Non-Perturbative QCD: No longer in the Shadow of QED

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ABSTRACT:
This presentation is a rapid summary of recent work dating from 2010 to 2018 by the QFT theoreticians HMF, YG, TG, RH, and two of my most recent grad students Y-MS, and PT. The emphasis here is on ideas, rather than specific calculations; all references can be supplied as desired.

1. INTRODUCTION

Our starting point was, of course, QED, a Theory we have "all grown up with", and we appreciate its superb perturbation agreement with experimental data.

We followed the QED example of writing a Lagrangian, in which the charged lepton fields were replaced by quark fields, and the photon field by gluon fields, with due inclusion of appropriate color coordinates. Of course, QCD contains an additional, fundamental gluon-gluon interaction; but did this not just mean that QCD is simply a "more complicated" version of QED? In fact, as we now understand, they are completely different entities, requiring different approaches, and different forms of Renormalization.

15th Workshop on Non-Perturbative QCD, Paris 2018
Julian Schwinger Centennial Conference 2018

2. A Functional Approach

Our starting point was the Schwinger QCD Generating Functional. By a simple, exact, functional rearrangement, it can be converted into a functional statement which naturally separates into two parts: an "Internal" part, which contains all the quantum details of any and every process describing the interactions of relevant quarks and gluons; and an "External" part, whose sole function is to "bring down", by functional differentiation of a "gluon source function", any desired number of well-defined "external" gluons into the problem.

But now consider the Physics of the problem. Just as one cannot measure precise 4-momenta of an asymptotic quark, because they only appear as hadronic bound-states, and hence their transverse position/momentum is always fluctuating, so one cannot measure the position/momentum of a gluon emitted or absorbed by a transverse fluctuating quark. Clearly, it is impossible to measure the precise coordinates of a gluon, or of any finite number of gluons, or how to distinguish any such group from another.

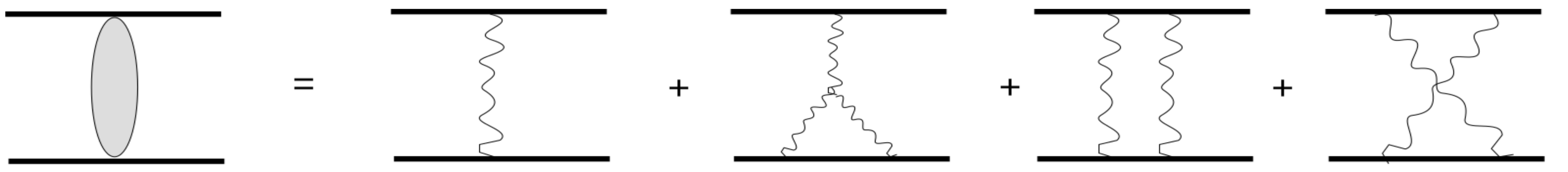
And this begs the question: Why is that meaningless gluon source function still present? The only answer must be that it is a leftover, a hangover from our QED-based forms, and it does not belong there. One can never measure a single gluon, as one can a single photon, and retaining that gluon source function as an essential part of any calculation is absurd. What to do? Simply add the physically-correct assumption that there can never be free, measurable gluons, and set that External gluon source equal to zero.

3. Gauge-invariance, Gluon Bundles, and Effective Locality

Three separate, new, exact results are now possible. Combining the functional solutions written a half-century ago by Fradkin, and those of a few decades ago by Halpern, which forms are all Gaussian in their gluon field dependence, the needed functional operations upon the "internal" gluon fields can be done exactly and immediately, and yield:

A) A gauge-independent result, with all gauge-dependent gluon propagators canceling exactly.

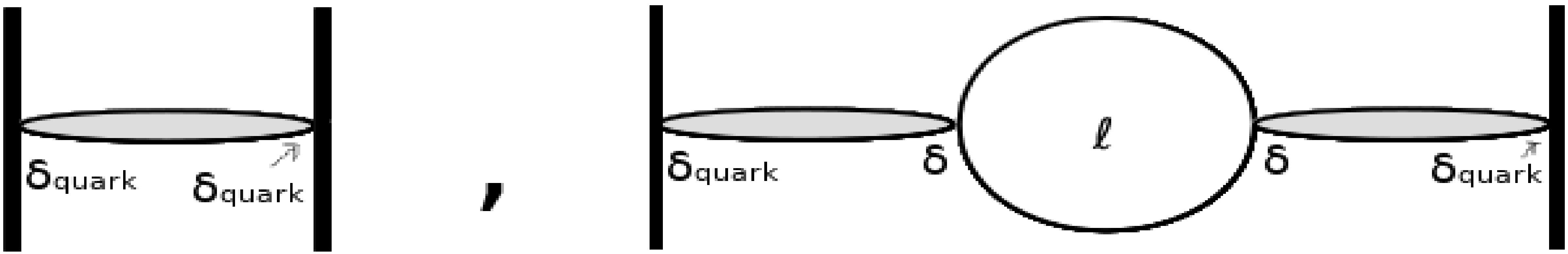
B) A new entity appears, the sum over an infinite number of gluons, for each 'piece' of the resulting interactions, which sum we have called a "**Gluon Bundle**" (GB). It is to be emphasized that no individual gluons now appear; they have all been summed in this non-perturbative formulation. The "radiative corrections" which do now appear are of two types: GBs exchanged between two quarks of the same or different hadrons, and closed-quark-loops (CQLs) which are supported by GBs coming directly from quarks, or grouped together by effective "chains" in a most general way.



C) The third exact and most simplifying property, to which we have given the name of Effective Locality (EL), is the appearance of local space-time-plus-color-index restrictions between GBs and CQLs. And these restrictions have the effect of replacing a remaining functional integral by sets of ordinary integrals, so that complicated lattice-gauge evaluation is not necessary; rather, as we have found in our analysis of high-energy, elastic pp scattering, simple approximations to ordinary integrals are sufficient, with perhaps the occasional use of a laptop.

4. Renormalization of Non-Perturbative, Gauge-Invariant QCD

The basic building blocks of QCD 'radiative corrections' are GBs and CQLs, which interact with quarks of the same or different hadrons, as indicated by the following figures,



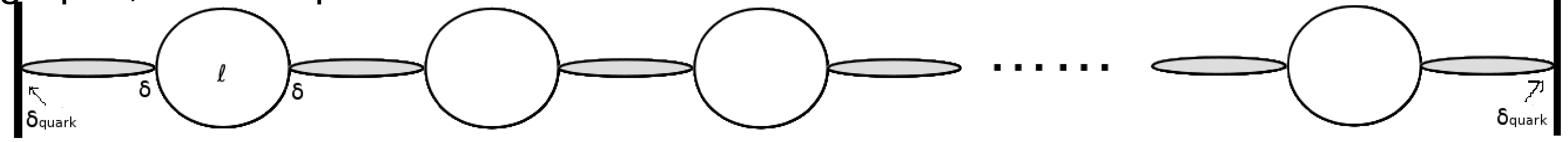
Because GBs can be attached to CQLs in so many ways, this can be a daunting problem. But now enters the question of Renormalization, not of gluons but of GBs, and which can be chosen to simplify the analysis tremendously.

In the original Halpern representation, space-time is broken up into n tiny, 4-volumes of size δ^4, such that the limits $n \to \infty$, $\delta \to 0$, reproduces the total volume of space-time. After an appropriate variable change, one finds that each GB is multiplied by a factor of δ; and it is convenient to visualize each GB as having one factor of δ at each of its ends. Then, we shall assume that the δ

at that end of a GB which is attached to a quark that is bound, or about to be bound into a hadron, δ_{quark} is not allowed to vanish, but is replaced by a real, finite $\delta(E)$, to be introduced phenomenologically. However, the δ at the end of a GB which is attached to a CQL will be allowed to vanish.

For clarity, consider a one-loop diagram, as above, with the loop suspended between two GBs, each of which is attached to a quark, so that a net factor of δ^2 multiplies that loop. But each CQL always contains a log UV divergence, which we'll call ℓ. Since δ is to vanish, and ℓ is to diverge, we define their product $\delta^2 \ell = \kappa$ to be a real, finite constant, whose numerical value is to be determined subsequently - from one bit of the experimental data, and then used without change to fit all of the remaining QCD data in that energy range. [It is important to know that this phenomenological choice of energy dependence $\delta(E)$ which appears is simply a result of the approximations which we have made, but is not really necessary; an exact calculation of elastic pp scattering is a 6-body quark problem, in which certain energy aspects, involving 6 sets of Meijer G-functions, have been simplified to those of a 2-body amplitude.]

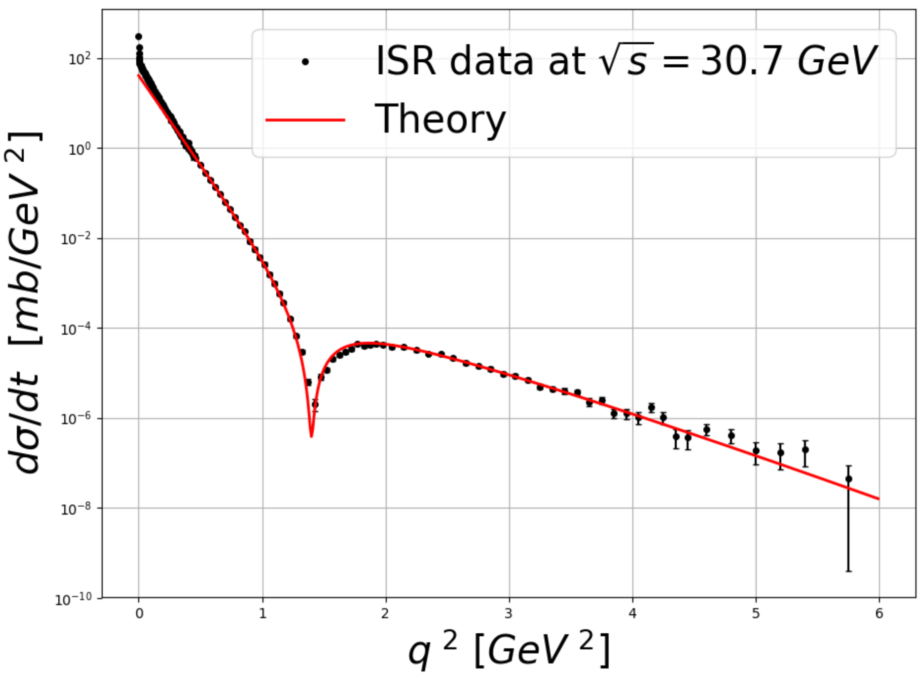
This definition of κ is not unique, but it has the great advantage that chain-loop graphs, for example

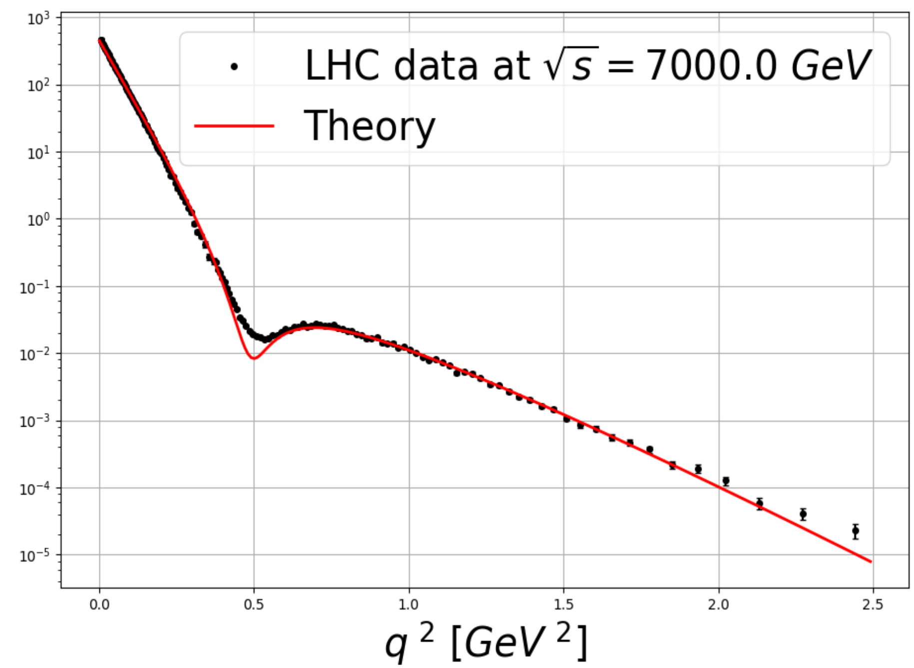


which transfer momentum, produce a finite contribution to their sum. And most importantly, all other non-chain loop graphs of the Functional Cluster Expansion vanish. With this definition of GB renormalization, only the chain-loop graphs survive, and the essentially geometric sum of all their contributions may be used to define a "finite color-charge renormalization". Of course, there are also GB contributions connecting the quarks of different hadrons.

5. Summary

Our first application of the above techniques has been to reproduce the ISR, LHC elastic pp scattering data, from ~20Gev to 13 Tev.





Proton proton elastic scattering differential cross section as a function of q^2 momentum transfer

6 References

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