Linear Sigma EFT for Nearly Conformal Gauge Theories

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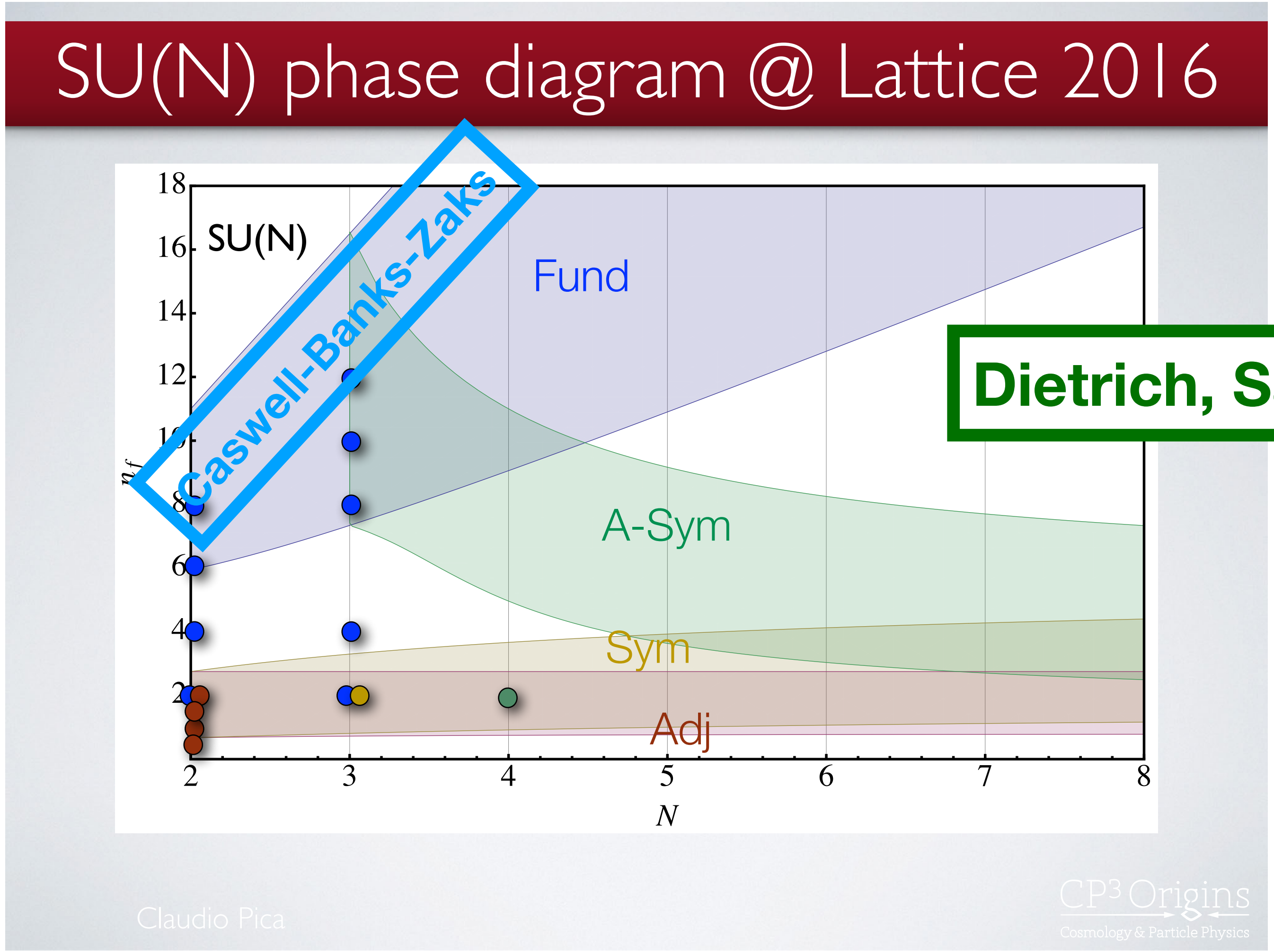
Outline

- Low Energy Physics near Conformal Windows
- Generalized Linear Sigma Model EFT
- Future Directions

Conformal Windows



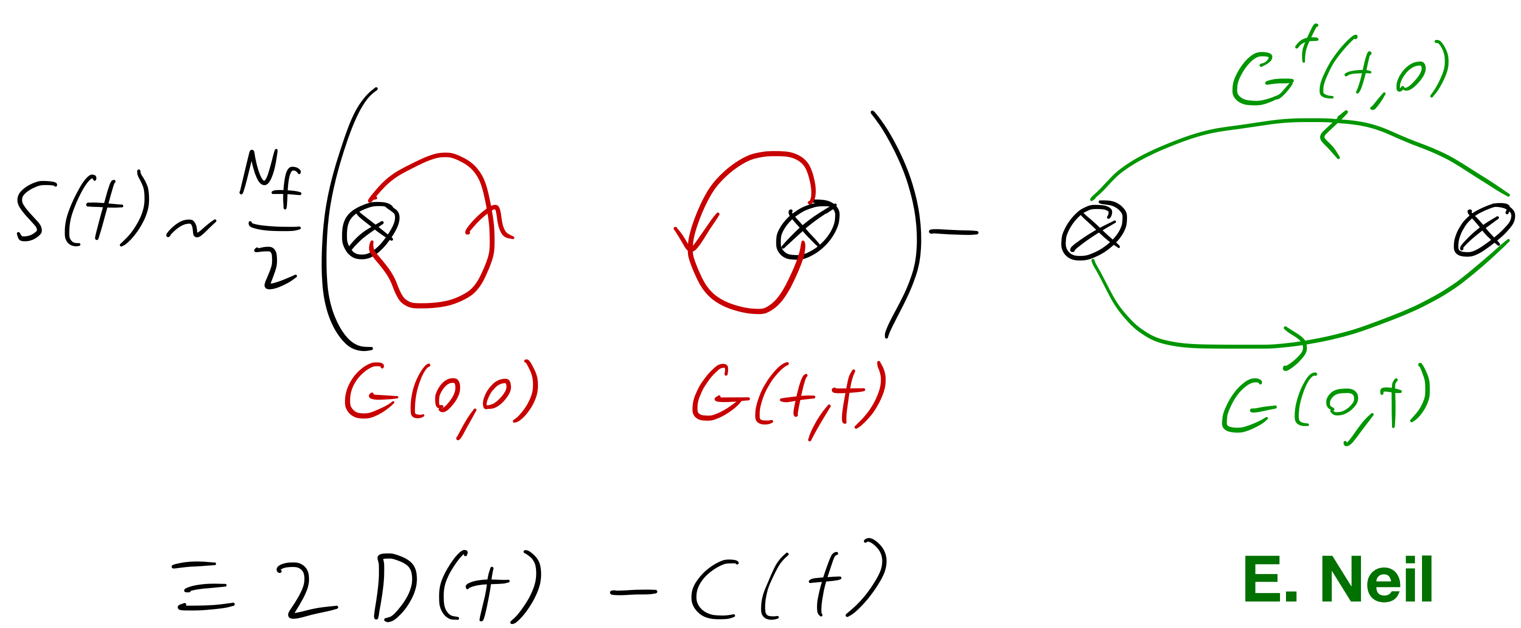
Dietrich, Sannino, Tuominen '05

- Shown estimate of lower edge (strongly coupled IRFP) is unreliable.
- Just below window is near-conformal region. A light scalar (pseudo-dilaton) might exist in this region [Yamawaki, Bando, Matutumo '86]
- Why are light scalars interesting? M_{Higgs} ~ (1/2) vev. So, a light composite scalar could be a composite Higgs candidate.

Hadron Mass Inequalities: Can the scalar be lighter than the pion?

- Rigorous hadron mass inequalities exist for flavored mesons in confining vector-like theories [Weingarten ('83), Witten ('83), Nussinov ('83), Detmold ('14)]

- In particular, $M_\pi \leq M_{a_0}$, so for σ/f_0 to be as light as π the valence-disconnected diagram in correlator must dominate. Can this feature be related to dilaton?

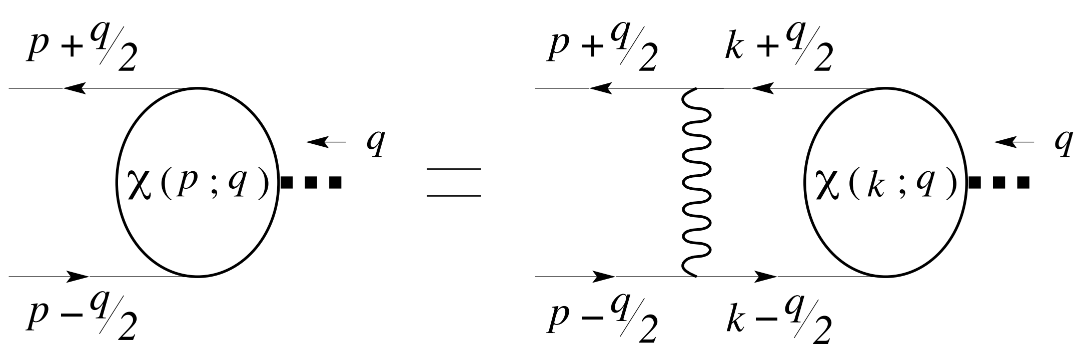


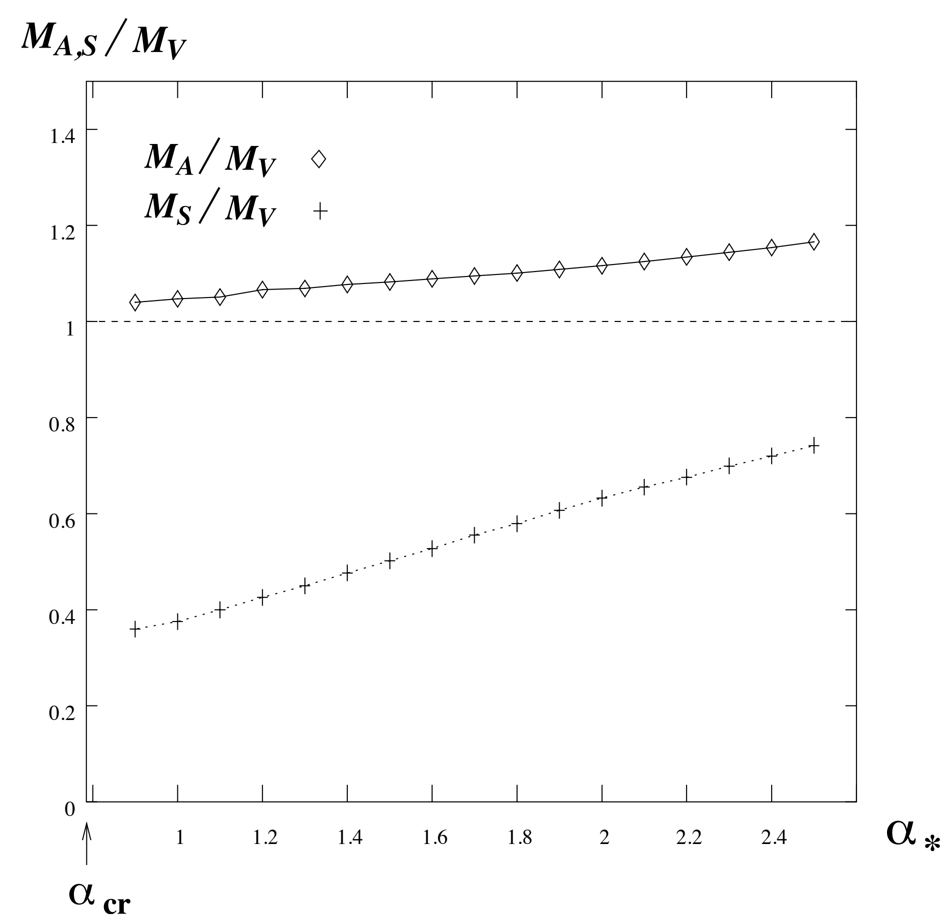
E. Neil

Other conjectured features of near-conformal theories

- Solving Schwinger-Dyson and Bethe-Salpeter equations suggests parity-doubled spectrum and light *flavored* scalar a_0 [Shrock, Kurachi ('06)]

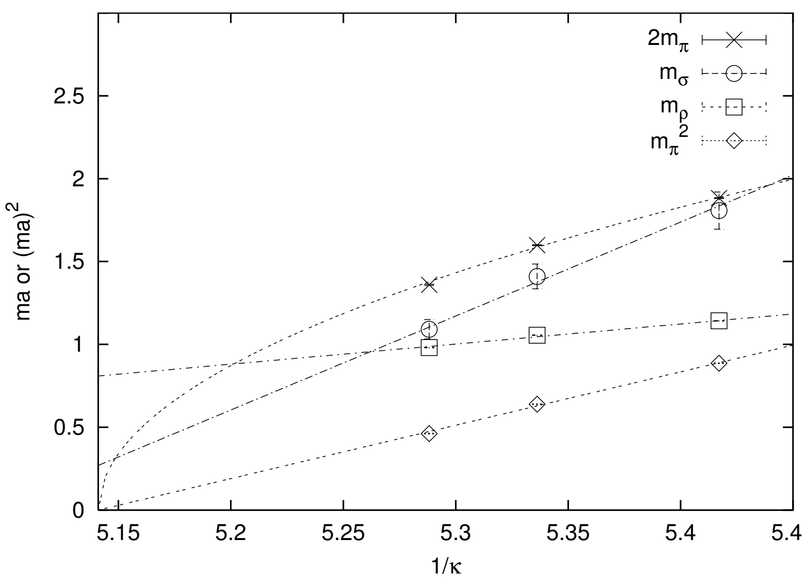
- Parity doubling is good for small S parameter but if the flavored scalar is really this light, shouldn't it be included in any EFT?



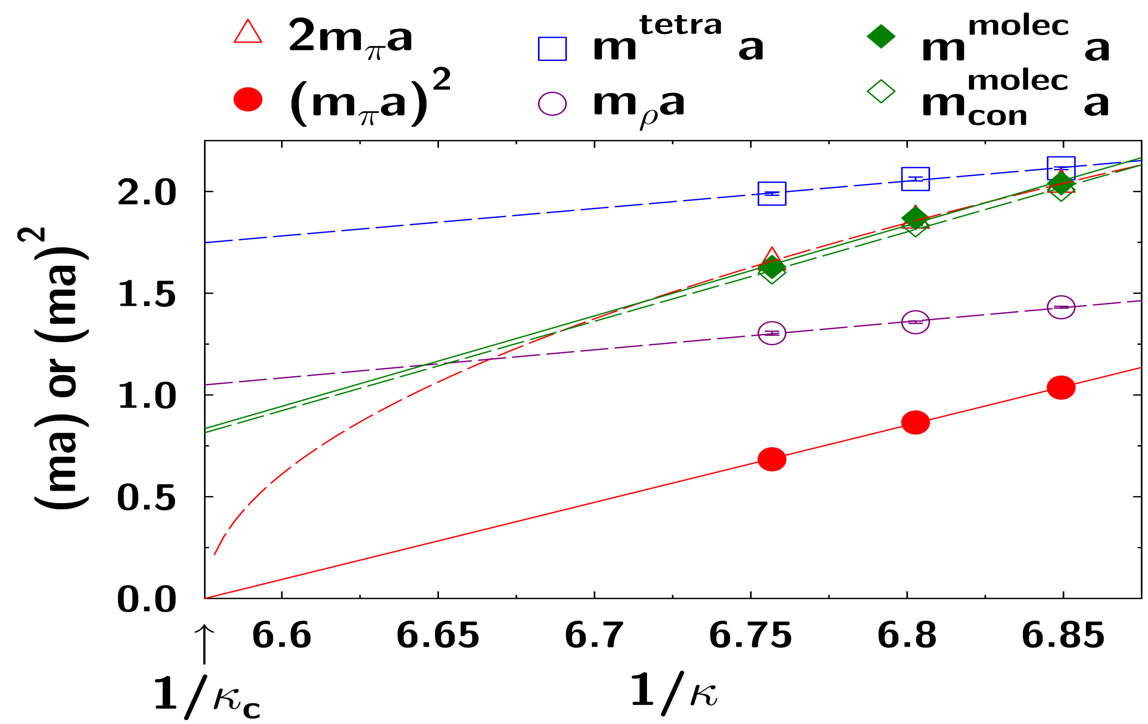


Scalar Sector of QCD

- Some heavy quark results from lattice SCALAR collaboration:

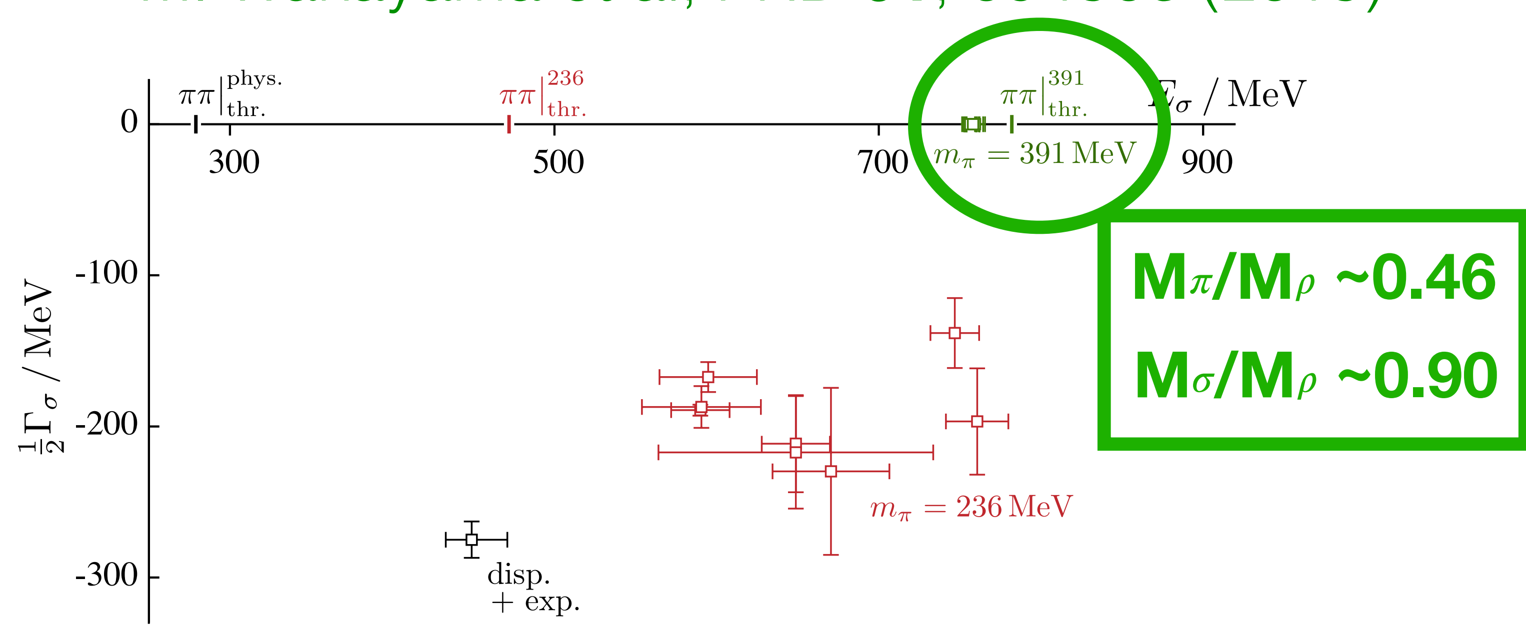


T. Kunihiro et al, PRD **70**, 034504 (2004)



M. Wakayama et al, PRD **91**, 094508 (2015)

- New result from Hadron Spectrum Collaboration



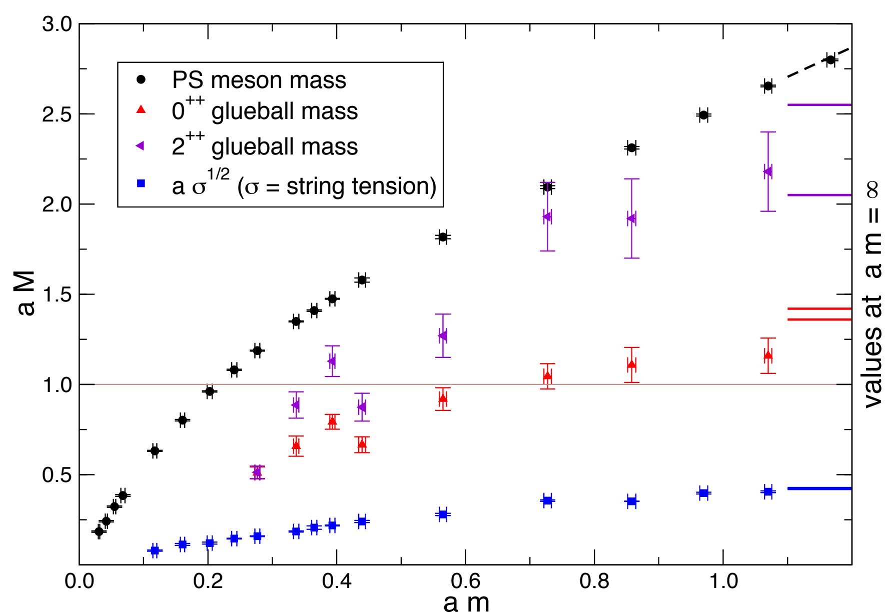
- Bottom line: $M_\sigma \sim M_\rho$.

R. Briceno et al, PRL **118**, 022002 (2017)

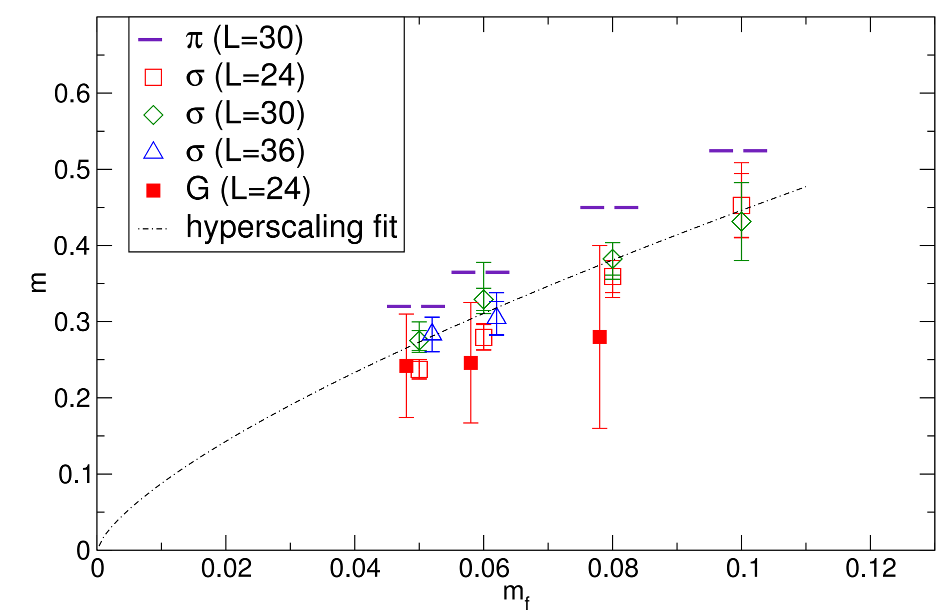
Theories with Light Scalars

- Mass-deformed IRFP theories with very light scalars.

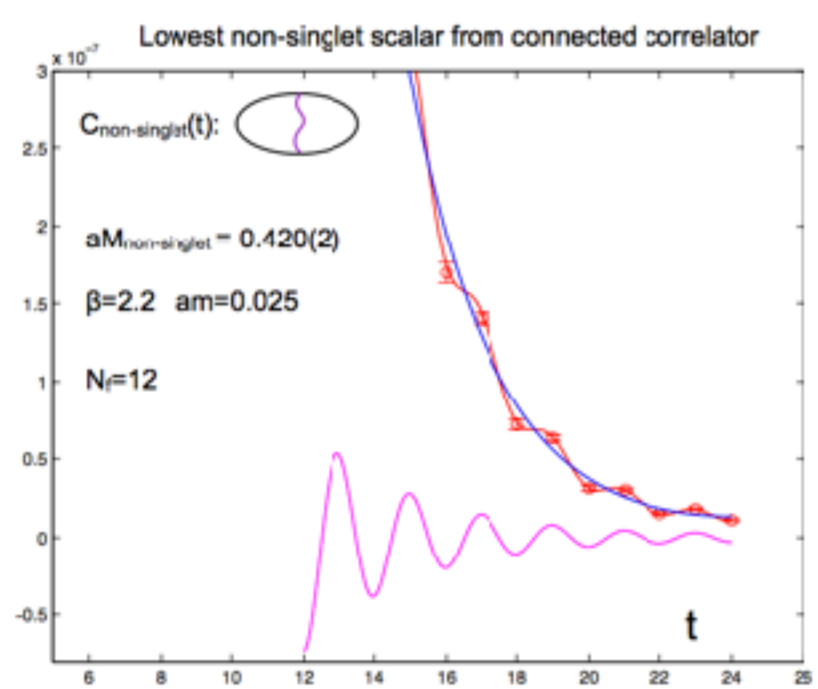
SU(2) N_f=2 adj (Edinburgh)
Phys. Rev. D 82, 014510 (2010)

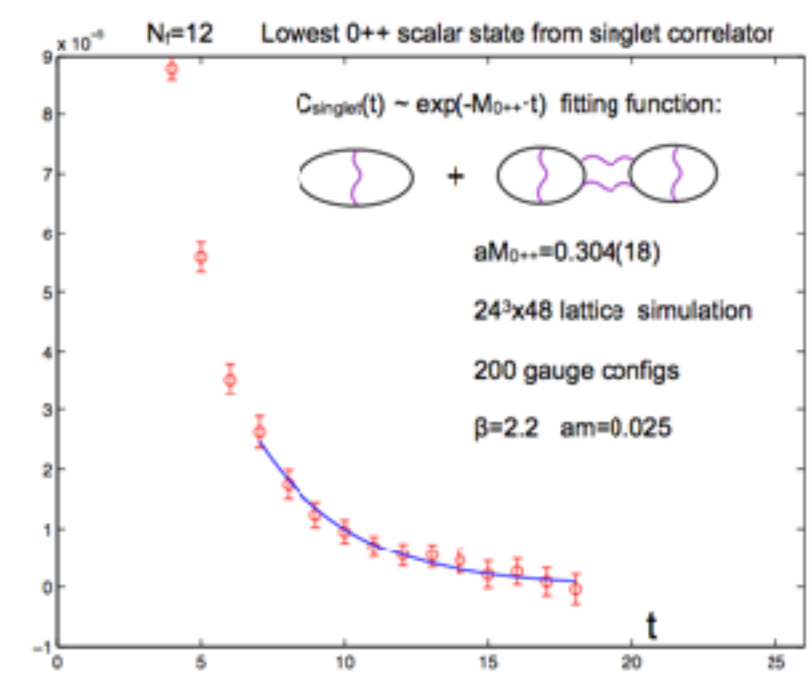


SU(3) N_f=12 fund (LatKMI)
Phys. Rev. Lett. 111, 162001 (2013)



SU(3) N_f=12 fund (LatHC)
USQCD White Paper 2013

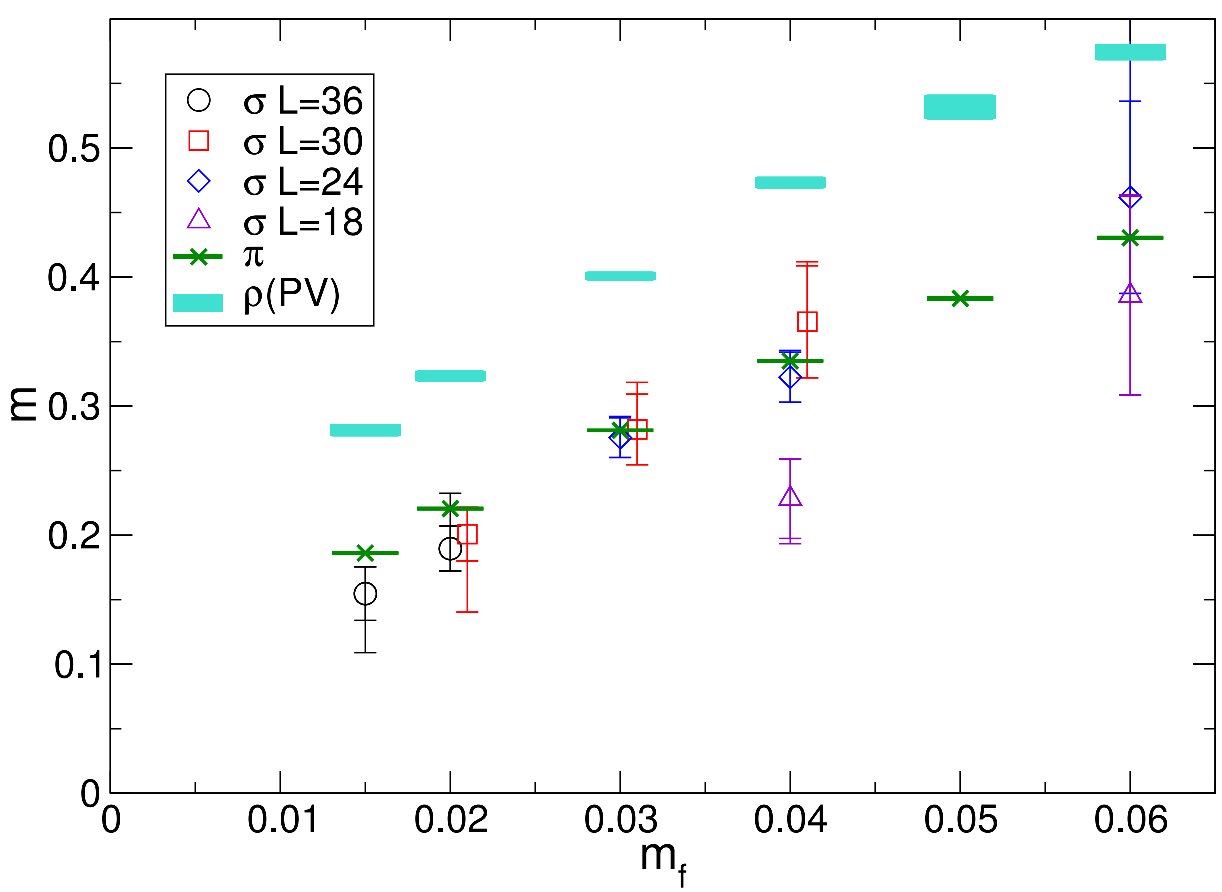




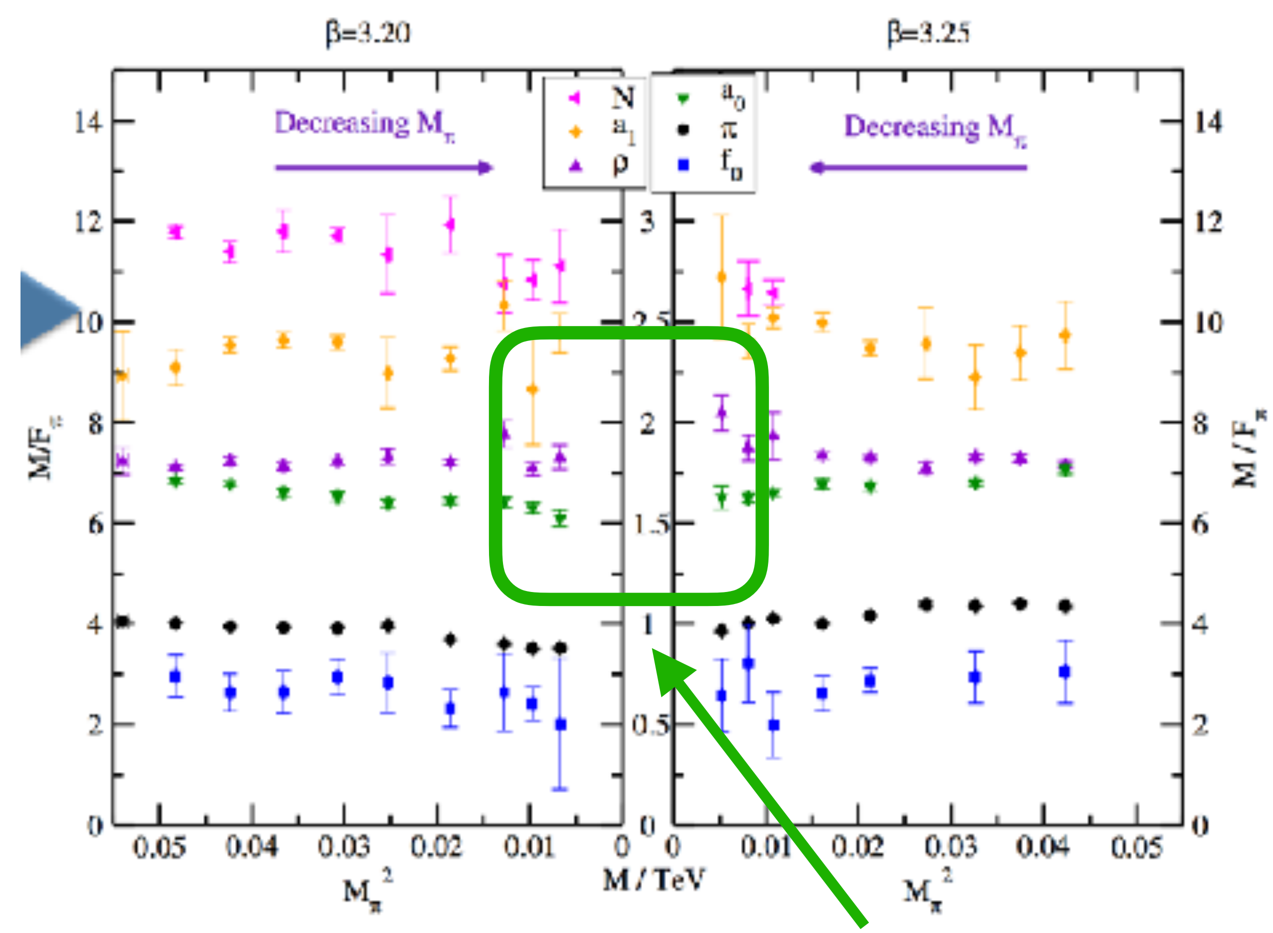
More Light Scalars

- Theories likely just below conformal window also have light scalars.

SU(3) N_f=8 fund
LatKMI (Nagoya)
Phys. Rev. D 89, 111502 (2014)



SU(3) N_f=2 sym
LatHC Collaboration
LATTICE 2015

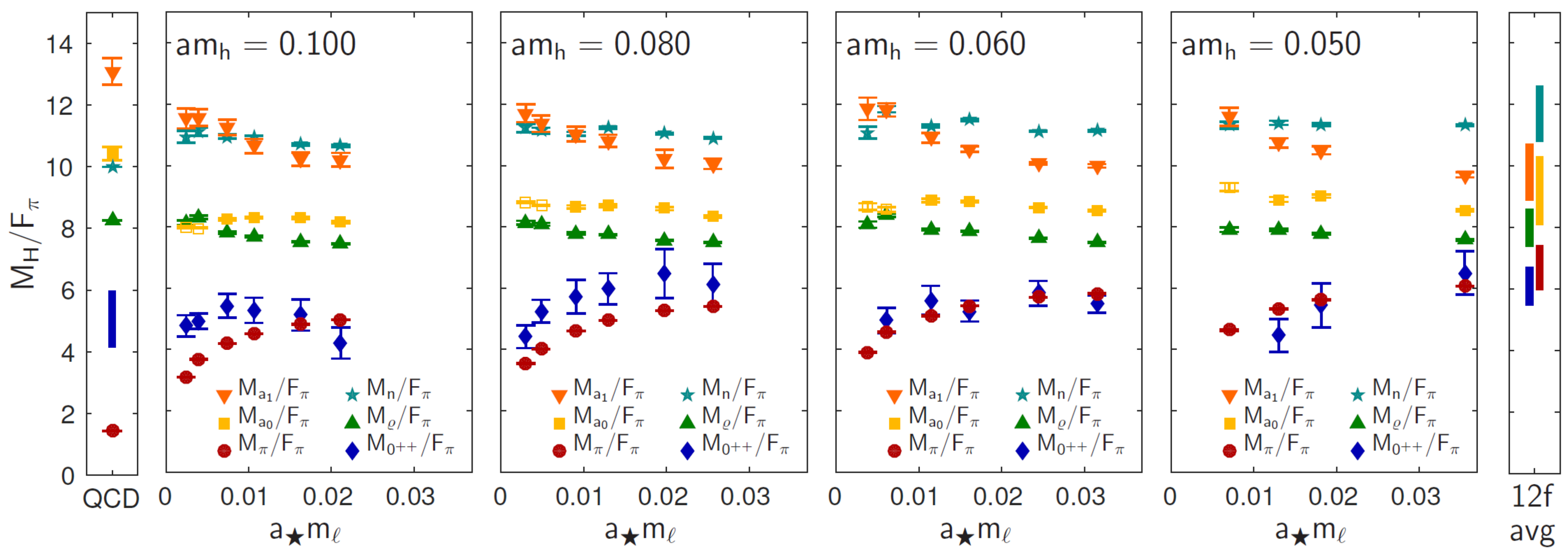


Note $M_{a_0} < M_\rho$

Mass-Split System: SU(3) $N_f=4+8$

- A mass-split system would be inside the conformal window if $m_h/m_l = 1$ as $m_l \to 0$.

- But, if $m_h/m_l \sim 5$ as $m_l \to 0$, the low energy theory may be outside the conformal window with light scalar.



Brower, Hasenfratz, Rebbi, Weinberg, Witzel (2016)

Lattice Strong Dynamics Collaboration

James Osborn
Xiao-Yong Jin

Joe Kiskis

Graham Kribs

David Schaich

Anna Hasenfratz
Ethan Neil
Oliver Witzel

Ethan Neil
Enrico Rinaldi

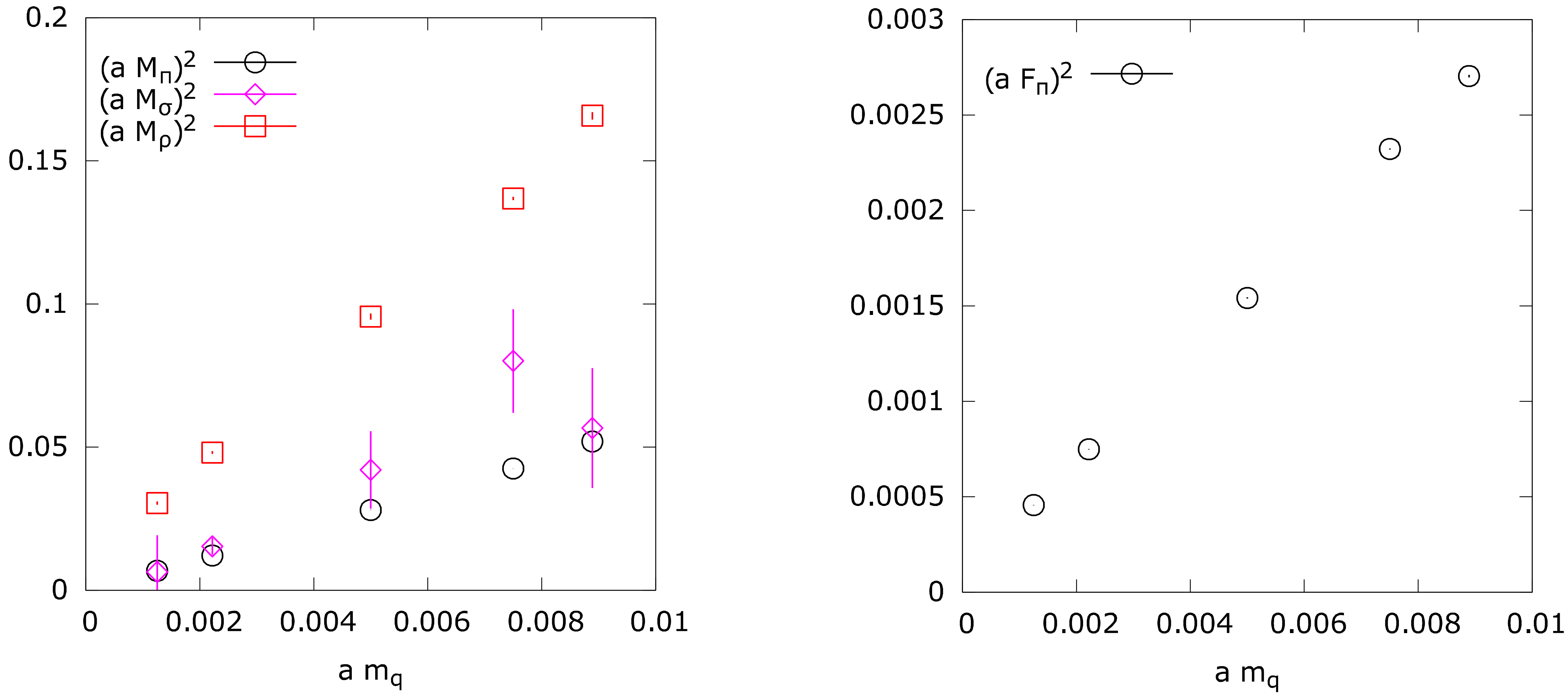
Richard Brower
Claudio Rebbi
Evan Weinberg

Michael Buchoff
Chris Schroeder
Pavlos Vranas

Tom Appelquist
George Fleming
Andy Gasbarro

with James Ingoldby (Yale)

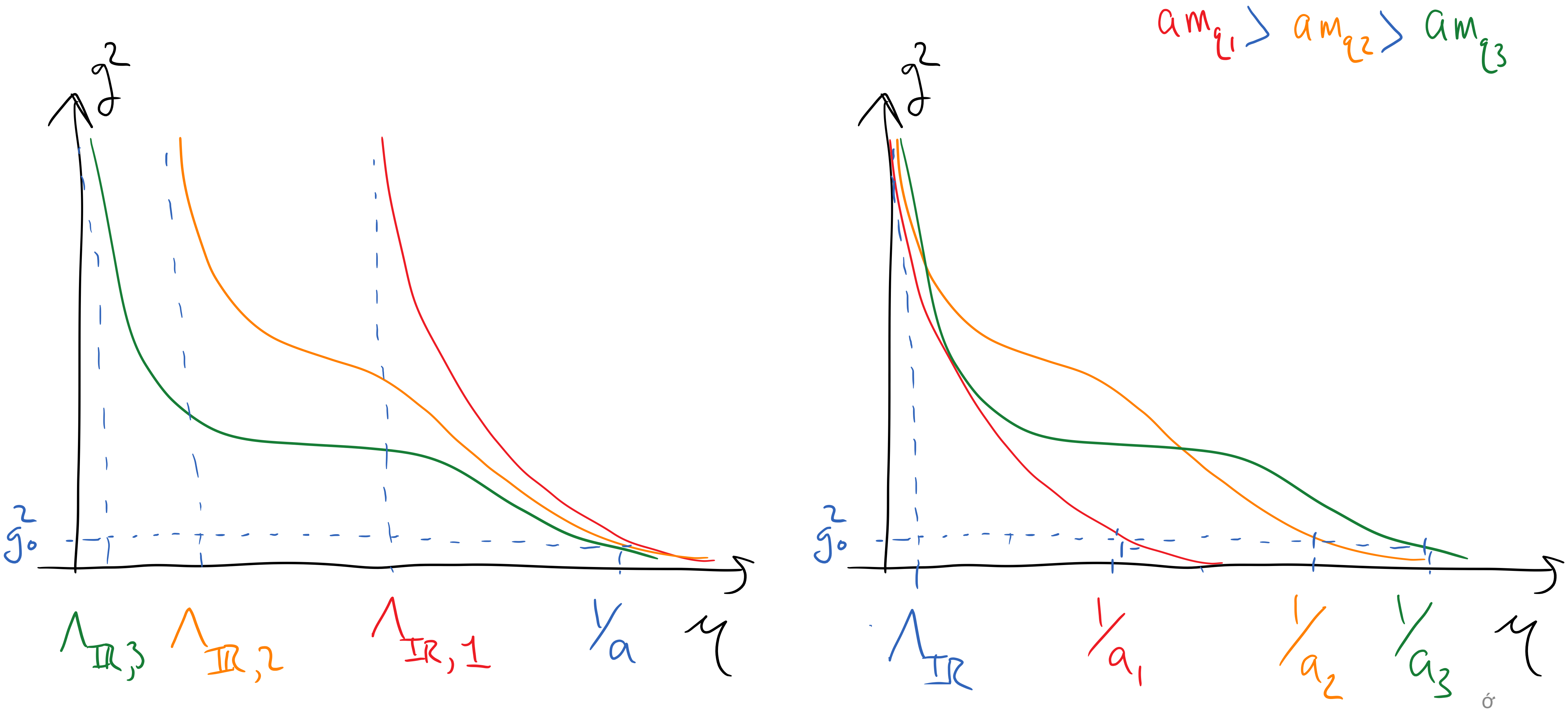
Finding an EFT for SU(3) N_f=8



- In LO χPT, $F_\pi(m_q) \sim f_\pi$. The lattice results show NLO >> LO for $F_\pi(m_q)$, but $M_\sigma \sim M_\pi << M_\rho$.

- **Notational convention**: chiral limit m_x , finite quark mass M_x

Near Conformal Effects

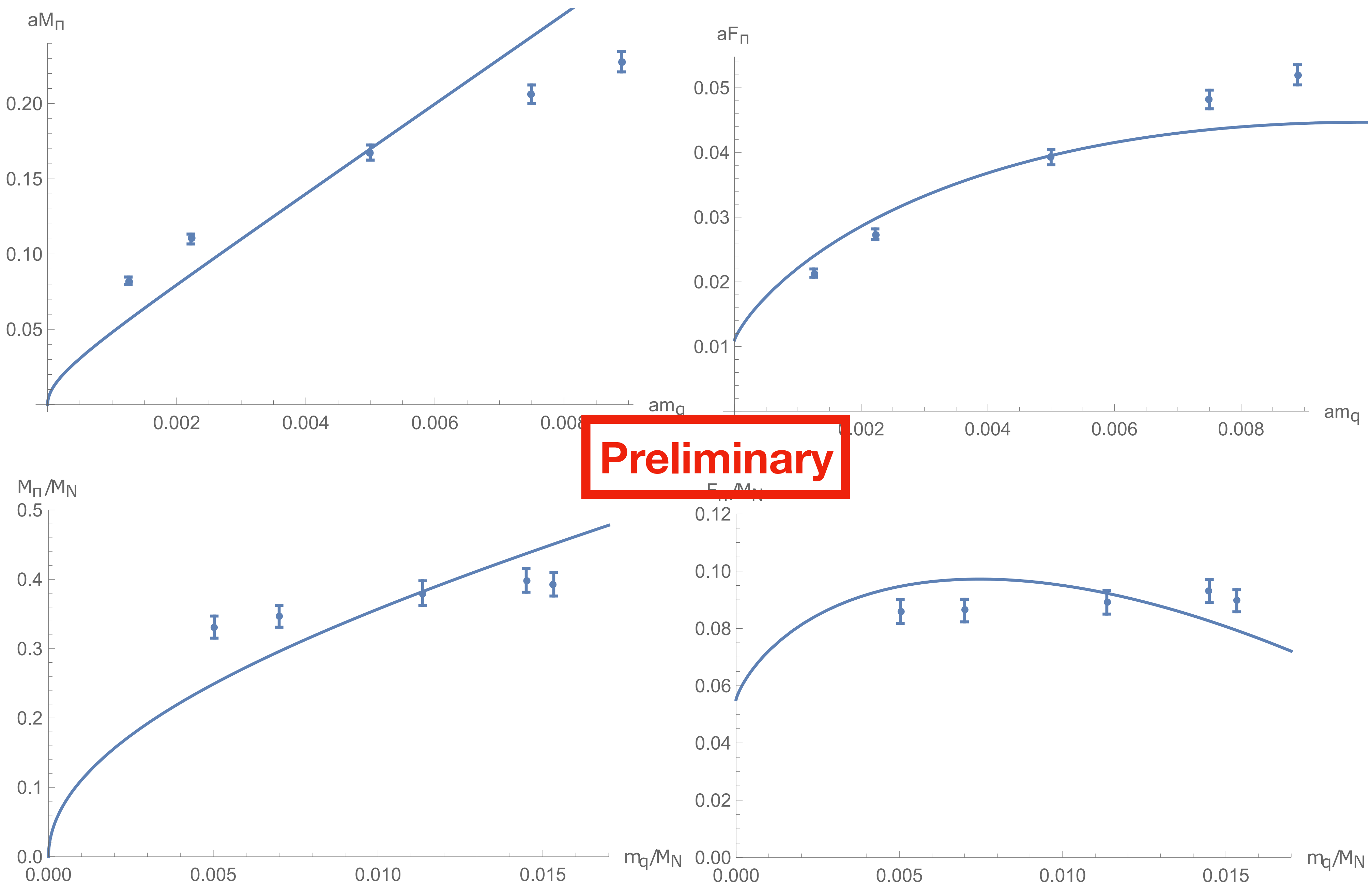


- Large slopes expected for IR quantities when plotted in bare lattice units $a\, m_q$.

SU(3) Nf=8 NLO Fits

Lattice units
χ^2/dof=29

Nucleon units
χ^2/dof=7



- Fitting in nucleon vs. lattice units relieves some tension, but NLO χPT still poor description of results.

Linear Sigma Model EFT

- Gell-Mann-Levy linear sigma model was early EFT for QCD. N_f=2 version isomorphic to O(4). Only π and σ included. Naively renormalizable as $\Lambda \to \infty$.
- N_f>2 requires additional dof: a_0, η'. Removing heavy η' means no longer renormalizable as $\Lambda \to \infty$.
- Very predictive as vev of σ tied to χSB.
- Can naturally incorporate light a_0 mesons.
- Just adding a scalar to chiral lagrangian much less predictive (many new LECs at LO).

LSM Fields and Lagrangian

- Bifundamental (Nf,Nf) transforms linearly under U(Nf)×U(Nf).

$$M(x) = \mathrm{s(x)} + \mathrm{ip(x)} \qquad \mathrm{s}(x) = \frac{\tilde{\sigma}(x)}{\sqrt{N_f}} + \tilde{a}_i(x)T_i \qquad \mathrm{p}(x) = \frac{\tilde{\eta}'(x)}{\sqrt{N_f}} + \tilde{\pi}_i(x)T_i$$

- Polar (non-linear) basis enables trivial decoupling of η'.

$$M(x) = \Sigma(x)S(x) \qquad \Sigma(\mathrm{x}) = \exp\left[\frac{i\sqrt{2}}{F_\pi}\left(\frac{\eta'(x)}{\sqrt{N_f}} + \pi_i(x)T_i\right)\right] \qquad S(x) = \frac{\sigma(x)}{\sqrt{N_f}} + a_i(x)T_i$$

- General Lagrangian (no explicit symmetry breaking)

$$\mathcal{L} = \frac{1}{2}\langle \partial_\mu M^\dagger \partial^\mu M \rangle - V_0(M) \qquad V_0 = \frac{\mu^2}{2}\langle M^\dagger M \rangle + \frac{\lambda_1}{4}\langle M^\dagger M \rangle^2 + \frac{N_f \lambda_2}{4}\left\langle \left(M^\dagger M\right)^2 \right\rangle$$

- Rewrite potential after SSB ($\mu^2 < 0$), easy to decouple a0.

$$V_0 = \frac{-m_\sigma^2}{4}\langle M^\dagger M \rangle + \frac{m_\sigma^2 - m_a^2}{8f^2}\langle M^\dagger M \rangle^2 + \frac{N_f m_a^2}{8f^2}\left\langle \left(M^\dagger M\right)^2 \right\rangle$$

Explicit Symmetry Breaking

Spurion $\chi = B\, m_q$, $B \sim \langle qq \rangle / f_\pi^2$

$$V_{SB} = -\sum_{i=1}^{9} \tilde{c}_i O_i(x),$$

Symbol	Operator	$\alpha \lesssim 1$	$1 < \alpha \lesssim 2$
O_1	$\langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	✓
O_2	$\langle M^\dagger M \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_3	$\langle (M^\dagger M)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X
O_4	$\langle \chi^\dagger M + M^\dagger \chi \rangle^2$	✓	X
O_5	$\langle \chi^\dagger \chi M^\dagger M \rangle$	✓	X
O_6	$\langle \chi^\dagger \chi \rangle \langle M^\dagger M \rangle$	✓	X
O_7	$\langle \chi^\dagger M \chi^\dagger M + M^\dagger \chi M^\dagger \chi \rangle$	✓	X
O_8	$\langle \chi^\dagger \chi \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_9	$\langle (\chi^\dagger \chi)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X

Relative size of χSB:

$$\frac{m_q B_\pi}{\Lambda^2} \sim \left(\frac{M_\sigma}{\Lambda}\right)^\alpha \ll 1$$

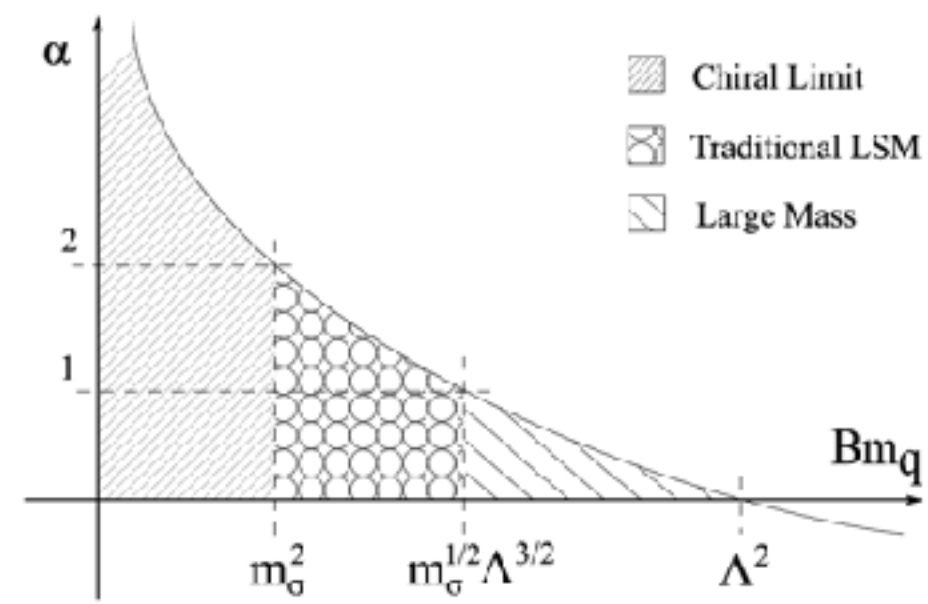
Estimate: $\Lambda \sim M_\rho$ when $M_\rho = 2\, M_\pi$

$$\frac{F^2}{f^2} = 1 + \frac{2}{m_\sigma^2}\left[2Bm_q\frac{f}{F} + 6Bm_q(c_2+c_3)F + 2B^2m_q^2(4c_4+c_5+c_6+2c_7) + 2B^3m_q^3\frac{c_8+c_9}{F}\right],$$

$$M_\pi^2 = 2Bm_q\frac{f}{F} + 2Bm_q(c_2+c_3)F + 8B^2m_q^2(c_4+c_7) + 2B^3m_q^3\frac{c_8+c_9}{F},$$

$$M_\sigma^2 = m_\sigma^2 + 6Bm_q\frac{f}{F} + 6Bm_q(c_2+c_3)F + 4B^2m_q^2(4c_4+c_5+c_6+2c_7) + 6B^3m_q^3\frac{c_8+c_9}{F},$$

$$M_a^2 = m_a^2\frac{F^2}{f^2} + 4Bm_q\frac{f}{F} + 8Bm_qc_2F + 2B^2m_q^2(8c_4+c_5+c_6+2c_7) + 4B^3m_q^3\frac{c_8+c_9}{F}.$$



$M_\sigma^2 \geq 3\, M_\pi^2$?

$$3M_\pi^2 - M_\sigma^2 + m_\sigma^2 = 4B^2m_q^2(2c_4 - c_5 - c_6 + 4c_7),$$

- If m_σ is light ($\sim f_\pi$) new kinematic regimes are opened in the linear sigma model. These new regimes match lattice results.

Formalism Paper Imminent

Linear Sigma EFT for Nearly Conformal Gauge Theories

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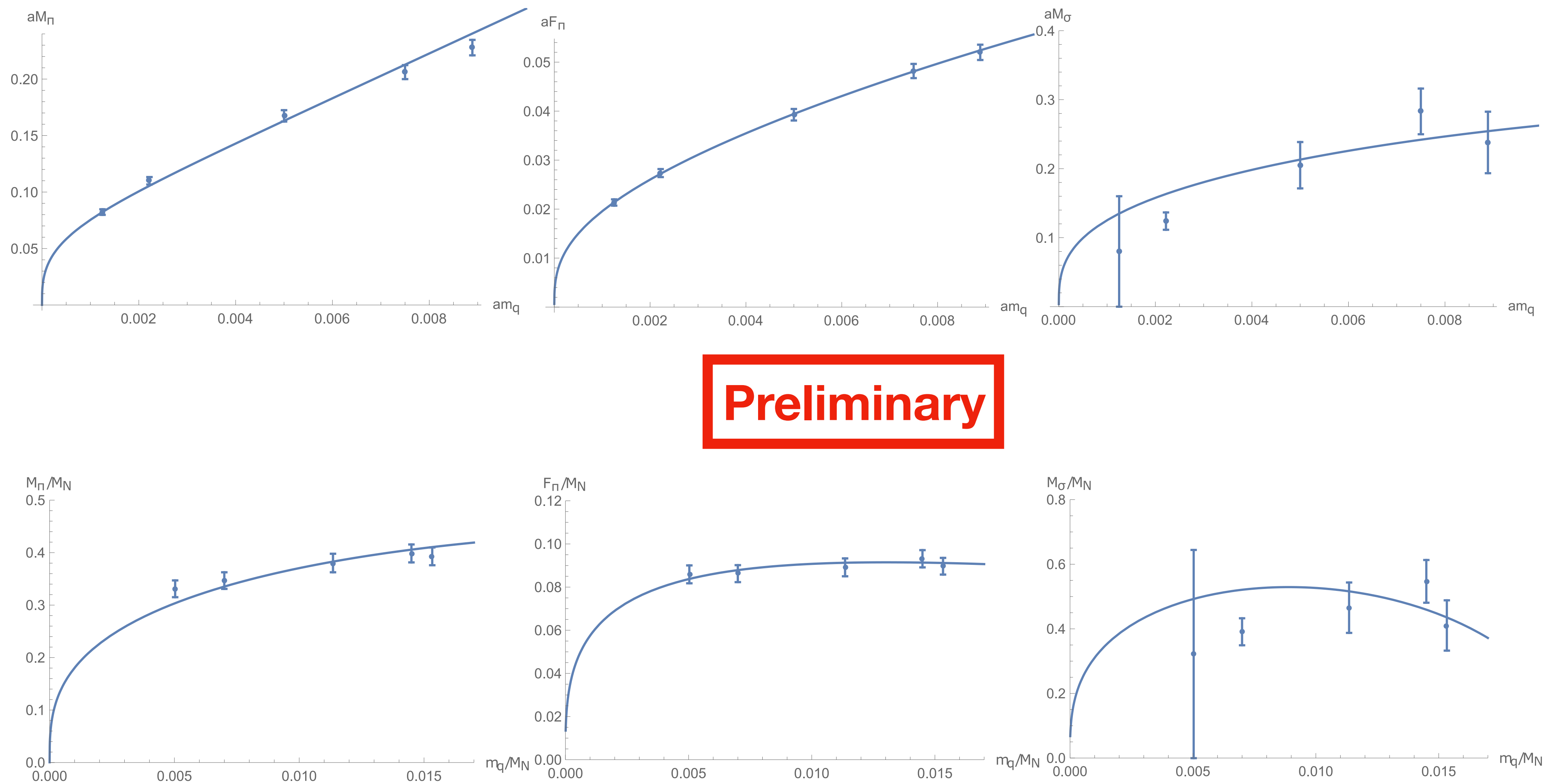
SU(3) Nf=8 LSM9 LO Fits

Lattice units
χ^2/dof=1.30

Nucleon units
χ^2/dof=1.39



- LSM with 9 LO breaking terms, required when $M\sigma \sim M\pi$, so far is good description of lattice results.

- Further analysis continues including ⟨qq⟩ and I=2 $a_{\pi\pi}$ scattering, plus more statistics for lattice results.

Future Directions

- Linear Sigma Model EFT has new kinematic regimes when $m_\sigma \sim f_\pi$. Explains how lattice results can evade inequality $M_\sigma^2 \geq 3\, M_\pi^2$.

- Working now on breaking flavor symmetry $SU(8) \to SU(2)\times SU(6)$: $m_2=0$, $m_6\neq0$. Closer to Higgs phenomenology.

- How heavy can we make 60 PNGBs keeping scalar light? Non-trivial given extra operators at LO.