NP-QCD, Paris, 14.06.18

Spontaneous CP breaking & the axion potential: an effective Lagrangian approach

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based on 1709.00731
(with P. Di Vecchia, G.C. Rossi & S. Yankielowicz)
Related work:
D. Gaiotto, Z. Komargodski, N. Seiberg, 1708.06806
(as well as older papers by A. Smilga and by M. Creutz)

Outline

- U(1)-CP problems & their mutually exclusive solutions
- L_{eff} @ small m/Λ & $1/N$ (w/ mN/Λ fixed)
- CP-viol. @ arbitrary θ: two non-viable solutions.

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- Spontaneous CP violation in QCD @ $\theta = \pi$
- $N_f = 1$, $N_f > 1$

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- L_{eff} w/ axion & simult. resol. of U(1) & CP problems
- Relevance of QCD @ $\theta \neq 0$ for axion potential
- Relevance of above @ $0 < T < T_{dec} \sim T_{ch}$?

PART 1

U(1)-CP problems in QCD and their mutually exclusive solutions (a reminder of old stuff)

- Weinberg's 1973 argument for automatic CP
- Weinberg's formulation of U(1) problem
- Instantons may solve U(1) but reintroduce CP

Weinberg's 1973 argument for automatic CP in QCD

Redefining the quark fields via a $U(N_f) \times U(N_f)$ transformation we can rewrite the original mass term (coming from EW breaking and generically CP-violating) in an explicitly CP-conserving form:

$$L_{QCD} = \cdots + \bar{\psi}_{iR} m_{ij} \psi_{jL} + \text{h. c.} \Rightarrow \cdots + \sum_{i=1}^{N_f} m_i \bar{\psi}'_i \psi'_i$$

Weinberg knew about the ABJ anomaly but at that time a term proportional to FF_{dual} was considered irrelevant.

Weinberg's formulation of U(1) problem

That same assumption about the irrelevance of the topological charge density was at the origin of the U(1) problem:

1. Why is the η much heavier than the pion?
2. Why is the η' much heavier than the other eight PNG bosons?
3. Why are the lightest PS (as opposed to V) mesons in approximately unmixed SU(3) reps?

Topological charge may solve U(1) but then reintroduces CP

Instantons (more generally topological charge fluctuations) may solve the U(1) problem (see below) but then falsify Weinberg's 1973 argument by reintroducing the danger of CP violation in the strong interactions (violating bounds on D_n)

Weinberg's argument still allows to lump all the CP-violation in a single (but now relevant) term:

$$L_{QCD} = \cdots - \bar{\theta} \frac{g^2}{32\pi^2} F^a \tilde{F}^a \equiv \cdots - \bar{\theta} Q$$

$$\bar{\theta} \equiv \theta + \arg \det m$$

L_{eff} @ small m/Λ & 1/N (w/ mN/Λ fixed)

(Di Vecchia and GV, Rosenzweig et al. Nath & Arnowitt, Witten…~1980)

Spontaneous breaking: $U(N_f)_L \otimes U(N_f)_R \to U(N_f)_V$

$$U^\dagger U = \frac{F_\pi^2}{2} \qquad U = \frac{F_\pi}{\sqrt{2}} e^{i\sqrt{2}\Phi/F_\pi} \quad ; \quad \Phi = \Pi^a T^a_{ij}$$

$$L = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F_\pi}{2\sqrt{2}}\mathrm{Tr}\left[\mu^2(U + U^\dagger)\right]$$

$$+\frac{Q^2}{2\chi_{YM}} + \frac{i}{2} Q \,\mathrm{Tr}\left[\log U - \log U^\dagger\right] - \theta Q\,.$$

$$\mu^2_{ij} = \mu^2_i \delta_{ij} = -2m_i \langle\bar\psi\psi\rangle F_\pi^{-2}\delta_{ij} \qquad \int d^4x Q(x) = \nu$$

NB: we have put all CPV in the θ angle

The μ_i^2 are nothing but the PNGB masses in the absence of anomaly effects (hence in presence of U(1) problem)

$$M_{ij}^2 = \frac{1}{2}(\mu_i^2 + \mu_j^2) \; ; \; i,j = 1,2,\ldots,N_f$$

For i ≠ j these are the physical masses (e.g. $\pi^\pm$)

The $U(1)_A$ anomaly is implemented in:

$$L = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F_\pi}{2\sqrt{2}}\mathrm{Tr}\left[\mu^2(U + U^\dagger)\right]$$
$$+\frac{Q^2}{2\chi_{YM}} + \frac{i}{2}Q\,\mathrm{Tr}\left[\log U - \log U^\dagger\right] - \theta Q\,.$$

through the $U(1)_A$ transformation:

$$\frac{i}{2}\mathrm{Tr}\left(\log U - \log U^\dagger\right) \rightarrow \frac{i}{2}\mathrm{Tr}\left(\log U - \log U^\dagger\right) + 2\alpha N_f Q$$

The term $Q^2/(2\chi_{YM})$ corresponds to the crucial assumption that the topological susceptibility in pure YM theory is non vanishing at large N (and of order Λ^4).

By now lattice calculations have given strong evidence that this is the case.

Furthermore, the numerical value of χ_{YM} is in (even too good an) agreement with the phenomenologically preferred value (GV '79): $\chi_{YM} \sim (180\ \text{MeV})^4$

Focusing on the "Cartan" PNGB and introducing

$$\phi_i = -\sqrt{2}\frac{\Phi_{ii}}{F_\pi}$$

we get, after integrating out the heavy field Q,

$$V(\phi_i) = -\frac{F_\pi^2}{2}\sum_{i=1}^{N_f}\mu_i^2\cos\phi_i + \frac{\chi_{YM}}{2}\left(\theta - \sum_{i=1}^{N_f}\phi_i\right)^2$$

whose stationary points are the solutions of

$$\mu_i^2\sin\phi_i - a\left(\theta - \sum_{j=1}^{N_f}\phi_j\right) = 0 \quad ; \quad i = 1, \dots, N_f \qquad a = \frac{2\chi_{YM}}{F_\pi^2}\,.$$

They depend on θ and on $\epsilon_i = \frac{\mu_i^2}{a} = -\frac{m_i\langle\bar{\psi}\psi\rangle}{\chi_{YM}} = O(\frac{mN}{\Lambda})$

Fluctuations around a given solution are described by the effective action:

$$L = -V(\hat{\phi}_i) + \frac{1}{2}\mathrm{Tr}\left(\partial_\mu \hat{U} \partial^\mu \hat{U}^\dagger\right) + \frac{F_\pi^2}{2}\mathrm{Tr}\left[\mu^2(\theta)\left(\cos\left(\frac{\sqrt{2}}{F_\pi}\hat{\Phi}\right) - 1\right)\right] - \frac{a}{2}\left[\mathrm{Tr}\left(\hat{\Phi}\right)\right]^2$$

$$+\chi_{YM}(\theta - \sum_{j=1}^{N_f}\hat{\phi}_i)\mathrm{Tr}\left[\sin\left(\frac{\sqrt{2}}{F_\pi}\hat{\Phi}\right) - \frac{\sqrt{2}}{F_\pi}\hat{\Phi}\right]$$

$$+\frac{1}{2\chi_{YM}}\left[\hat{Q} - \chi_{YM}\frac{\sqrt{2}}{F_\pi}\mathrm{Tr}\,\hat{\Phi}\right]^2 ,$$

$$a = \frac{2\chi_{YM}}{F_\pi^2} .$$

where $\mu_{ij}^2(\theta) \equiv \mu_i^2 \cos\hat{\phi}_i \delta_{ij}$.

Solving U(1) problem

CP-violating term

For equal quark masses WV formula: $M_s^2 = N_f a$

CPV @ arbitrary θ: two non-viable solutions

If we want to kill the CP-violating term we need

$$\chi_{YM}(\theta - \sum_{i=1}^{N_f}\hat{\phi}_i)\mathrm{Tr}\left[\sin\left(\frac{\sqrt{2}\hat{\Phi}}{F_\pi}\right) - \frac{\sqrt{2}\hat{\Phi}}{F_\pi}\right] = 0$$

where

$$\mu_i^2 \sin\phi_i - a\left(\theta - \sum_{j=1}^{N_f}\phi_j\right) = 0 \quad ; \quad i = 1, \ldots \qquad a = \frac{2\chi_{YM}}{F_\pi^2} \, .$$

For generic θ only two possibilities

1. $\chi_{YM} = 0$ but then we have a U(1) problem
2. $\mu_i = 0$ (for at least one i) but it's bad for CA

We are forced to have θ = 0 or π.

PART 2

Spontaneous CP violation in QCD @ $\theta = \pi$

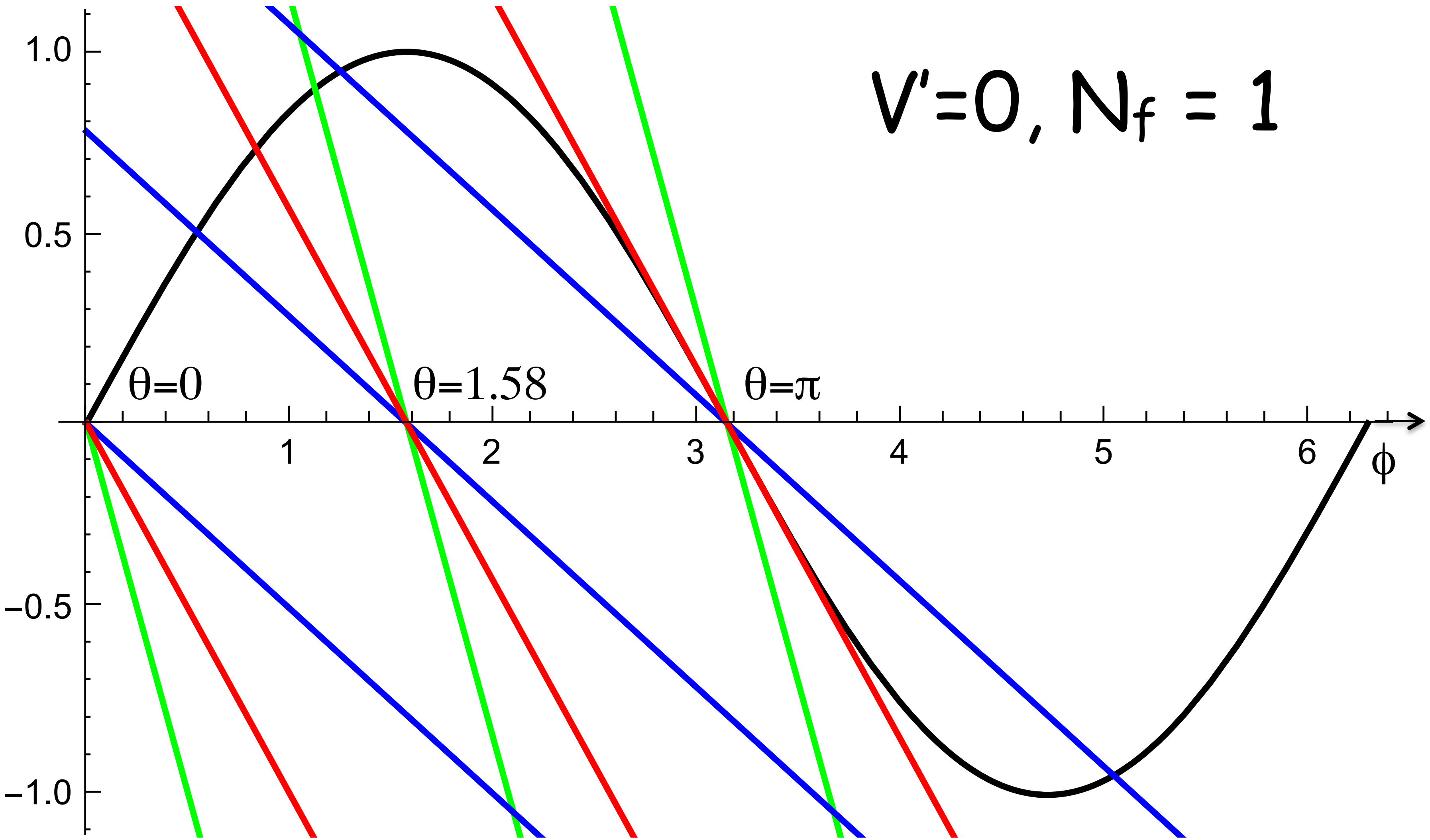
$N_f = 1$, $N_f > 1$

$N_f = 1$

$$\frac{V}{aF_\pi^2} = -\epsilon\cos\phi + \frac{1}{2}(\theta - \phi)^2 \; ; \; \epsilon \equiv \frac{\mu^2}{a}$$

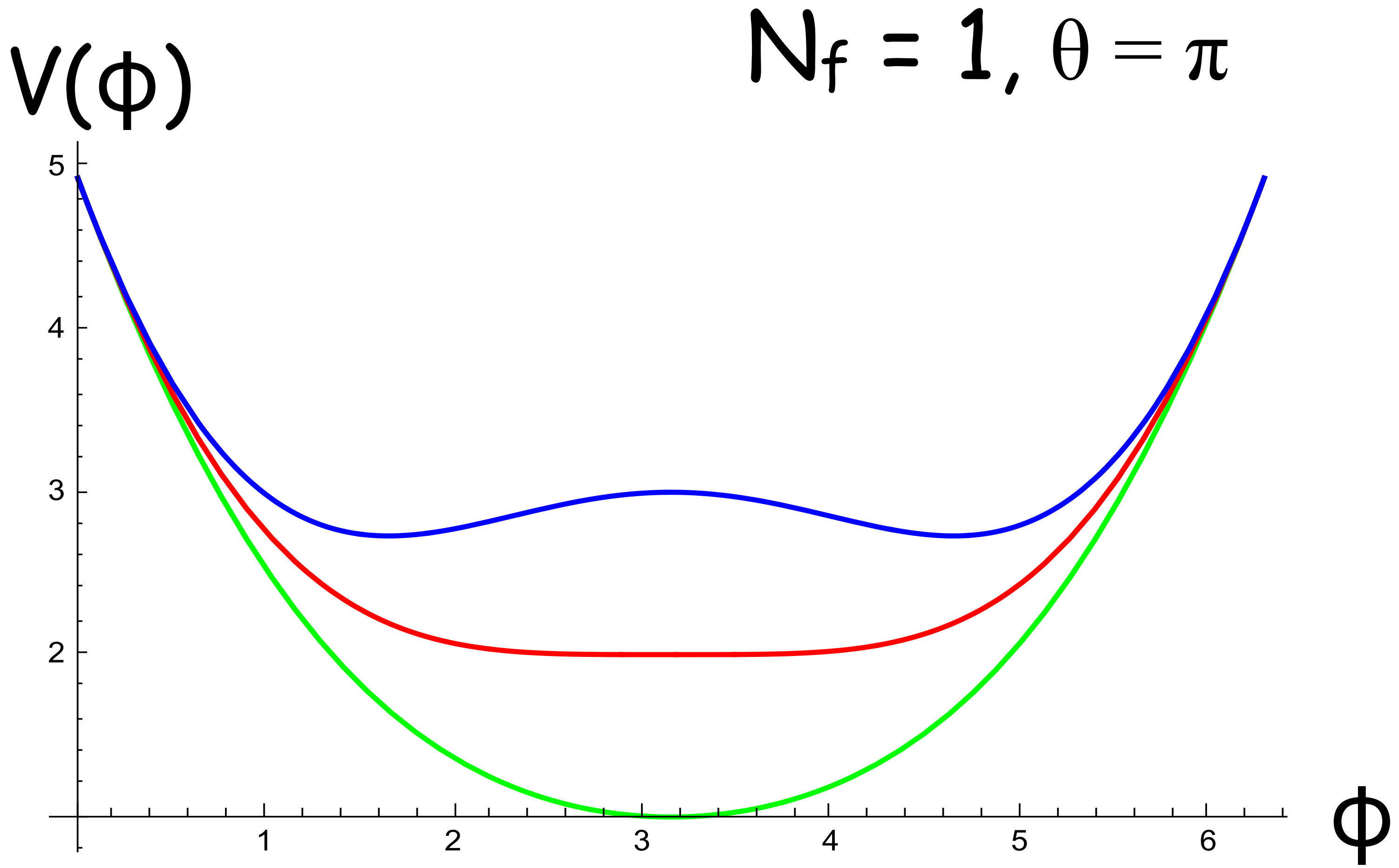
$$\frac{V'}{aF_\pi^2} = \epsilon\sin\phi - \theta + \phi \; ; \; \frac{V''}{aF_\pi^2} = \epsilon\cos\phi + 1$$

The structure of the solutions depends crucially on the value of ε.

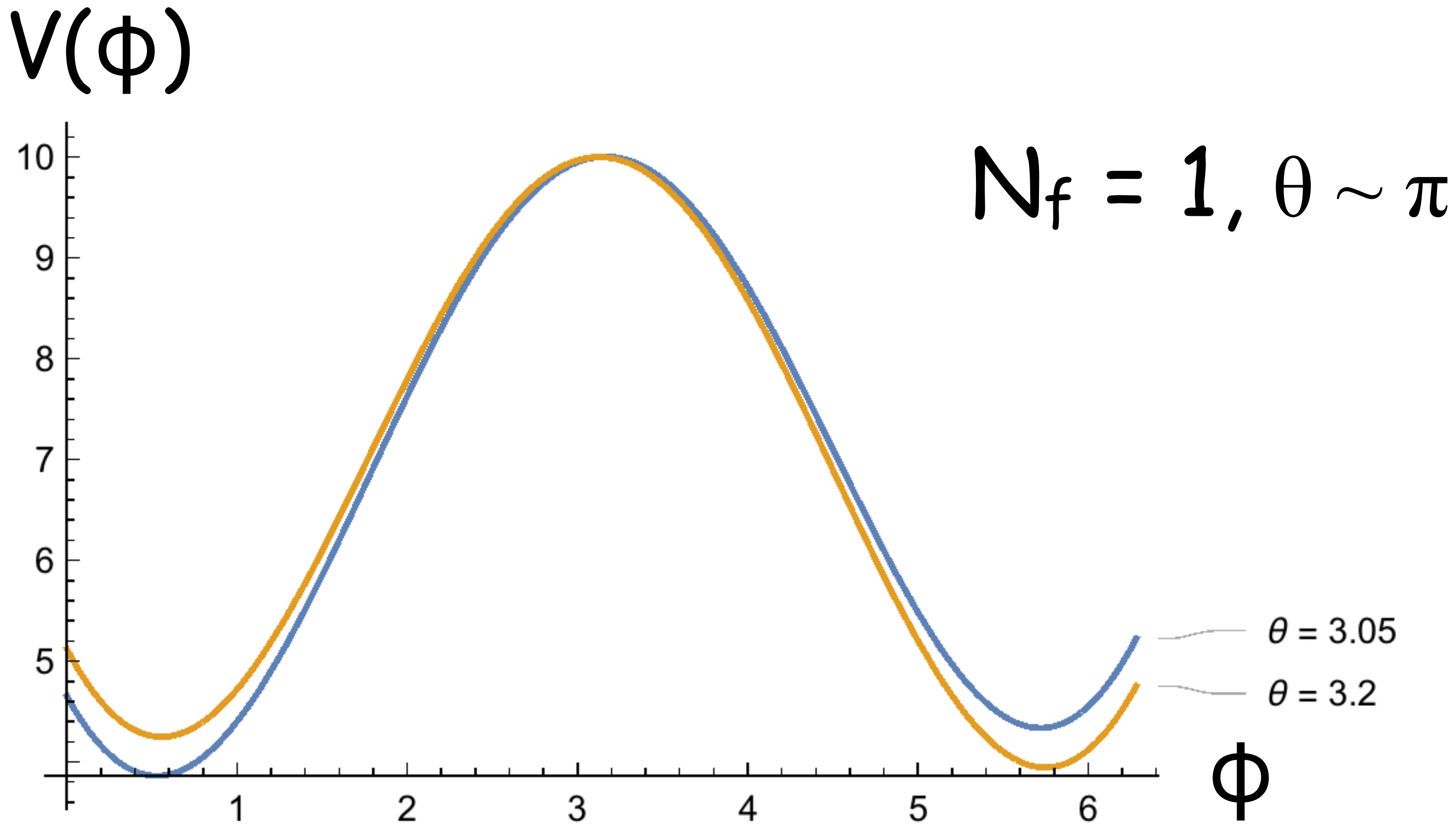


$\varepsilon < 1$:green lines ; $\varepsilon > 1$:blue lines ; $\varepsilon = 1$:red lines

Alternatively, we can look at $V(\phi)$ for $\theta \sim \pi$



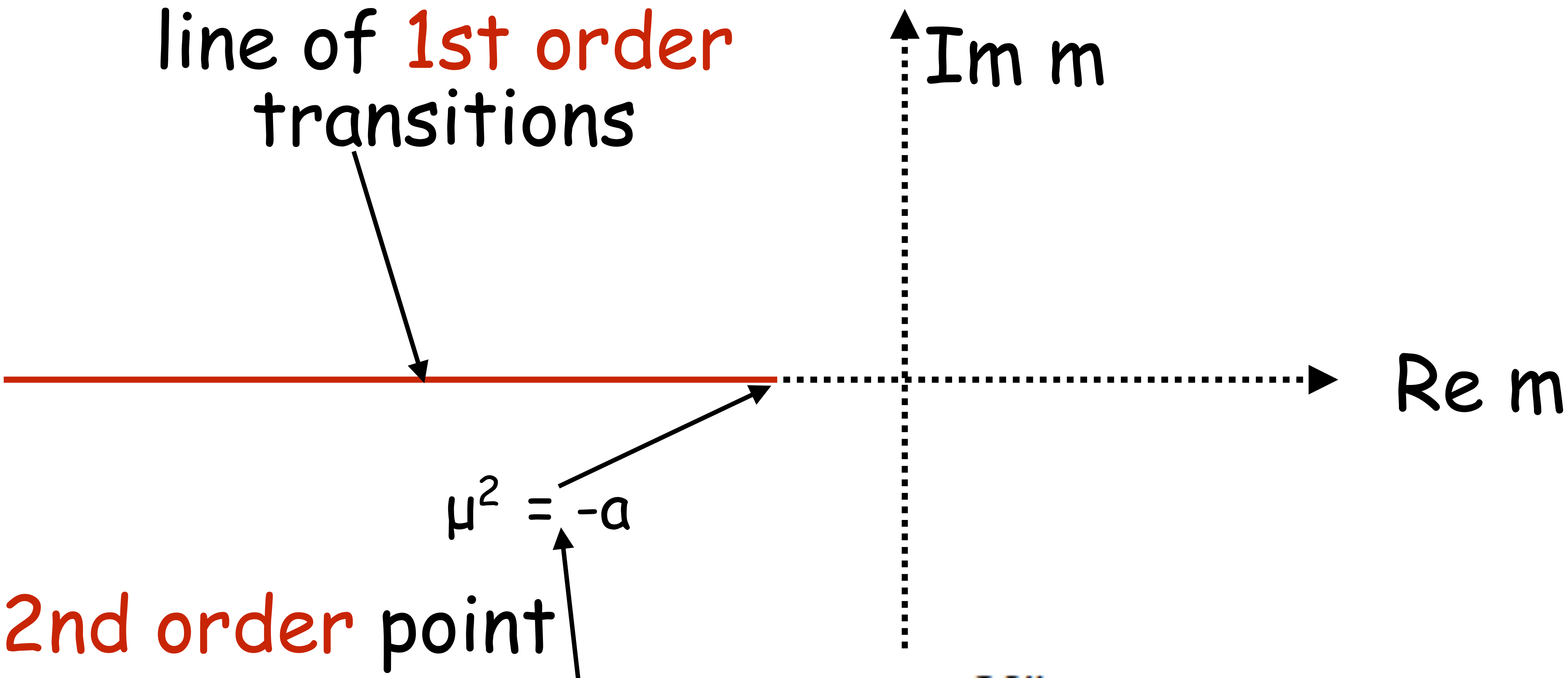
$\varepsilon < 1$: green curve ; $\varepsilon > 1$: blue curve ;
$\varepsilon = 1$:red curve



ground state **jumps** by a finite amount as one goes through $\theta = \pi$

Complex-m plane

($\theta = 0$, CPV lumped in arg m)
as emphasized by GKS



w/ massless PNGB since $\frac{V''}{a} = \epsilon \cos\phi + 1$ -> 0

A related phenomenon: @ 2nd-order point, χ_{QCD} diverges. In general

$$\chi_{QCD} = \frac{\chi_{YM}}{1 + a\sum_{i=1}^{N_f} \frac{1}{\mu_i^2(\theta)}} = \left(\chi_{YM}^{-1} - \sum_{k=1}^{N_f} (m_i \langle \bar{\psi}\psi \rangle)^{-1}\right)^{-1}$$

showing that χ_{QCD} -> 0 if any one of the quarks is massless. Here, instead, no quark is massless but a physical PNGB is.

$N_f > 1$

Only slightly more complicated.

The 2nd-order critical point is given by

$$\sum_i \frac{1}{\mu_i^2(\theta=\pi)} + \frac{1}{a} = 0$$

i.e. in practice

$$\sum_{i\neq 1} \frac{1}{\mu_i^2} + \frac{1}{a} = \frac{1}{\mu_1^2} \; ; \; \mu_1^2 \leq \mu_i^2$$

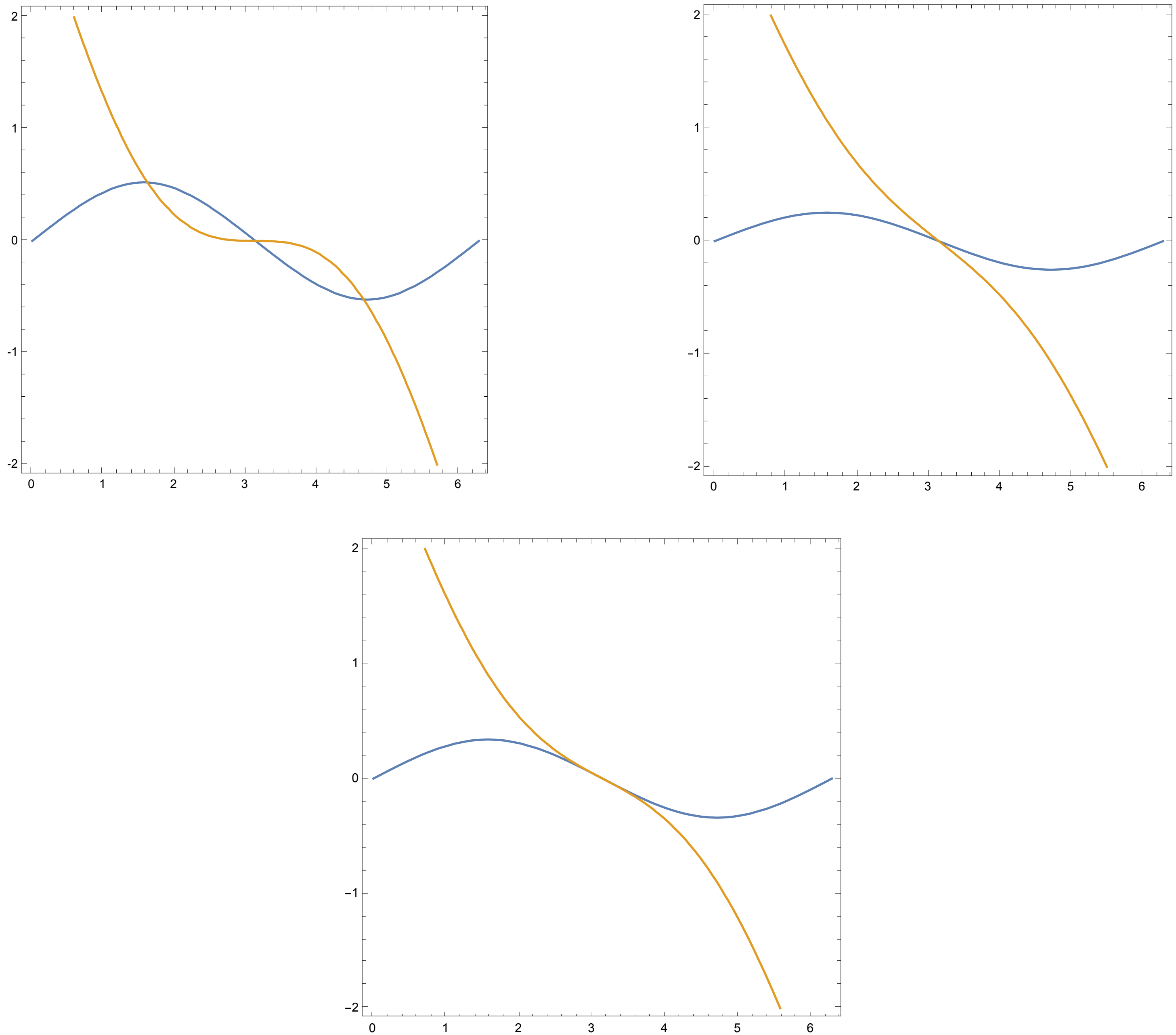
noticed by Creutz ('04) for $N_f=3$, $a \gg \mu_i^2$

CP (un)broken if l.h.s. (<) > r.h.s.

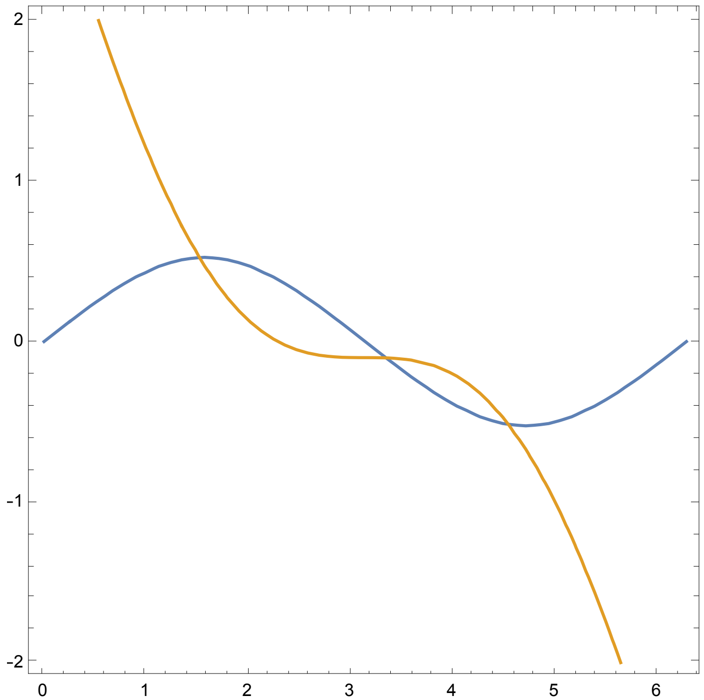
In real QCD at T=0, CP is unbroken at $\theta = \pi$ (but excluded by CA)

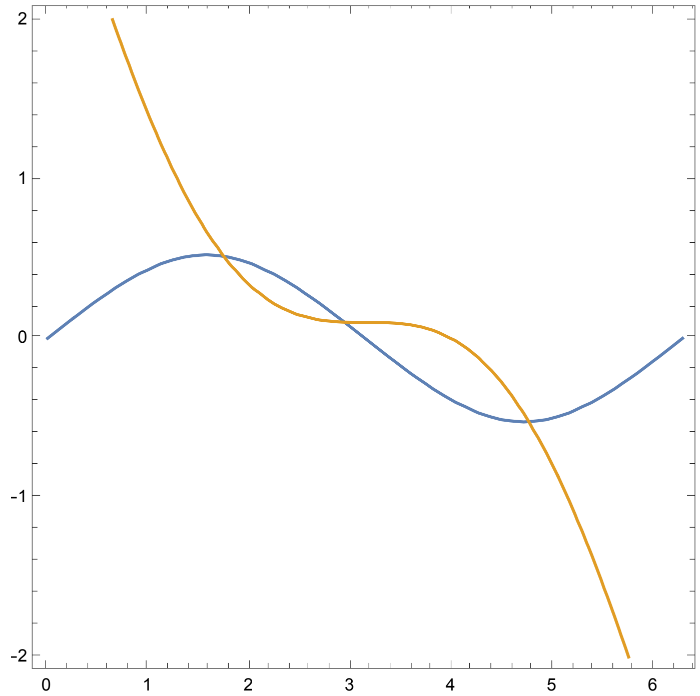
$N_f = 2, \theta = \pi, m_d = 2\, m_u$

(for different values of μ_1^2/a)



$N_f = 2,\ \theta \sim \pi\ ,\ m_d = 2\ m_u$





A very special case: $N_f = 2, \theta = \pi$, $m_d = m_u$

Looks straightforward: we are always in the CP-broken situation except if we send $\varepsilon_1 = \varepsilon_2$ to zero.

However, as first noted by Smilga ('99), at $\theta = \pi$ the potential develops a flat direction @ $O(\varepsilon)$ (with $\phi_1 + \phi_2 = \pi$)

$$V(\phi_i) = -\frac{F_\pi^2}{2}\sum_{i=1}^{N_f}\mu_i^2\cos\phi_i + \frac{\chi_{YM}}{2}\left(\theta - \sum_{i=1}^{N_f}\phi_i\right)^2$$

In order to lift the flat direction we need to go to $O(\varepsilon^2)$ which looks beyond control.

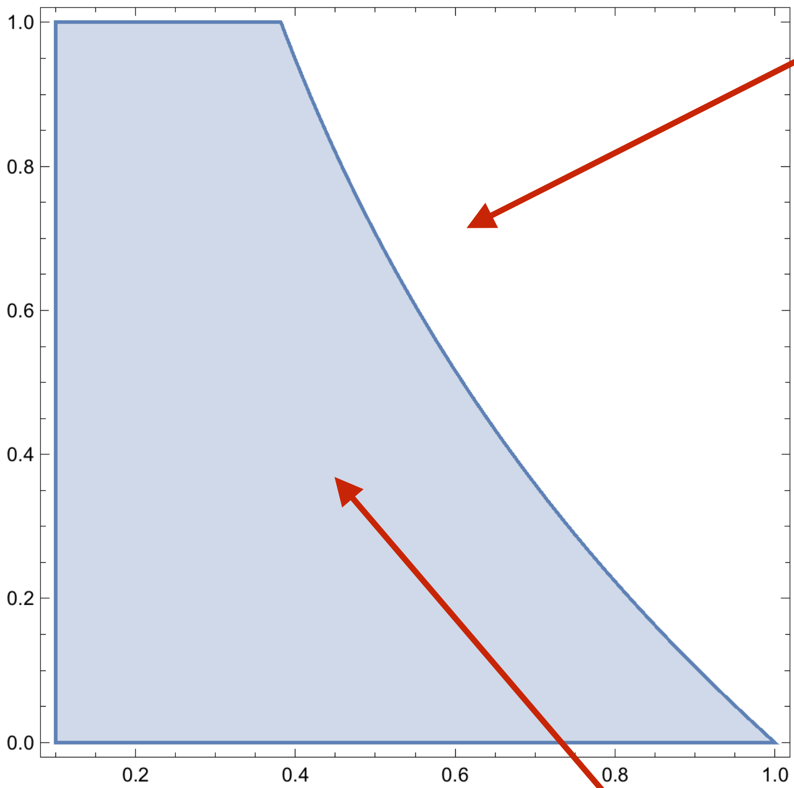
Smilga does it within a Skyrmion model and argues that CP is broken.

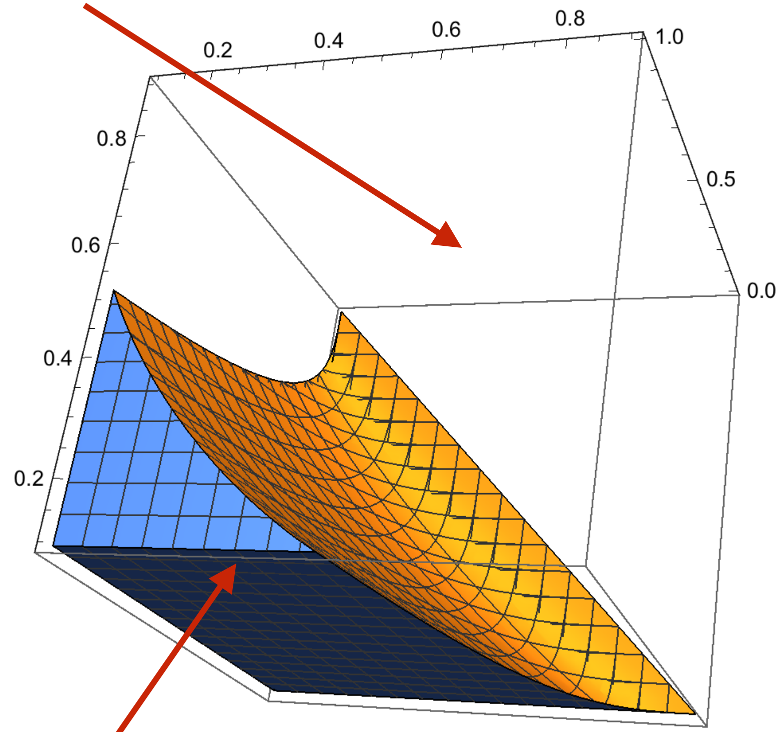
We can do better(?) since the terms we neglect are down by at least a 1/N factor.

We conclude in favor of CP breaking at sufficiently large N.

Critical (hyper)-surface in general case

broken CP





$N_f = 2$ N_f=2

unbroken CP

N_f=3 $N_f = 3$

hor. axes = mass ratios,
vertical is D = det(μ^2/a)

Our results satisfy “decoupling“ in a stronger-than-usual sense.

If a quark becomes heavier than Λ/N the problem reduces to the one in which the “heavy“ quark is removed.

=> smooth connection with YM (where there are interesting results by GKKS)?

PART 3

Adding a generic axion to QCD

An axion solves the strong CP problem exactly like a massless quark would do.

The NGB corresponding to the $U(1)_{PQ}$ would be exactly massless if the anomaly were ineffective.

Phenomenological problems force scale of $U(1)_{PQ}$ breaking to be sufficiently large

Simultaneous resolution of U(1) & CP problems with axion in L_{eff} language

(see e.g. Di Vecchia & Sannino, 1310.0954)

$$L = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2}\partial_\mu N \partial^\mu N^\dagger + \frac{F_\pi}{2\sqrt{2}}\mathrm{Tr}\left(\mu^2(U + U^\dagger)\right) + \frac{Q^2}{2\chi_{YM}}$$

$$+\frac{i}{2}Q\left[\log U - \log U^\dagger + \alpha_{PQ}\left(\log N - \log N^\dagger\right)\right] - \theta Q\,.$$

$$L = \frac{1}{2}\sum_{i=1}^{N_f}\partial_\mu v_i \partial^\mu v_i + \frac{F_\pi^2}{2}\sum_{i=1}^{N_f}\mu_i^2 \cos\left(-\phi_i + \frac{\sqrt{2}}{F_\pi}v_i\right) + \frac{Q^2}{2\chi_{YM}}$$

$$+\frac{1}{2}(\partial_\mu \alpha)^2 - Q\left(\theta - \sum_{i=1}^{N_f}\phi_i - \beta + \frac{\sqrt{2}}{F_\pi}\sum_{i=1}^{N_f}v_i + \frac{\alpha_{PQ}\sqrt{2}}{F_\alpha}\sigma\right),$$

$$N = \frac{F_\alpha}{\sqrt{2}}\mathrm{e}^{i\frac{\sqrt{2}}{F_\alpha}\alpha} \qquad\qquad \alpha = \sigma - \frac{\sqrt{2}\alpha_{PQ}}{F_\alpha}\beta$$

NB: $U(1)_{PQ}$ is only broken expl. by the anomaly!

Upon integrating out Q we get:

$$V(\phi_i,\beta) = -\frac{F_\pi^2}{2}\sum_{i=1}^{N_f}\mu_i^2\cos\phi_i + \frac{\chi_{YM}}{2}\left(\theta - \sum_{i=1}^{N_f}\phi_i - \beta\right)^2$$

and therefore:

$$-\frac{F_\pi^2}{2}\mu_i^2\sin\phi_i + \chi_{YM}(\theta - \sum_i \phi_i - \beta) = 0$$

$$\theta - \sum_i \phi_i - \beta = 0 \qquad \beta = \theta \ , \ \phi_i = 0$$

The mass^2-matrix of fluctuations reads

$$b \equiv \frac{\alpha_{PQ} F_\pi}{F_\alpha} \ll 1$$

$$A = \begin{pmatrix} b^2 a & ba & ba & ba & \dots & ba \\ ba & \mu_1^2 + a & a & a & \dots & a \\ ba & a & \mu_2^2 + a & a & \dots & a \\ \dots & \dots & \dots & \dots & \dots & \dots \\ ba & a & a & a & \dots & \mu_{N_f}^2 + a \end{pmatrix}$$

from which we get:

$$\det\left(p^2\delta_{ij} - A_{ij}\right) = p^2 \prod_{i=1}^{N_f}(p^2 - \mu_i^2)\left[1 - a\left(\sum_{i=1}^{N_f}\frac{1}{p^2-\mu_i^2} + \frac{b^2}{p^2}\right)\right]$$

$$= \prod_{i=1}^{N_f+1}\left(p^2 - M_i^2\right)\,, \qquad \det A = ab^2\prod_{i=1}^{N_f}\mu_i^2 = \prod_{j=1}^{N_f+1} M_j^2$$

$$M^2_{axion} = \frac{b^2}{\frac{1}{a} + \sum_{i=1}^{N_f}\frac{1}{\mu_i^2}}$$

=> standard expression if $a \gg \mu_1^2, \mu_2^2$

Why is QCD @ $\theta \sim \pi$ relevant for axion potential?

β is like a dynamical θ.

There is periodicity 2π in both.

Considering the axion potential near the boundary of its periodicity range is analogous to studying QCD for $\theta \sim \pi$

Standard axion potential (PdV&GV) is obtained by integrating out the PNGB in the chiral limit ($\varepsilon_i \ll 1$)

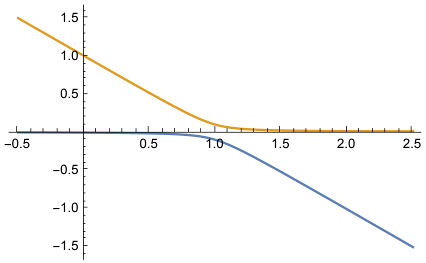
For the realistic case of two non-degenerate light quarks it takes the (periodic and smooth) form:

$$V_{axion}(\sigma) = -F_\pi^2 \sqrt{(\mu_1^2+\mu_2^2)^2 - 4\mu_1^2\mu_2^2 \sin^2\left(\frac{\alpha_{PQ}\sigma}{\sqrt{2}F_\alpha}\right)} + \mathrm{O}(\mu_i^2/a)$$

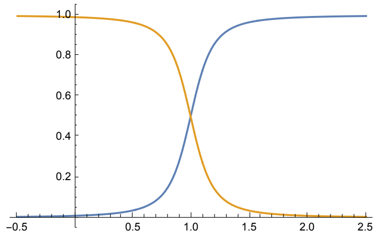
However we have seen that, at (near) the 2nd-order phase trans. points, a PNGB becomes massless (very light) at $\theta = \pi$.

Integrating it out becomes tricky. There is also strong mixing between the (bare) axion and the light PNGB…

Eigenvalues $N_f = 1$ Eigenvectors



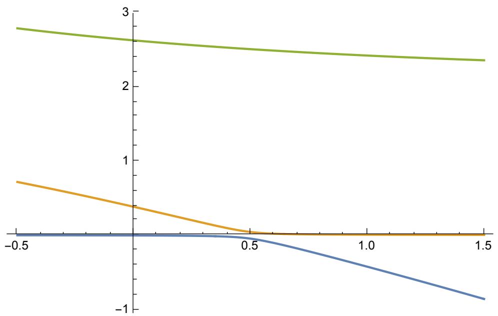
μ^2/a



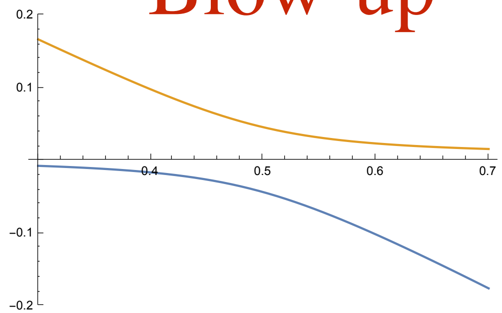
μ^2/a

$N_f = 2$

Eigenvalues

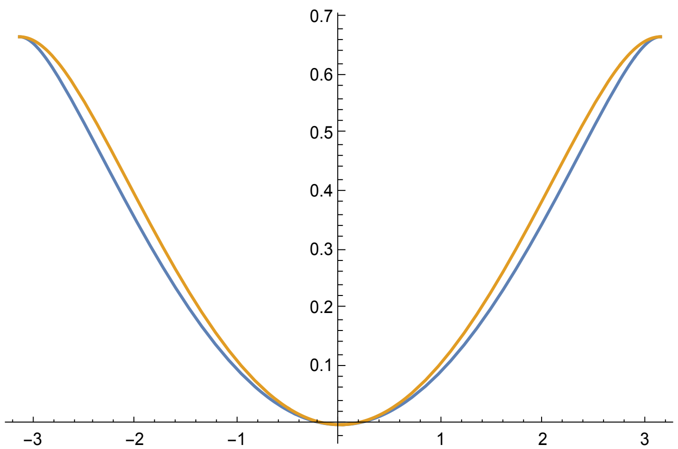


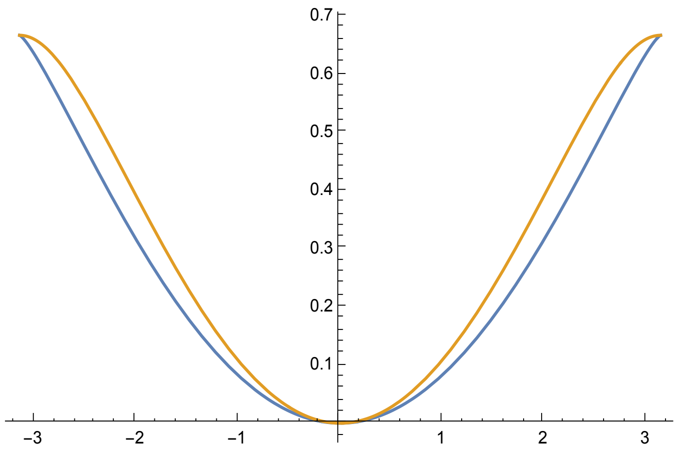
Blow-up

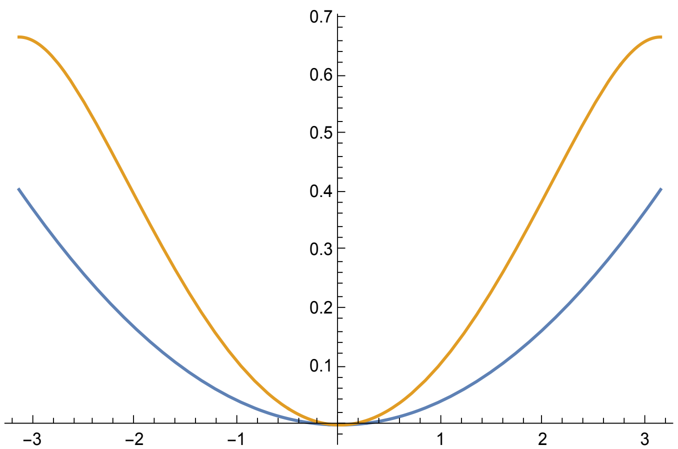


The nature of the tachyonic state changes abruptly along the critical point

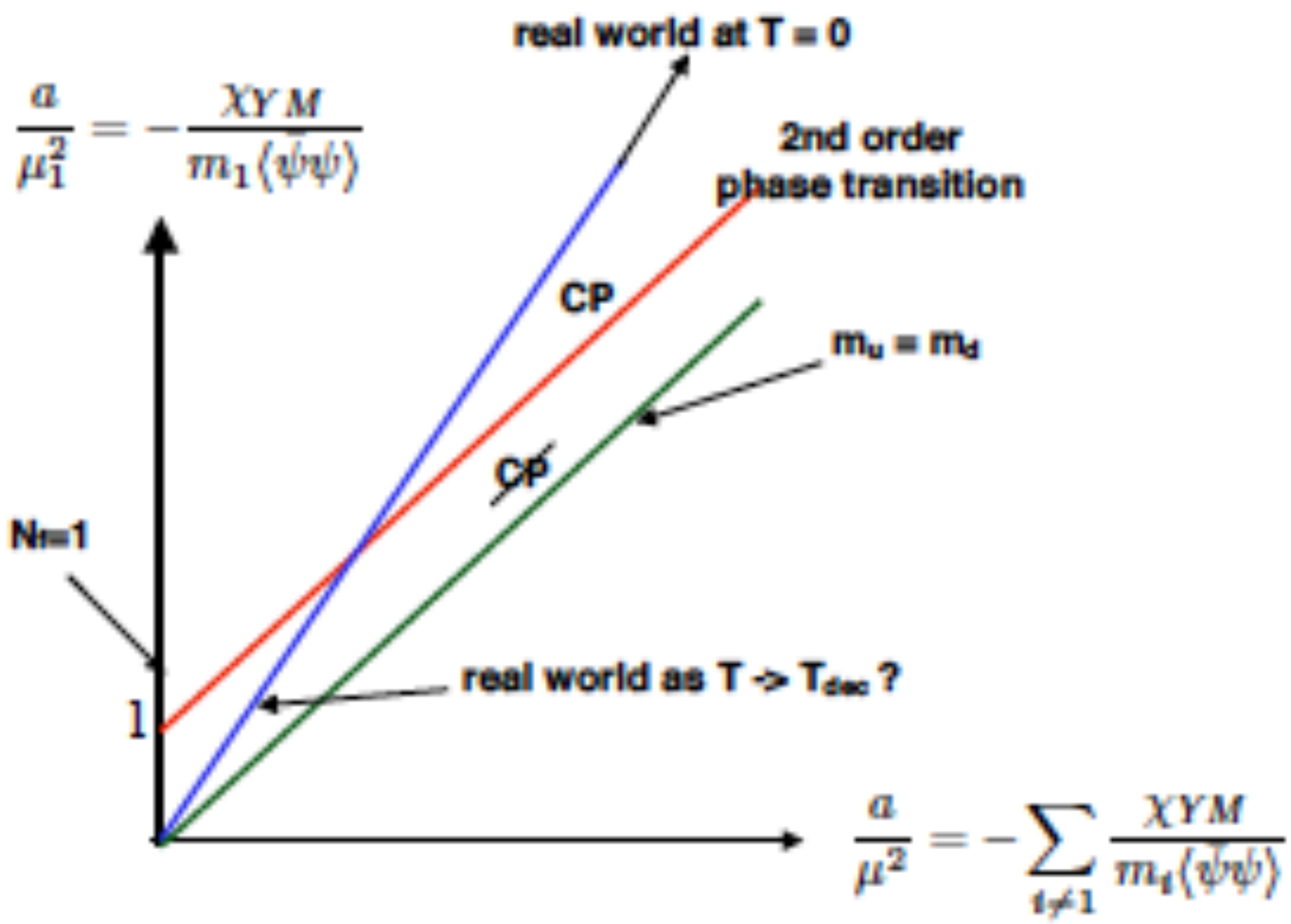
In the vicinity and beyond such critical point the axion potential is strongly modified at its boundary (develops kink)







Relevance of above @ 0 < T < T_{dec} ~ T_{ch} ?



Need dedicated lattice calculations measuring, at the same time, the quenched quark condensate and χ_{YM} as a function of T near Tc

Summary

- There is either a U(1) or a strong CP problem in QCD. All evidence is in favor of the latter

- QCD at $\theta \sim \pi$ shows a very rich structure of possible phase transitions. These can be studied very explicitly in the small m, small 1/N limit (with mN/Λ fixed) by effective Lagrangian techniques

- When the strong-CP problem is solved a la PQ the properties of QCD near $\theta \sim \pi$ have a bearing on the properties of the axion potential near the boundary of its periodicity interval.

- More lattice results are necessary before deciding whether important modifications of the standard axion potential are needed as one approaches T_c