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Thermalization in a confining gauge theory

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Bibliography

T. Ishii (Crete), E. Kiritsis (APC+Crete), C. Rosen (Crete)
arXiv: 1503.07766[hep-th]

and previous work with the Brussels group

B. Craps (Vrije U., Brussels), E. Kiritsis (APC+Crete), C. Rosen (Crete), A. Taliotis, J. Vanhoof, H. Zhang (Vrije U., Brussels)
arXiv: 1311.7560[hep-th]

Introduction

- The process of thermalization in QFT is poorly understood even today.

- The problem has been brought forward with the heavy-ion collisions at RHIC and CERN.

- The data indicate rapid thermalization of the initial energy density and the formation of a quark gluon plasma.

- The thermalization time is an order of magnitude smaller than what was expected at RHIC and seems even smaller at LHC.

- There are even hints that a kind of hydrodynamics is valid before even thermalization.

- The theory (QCD) is in a strongly coupled regime for most of the energy range of the experiments.

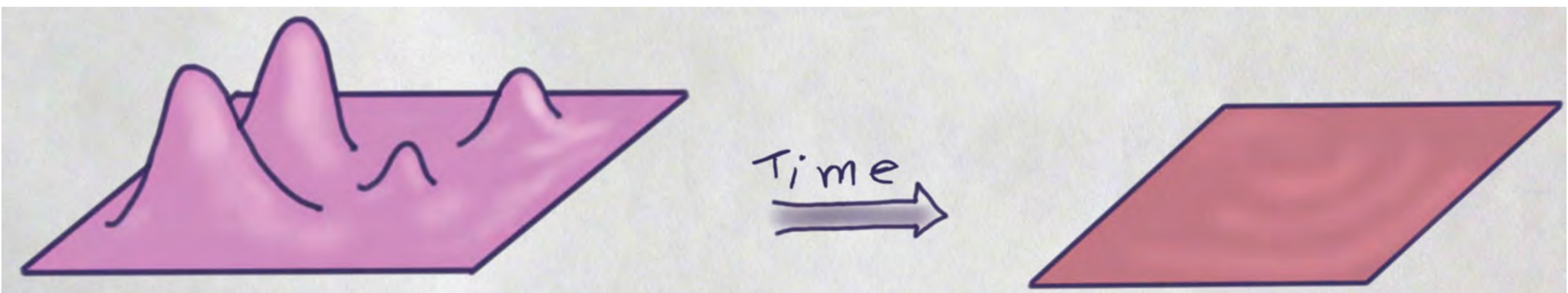
- The challenge is to understand thermalization in this context and more generally.

The theoretical setup for thermalization

• We would like to consider the theory initially in its vacuum state and then perturb it by time and space dependent coupling constants. This provides localized perturbations in space and time.

• There is a simplification that is important in order to manage the technical problem today: take the perturbation to be space independent:

$$L_{QFT} + \int d^4x \ f_0(t) \ O(x) \qquad \to \qquad \nabla^t \langle T_{tt} \rangle = \dot{f}_0 \ \langle O \rangle$$



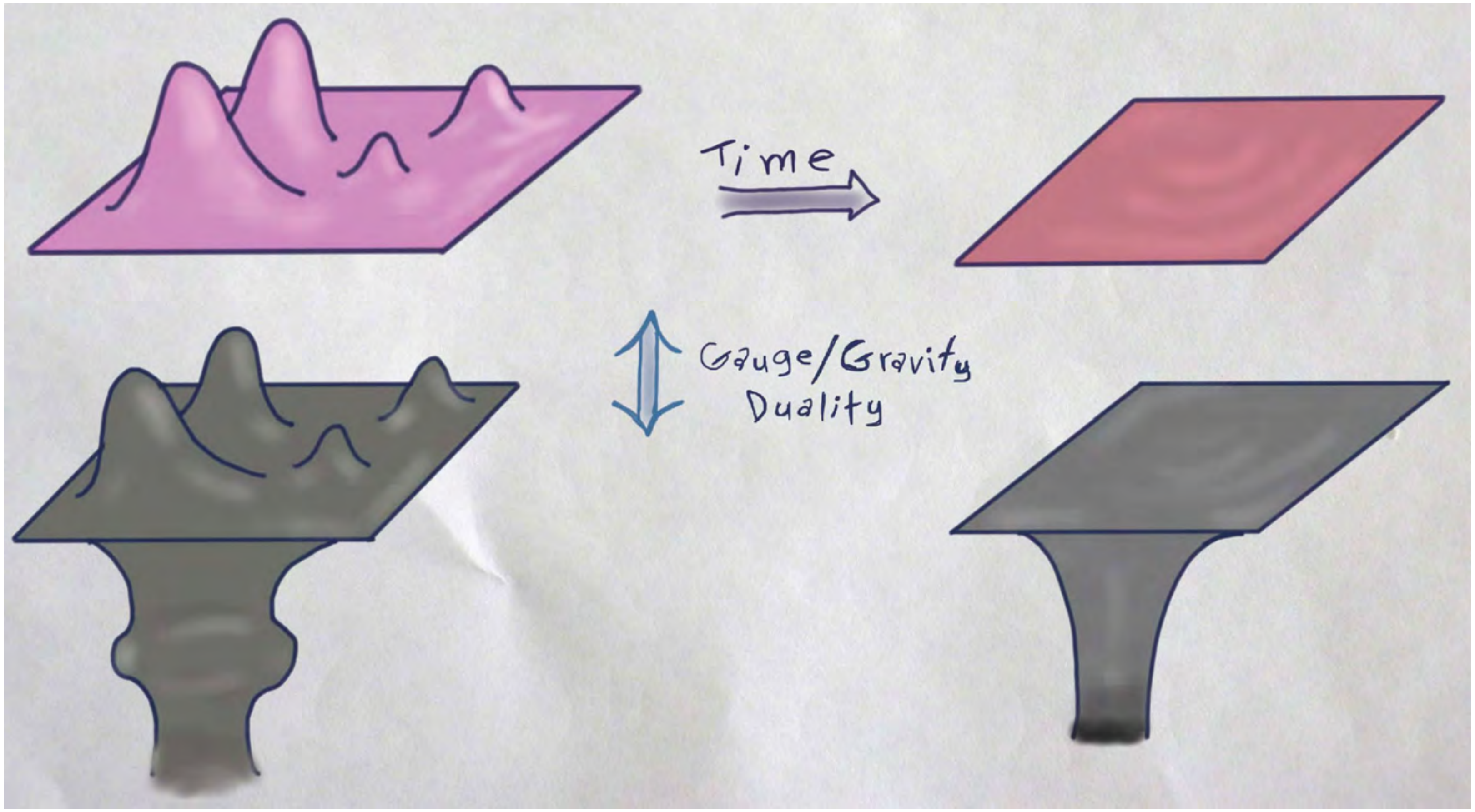
• The approach to equilibration is controlled by the expectation values $\langle T_{tt} \rangle (t)$, $\langle O \rangle (t)$.

• We expect that if the system thermalizes then

$$\langle O \rangle (t \to \infty) \quad \to \quad Tr[\rho_{\text{thermal}} \ O(x)]$$

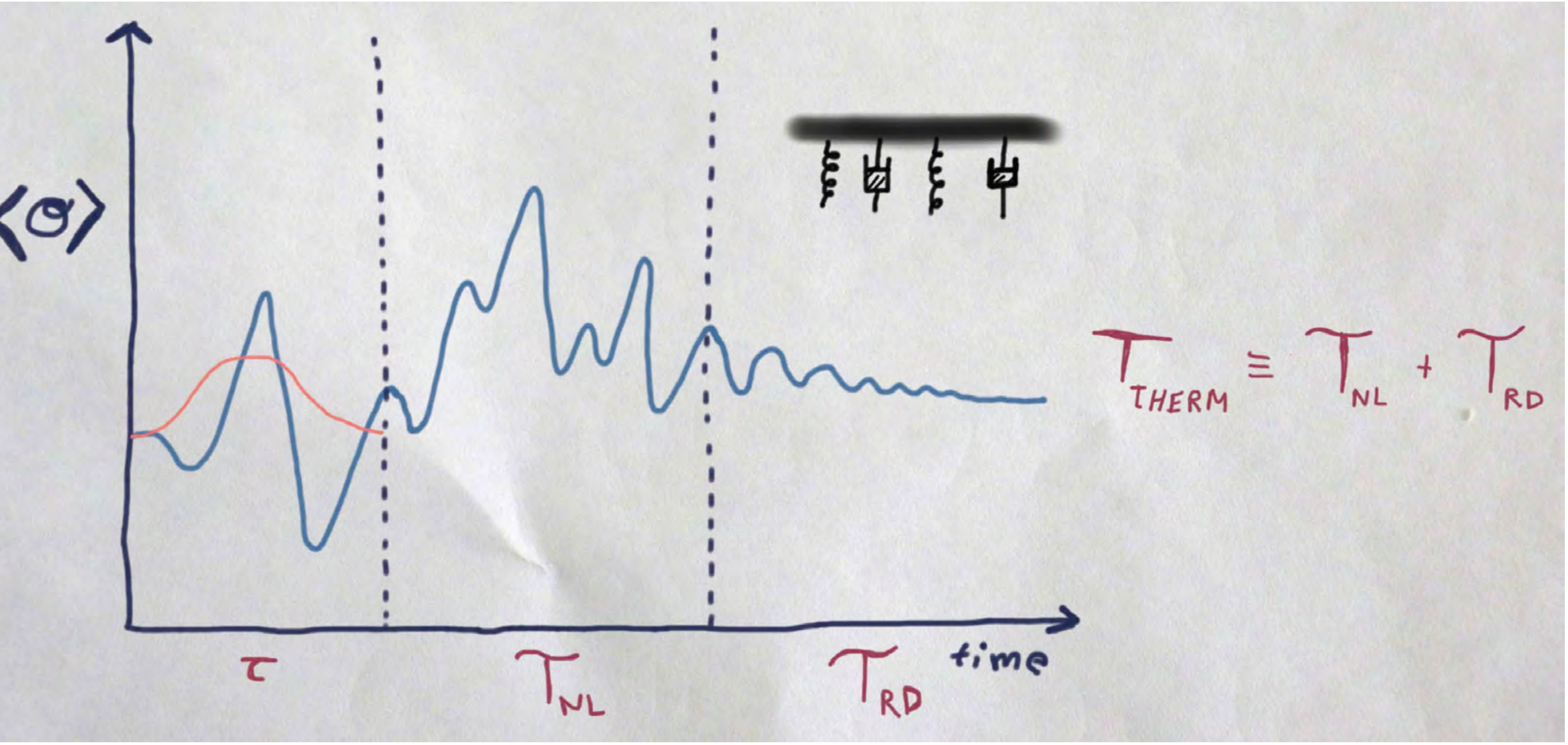
Thermalization at strong coupling

• To calculate the observables at strong coupling we will assume the holographic (AdS/CFT) correspondence (aka gauge/gravity duality).



• Thermalization corresponds to black hole formation in the bulk spacetime.

Expected characteristic scales



- There are three possible characteristic times involved.

Thermalization calculations

• The first numerical calculations involved AdS (CFT) and an existing bulk black hole that was perturbed by boundary data.

Chessler+Yaffe (2008)

• Analytical techniques using a small amplitude expansion for a massless scalar field in AdS were developed. There were two parameters, a dimensionless amplitude ϵ for the perturbing scalar field and a characteristic perturbation time δt.

Minwalla+Battacharyya (2009)

• The analysis indicated that in the limit of $\epsilon \to 0$ and in the infinite volume limit (Poicaré AdS) there is instant thermalization always (formation of a black hole).

• In the finite volume case (global AdS), they also found a regime in which a black hole was not formed (in the first attempt).

$$x \equiv \frac{\delta t}{R}$$

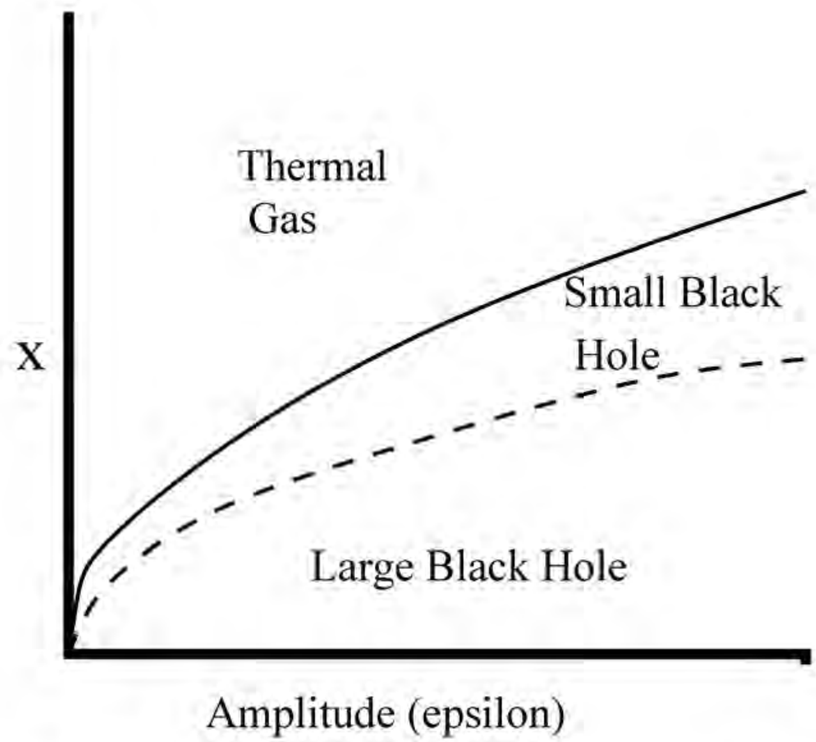


Figure 2: The 'Phase Diagram' for our dynamical stirring in global AdS. The final outcome is a large black hole for $x \ll \epsilon^{\frac{2}{d}}$ (below the dashed curve), a small black hole for $x \ll \epsilon^{\frac{1}{d-1}}$ (between the solid and dashed curve) and a thermal gas for $x \gg \epsilon^{\frac{1}{d-1}}$. The solid curve represents non analytic behaviour (a phase transition) while the dashed curve is a crossover.

Minwalla+Battacharyya (2009)

Non-conformal/Confining QFTs

• We expect similarities between a conformal (scale-invariant) gauge theory and a non-conformal gauge theory.

• The physical interest is in non-conformal confining gauge theories like YM or QCD.

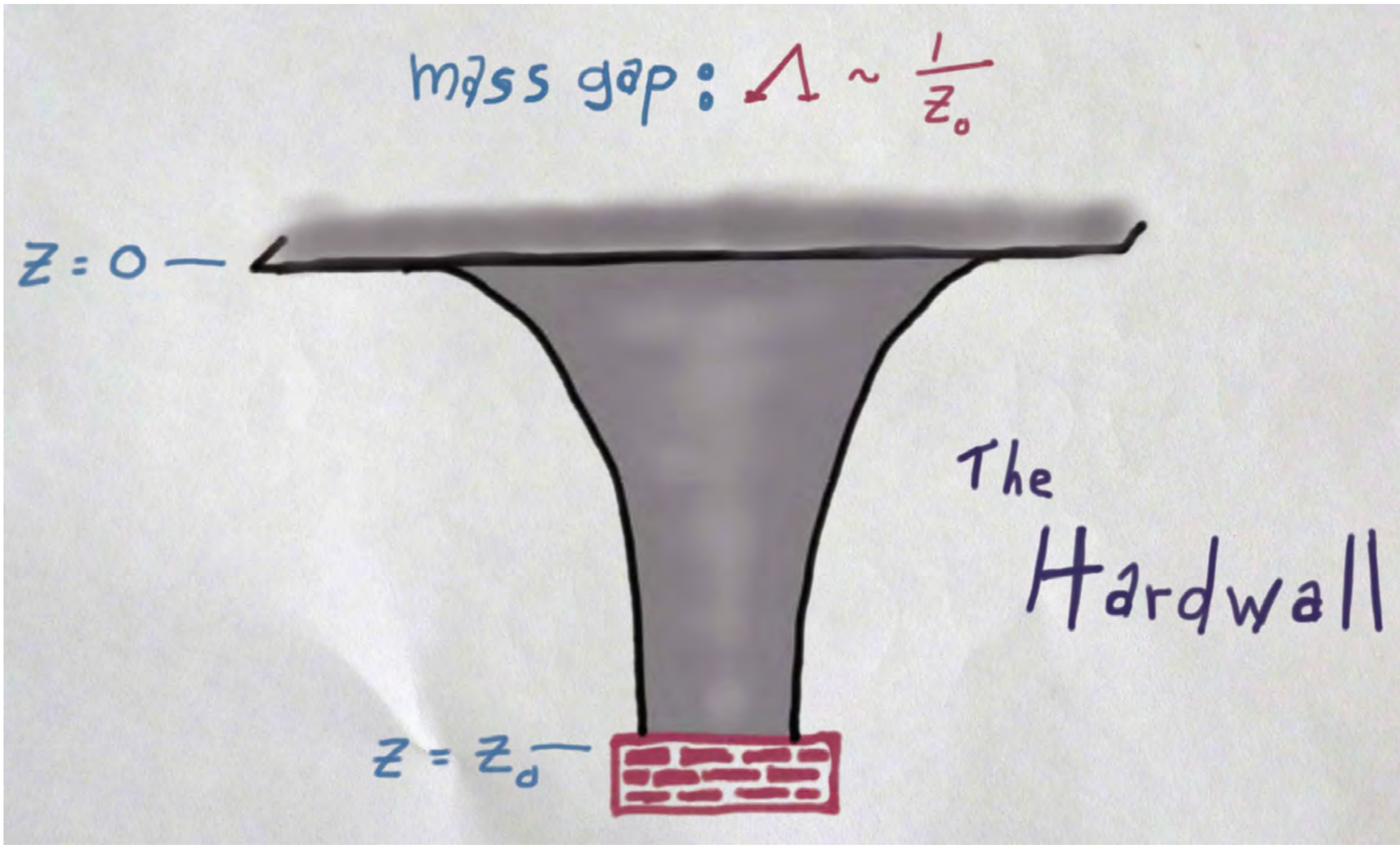
• We expect also important differences in a confining gauge theory (like YM) that has a non-trivial scale, Λ_{QCD}.

• The goal therefore is to go beyond AdS and study confining holographic theories.

The Hard Wall Model

• There is a very simple holographic model for a confining gauge theory: The hard wall model.

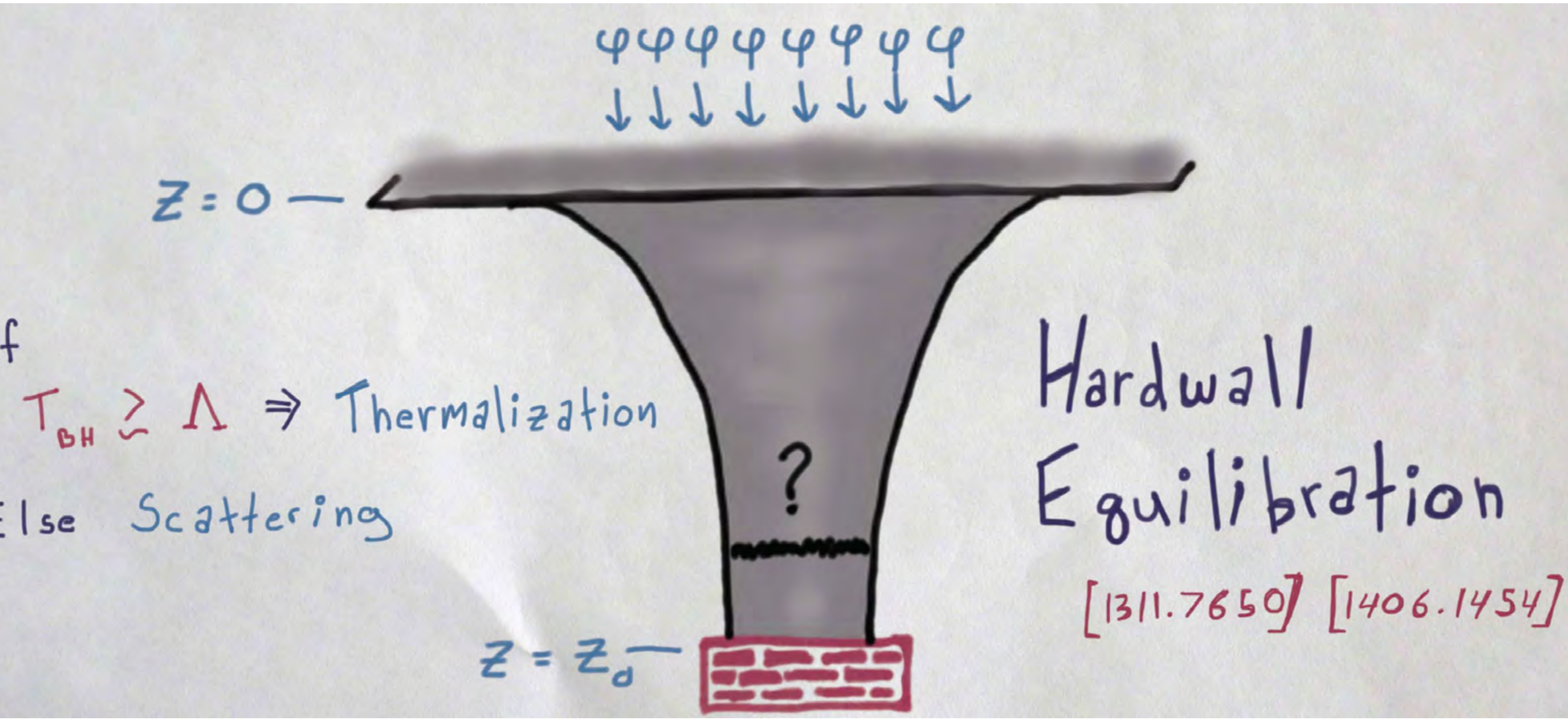
Polchinski+Strassler



• The hard wall induces an IR scale and confinement.

• In this simple model we can obtain analytic solutions for sufficiently small perturbations.

Craps+Kiritsis+Rosen+Taliotis+Vanhoof+Zhang



The analytic treatment indicates that

• When $\epsilon \gg (\Lambda\delta t)^2$ an AdS-Schwarzschild black brane is formed in the bulk, with event horizon size $r_h \simeq \frac{\sqrt{\epsilon}}{\delta t}$.

• When $\epsilon \ll (\Lambda\delta t)^3$, no black hole is formed in the first scattering period.

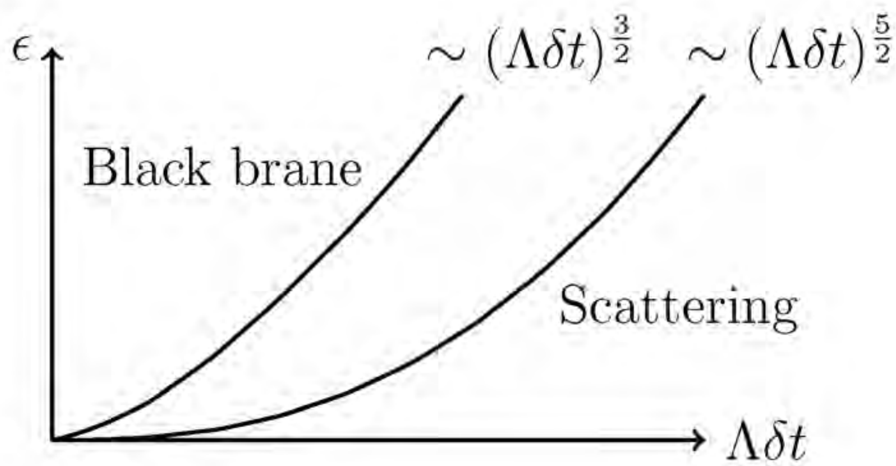
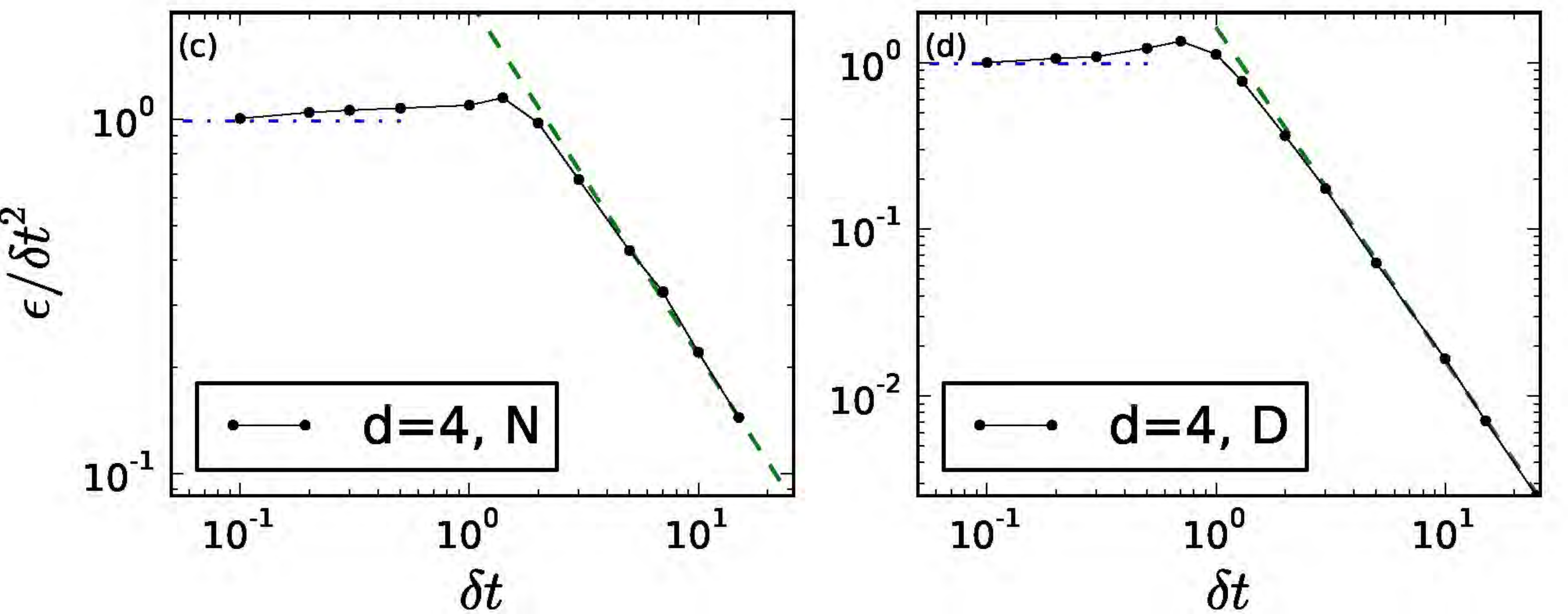


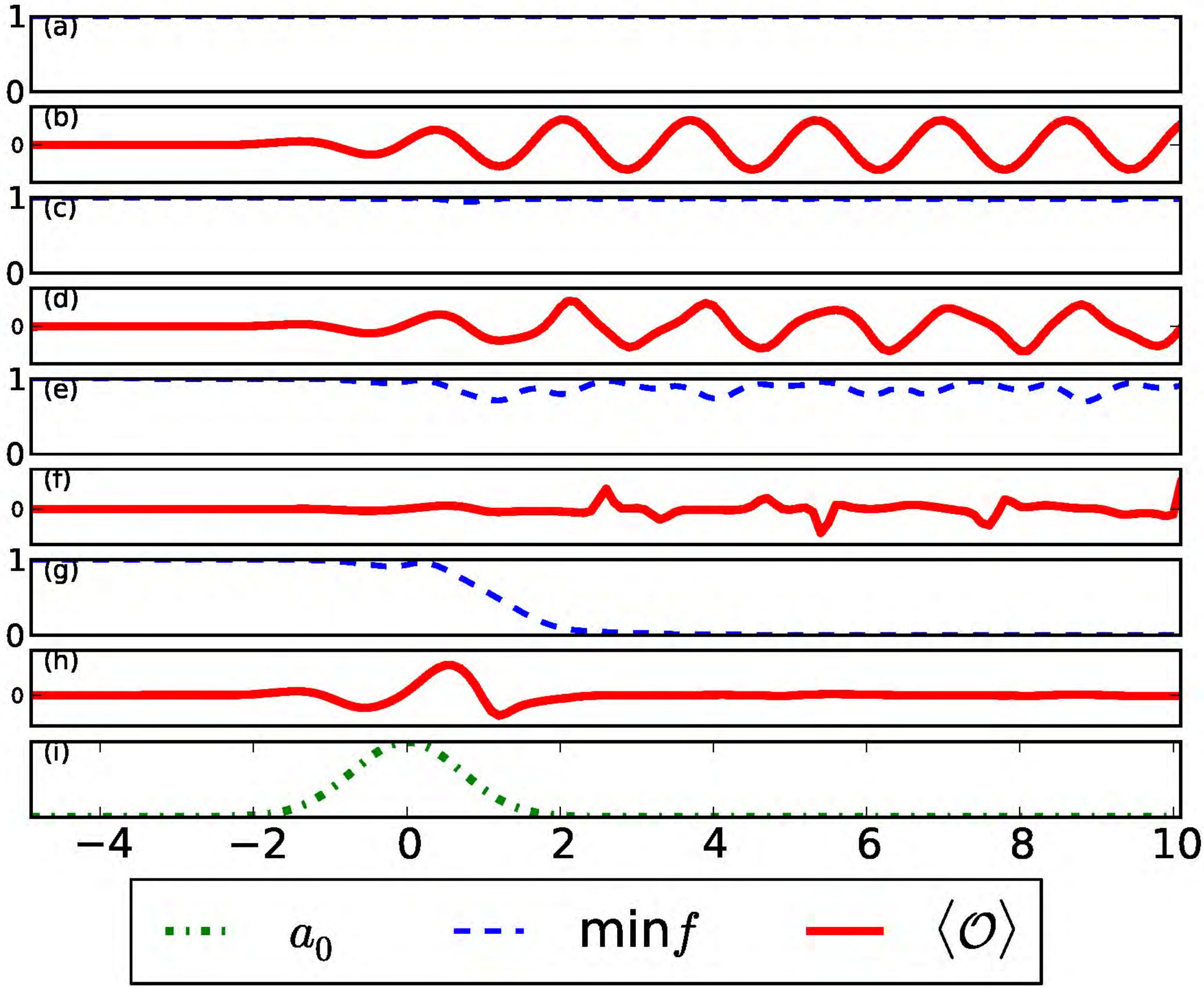
Figure 2: *Different phases depending on the amplitude ϵ, the injection time δt and the location of the hard wall Λ.*

The non-linear analysis

- Numerical solutions confirm these expectations.

Craps+Lindgren+Taliotis+Vanhoof+Zhang




(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
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1
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a_0
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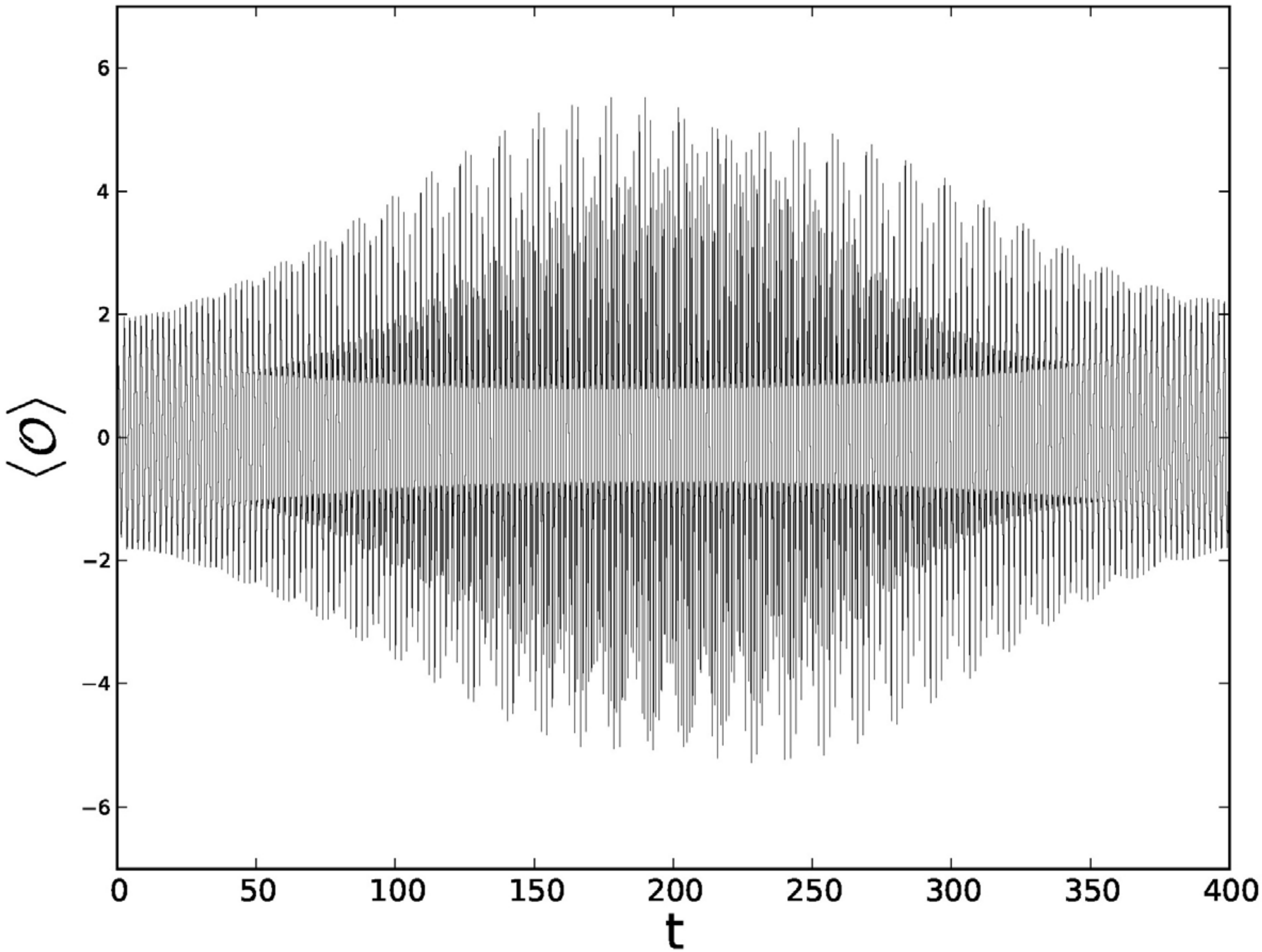
$\langle \mathcal{O} \rangle$



6
4
2
0
−2
−4
−6
⟨O⟩
0
50
100
150
200
250
300
350
400
t

Improved Holographic QCD

• We would like to move closed to holographic YM using models that are more realistic than the hard-wall model.

• So far the perturbations were generated by a scalar without a potential. Although there is such a scalar in holographic YM (the axion, dual to the θ angle) it is not realistic enough.

• Holographic models were developed that describe with rather good accuracy the strong coupling physics of YM theory.

Gursoy+Kiritsis+Nitti, Gubser+Nellore

• They contain the fields dual to the most important YM operators

$$\phi \quad \Leftrightarrow \quad Tr[F^2]$$

$$g_{\mu\nu} \quad \Leftrightarrow \quad T_{\mu\nu}$$

• The gravitational action is the Einstein Dilaton action with a potential.

$$S_{IHQCD} = M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 - V(\phi) \right]$$

• The potential is in one-to-one correspondence with the YM β-function.

• At large values of ϕ, $V \sim \sqrt{\phi}\; e^{\frac{4}{3}\phi}$.

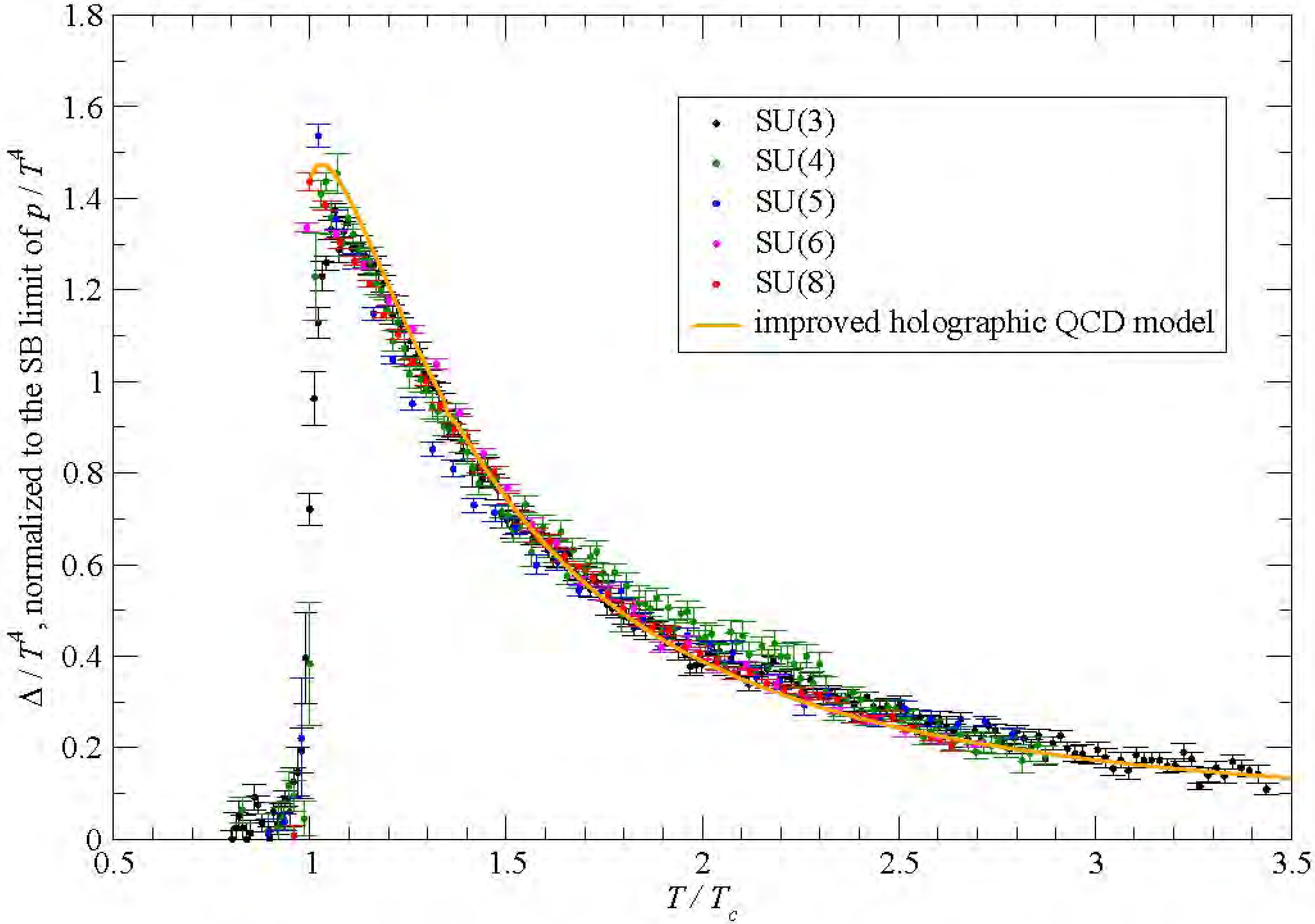


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Panero

Black holes in Improved Holographic QCD

Gursoy+Kiritsis+Mazzanti+Nitti

• Black holes can be characterized by their Hawking Temperature (two to one map), or the value of the scalar (dilaton) at the horizon $\lambda_H = e^{\phi_H}$ (one to one map).

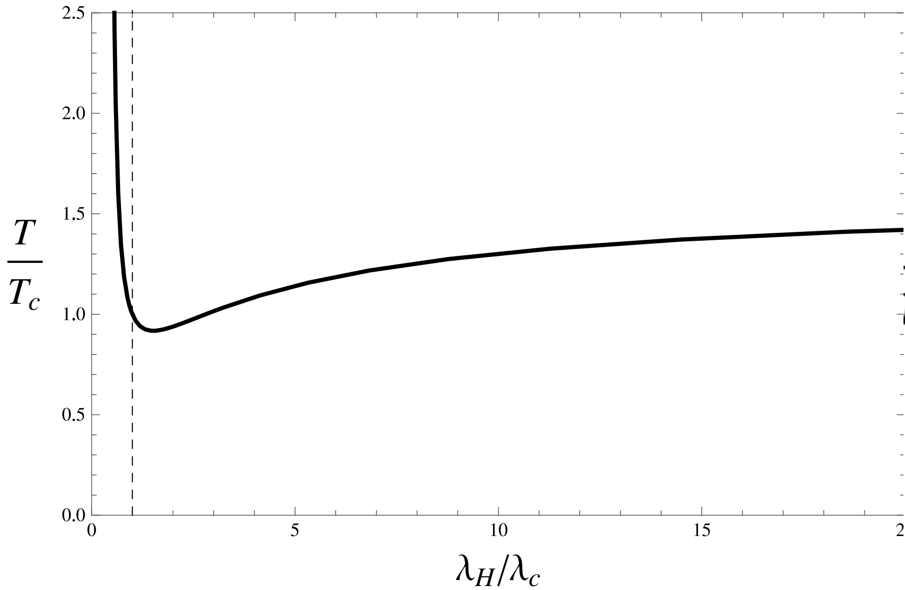
• There are two black hole branches

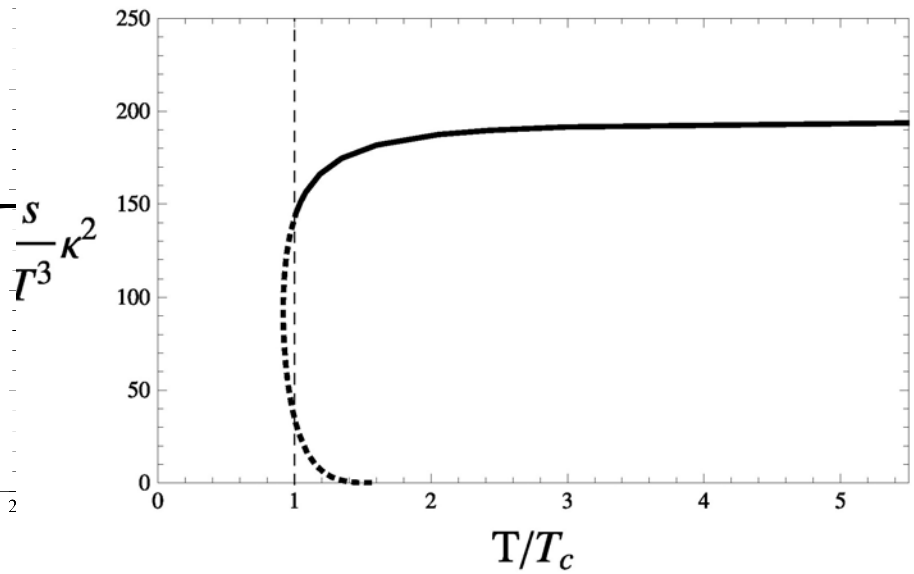
(a) The large black hole branch.

(b) The small black hole branch. They are thermodynamically unstable with negative specific heat. As the horizon size vanishes the small black hole turns into the vacuum state solution.

• There is a minimum temperature $T > T_0$, that separates the large from the small black hole branch.

• The first order (deconfining) phase transition to the deconfined (black hole phase) happens at $T_c > T_0$ inside the large black hole branch.





Plots of the temperature scaled by the critical temperature as a function of λ_H/λ_c (left) and the entropy density scaled by the third power of the temperature as a function of T/T_c (right). The rightmost plot becomes "dotted" as one passes through the phase transition by lowering the temperature from above. This is meant to indicate that in the field theory, this low temperature phase is governed by the thermal gas solutions, whose entropy is subleading in the number of colors N_c.

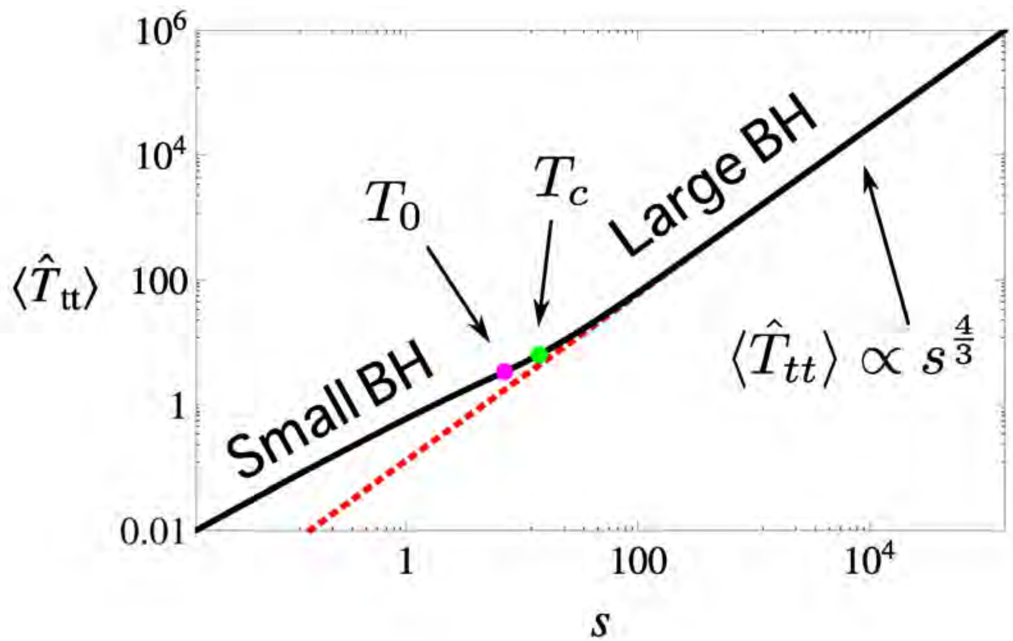


Figure 6. The energy density $\langle \hat{T}_{tt} \rangle$ as a function of entropy s, in units of f_0 and with $\kappa = 1$. The asymptotic behaviors are $\langle \hat{T}_{tt} \rangle \propto s^{4/3}$ and $\langle \hat{T}_{tt} \rangle \propto s\sqrt{-\ln s}$ in the limits of very large and very small black holes, and a fit for the former is plotted with a red dotted line. The green and magenta dots mark the locations of the first order phase transition at $T = T_c$ and the division between small and large black holes at $T = T_0$, respectively.

Quench dynamics

- The quench profile is:

$$f_0(v) = \tilde{f}_0 - \delta f_0 \, e^{-\frac{v^2}{2\tau^2}}$$

- For numerical simplicity we start with the theory in a thermal state that corresponds to the small black hole branch.

- The "smallest" the initial black hole, the closest we are to the initial (confining) ground state of the theory.

We find the following:

- The characteristic time associated with the intermediate non-linear regime is negligible compared to τ and T_{RD}.

- This seems to be a generic occurrence in holography/gravity and a clean explanation is lacking.

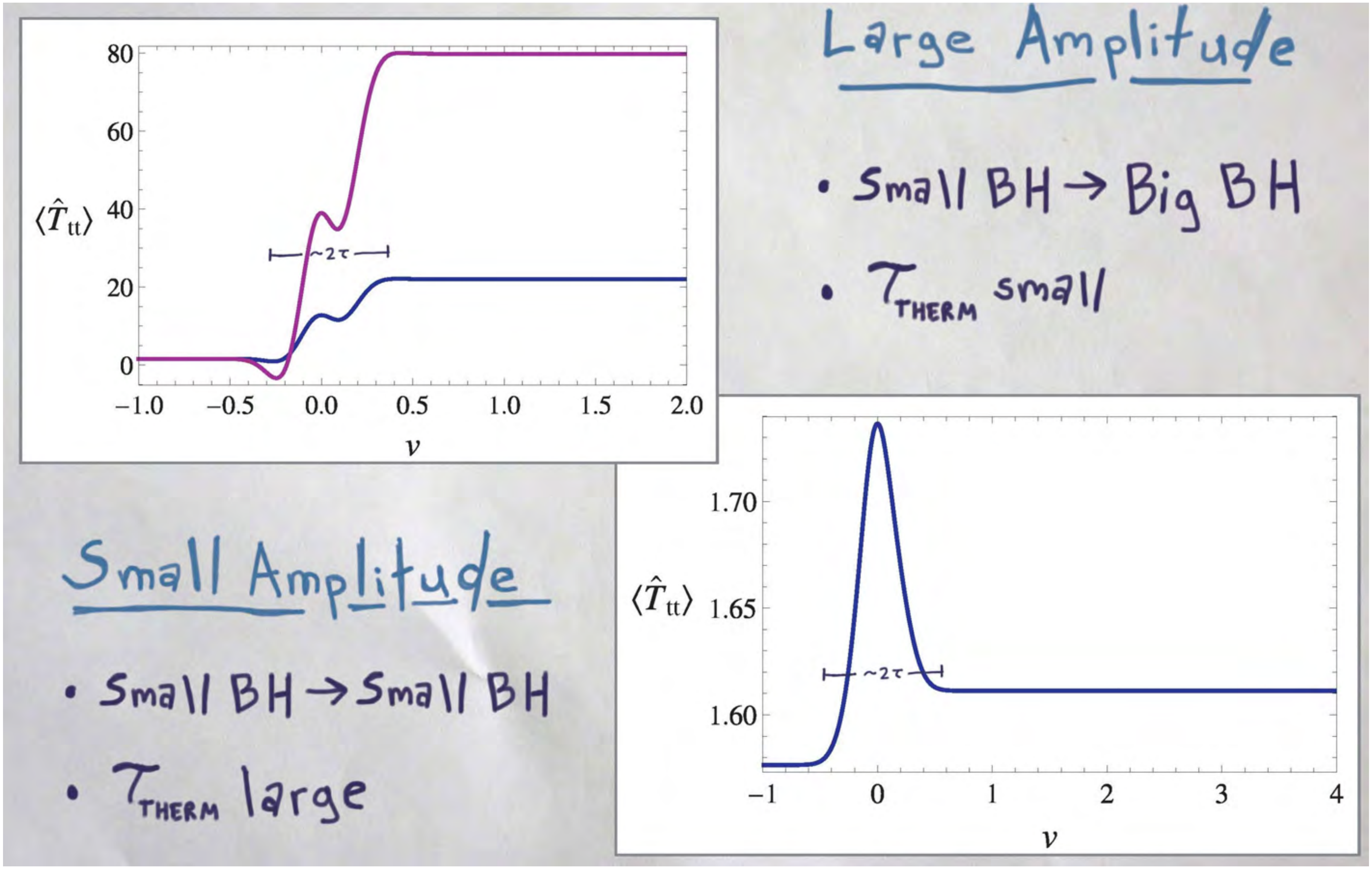
- This is probably due to the nature of gravity: in most cases non-linearities cross the horizon before they fully develop.

- Therefore

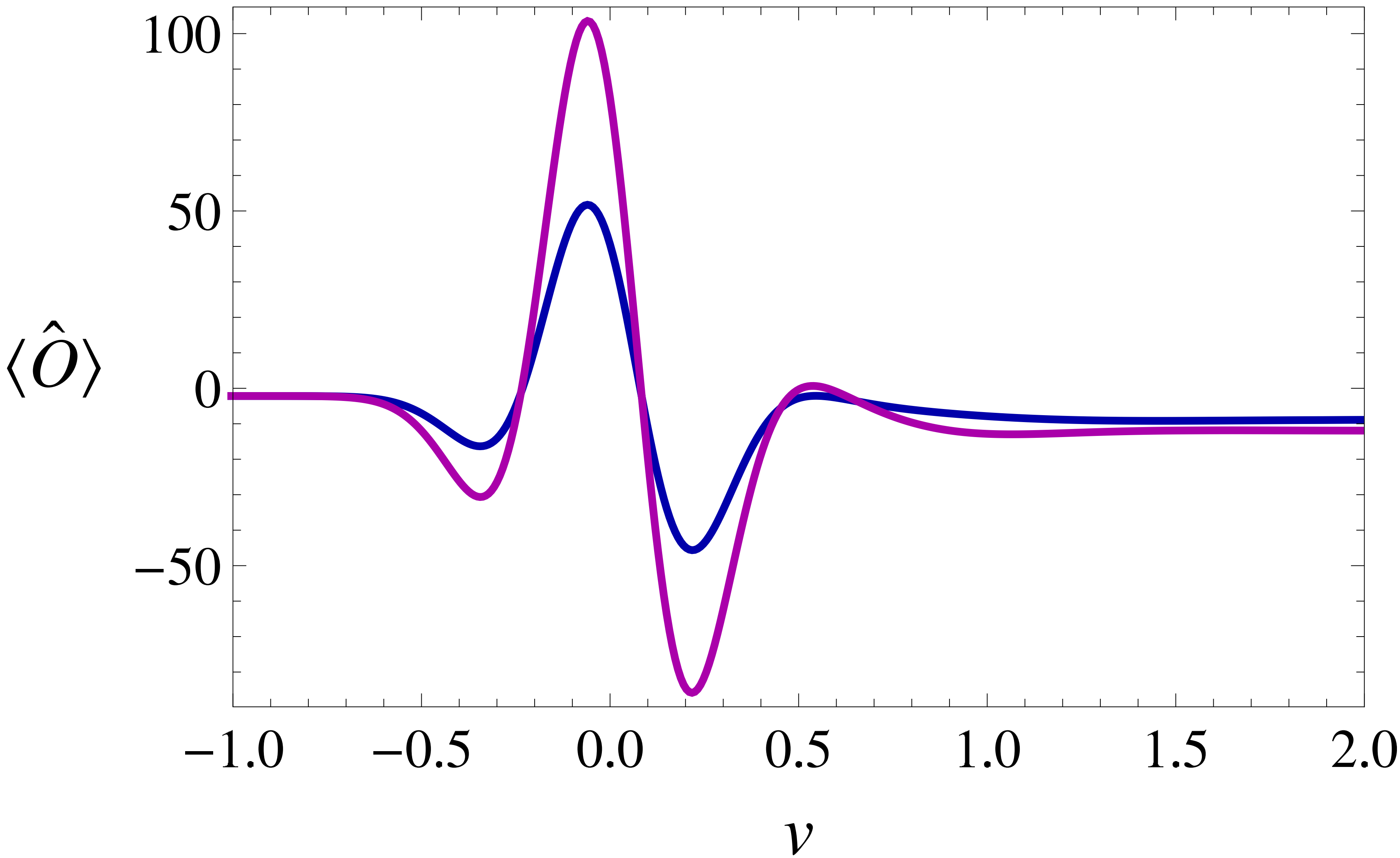
$$T_{\text{thermalization}} \quad \simeq \quad \frac{1}{\Gamma}$$

where Γ is the imaginary part of the lowest quasinormal mode.

- For adiabatic perturbations, $\tau \gg 1$ the system does NOT oscillate but goes continuously to the final-state black hole.

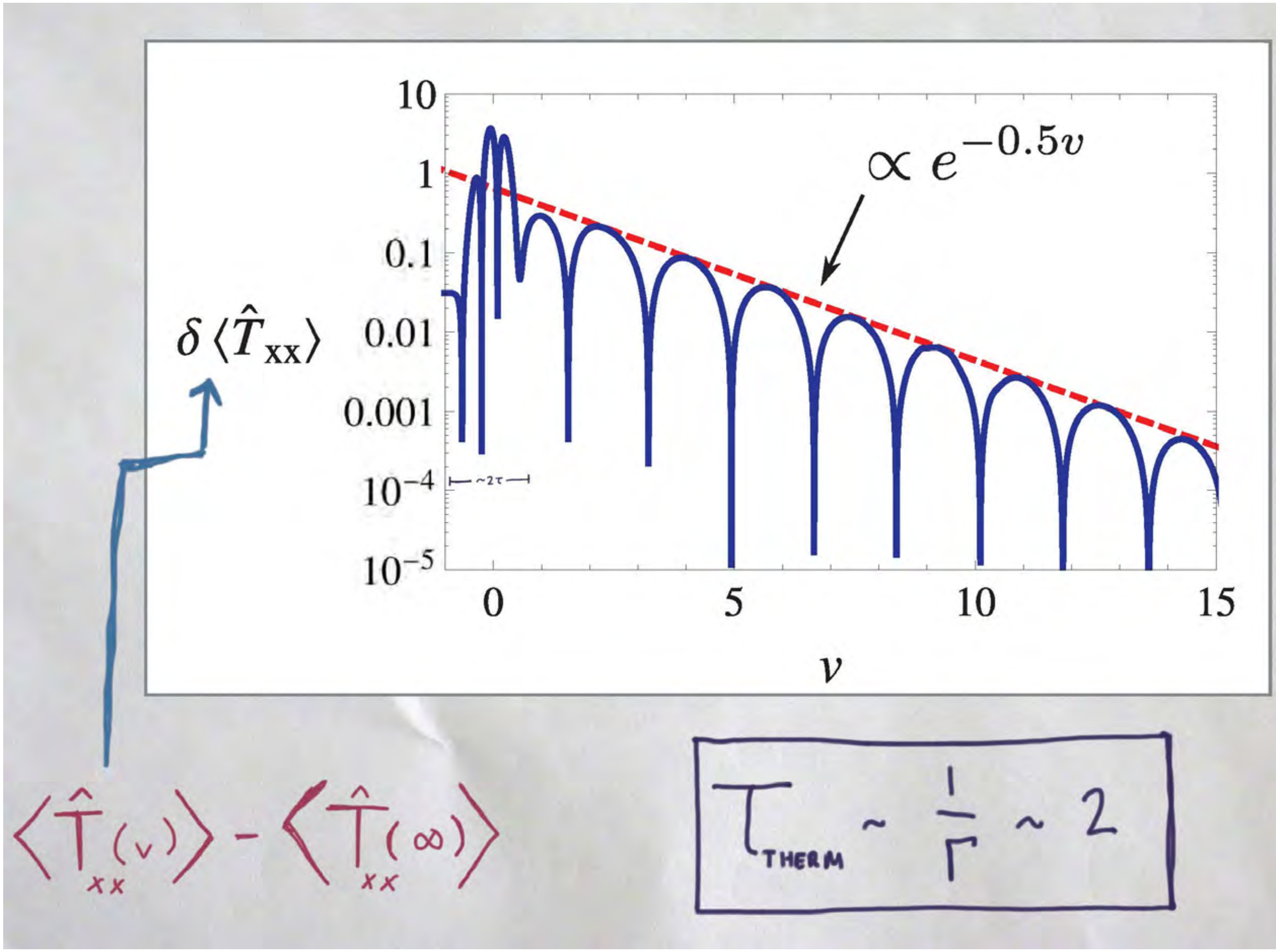

Large Amplitude
• Small BH → Big BH
• τ_{THERM} small
Small Amplitude
• Small BH → Small BH
• τ_{THERM} large
$\langle \hat{T}_{tt} \rangle$
v
~2τ
80
60
40
20
0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
1.70
1.65
1.60
−1
0
1
2
3
4

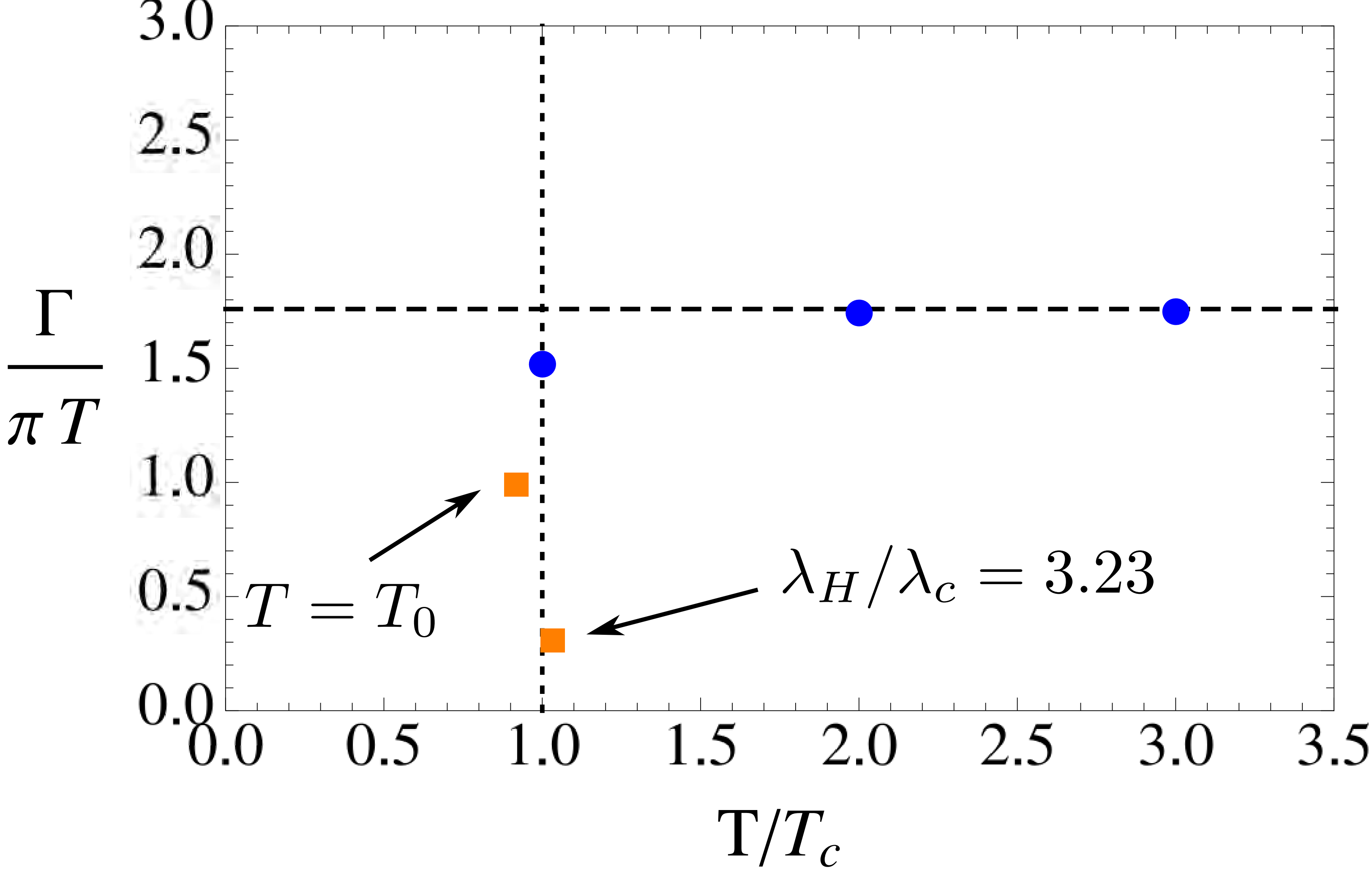
Large amplitude quench



The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively.

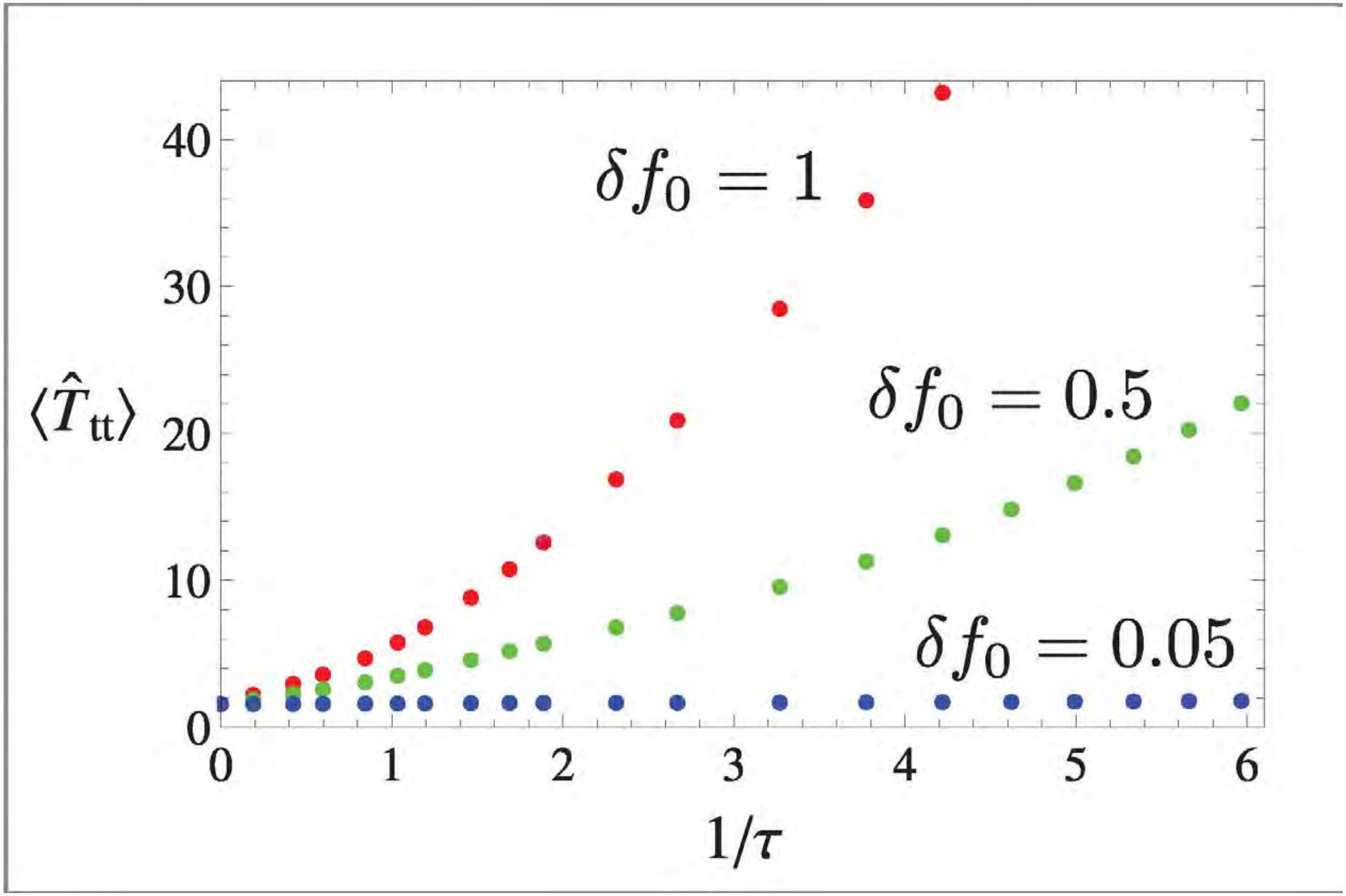
The ring-down phase



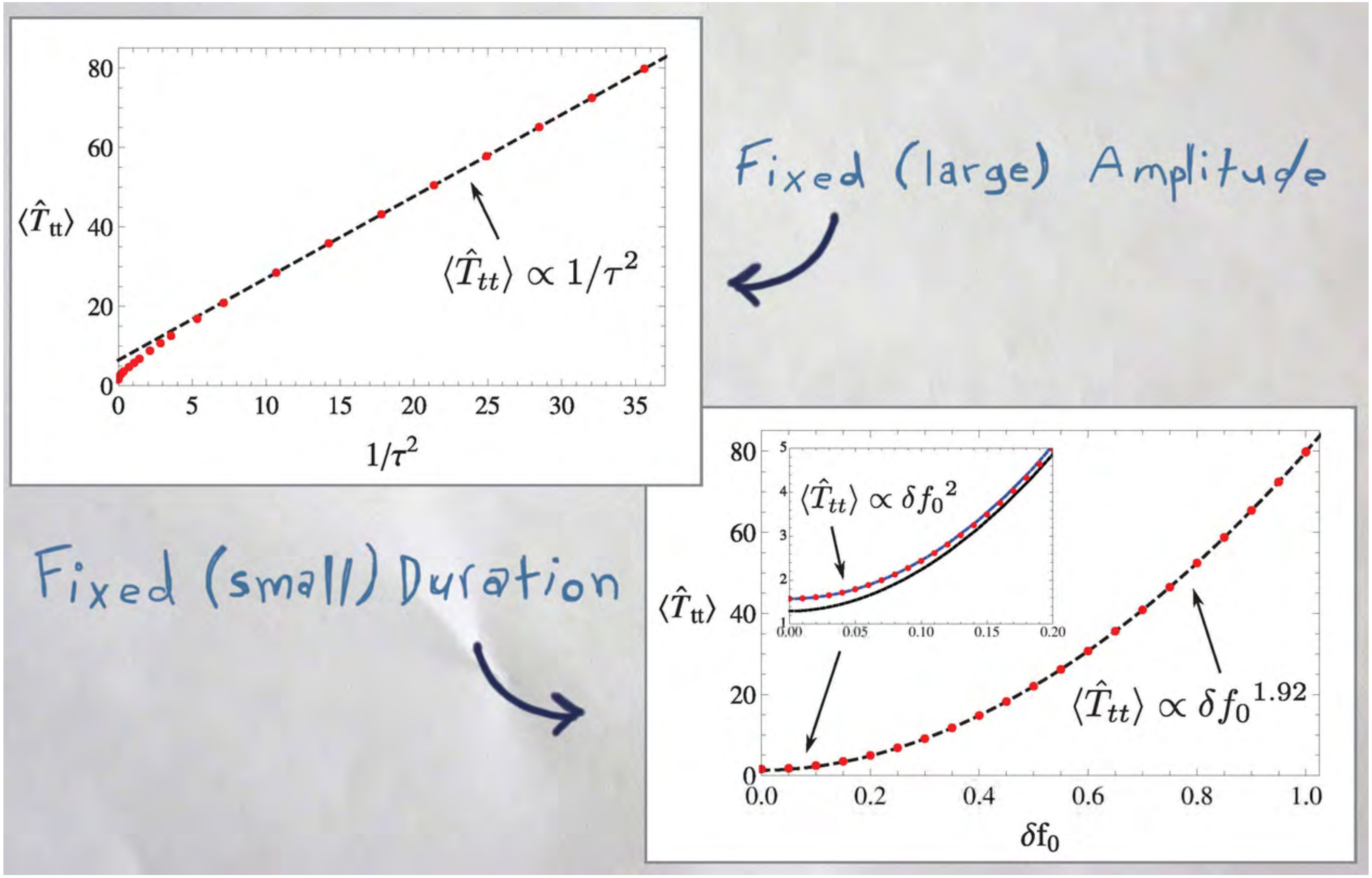


The temperature dependence of the decay width Γ for the lowest lying scalar quasi-normal mode in several states of our theory. The blue circles are large black branes whose temperature is an integer multiple of T_c. The orange squares correspond to the minimum temperature black brane (top) and the smallest black hole we perturb in our study (bottom). The ratio $\Gamma/\pi T$ approaches 1.75953 (the dashed line) at high temperatures, which coincides with the expected value for perturbations of AdS_5 Schwarzschild by a dimension 3 scalar operator

The numerical data

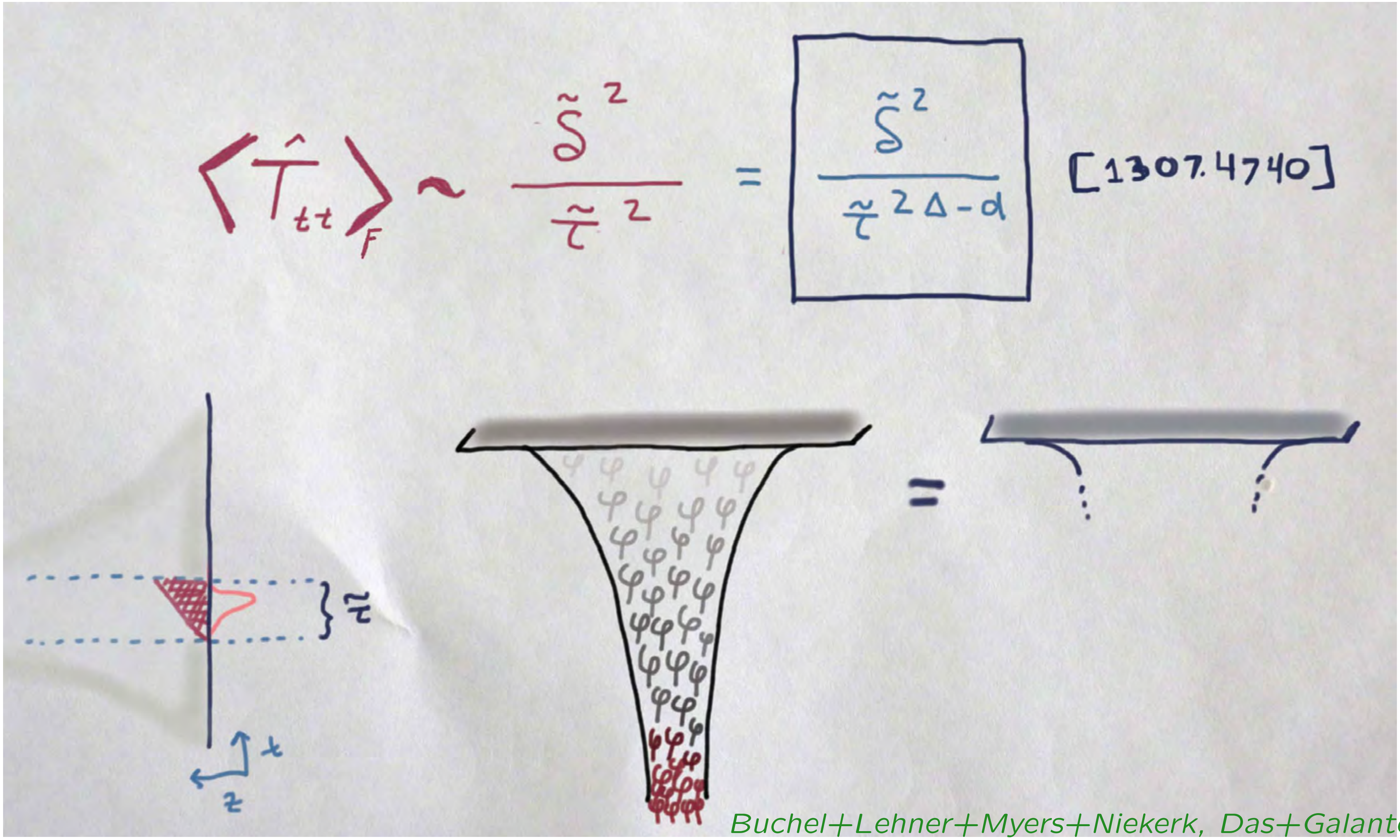


Scaling



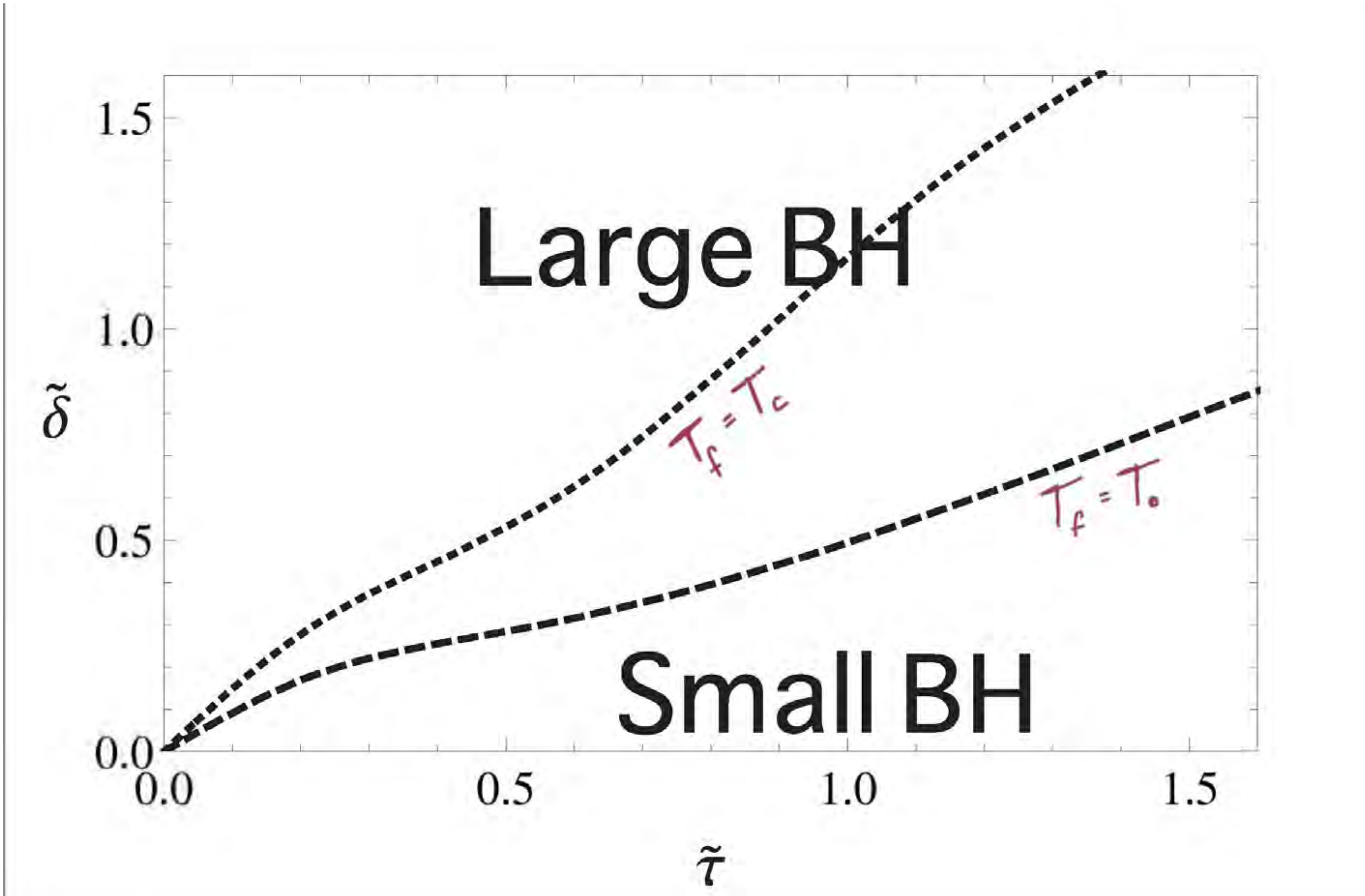
Fixed (large) Amplitude
Fixed (small) Duration
$\langle\hat{T}_{tt}\rangle$
$1/\tau^2$
$\langle\hat{T}_{tt}\rangle \propto 1/\tau^2$
δf_0
$\langle\hat{T}_{tt}\rangle \propto \delta f_0{}^2$
$\langle\hat{T}_{tt}\rangle \propto \delta f_0{}^{1.92}$

Fast Quenches

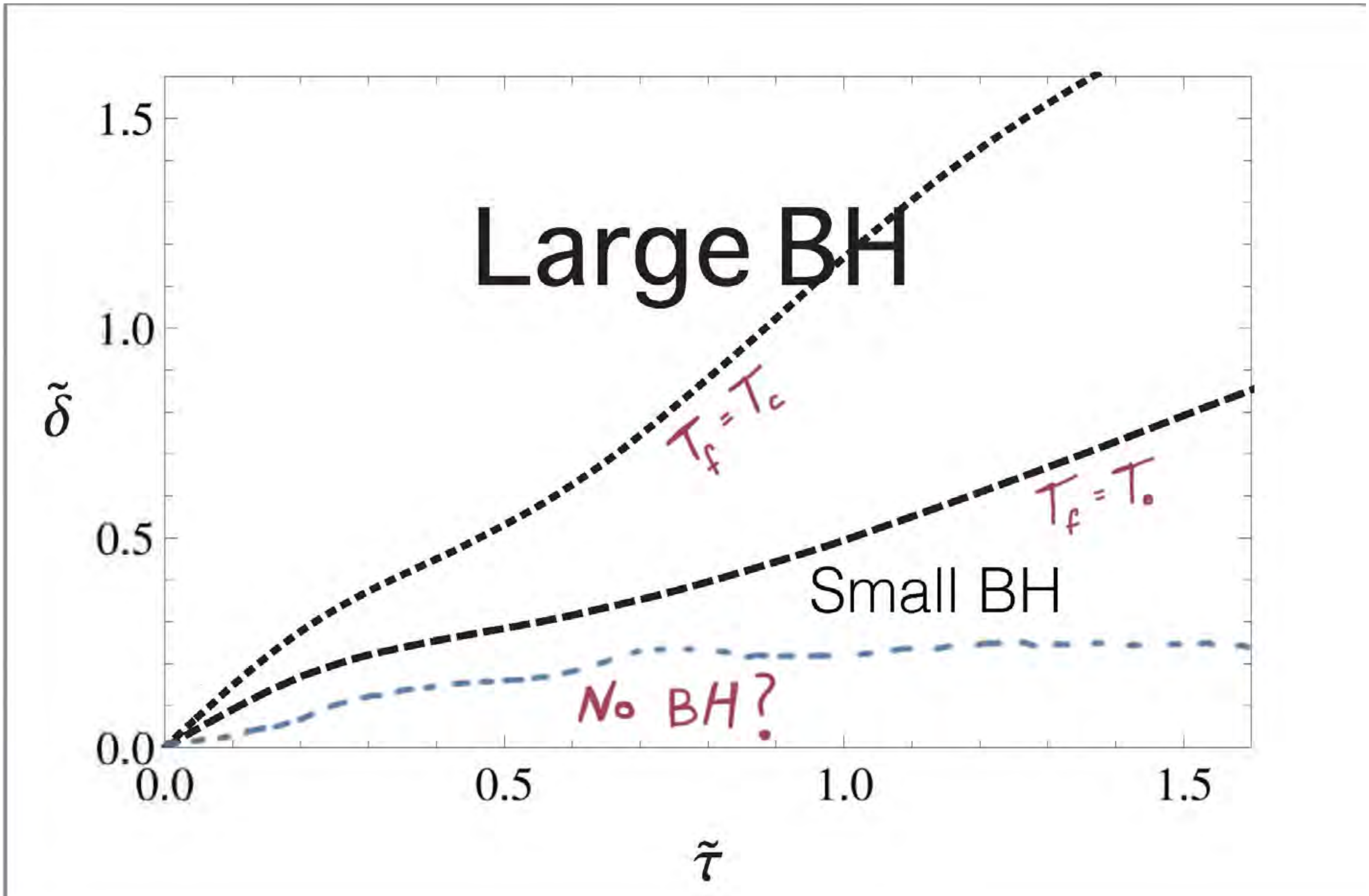


Buchel+Lehner+Myers+Niekerk, Das+Galante+Myers

The phase diagram



The full phase diagram



Conclusions and Outlook

• Rapid Transitions to the linear regime: The confinement scale plays little role for perturbations with $\frac{\langle T_{tt}\rangle}{\Lambda^4} \gtrsim 1$

• Universality and scaling in the abrupt quench limit: fast processes are only sensitive to (static) UV behavior.

• Other non-perturbative scaling regimes found, and need to be understood.

• The results are compatible (by extension) with an almost instant thermalization in QCD processes.

• The extension to the initial state being the ground state should be analyzed. A Choptuik-like phase transition is expected, but a different Choptuik exponent is expected.

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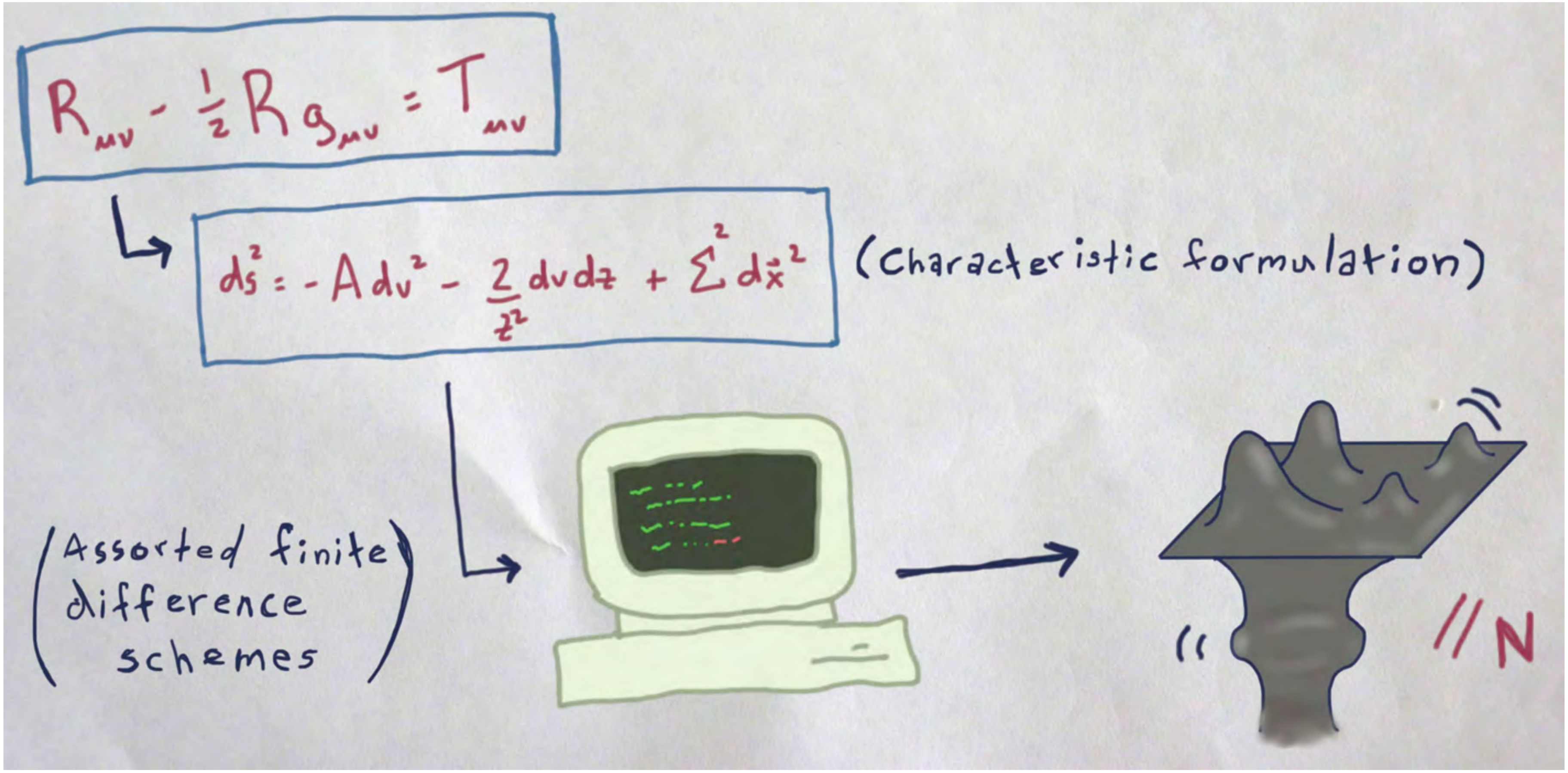
THANK YOU

The solution procedure



The turbulent AdS instability

• Numerical studies indicated that that in global AdS, (spherically symmetric) massless scalar perturbations always lead to instability.

Bizon+Rostorowski (2011)

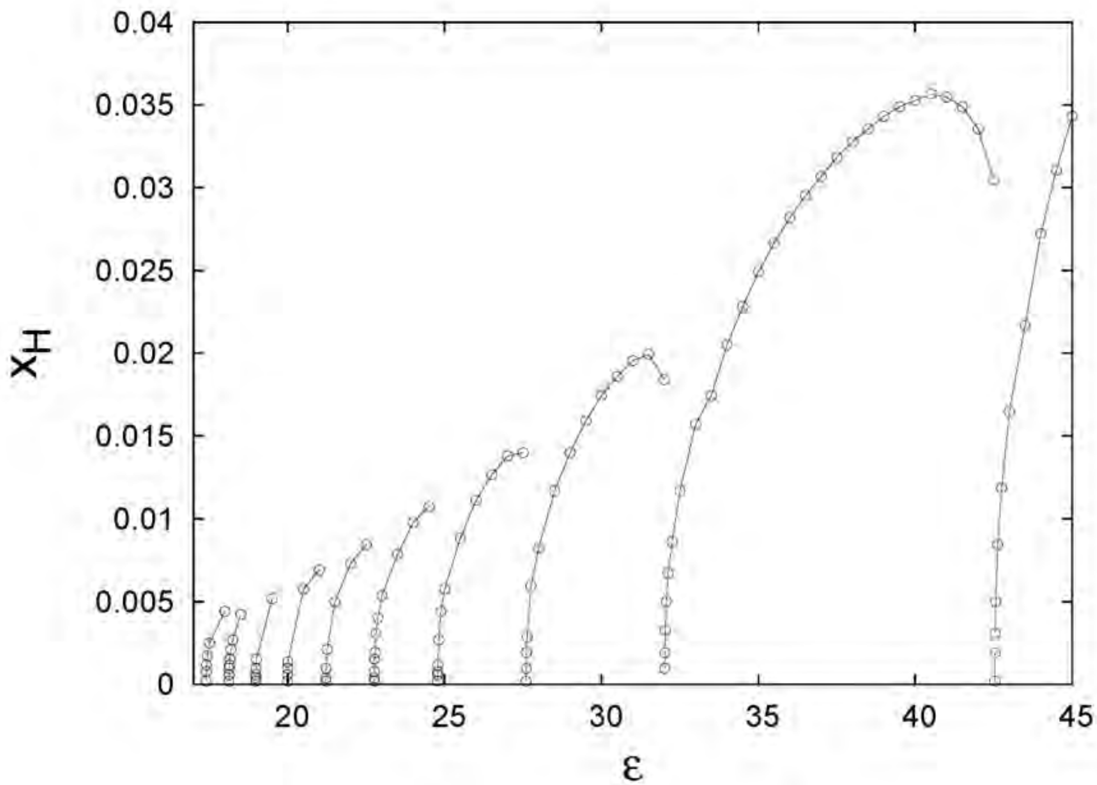


FIG. 1: Horizon radius vs amplitude for initial data (9). The number of reflections off the AdS boundary before collapse varies from zero to nine (from right to left).

• In this study, the width δt was kept fixed and the amplitude ϵ was varied.

• In the beginning of each curve, Choptwick scaling was observed with the characteristic exponent of flat space (0.37).

• The black hole formation was interpreted as a generic turbulent instability of AdS, and a resonance picture was developed to qualitatively explain it.

, Bizon+Rostorowski (2011)

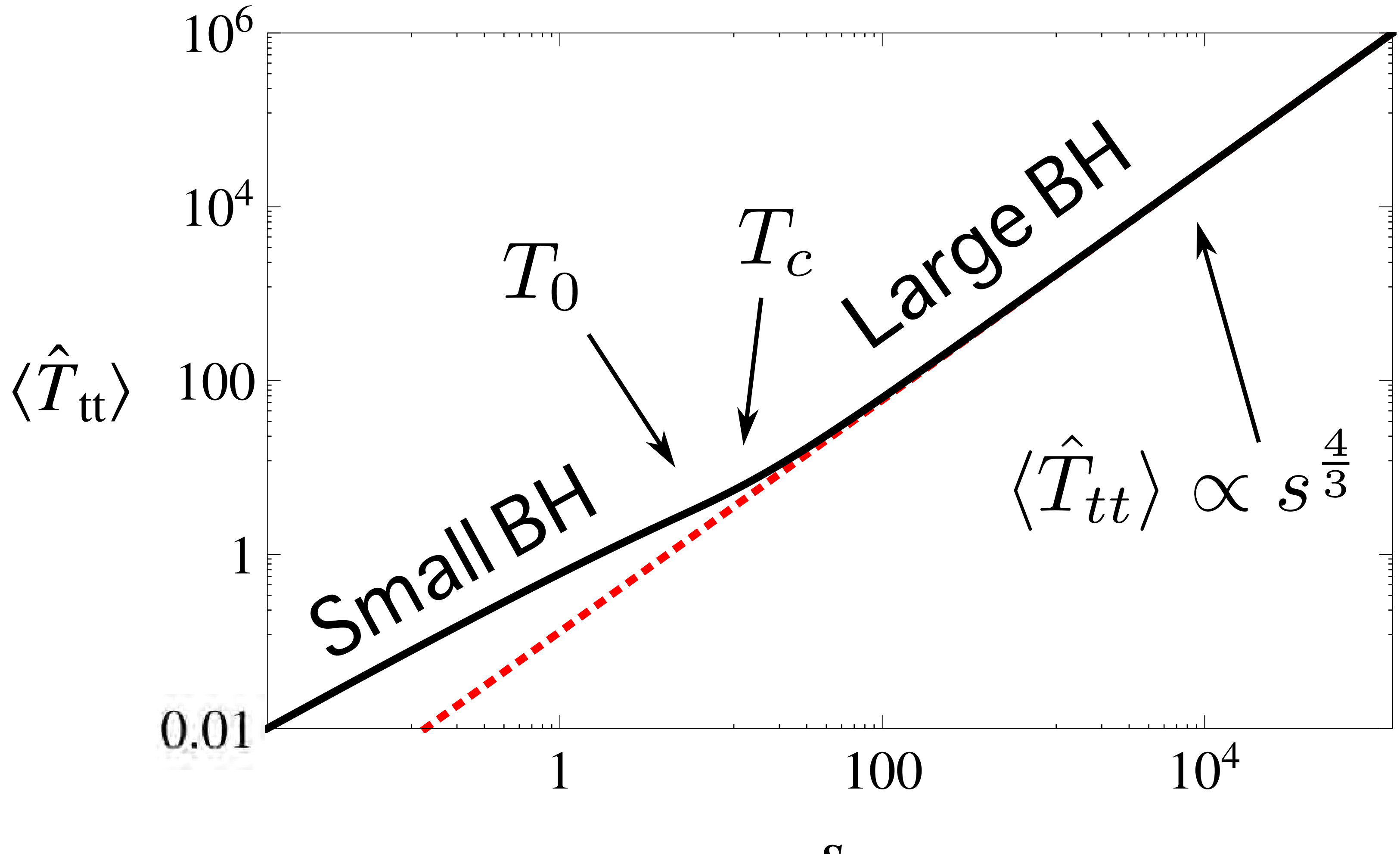
• Further studies explored other regions of the $(\delta t, \epsilon)$ phase diagram and the picture become complicated as there were also islands of stability in the parameter space.

Dias+Horowitz+Marolf+Santos (2012)

Maliborski+Rostworowski (2013), Buchel+Liebling+Lehner (2013)

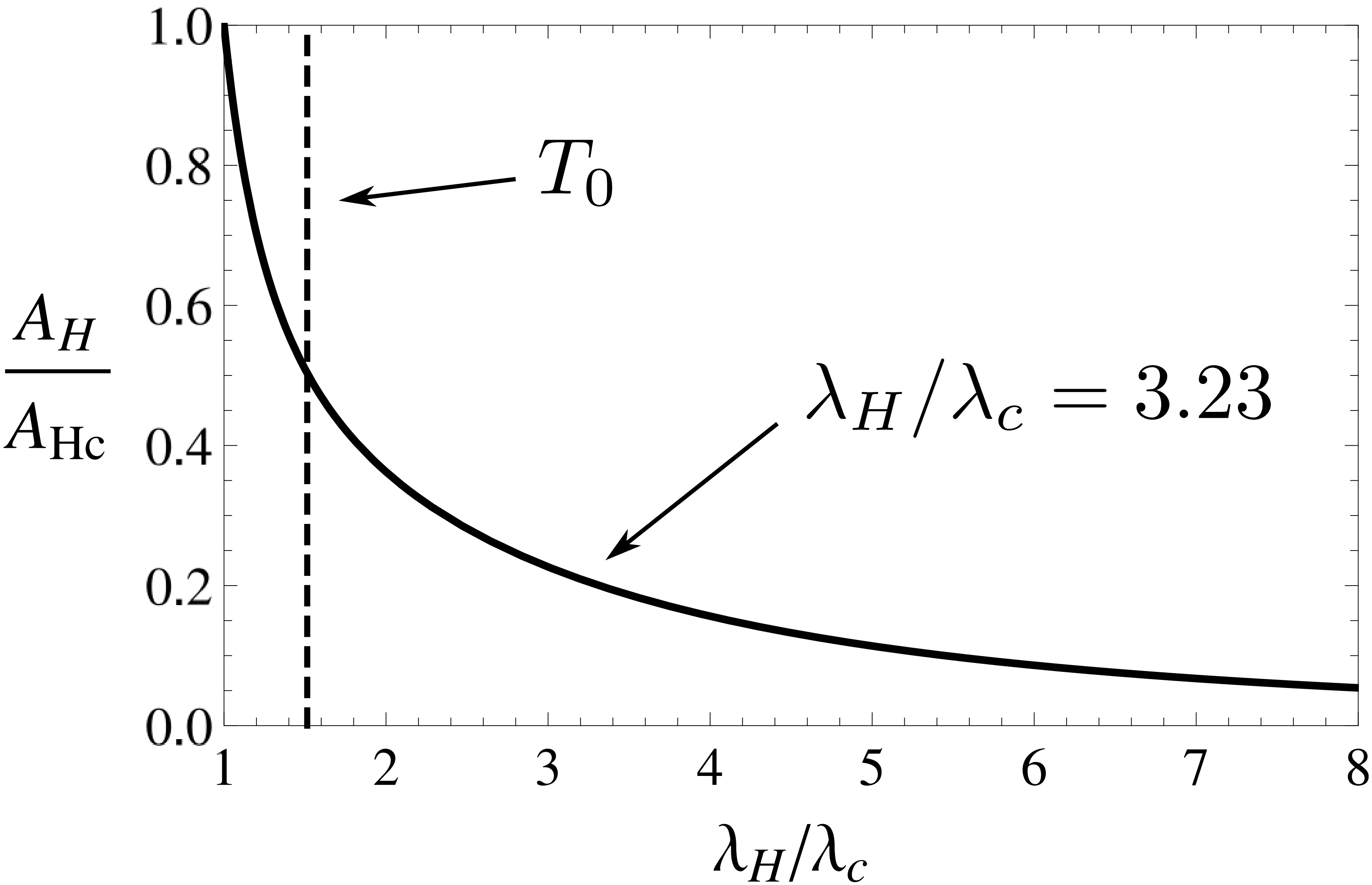
• There is no consensus so far on what happens in AdS for the whole phase diagram.

The thermodynamic functions



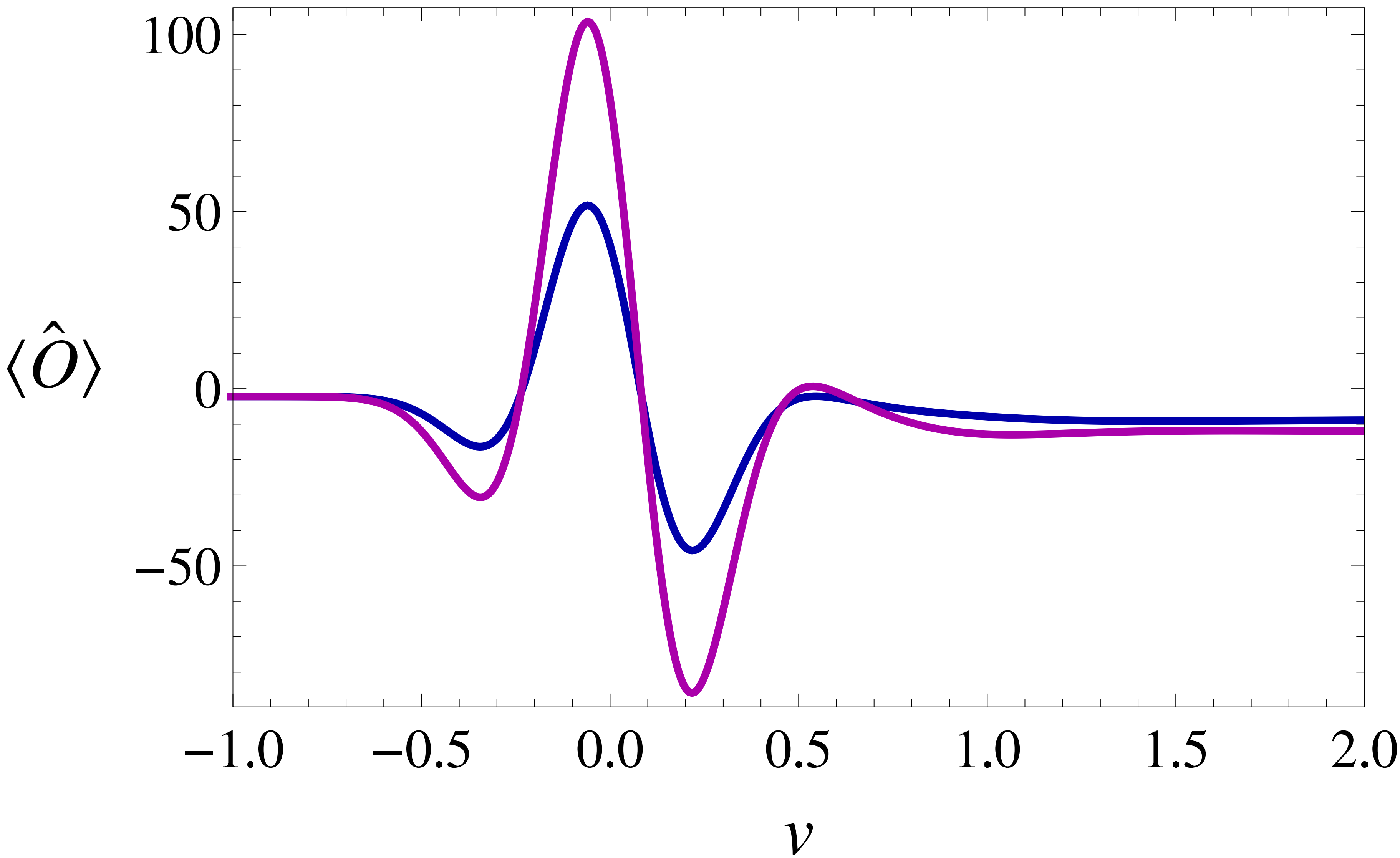
The energy density $\langle \hat{T}_{tt} \rangle$ as a function of entropy s, in units of f_0 and with $\kappa = 1$. The asymptotic behaviors are $\langle \hat{T}_{tt} \rangle \propto s^{4/3}$ and $\langle \hat{T}_{tt} \rangle \propto s\sqrt{-\ln s}$ in the limits of very large and very small black holes, and a fit for the former is plotted with a red dotted line. The green and magenta dots mark the locations of the first order phase transition at $T = T_c$ and the division between small and large black holes at $T = T_0$, respectively.

The small black hole initial states

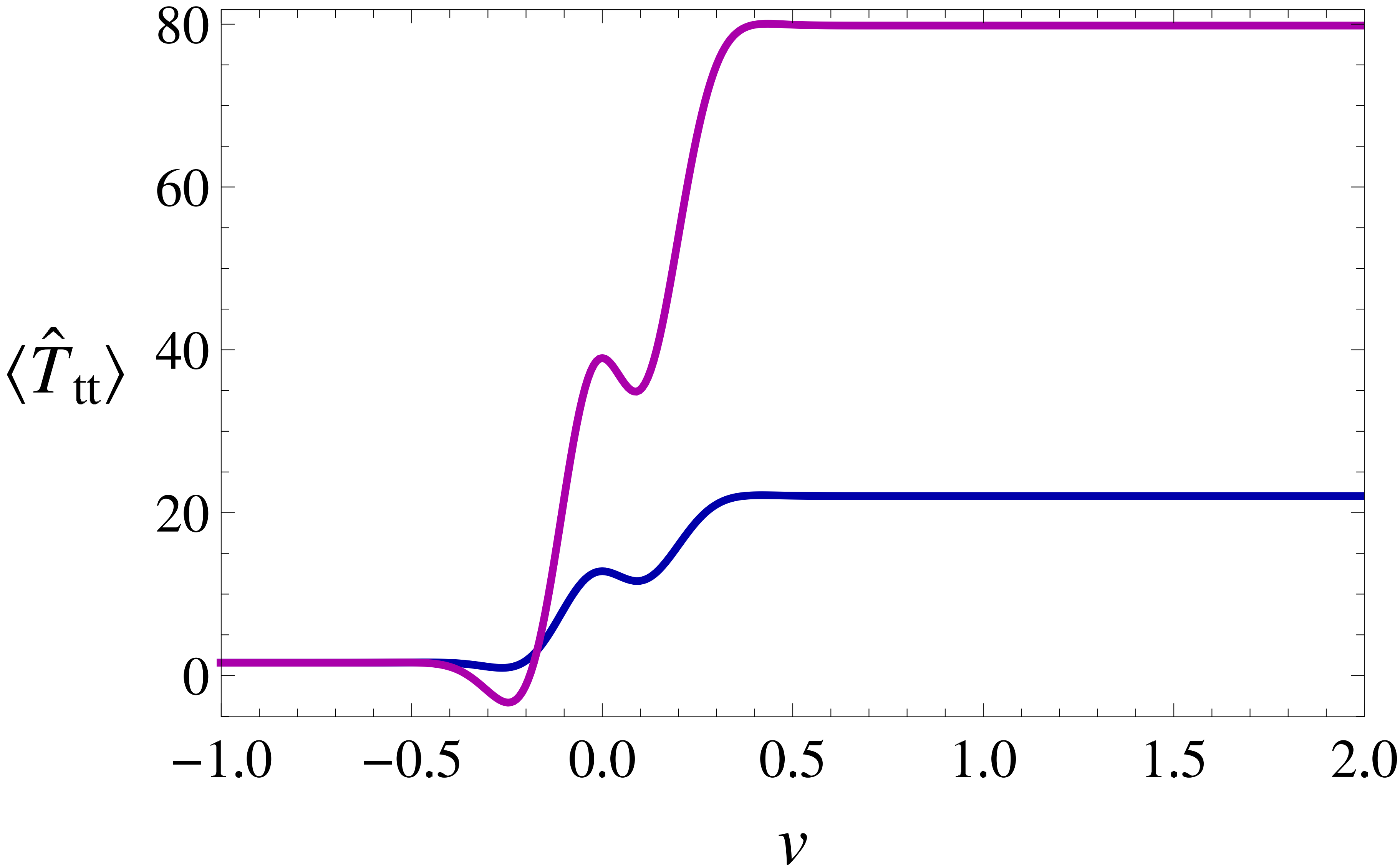


The area density of small black holes compared with that at the phase transition, A_{Hc}. The red dot marks the location in our space of solutions of the smallest black hole we perturb in this study. The dashed line indicates the location of the small and large black hole transition at $T = T_0$. As these are static black brane solutions, the apparent and event horizons coincide.

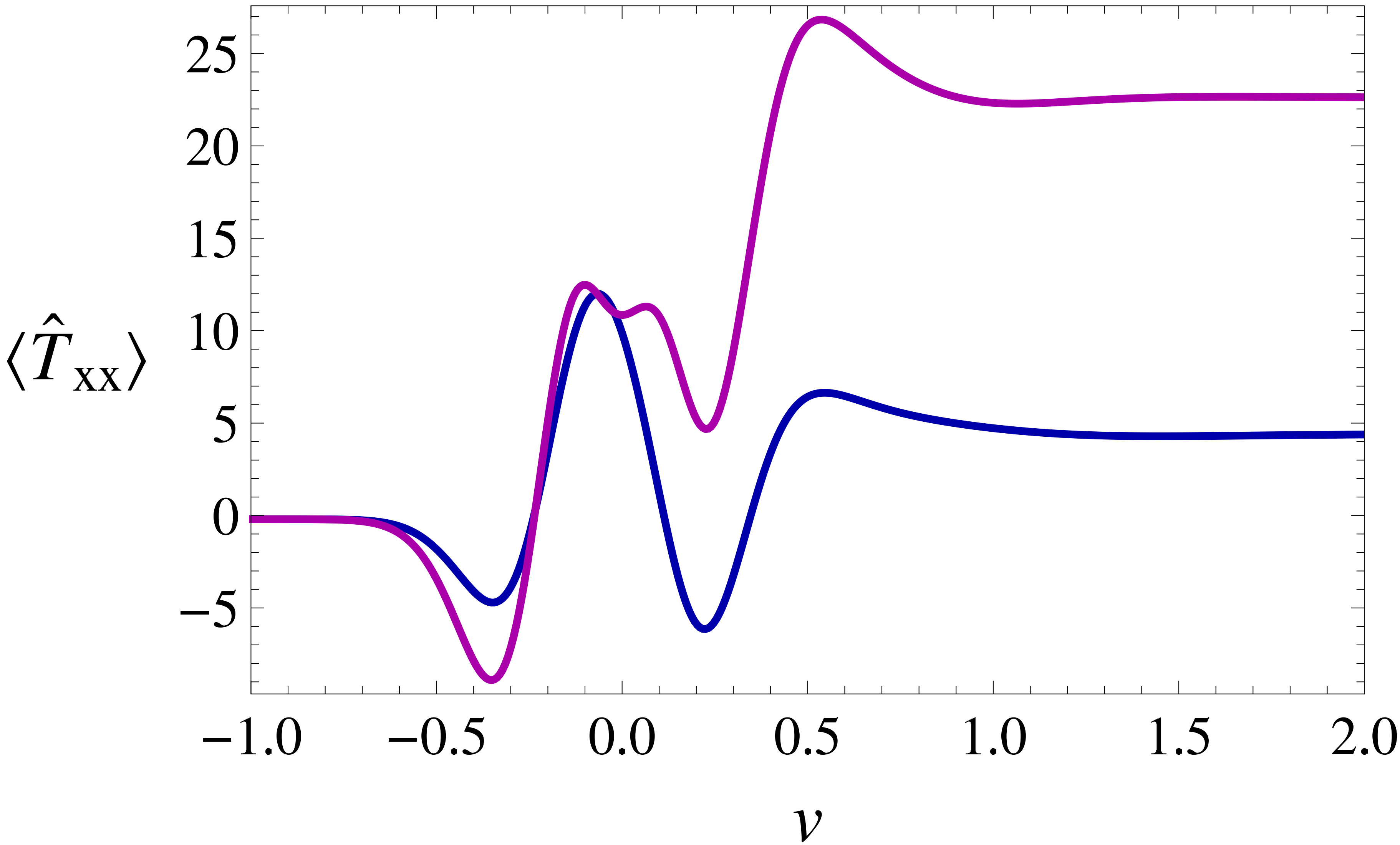
Large amplitude quench



The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively.

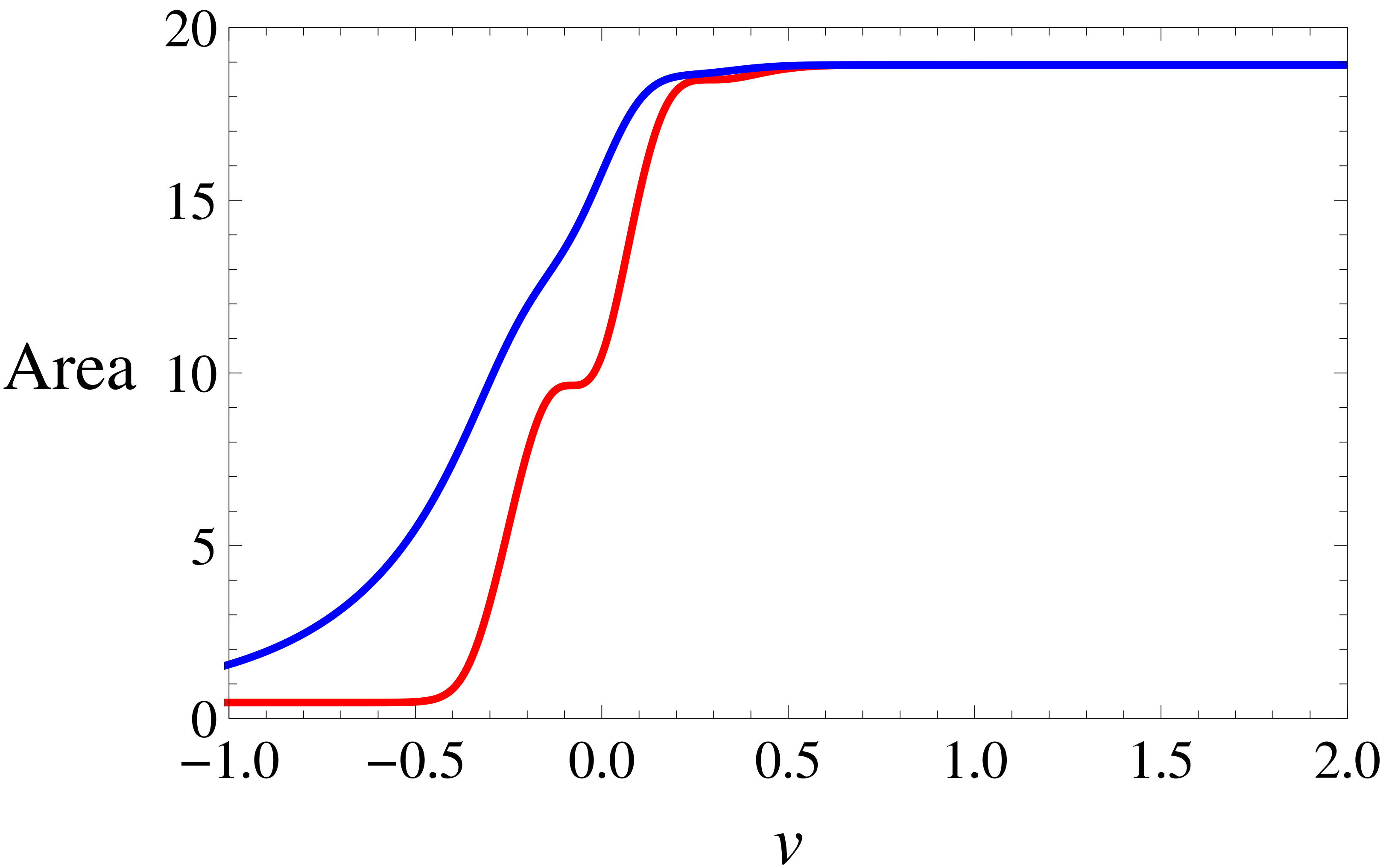


The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively. In the case of the larger amplitude quench, it is interesting to note that the energy density appears to be driven below the ground state energy density (i.e. negative) in the first moments of the quench.



The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively.

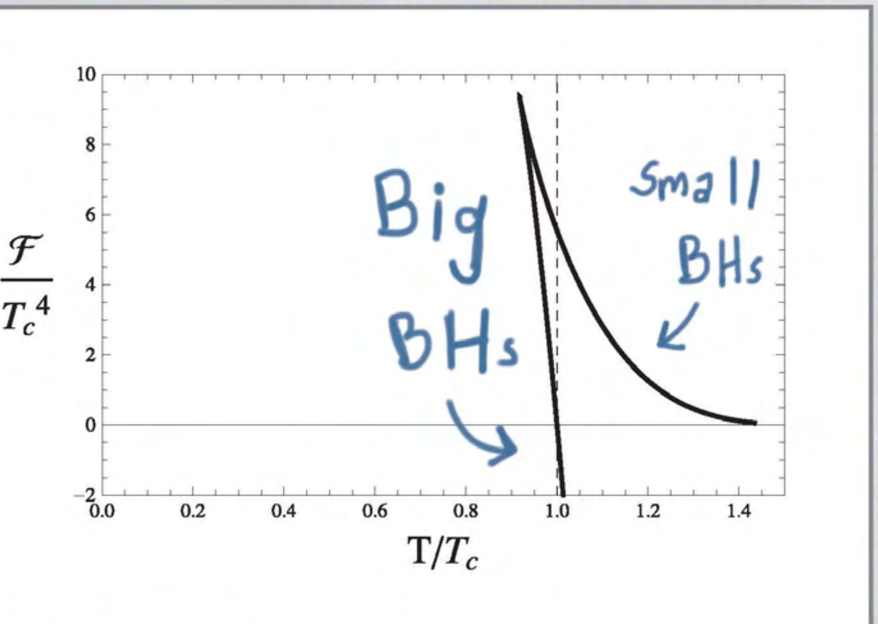
The evolution of horizons

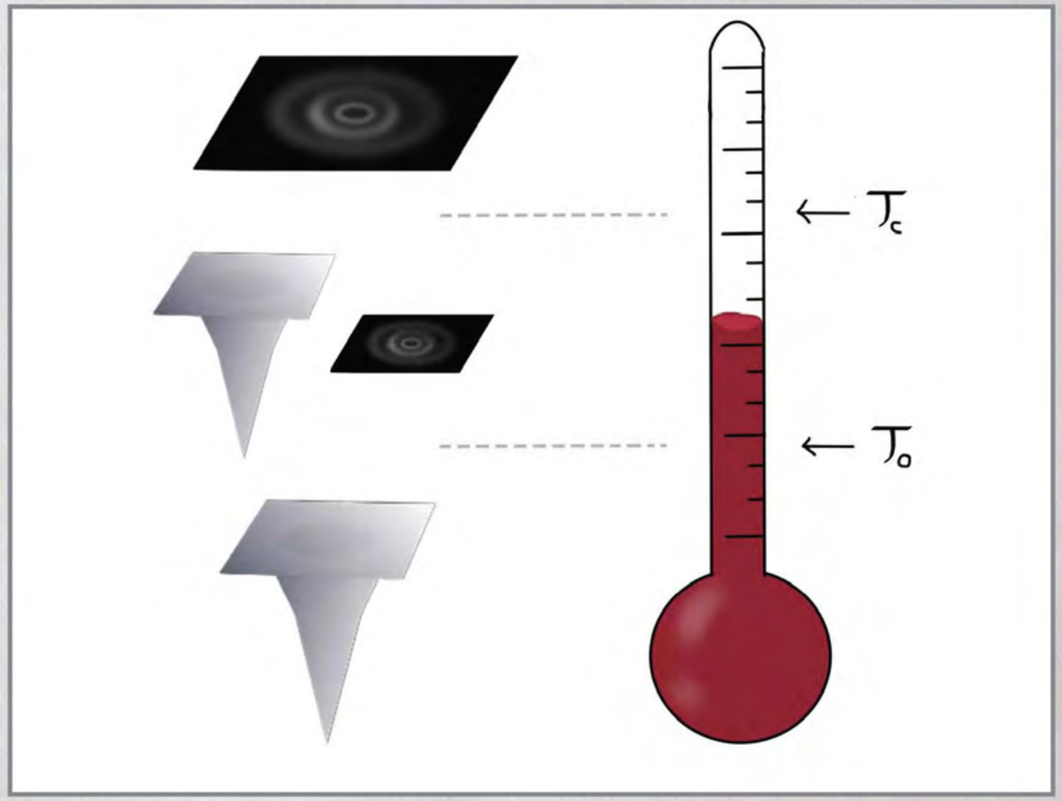


Example of the time evolution of the apparent (red) and event (blue) horizons. The event horizon coincides with the apparent horizon when the bulk solution is static, at $v \to \pm\infty$.

Black holes

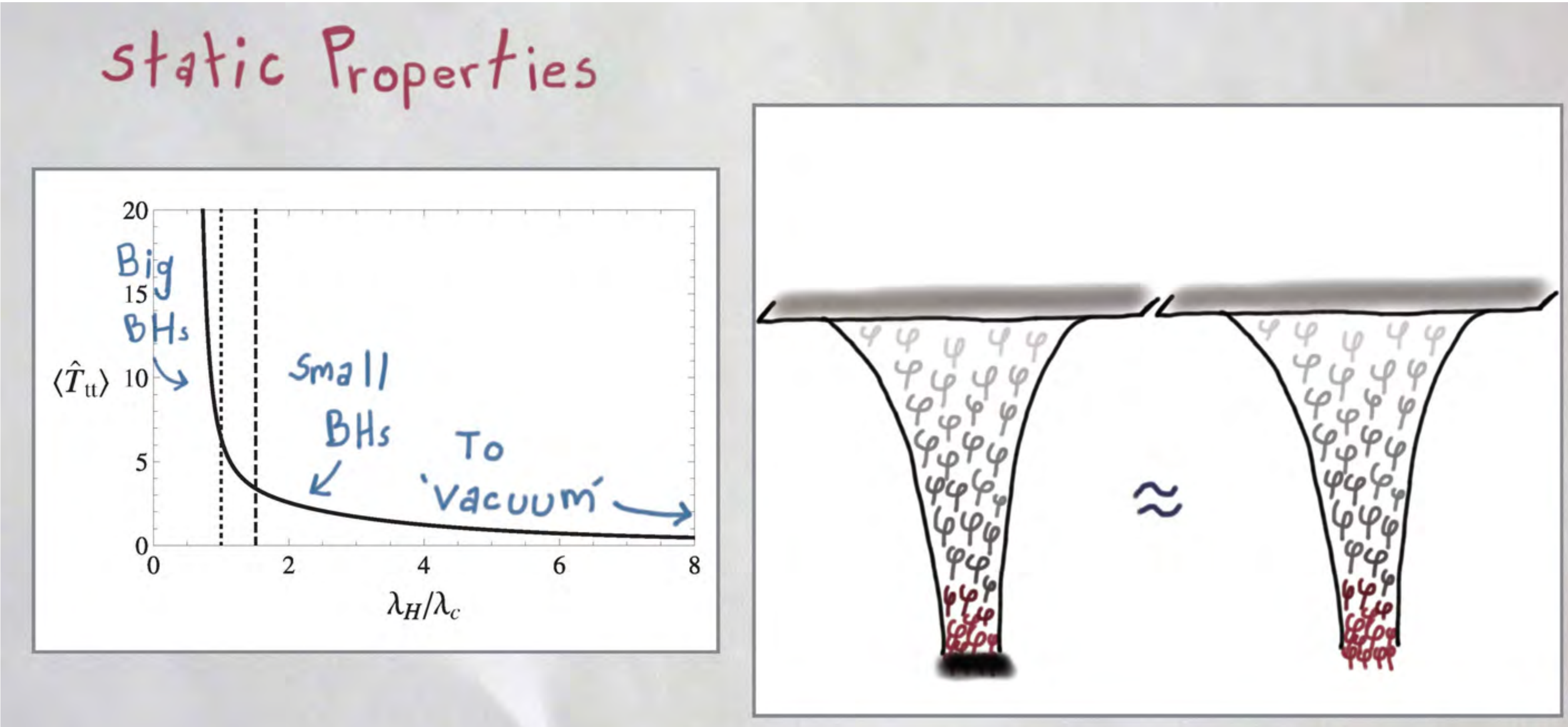
static Properties





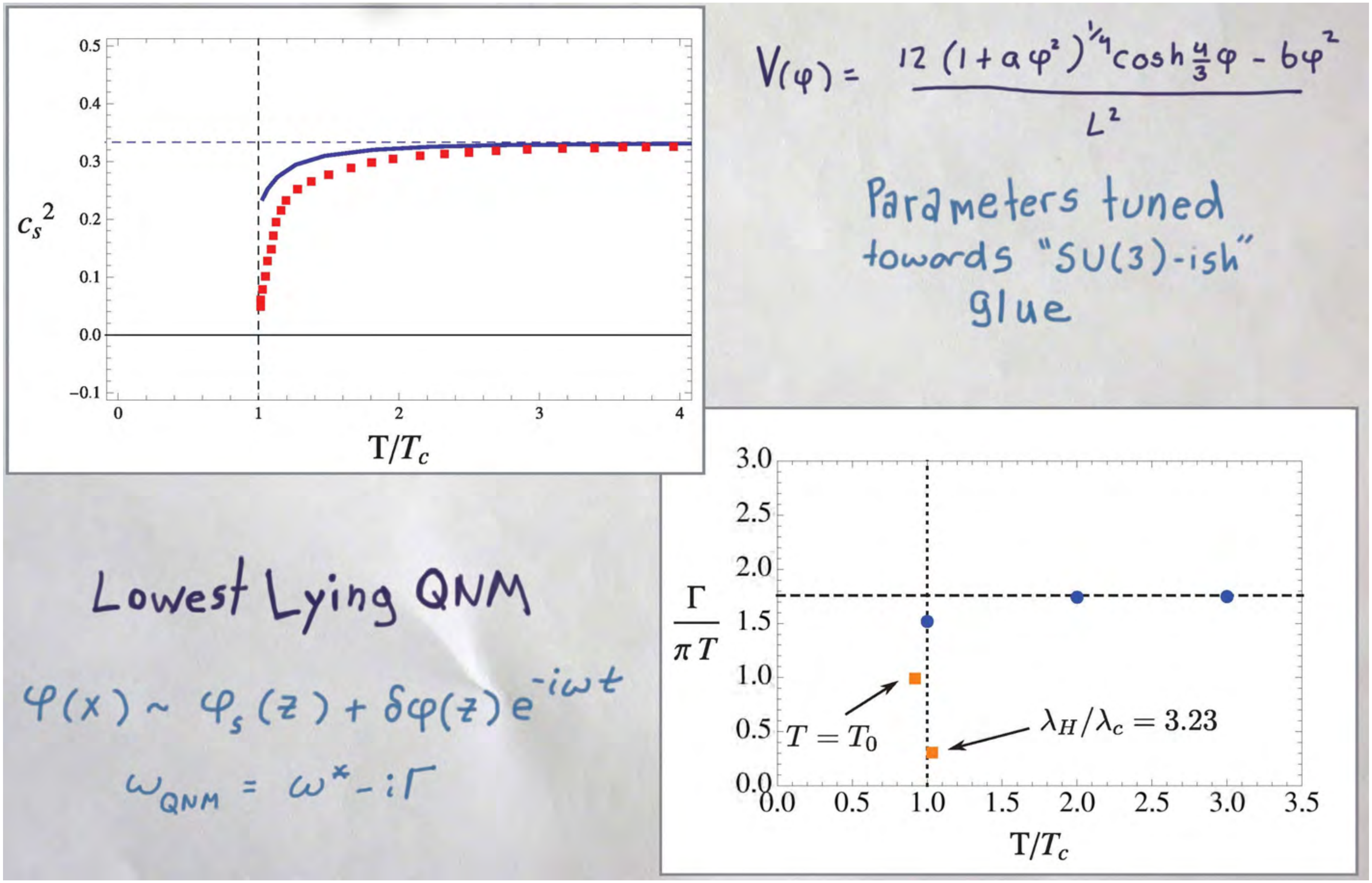
$$C_v = -T \frac{\partial^2 F}{\partial T^2} \Rightarrow$$ Small BHs locally unstable

•



static Properties
20
15
10
5
0
0
2
4
6
8
$\langle \hat{T}_{tt} \rangle$
λ_H/λ_c
Big BHs
Small BHs
To 'vacuum'
≈

Large Black holes



Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 2 minutes
- The theoretical setup for thermalization 4 minutes
- Thermalization at strong coupling 5 minutes
- Expected characteristic scales 7 minutes
- Thermalization calculations 10 minutes
- Non-conformal/Confining QFTs 11 minutes
- The Hard Wall Model 14 minutes
- The Non-linear analysis 17 minutes
- Improved Holographic QCD 20 minutes
- Black holes in Improved Holographic QCD 24 minutes
- Quench dynamics 27 minutes

- The solution procedure 39 minutes
- The turbulent instability of AdS 41 minutes
- The thermodynamic functions 43 minutes
- The small black hole initial states 44 minutes
- Large amplitude Quench 47 minutes
- The evolution of horizons 49 minutes
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