and emerging space

2. Implementation of AdS/DL and emerging space

2-1 **Emergent geometry in deep learning**

2-2 **Can AdS Schwarzschild be learned?**

2-3 **Emergent space from real material?**

2-4 **Numerical experiment summary**

2-5 **Machines learn…, what do we learn?**

2-1 Emergent geometry in deep learning

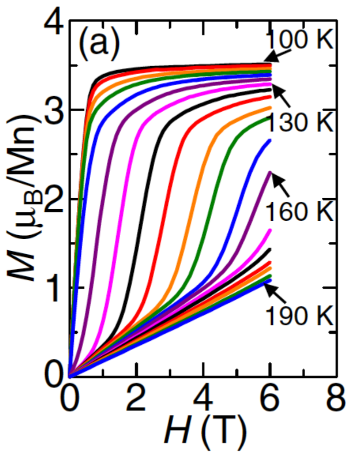
Experiment 1: "Can AdS Schwarzschild be learned?"

1) Use AdS Schwarzschild and generate input data.
2) Prepare network with unspecified metric.
3) Let the network learn it by the data.
4) Check if AdS Schwarzschild is reproduced.

Experiment 2: "Emergent space from real material?"

1) Use material experimental data.
   Ex) Magnetization curve of
   strongly correlated material
2) 3) (same as above.)
4) Watch how space emerges!

$Sm_{0.6}Sr_{0.4}MnO_3$



2-2 Exp1: Can AdS Schwarzschild be learned?

1) Use AdS Schwarzschild and generate input data.
2) Prepare network with unspecified metric.
3) Let the network learn it by the data.
4) Check if AdS Schwarzschild is reproduced.

$$\partial_\eta^2 \phi + h(\eta)\partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$$

$$h(\eta) = 3\coth(3\eta) \qquad V[\phi] = -\phi^2 + \frac{1}{4}\phi^4$$

AdS Schwarzschild metric
in the unit of AdS radius $L = 1$

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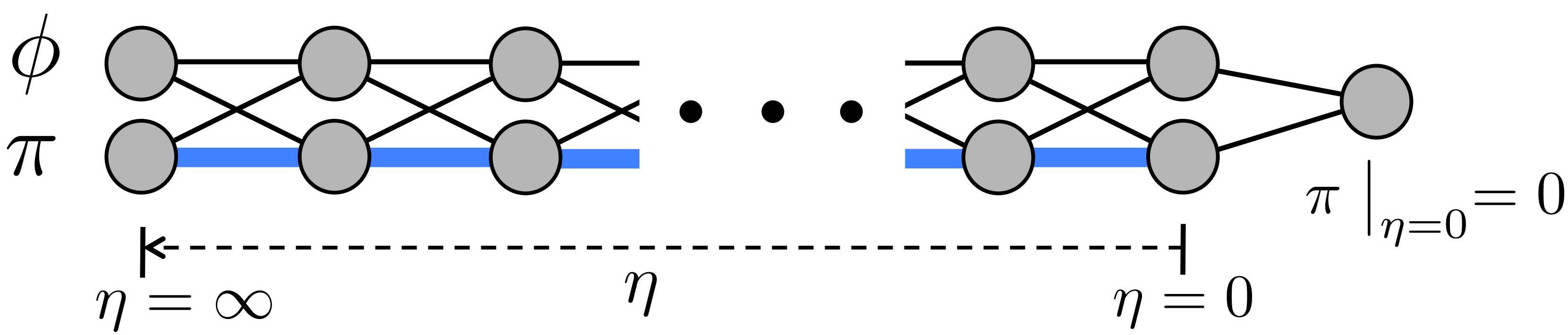
$$\begin{cases} \phi(\eta+\Delta\eta) = \phi(\eta) + \Delta\eta\,\pi(\eta) \\ \pi(\eta+\Delta\eta) = \pi(\eta) + \Delta\eta\left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta\phi(\eta)}\right) \end{cases}$$



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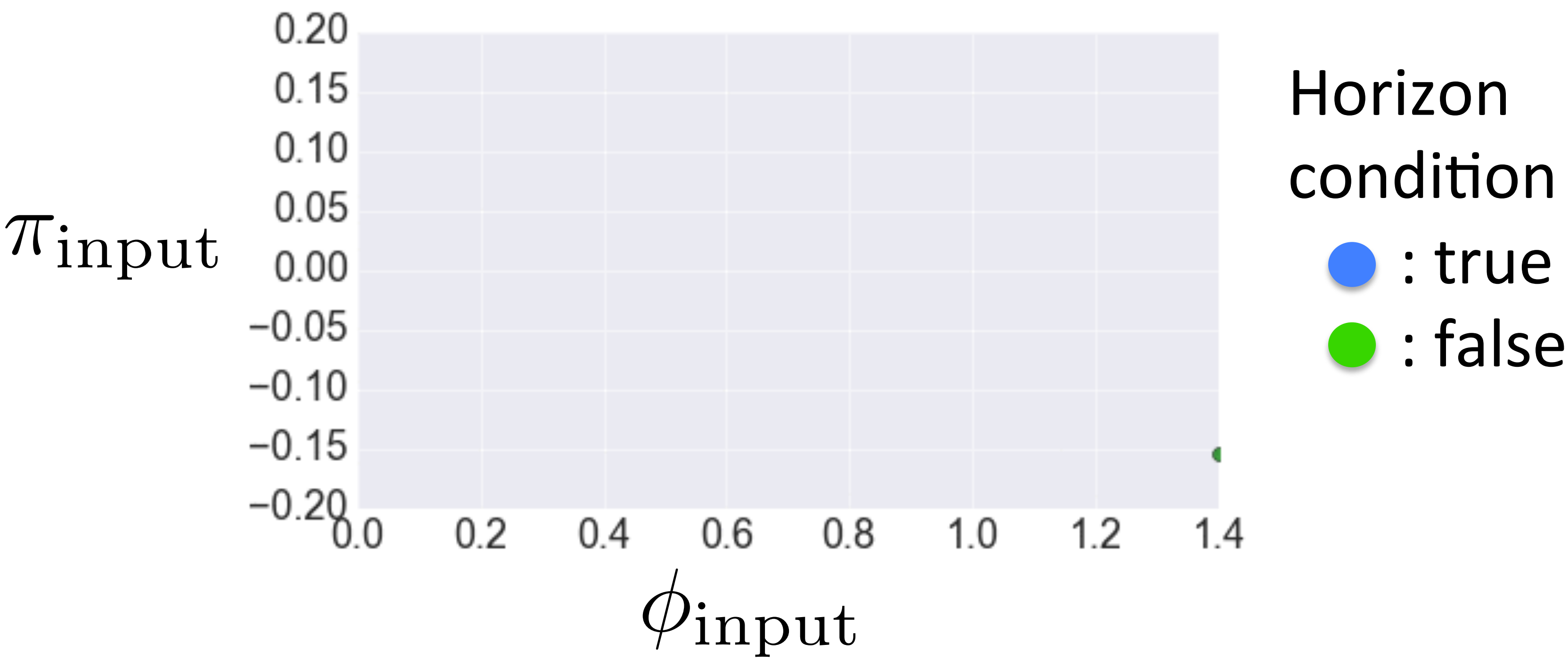

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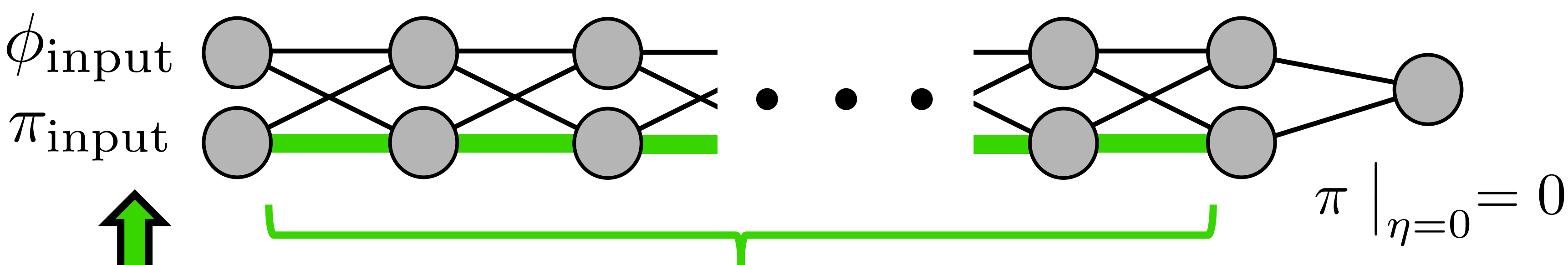
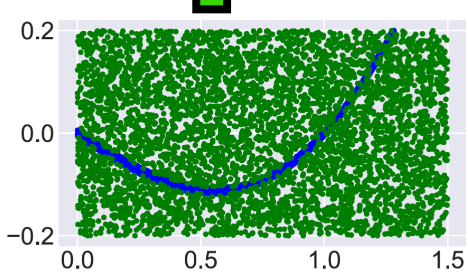


Unspecified metric
(10 layers, to be trained)



Generated data from AdS Schwarzschild
(10000 data points)

2-2 Exp1: Can AdS Schwarzschild be learned?

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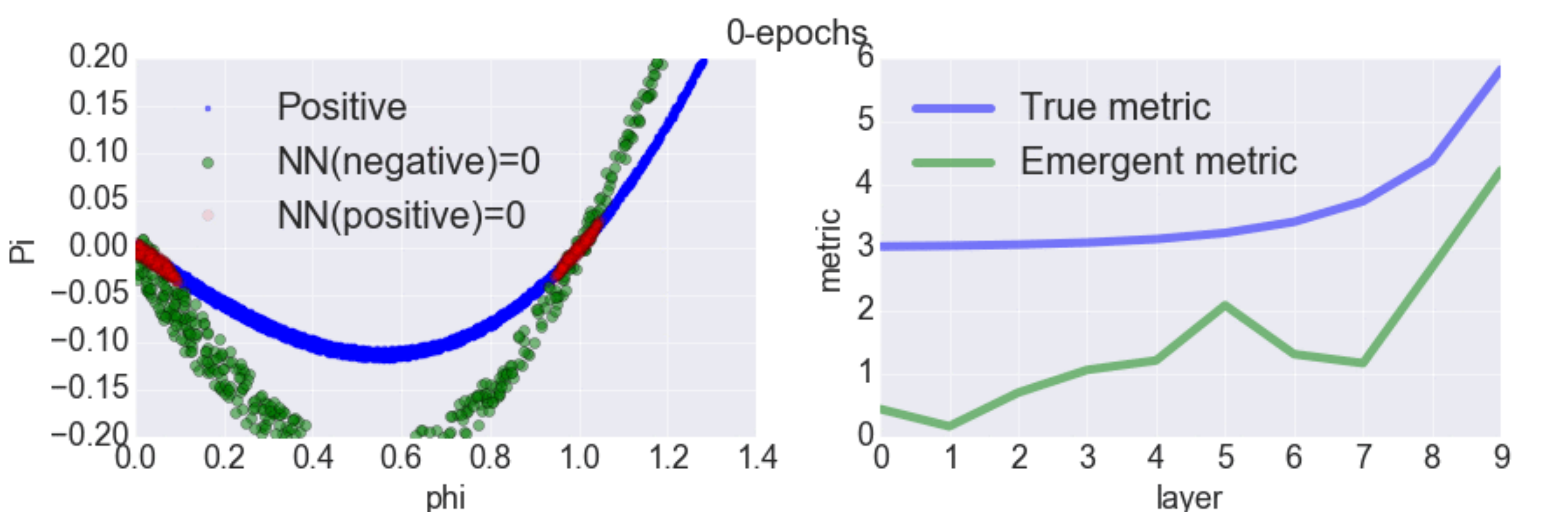

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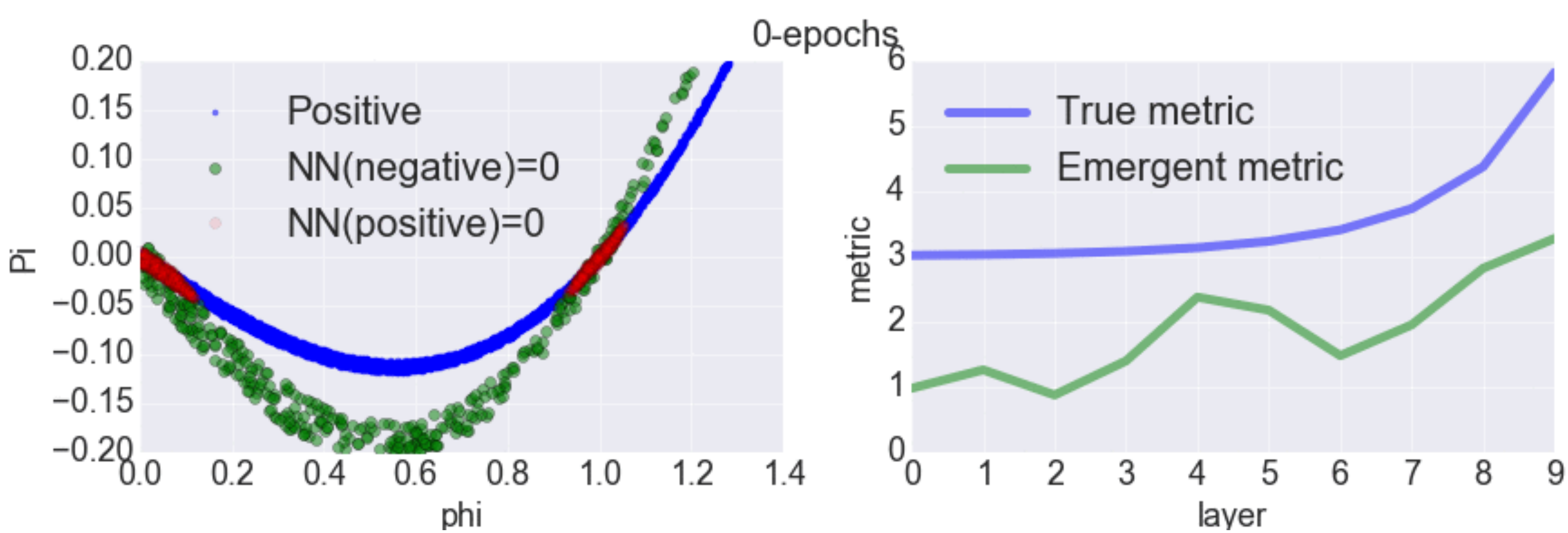

With a regularization

2-1 Emergent geometry in deep learning

Experiment 1: "Can AdS Schwarzschild be learned?"

1) Use AdS Schwarzschild and generate input data.
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Experiment 2: "Emergent space from real material?"

1) Use material experimental data.
   Ex) Magnetization curve of
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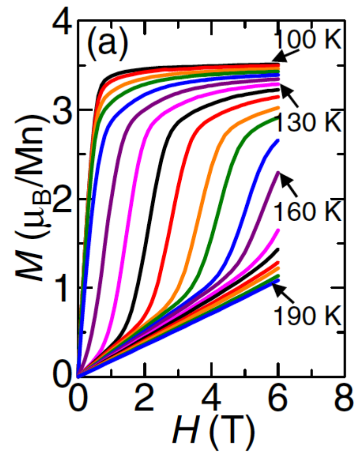
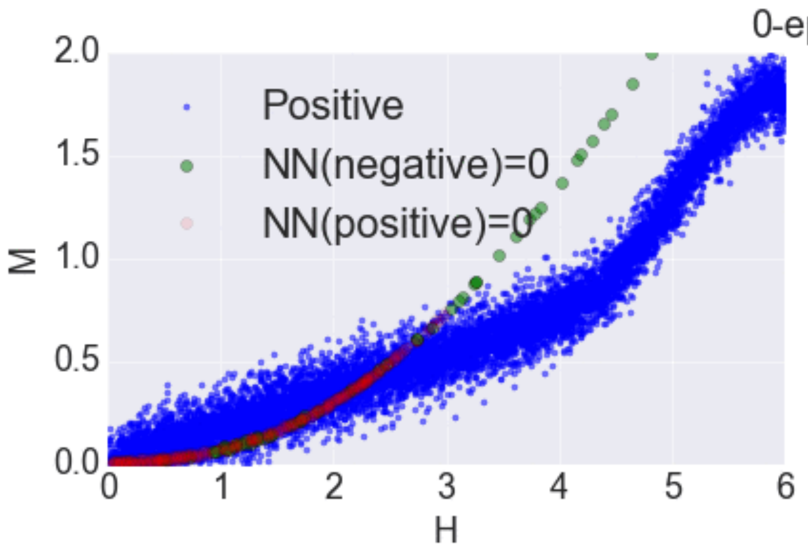
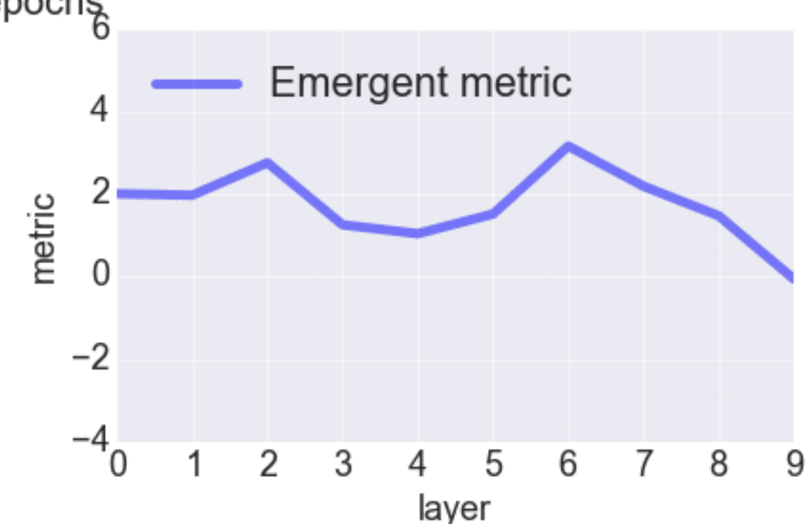
2) 3) (same as above.)
4) Watch how space emerges!

$Sm_{0.6}Sr_{0.4}MnO_3$



2-3 Exp2: Emergent space from real material?





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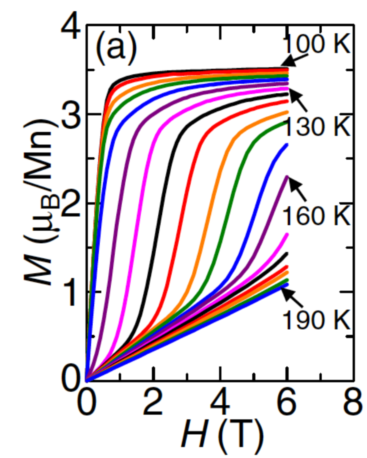
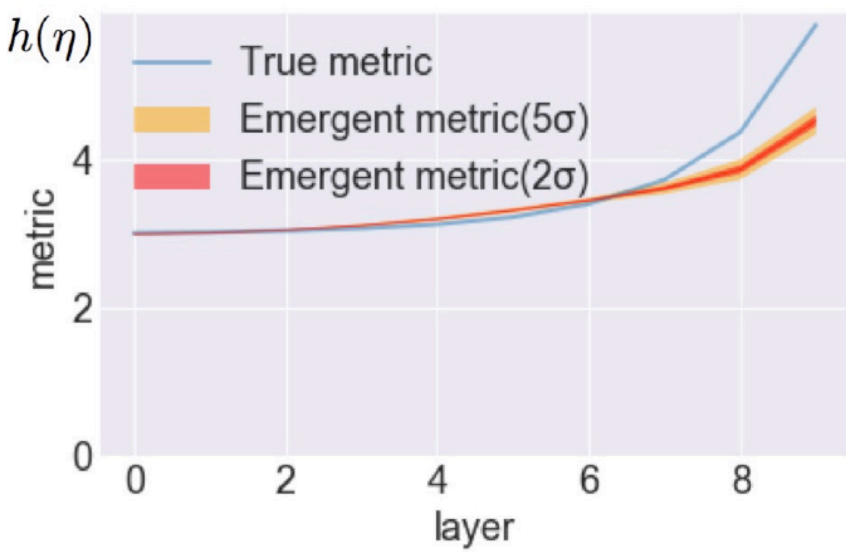


2-4 Numerical experiment summary

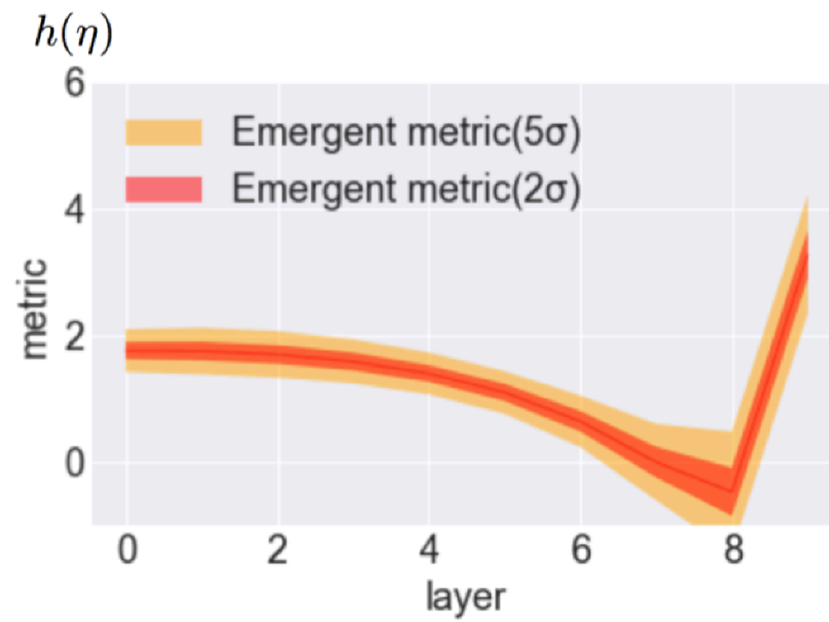
<u>Experiment 1</u>

AdS Schwarzschild is successfully learned.



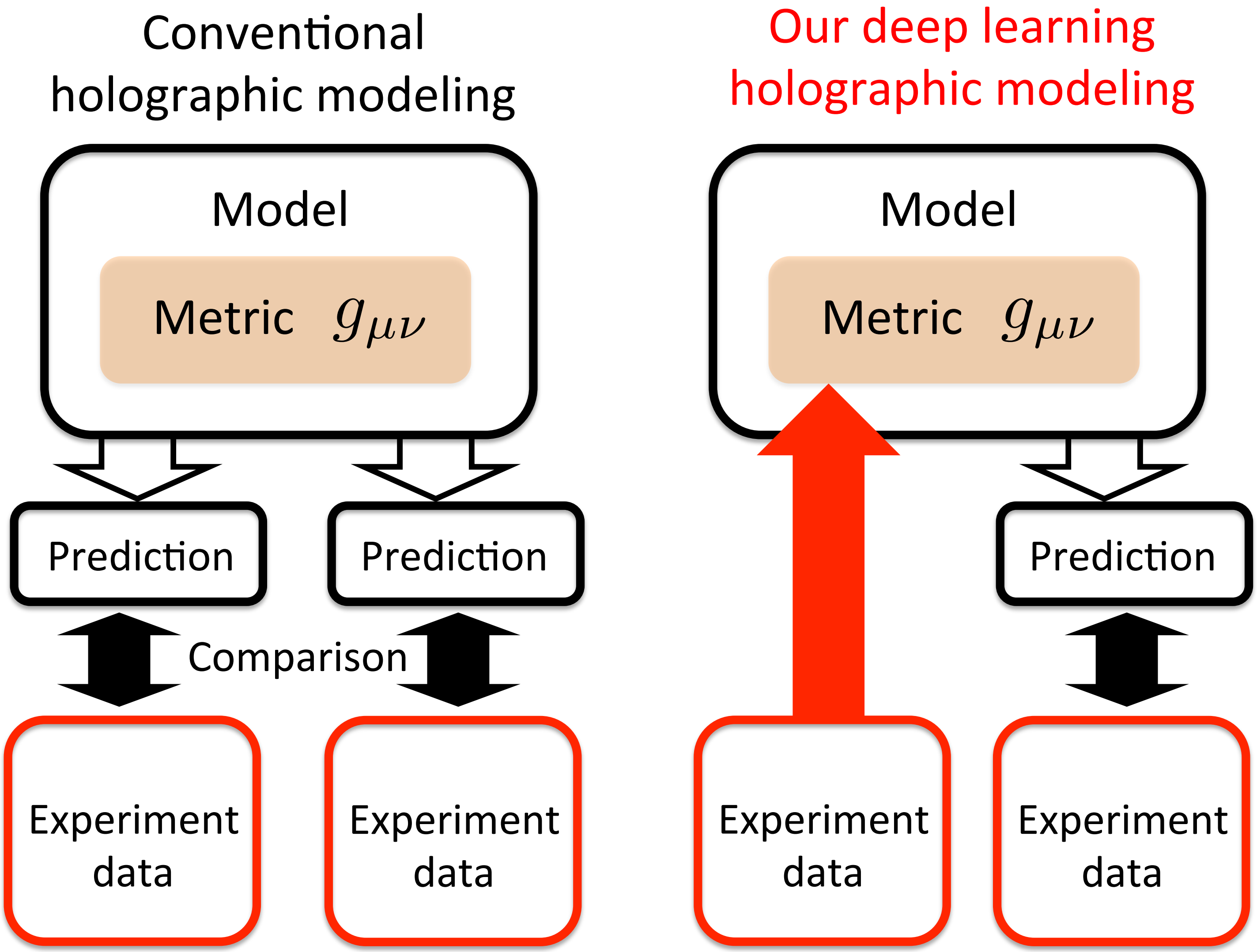
<u>Experiment 2</u>

Experimental data is explained by emergent space.



2-5

Machines learn..., what do we learn?



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