Emergent Gauge Theory

(Based on work with JiaHui Huang, Minkyoo Kim, Laila Tribelhorn and Jaco Van Zyl)

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Problem Statement

We will study excitations of the LLM geometries.

Lin, Lunin, Maldacena, [hep-th/0409174]

$$\begin{aligned} ds^2 &= -y(e^G + e^{-G})(dt + V_i dx^i)^2 + \frac{1}{y(e^G + e^{-G})}(dy^2 + dx^i dx^i) \\ &\quad + ye^G d\Omega_3 + ye^{-G} d\tilde{\Omega}_3 \end{aligned} \tag{1}$$

$$z = \frac{1}{2}\tanh(G) \qquad y\partial_y V_i = \epsilon_{ij}\partial_j z \qquad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \tag{2}$$

$$\partial_i\partial_i z + y\partial_y \frac{\partial_y z}{y} = 0. \tag{3}$$

Regularity forces $z = \pm\frac{1}{2}$ on $y = 0$ plane.

Problem Statement

$$\partial_i \partial_i z + y \partial_y \frac{\partial_y z}{y} = 0. \quad (4)$$

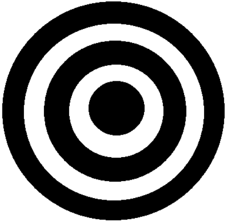


Figure: A coloring of the LLM plane determines z. The black regions are sources of five form flux.

The geometries are 1/2 BPS and are the result of backreaction from a condensate of giant graviton branes.

Problem Statement

The CFT operators dual to the LLM geometries are known - they are given by Schur polynomials $\chi_B(Z)$.

Corley, Jevicki, Ramgoolam, [hep-th/0111222]

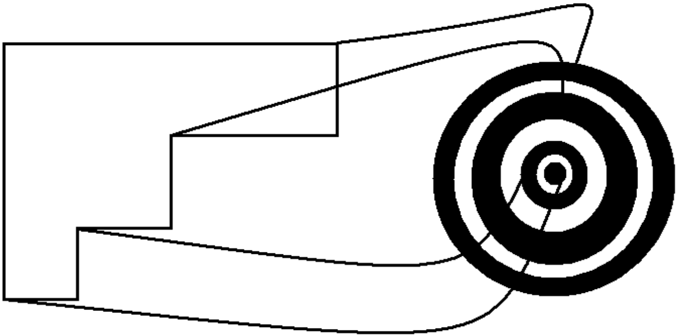


Figure: The coloring of the LLM plane determines both z and R.

Excitations are described by adding boxes to B.

Problem Statement

The LLM geometries are 1/2 BPS and result from backreaction of a condensate of giant graviton branes. We will study excitations of these geometries, using the CFT. This entails studying correlators of operators with a dimension $\sim N^2$.

The excitations we study are open string excitations of the underlying giant graviton branes. The dynamics of the open strings will give rise to an emergent gauge theory. In how much detail can we explore this emergent theory?

Balasubramanian, Berenstein, Feng and Huang, [hep-th/0411205]

We will argue that the emergent gauge theory is $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$.

Strategy

Our goal is to argue that the emergent gauge theories are $\mathcal{N} = 4$ super Yang-Mills theories. We do this in three steps

1. Construct a bijection between operators in planar $\mathcal{N} = 4$ super Yang-Mills and the planar limit of emergent gauge theory.
2. Argue that planar three point functions in planar emergent gauge theory vanish, so OPE coefficients match.
3. Argue that the spectrum of anomalous dimensions matches the spectrum of $\mathcal{N} = 4$ super Yang-Mills.

Approach

Due to the enormous number of fields involved summing the planar diagrams *does not* give an accurate description of the large N dynamics. Not known if there is a class of diagrams that can be summed.

Idea is to use group theory to use permutation symmetries in the problem in a novel way. Operators are labeled by representations of groups and of subgroups, $\chi_{R,(r,s)\alpha\beta}(Z,Y)$, $r \vdash n_z$, $s \vdash n_Y$, $R \vdash n_Z + n_Y$

This reduces the problem to linear algebra. For example, computing 2 point functions is reduced to finding projectors, multiplying them and taking a trace.

$$\langle \chi_{R,(r,s)\alpha\beta} \chi^{\dagger}_{T,(t,u)\gamma\delta} \rangle = \frac{\delta_{RT}\delta_{rt}\delta_{su}\delta_{\alpha\gamma}\delta_{\beta\delta}\mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s}$$

Bhattacharyya, Collins, dMK, [arXiv:0801.2061 [hep-th]]

Background independence

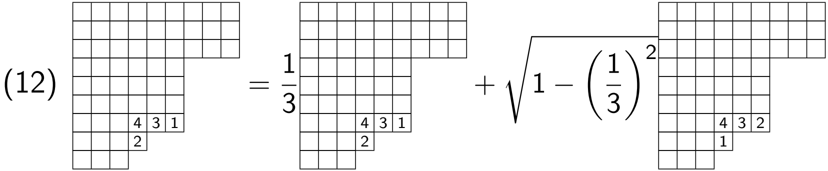
Classify ingredients of the computation as background independent / dependent.

Background independent: something that takes the same value on any inward pointing corner or even in the absence of a background, i.e. planar limit of original CFT. Take the same value regardless of which collection of branes are excited, hence the name "background independent".

Background dependent: does depend on the collection of branes we excite.

Background independent

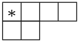
$$(12)\ \begin{array}{|c|c|c|}\hline 4&3&1\\\hline 2\\\cline{1-1}\end{array} = \frac{1}{3}\ \begin{array}{|c|c|c|}\hline 4&3&1\\\hline 2\\\cline{1-1}\end{array} + \sqrt{1-\left(\frac{1}{3}\right)^2}\ \begin{array}{|c|c|c|}\hline 4&3&2\\\hline 1\\\cline{1-1}\end{array}$$



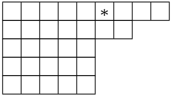
Background dependent

There are two background dependent quantities that will plays a role:

The factor of a box:



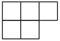
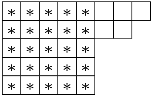
$\rightarrow \quad N$



$\rightarrow \quad N+5$

All N dependence comes from factors of the excitation.

Ratios of hooks

$R =$  $\qquad +R =$ 

$$\frac{\text{hooks}_{+R}}{\text{hooks}_{+r}} = \alpha \frac{\text{hooks}_R}{\text{hooks}_r}$$

Bijection

Operators in the planar $\mathcal{N} = 4$ super Yang-Mills (dimension J obeys $\frac{J^2}{N} \ll 1$ - this means that J has at most order $\sqrt{N}$ boxes)

$$O_A = \sum_{R,r,s,\alpha,\beta} a^{(A)}_{R,(r,s),\alpha,\beta} \chi_{R,(r,s)\alpha\beta}(Z, Y, X, \cdots)$$

Operator in the planar emergent gauge theory

$$O_{+A} = \sum_{R,r,s,\alpha,\beta} a^{(A)}_{R,(r,s),\alpha,\beta} \chi_{+R,(+r,s)\alpha\beta}(Z, Y, X, \cdots)$$

The coefficients appearing in the two sums are the same.

Correlators

Planar correlator:

$$\langle O_A(x_1)O_B(x_2)^\dagger\rangle = \sum_{R,r,s,\alpha} \frac{a^{(A)}_{R,(r,s),\alpha,\beta}a^{(B)*}_{R,(r,s),\alpha,\beta}\mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s} \frac{1}{|x_1-x_2|^{2J}}$$

Emergent gauge theory corelator:

$$\langle\cdots\rangle_B = \frac{\langle\cdots\rangle}{f_B}|x_1-x_2|^{2|B|}$$

$$\langle O^{(B)}_A(x_1)O^{(B)}_B(x_2)^\dagger\rangle_B = \sum_{R,r,s,\alpha} \frac{a^{(A)}_{R,(r,s),\alpha,\beta}\, a^{(B)*}_{R,(r,s),\alpha,\beta}\,\mathrm{hooks}_{+R}\, f_{+R}}{f_B\,\mathrm{hooks}_{+r}\mathrm{hooks}_s} \frac{1}{|x_1-x_2|^{2J}}$$

Correlators

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle = F_{AB}(N) \frac{1}{|x_1 - x_2|^{2J}}$$

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle_B = F_{AB}(N_{\rm eff}) \frac{1}{|x_1 - x_2|^{2J}} \left(1 + O\left(\frac{1}{N}\right)\right)$$

Planar three point functions of CFT single traces vanish. Planar three point functions of emergent gauge theory single traces vanish. OPE coefficients of both agree.

Anomalous dimensions

$$DO_{+R,(+r,s)\mu_1\mu_2}(Z,Y) =$$

$$\sum_{T,(t,u)\nu_1\nu_2} N_{+R,(+r,s)\mu_1\mu_2;+T,(+t,u)\nu_1\nu_2} O_{+T,(+t,u)\nu_1\nu_2}(Z,Y)$$

where

$$N_{+R,(+r,s)\mu_1\mu_2;+T,(+t,u)\nu_1\nu_2} = -\frac{g_{YM}^2}{8\pi^2}\sum_{+R'}\frac{c_{+R,+R'}d_{+T}nm}{d_{+R'}d_{+t}d_u(n+m)}$$

$$\times\sqrt{\frac{f_{+T}\mathrm{hooks}_{+T}\mathrm{hooks}_{+r}\mathrm{hooks}_s}{f_{+R}\mathrm{hooks}_{+R}\mathrm{hooks}_{+t}\mathrm{hooks}_u}}$$

$$\times\mathrm{Tr}\left(\left[(1,m+1),P_{+R,(+r,s)\mu_1\mu_2}\right]I_{+R'+T'}\left[(1,m+1),P_{+T,(+t,u)\nu_2\nu_1}\right]I_{+T'+R'}\right)$$

De Comarmond, dMK, Jefferies, [arXiv:1012.3884 [hep-th]]

In the end: $N \to N_{\mathrm{eff}}$

Weak coupling tests

Evidence supporting the above result:

1. Explicit computations of anomalous dimensions of the emergent gauge theory, to two loops, agree with $N \to N_{\text{eff}}$ rule.
2. Using $su(2|2)$ symmetry two magnon S-matrix is determined. Agrees up to two loops with a computation performed in the emergent gauge theory.

dMK, Mathwin, van Zyl, [arXiv:1601.06914 [hep-th]]

Strong coupling tests

Solution to NG in LLM: $\phi = \sigma - \tau$, $t = \kappa\tau$, $r = r(\sigma)$. Eqn of motion can be integrated once

$$r'(\sigma) = \frac{\kappa r\sqrt{1 - \frac{r^2}{C^2}}}{\sqrt{(1-\kappa)^2 h^4(r) r^2 - (\kappa - (1-\kappa)V_\phi(r))^2}}$$

C is an integration constant.

$$E = \frac{\sqrt{\lambda}}{2\pi}\int_{\sigma_{\rm min}}^{\sigma_{max}} d\sigma \frac{\partial L_{NG}}{\partial \dot{t}} \qquad J = \frac{\sqrt{\lambda}}{2\pi}\int_{\sigma_{\rm min}}^{\sigma_{max}} d\sigma \frac{\partial L_{NG}}{\partial \dot{\phi}},$$

$$E - J = \frac{\sqrt{\lambda}}{\pi} r_0 \sin\left(\frac{p}{2}\right) - 4\frac{\sqrt{\lambda}}{\pi} r_0 \sin^3(\frac{p}{2}) e^{-2\left(\frac{\pi}{\sqrt{\lambda} r_0 \sin(\frac{p}{2})}+1\right)} + \cdots$$

Net effect $\sqrt{\lambda} \to \sqrt{\lambda} r_0$ which is $N \to N_{\rm eff}$.

dMK, Kim, Van Zyl, arXiv:1802.01367 [hep-th]

Summary

We have studied the emergent gauge theory that arises by exciting the branes that backreact to produce the LLM geometry.

Using group representation theory techniques we have managed to learn enough about the dynamics to suggest its an $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\text{eff}})$.

Passes weak and strong coupling tests.

New holographic dualities and decoupling limits?