Tensor Field Theory in the large N limit

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1. What are tensor field theories?

2. Large N limit

3. Strongly coupled fixed point at large N

Vector $\mathcal{O}(N)$ model

Field theory with:

- a vector field $\phi_a(x)$, $x \in \mathbb{R}^d$, $a = 1, \dots N$
- an action S invariant under a global change of basis

$$S = \frac{1}{2} \int_x \sum_a \phi_a(x)(-\Delta + m^2)\phi_a(x) + \frac{\lambda}{4} \int_x \left(\sum_a \phi_a(x)\phi_a(x) \right)^2$$

Tensor field theories

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- a **tensor field** $\phi_{a_1 \dots a_r}(x)$ of rank at least 3
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Strongly coupled infrared fixed point. Can be studied at all orders in the relevant and marginal couplings.

The $O(N)^3$ tensor model

$$\phi_{a'_1 a'_2 a'_3}(x) = \sum O^{(1)}_{a'_1 a_1} O^{(2)}_{a'_2 a_2} O^{(3)}_{a'_3 a_3} \; \phi_{a_1 a_2 a_3}(x)$$

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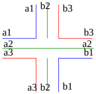
- the "tetrahedron"

$$\mathcal{T}[\phi] = \int_x \sum \phi_{a_1 a_2 a_3}(x) \phi_{a_1 b_2 b_3}(x) \phi_{b_1 a_2 b_3}(x) \phi_{b_1 b_2 a_3}(x)$$



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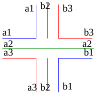
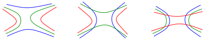


- the "pillow"

$$\mathcal{P}[\phi] = \frac{\int_x \sum \left(\phi_{p_1 a_2 a_3}(x) \phi_{q_1 a_2 a_3}(x) \right) \left(\phi_{p_1 c_2 c_3}(x) \phi_{q_1 c_2 c_3}(x) \right) + \dots}{3}$$



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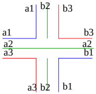

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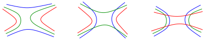
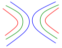


- the "double trace":

$$\mathcal{D}[\phi] = \int_x \sum \Big(\phi_{a_1 a_2 a_3}(x)\phi_{a_1 a_2 a_3}(x)\Big)\Big(\phi_{b_1 b_2 b_3}(x)\phi_{b_1 b_2 b_3}(x)\Big)$$



The melonic large N limit

$$Z = \int [d\phi] \; e^{-\frac{1}{2}\int \phi p^2 \phi} \; e^{-V[\phi]} \;, \qquad V[\phi] = \frac{m^2}{2}\mathcal{M}[\phi] + \frac{\lambda_t}{4N^{3/2}}\mathcal{T}[\phi] + \frac{\lambda_p}{4N^2}\mathcal{P}[\phi] + \frac{\lambda_d}{4N^3}\mathcal{D}[\phi]$$

$-\Gamma[\varphi]$ generating functional of amputated 1PI graphs:

$$\Gamma[\varphi] = -\frac{1}{2}\int \varphi\Sigma\varphi + \frac{\Gamma_t}{4N^{3/2}}\mathcal{T}[\varphi] + \frac{\Gamma_p}{4N^2}\mathcal{P}[\varphi] + \frac{\Gamma_d}{4N^3}\mathcal{D}[\varphi] + \dots$$

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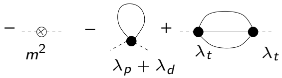

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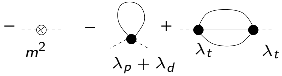
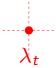
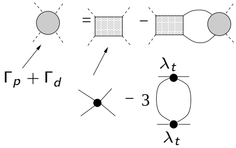
$\Sigma =$ 

$\Gamma_t =$ 



Renormalization as a discrete iteration

$$Z = \int d\mu_C[\phi] \; e^{-V^{(n)}[\phi]} \; , \qquad C = \frac{e^{-p^2}}{p^2} \; , \qquad \text{Choose an } M > 1$$

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- split the covariance $C = p^{-2} e^{-M^2 p^2} + p^{-2}\chi(p)$ in background $\tilde{C}$ and fluctuation Π:

$$Z = \int d\mu_C[\phi]\, e^{-V^{(n)}[\phi]} = \int d\mu_{\tilde{C}}[\psi] \int d\mu_{\Pi}[\zeta]\, e^{-V^{(n)}[\psi+\zeta]} = \int d\mu_{\tilde{C}}[\psi]\, e^{-\Gamma[\psi]}$$

- Z_n wave function $\Gamma[\psi] = \frac{Z_n - 1}{2} \int \psi p^2 \psi + \Gamma^0[\psi]$ modify covariance $Z = \int d\mu_{\frac{\tilde{C}}{Z_n}}[\psi] e^{-\Gamma^0[\psi]}$

- rescale the field $R[\psi](x) = M^{1-d/2} Z_n^{-1/2} \psi\left(M^{-1}x\right)$ get $V^{(n+1)}[\psi]$:

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Field dimension $\Delta_{\phi,n} = \frac{1}{2}\left(d - 2 + \frac{\ln Z_n}{\ln M}\right)$ RG transformation $(m_n^2, \lambda_n) \to (m_{n+1}^2, \lambda_{n+1})$

Lyapunov exponents at a fixed point

A one dimensional iteration $x_{n+1} = f(x_n)$ behaves near a fixed point $x_\star = f(x_\star)$ like:

$$x_n = x_\star + M^{n\Delta}(x_0 - x_\star) \ , \qquad \Delta = \frac{\ln f'(x_\star)}{\ln M} = \begin{cases} \text{relevant} & \Delta > 0 \\ \text{marginal} & \Delta = 0 \\ \text{irrelevant} & \Delta < 0 \end{cases}$$

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The Wilson Fisher fixed point

$\frac{\lambda}{4!}\phi^4$ perturbation, $4-\epsilon$ dimensions, first order in λ

$$Z_n = 1 + \frac{1}{6(4\pi)^4}\lambda_n^2 \ln M \ , \qquad \lambda_{n+1} = \lambda_n + \epsilon\lambda_n \ln M - \frac{3}{(4\pi)^2}\lambda_n^2 \ln M \ ,$$

$$m_{n+1}^2 = M^2\left[m_n^2 + \frac{1}{2}\lambda_n \int \frac{d^4q}{(2\pi)^4} \, \frac{\chi(q)}{q^2 + m_n^2\chi(q)}\right]$$

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Fixed point $(\lambda_\star = \epsilon\frac{(4\pi)^2}{3}, m_\star^2 \sim -\epsilon)$, with Lyapunov exponents:

$$\Delta_\phi = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{108} \ , \qquad \Delta_\lambda = -\epsilon \ , \qquad \Delta_{m^2} = 2 - \frac{\epsilon}{3}$$

The RG equations in tensor field theory

$$Z_n = 1 + \frac{\tilde\lambda_{t;n}^2}{Z_n^3} I^1_{m^2_{n+1}} \ , \qquad m^2_{m+1} = \frac{M^2}{Z_n}\left[m_n^2 + \frac{\tilde\lambda_{p;n} + \tilde\lambda_{d;n}}{Z_n} T_{m^2_{n+1}} - \frac{\tilde\lambda^2_{t;n}}{Z_n^3} I^0_{m^2_{n+1}} \right] ,$$

$$\tilde\lambda_{t;n+1} = \frac{M^{4-d}}{Z_n^2} \tilde\lambda_{t;n} \ ,$$ exact equation even including all the radiative corrections

$$\tilde\lambda_{p;n+1} = \frac{M^{4-d}}{Z_n^2} \tilde A_{p;n} = \frac{M^{4-d}}{Z_n^2} \frac{\tilde\lambda_{p;n} - 3\frac{\tilde\lambda^2_{t;n}}{Z_n^2} D_{m^2_{n+1}}}{1 - \left(\frac{\tilde\lambda^2_{t;n}}{Z_n^4} S_{m^2_{n+1}} - \frac{1}{3}\frac{\tilde\lambda_{p;n}}{Z_n^2} D_{m^2_{n+1}} \right)} ,$$

$$\tilde\lambda_{d;n+1} = \frac{M^{4-d}}{Z_n^2} \tilde B_{d;n}$$

$$= \frac{M^{4-d}}{Z_n^2} \frac{\tilde\lambda_{d;n} + \left[2\left(\frac{\tilde\lambda^2_{t;n}}{Z_n^4} S_{m^2_{n+1}} - \frac{1}{3}\frac{\tilde\lambda_{p;n}}{Z_n^2} D_{m^2_{n+1}} \right) - \frac{\tilde\lambda_{d;n}}{Z_n^2} D_{m^2_{n+1}} \right] \tilde A_{p;n}}{1 - \left(3\frac{\tilde\lambda^2_{t;n}}{Z_n^4} S_{m^2_{n+1}} - \frac{\tilde\lambda_{p;n} + \tilde\lambda_{d;n}}{Z_n^2} D_{m^2_{n+1}} \right)} .$$

I^0, I^1, T, D, S depend on M, d and m_{n+1} and are strictly positive

$$\Delta_{\phi;n} = \frac{1}{2}\left(d - 2 + \frac{\ln Z_n}{\ln M} \right) \ , \qquad \Delta^{\text{classical}}_{\partial^r \phi^m} = d - r - m\Delta_{\phi;n}$$

The flow of the tetrahedral coupling

The (exact) flow equation of the tetrahedral coupling $\tilde{\lambda}_{t;n+1} = \frac{M^{4-d}}{Z_n^2} \tilde{\lambda}_{t;n}$ is a fixed point equation (for any $\tilde{\lambda}_{t,\star}$) if

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Field dimension and classical Lyapunov exponents at $\star$:

$$\boxed{\Delta_{\phi;\star} = \frac{d}{4}}\,, \qquad \Delta^{\text{classical}}_{\partial^r\phi^m} = d - r - m\frac{d}{4}$$

The couplings not included $m \geq 6$ or $m = 4, r \geq 1$ or $m = 2, r \geq 4$ are (classically) irrelevant

The fixed point

$$Z_n = 1 + \frac{\tilde{\lambda}_{t;n}^2}{Z_n^3} I^1 \;, \qquad m_{m+1}^2 = \frac{M^2}{Z_n}\left[m_n^2 + \frac{\tilde{\lambda}_{p;n} + \tilde{\lambda}_{d;n}}{Z_n} T_{m_{n+1}^2} - \frac{\tilde{\lambda}_{t;n}^2}{Z_n^3} I^0 \right] \,,$$

$$\tilde{\lambda}_{t;n+1} = \frac{M^{4-d}}{Z_n^2}\,\tilde{\lambda}_{t;n} \;,$$

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$$\tilde{\lambda}_{d;n+1} = \frac{M^{4-d}}{Z_n^2}\,\tilde{B}_{d;n}$$

$$= \frac{M^{4-d}}{Z_n^2}\,\frac{\tilde{\lambda}_{d;n} + \left[2\left(\frac{\tilde{\lambda}_{t;n}^2}{Z_n^4} S - \frac{1}{3}\frac{\tilde{\lambda}_{p;n}}{Z_n^2} D \right) - \frac{\tilde{\lambda}_{d;n}}{Z_n^2} D \right] \tilde{A}_{p;n}}{1 - \left(3\frac{\tilde{\lambda}_{t;n}^2}{Z_n^4} S - \frac{\tilde{\lambda}_{p;n} + \tilde{\lambda}_{d;n}}{Z_n^2} D \right)} \,.$$

The fixed point

$$
\begin{aligned}
Z_\star &= M^{2-\frac{d}{2}} \;, \qquad \Delta_{\phi;\star} = \frac{d}{4} \\
\tilde\lambda_{t;\star} &= \pm\sqrt{\frac{Z_\star^4 - Z_\star^3}{I^1}} \;, \\
\tilde\lambda_{p;\star} &= \frac{3}{2}\tilde\lambda_{t;\star}^2 \frac{S}{Z_\star^2 D} \pm 3\sqrt{\frac{\tilde\lambda_{t;\star}^4}{4}\left(\frac{S}{Z_\star^2 D}\right)^2 - \tilde\lambda_{t;\star}^2} \;, \\
\tilde\lambda_{d;\star} &= \mp 3\sqrt{\frac{\tilde\lambda_{t;\star}^4}{4}\left(\frac{S}{Z_\star^2 D}\right)^2 - \tilde\lambda_{t;\star}^2} \pm \sqrt{\frac{9\tilde\lambda_{t;\star}^4}{4}\left(\frac{S}{Z_\star^2 D}\right)^2 - 3\tilde\lambda_{t;\star}^2} \\
m_\star^2 &= \frac{\frac{M^2}{Z_\star}\left[\frac{\tilde\lambda_{p;\star}+\tilde\lambda_{d;\star}}{Z_\star} T_\star - \frac{\tilde\lambda_{t;\star}^2}{Z_n^3} I^0\right]}{1-\frac{M^2}{Z_\star}}
\end{aligned}
$$

$$
\Delta_{\lambda_t} = -\frac{\ln(4Z_\star - 3)}{\ln M} < 0 \;, \qquad \Delta_{m^2} = \frac{d}{2} - \frac{\ln\left[1 + \frac{\lambda_{p;\star}+\lambda_{d;\star}}{Z_\star^2} D\right]}{\ln M}
$$

Take $\epsilon = 4 - d$ small

Fixed point

$$\tilde{\lambda}_{t;\star} = \pm\sqrt{\frac{\epsilon}{2}} \, , \qquad \tilde{\lambda}_{p,\star} = \pm 3\,\mathrm{i}\sqrt{\frac{\epsilon}{2}} \, , \qquad \tilde{\lambda}_{d,\star} = (\mp 3 \pm \sqrt{3})\,\mathrm{i}\sqrt{\frac{\epsilon}{2}}, \qquad m_\star \sim \mathrm{i}\sqrt{\epsilon}$$

Exponents:

$$\Delta_\phi = 1 - \frac{\epsilon}{4} \, , \qquad \Delta_{\lambda_t} = -2\epsilon \, , \qquad \Delta_{m^2} = 2 \pm \mathrm{i}\sqrt{6\epsilon}$$

Wilson Fisher: $\Delta_\phi = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{108} \, , \qquad \Delta_\lambda = -\epsilon \, , \qquad \Delta_{m^2} = 2 - \frac{\epsilon}{3}$

Conclusion

Tensor field theories (bosonic) have a strongly coupled fixed point with :

- $\Delta_\phi = \frac{d}{4}$
- $\Delta_{\lambda_t} < 0$ for $d < 4$
- can be studied at all orders in the (classically) marginal and relevant couplings.

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To do:

- include irrelevant couplings
- fermionic models
- ...