# *PT*-symmetric interpretation of double scaling in QFT

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*Washington University*

**12th Workshop on Nonperturbative QCD**
**Paris, June 2013**

# Dirac Hermiticity

$$H = H^{\dagger}$$ († means transpose + complex conjugate)

guarantees real energy and unitary time evolution but … is a **mathematical** and not a **physical** axiom of quantum mechanics

**Dirac Hermiticity can be generalized:**

**Replace Dirac Hermiticity by the *physical* and *weaker* condition of *PT* symmetry**

# Example of a non-Hermitian *PT*-symmetric Hamiltonian:

$$\boxed{H = p^2 + ix^3}$$

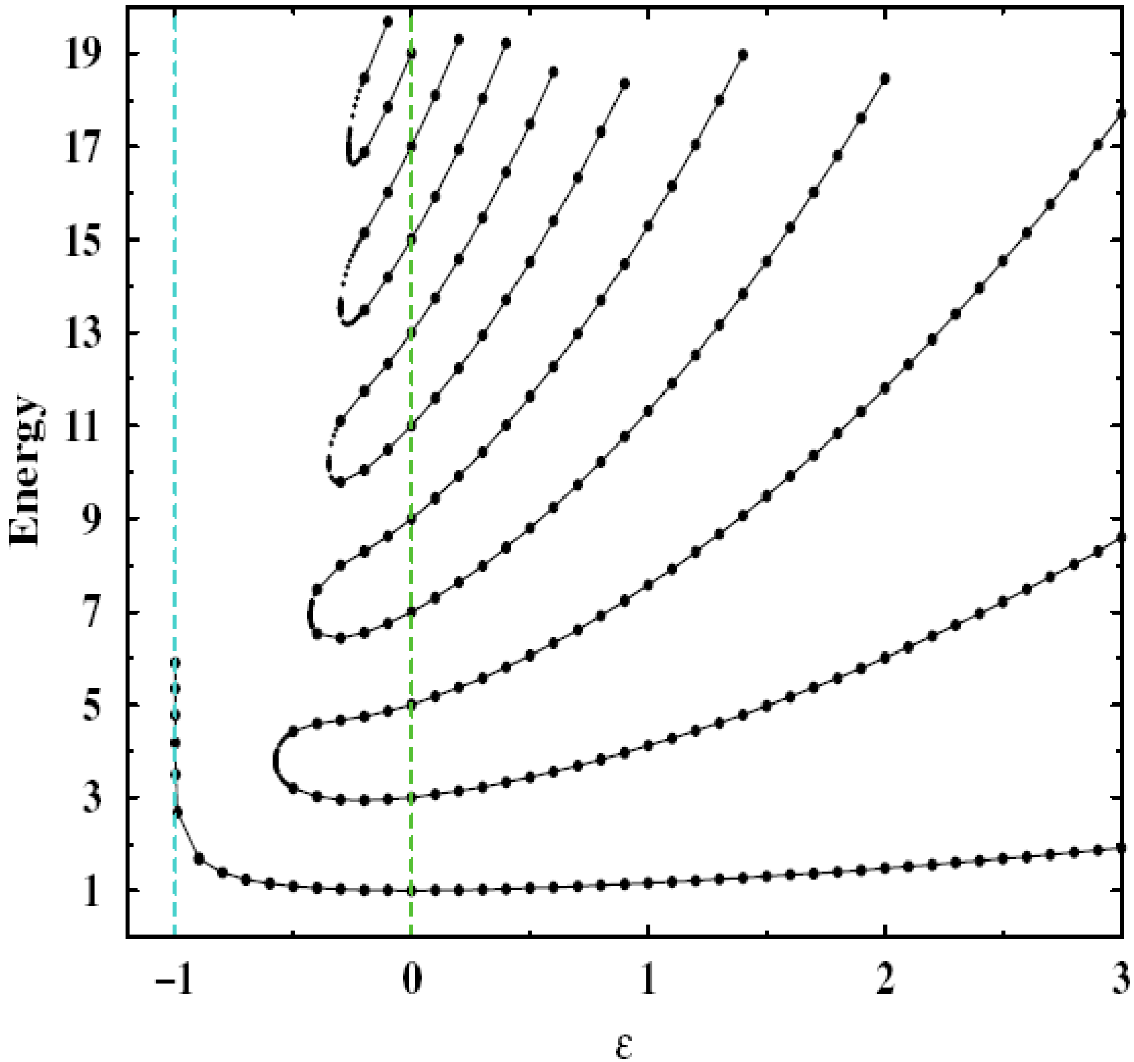
This Hamiltonian has
***PT* symmetry!**

***P*** = parity

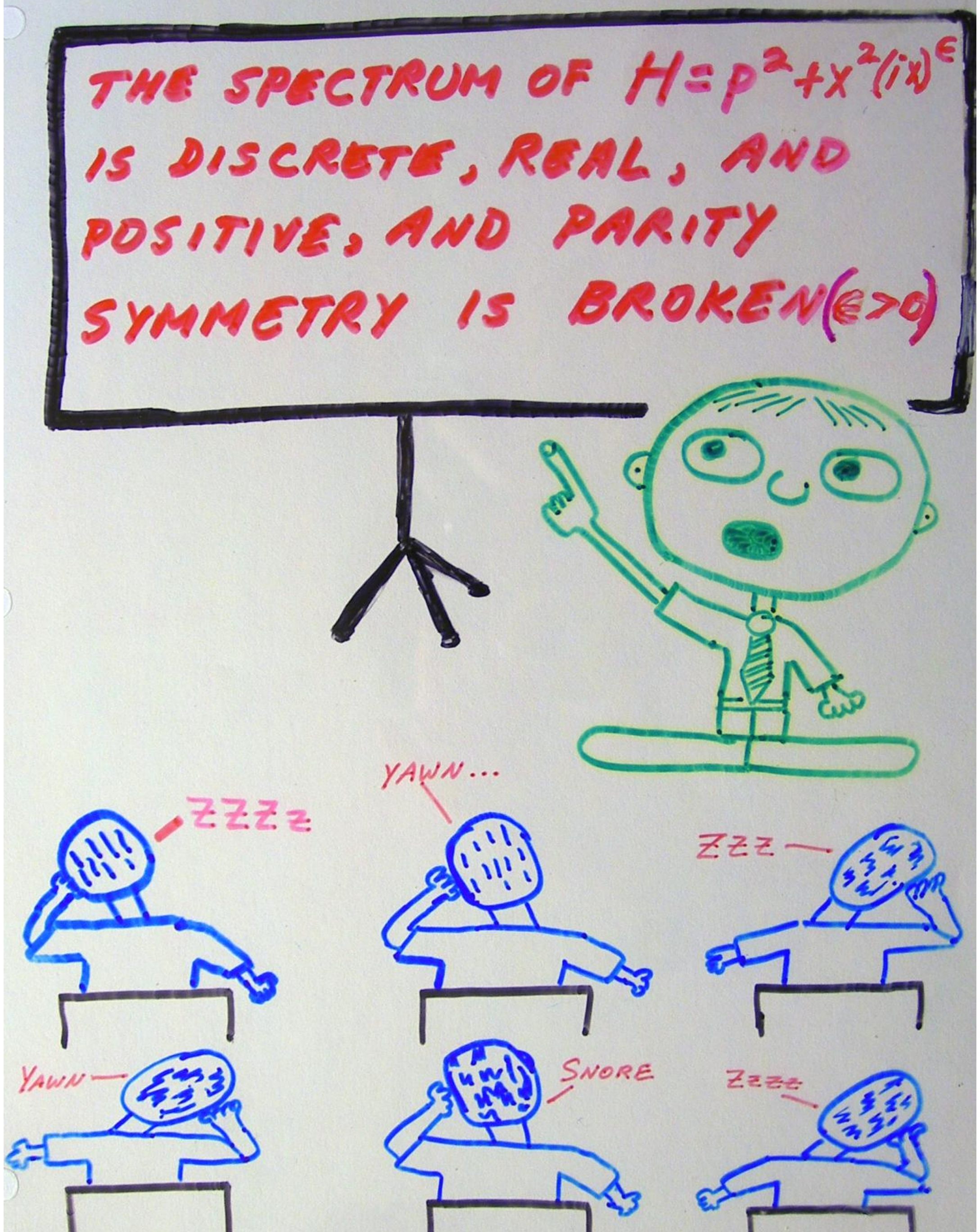
***T*** = time reversal

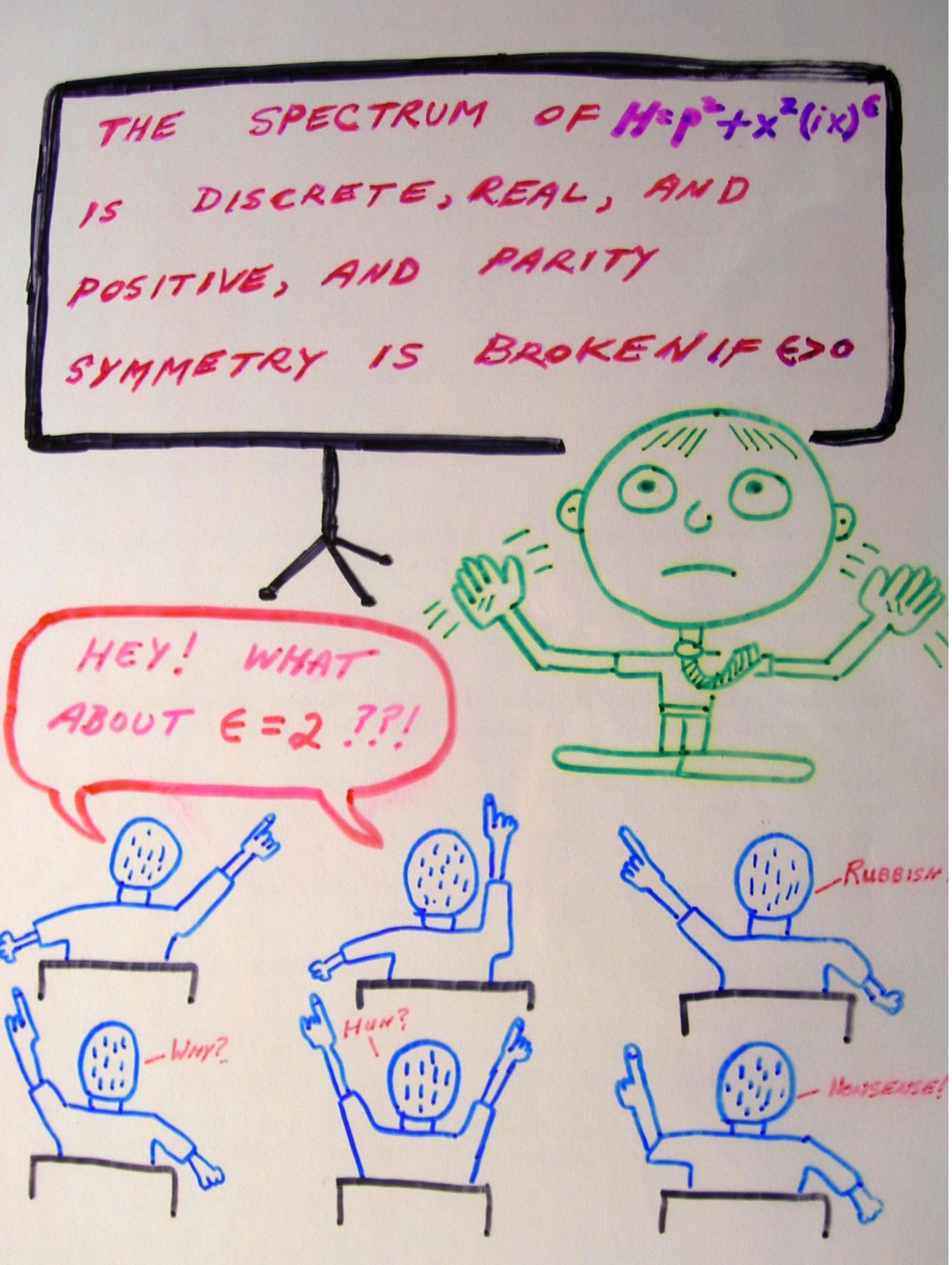
A *class* of ***PT***-symmetric Hamiltonians:

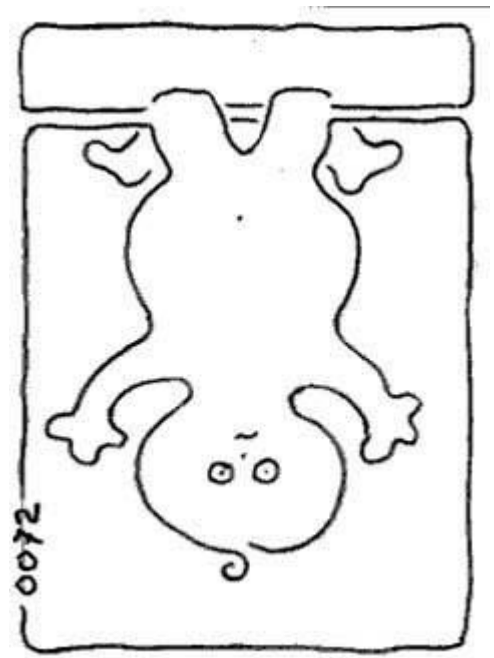
$$\boxed{H = p^2 + x^2(ix)^\varepsilon \qquad (\varepsilon \text{ real})}$$




CMB and S. Boettcher
*Physical Review Letters* **80**, 5243 (1998)


THE SPECTRUM OF $H=p^2+x^2(ix)^\epsilon$ IS DISCRETE, REAL, AND POSITIVE, AND PARITY SYMMETRY IS BROKEN ($\epsilon>0$)
ZZZZz
YAWN...
ZZZ
YAWN
SNORE
Zzzz



Upside-down potential having ***real positive discrete*** eigenvalues!

$$-x^4 \quad (!!)$$

# Some of my work on *PT* symmetry

- CMB and S. Boettcher, *Physical Review Letters* **80**, 5243 (1998)
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# *PT* papers (2008-2010)

- K. Makris, R. El-Ganainy, D. Christodoulides, and Z. Musslimani, *Phyical Review Letters* **100**, 103904 (2008)
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- C. Ruter, K. Makris, R. El-Ganainy, D. Christodoulides, M. Segev, and D. Kip, *Nature Physics* **6**, 192 (2010)
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- CMB and S. Klevansky, *Physical Review Letters* **105**, 031602 (2010)

# *PT* papers (2011-2012)

- Y. Chong, L. Ge, and A. Stone, *Physical Review Letters* **106**, 093902 (2011)
- Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. Christodoulides, *Physical Review Letters* **106**, 213901 (2011)
- P. Mannheim and J. O'Brien, *Physical Review Letters* **106**, 121101 (2011)
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- P. Dorey, C. Dunning, and R. Tateo, *Journal of Physics A* **40**, R205 (2007)
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# **Developments in *PT* Quantum Mechanics**

(Since 'official' beginning in 1998)

- Over fifteen international conferences on ***PT***
- Over 1000 published papers
- Many many ***many*** experimental results in last four years

ONR BAA Announcement Number: ONRBAA12-020

# BROAD AGENCY ANNOUNCEMENT (BAA)

## Fiscal Year (FY) 2013 Department of Defense Multidisciplinary Research Program of the University Research Initiative

## INTRODUCTION:

This publication constitutes a Broad Agency Announcement (BAA) as contemplated in Department of Defense Grant and Agreement Regulation (DODGARS) 22.315(a). A formal Request for Proposals (RFP), solicitation, and/or additional information regarding this announcement will not be issued. Request for the same will be disregarded.

The Office of Naval Research (ONR) will not issue paper copies of this announcement. The ONR and Department of Defense (DoD) agencies involved in this program reserve the right to select for award all, some or none of the proposals submitted in response to this announcement. The ONR and other participating DoD agencies provide no funding for direct reimbursement of proposal development costs. Technical and cost proposals (or any other material) submitted in response to this BAA will not be returned. It is the policy of ONR and the other participating DoD Services to treat all proposals as sensitive competitive information and to disclose their contents only for the purposes of evaluation.

The DoD Multidisciplinary University Research Initiative (MURI), one element of the University Research Initiative (URI), is sponsored by the DoD research offices: the Office of Naval Research (ONR), the Army Research Office (ARO), and the Air Force Office of Scientific Research (AFOSR) (hereafter collectively referred to as "DoD agencies").

Awards will take the form of grants. Therefore, proposals submitted as a result of this announcement will fall under the purview of the Department of Defense Grant and Agreement Regulations (DoDGARs).

## AFOSR FY2013 MURI TOPIC #15

Submit white papers and proposals to Air Force Office of Scientific Research

### Photonic Synthetic Matter

**Background:** The fundamental symmetries of parity and time are now being exploited to enable the spatial guiding and selection of propagating radiation, and could ultimately underpin a new generation of sophisticated, integrated photonic devices. Parity-Time (PT) Symmetric Materials is a class of theoretically conceived materials that does not exist in nature. Much like negative-index materials, it is based on an abstract set of mathematical properties governing electromagnetic wave propagation. It initially emerged within the context of quantum field theory as a novel theoretical construct. Mathematically speaking, a physical system exhibits Parity Time-symmetry provided that a physical trait of the system is invariant under the combined action of spatial and time reversal. In the past few years, the possibility of PT-symmetry was theoretically introduced and experimentally demonstrated (proof of principle) by several groups. One-dimensional PT-symmetric systems have been achieved by fabricating a material-system in which optical loss is judiciously balanced by optical gain via inversion symmetry. Suitably configured PT-symmetric materials will allow unusual control of how waves propagate through the materials. For example, PT concepts can provide new strategies to introduce gain in many optical metamaterials and plasmonics systems that have so far been plagued by losses. Scattering from PT structures can be appropriately engineered to induce an abrupt switch to a new state of behavior which can provide opportunities for designing new laser structures and, alternatively, coherent perfect absorbers or anti-lasers. Polymer processing will allow the fabrication of 1D (waveguides), 2D (Bragg arrays), and 3D (nano- and micro-scatterers and whispering gallery resonators) structures, which would be difficult to achieve with other materials. The flexibility of polymers is a valuable asset that allows, for example, the fabrication of structures that may conform to non-planar geometries or configurations such as the external surface of an aircraft. During this effort, the potential of PT-symmetry will be explored by conducting further theoretical studies of these structures through modeling and simulation and extending the PT-symmetry concepts beyond the optical regime. Unusual wave propagation control will be explored by extending the 1D demonstration to fabrication of complex 2D and 3D structures through advanced polymer processing techniques.

**Objective:** To explore and to achieve 1D, 2D, and 3D PT-Symmetric structures in the optical regime and to extend the PT-Symmetry concept beyond the optical domain.

**Research Concentration Areas:** (1) Theoretical studies involving both modeling and simulation methods to analyze the optical behavior of PT-symmetric systems in higher-dimensions and under vectorial or nonlinear conditions will be pursued. Exploration of the concepts and models beyond the optical regime to other quantum domains of open systems will also be undertaken. (2) Utilization of advanced self-assembly approaches such as engineered specific interactions, nano-domain phase separation control, and advanced multi-component fiber spinning processes to achieve multi-dimensional PT-symmetric systems. (3) Perform experiments to characterize these PT-symmetric systems and to validate theoretical predictions. Also explore how optical isolation can be enhanced with such materials in the context of photonic monolithic integration for next generation photonic monolithic circulates, like RF engineered semiconductor lasers.

**Resource Allocation**: It is anticipated that awards under this topic will be no more than an average of $1.5M per year for 5 years, supporting no more than 6 funded faculty researchers. Exceptions warranted by specific proposal approaches should be discussed with the topic chief during the white paper phase of the solicitation.

**Research Topic Chiefs:** Dr. Arje Nachman, AFOSR, 703-696-8427, arje.nachman@afosr.af.mil; Dr. Charles Lee, AFOSR, 703-696-7779, Charles.lee@afosr.af.mil

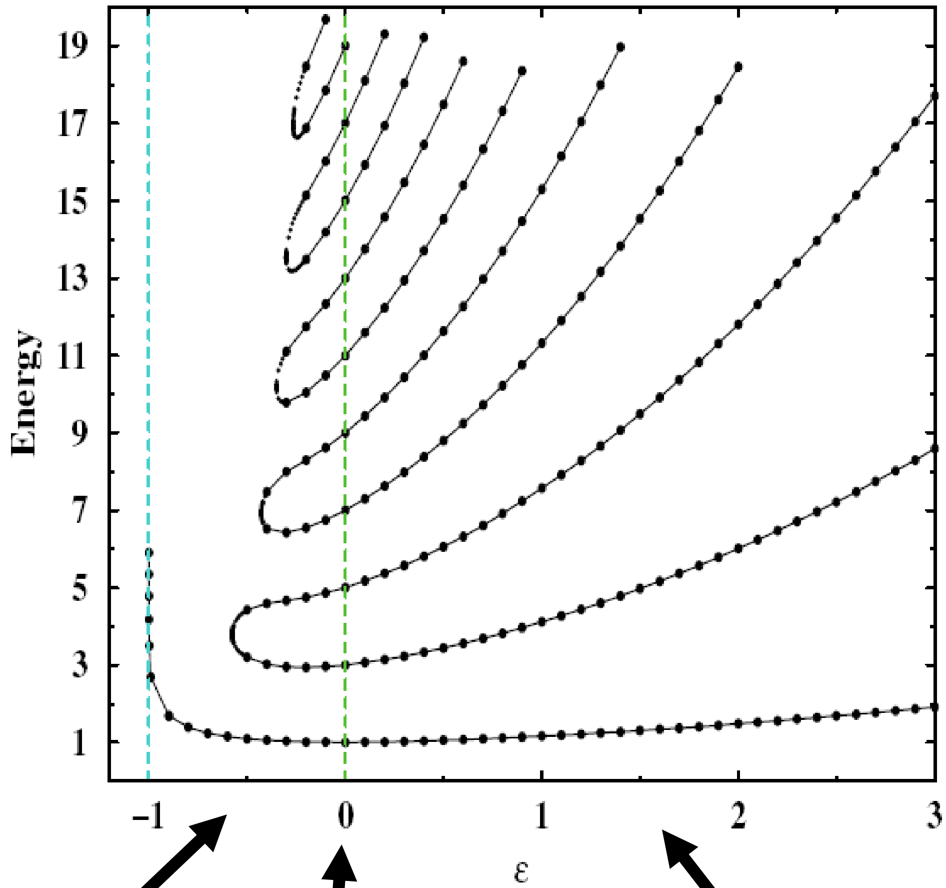
# Rigorous proof of real eigenvalues

Proof is difficult! Uses techniques from conformal field theory and statistical mechanics:

(1) Bethe ansatz
(2) Monodromy group
(3) Baxter T-Q relation
(4) Functional determinants

"***ODE/IM*** Correspondence"
P. Dorey, C. Dunning, and R. Tateo

$$H = p^2 + x^2(ix)^\varepsilon \quad (\varepsilon \text{ real})$$



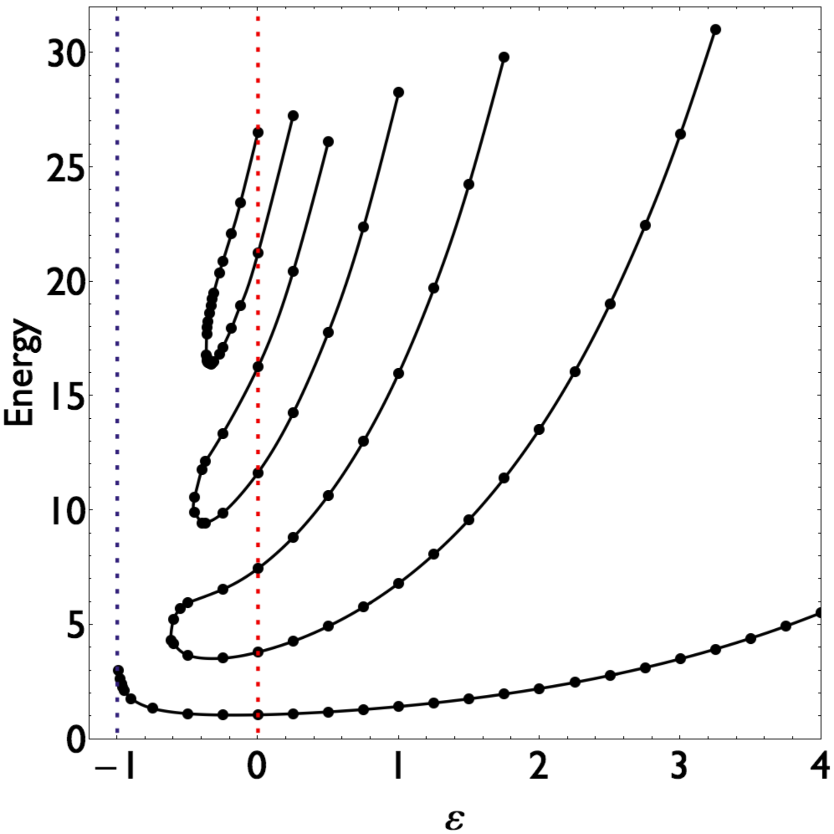

Region of ***broken***
***PT*** symmetry

***PT* Boundary**

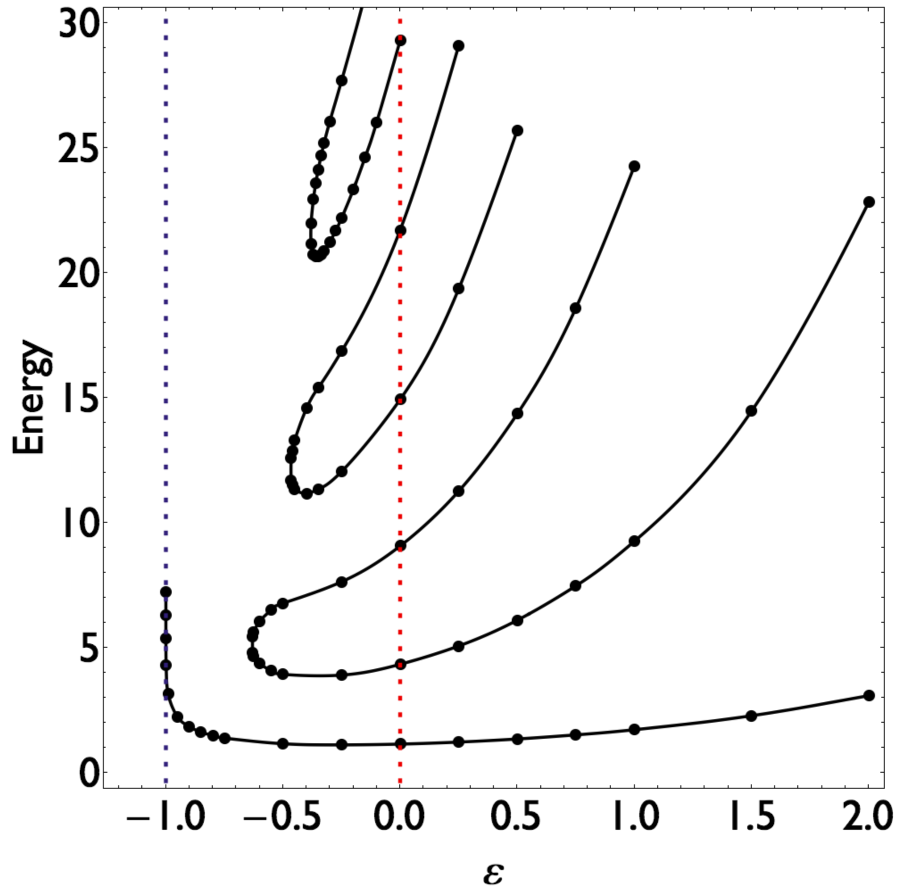
Region of ***unbroken***
***PT*** symmetry

$$H^{(2n)} = p^{2n} + x^2(ix)^\varepsilon \quad (\varepsilon \text{ real}; n = 1, 2, 3, \ldots)$$

*n*=2:




*n*=3:




CMB and D. Hook
Phys. Rev. A **86**, 022113 (2012)


**Broken *P*arro*T*** **Unbroken *P*arro*T***

# Broken *PT* symmetry in Paris

# Hermitian Hamiltonians: BORING!

Eigenvalues are always real – nothing interesting happens

# ***PT*-symmetric Hamiltonians: ASTONISHING!**

Transition between parametric regions of broken and unbroken ***PT*** symmetry...
Can be observed experimentally!

**At a physical level, *PT*-symmetric systems have balanced loss and gain and are thus intermediate between closed and open systems.**

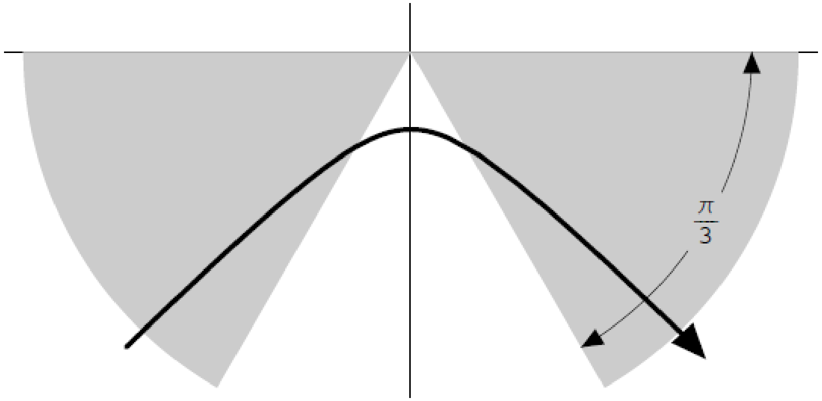
**Hermitian *H***


***PT*-symmetric *H***


**Non-Hermitian *H***

**At a mathematical level, we are extending conventional classical mechanics and Hermitian quantum mechanics into the complex plane…**

$-x^4$ theory defined in Stokes wedges:



# Complex plane


## The eigenvalues are real and positive, but is this quantum mechanics?

- Probabilistic interpretation??
- Hilbert space with a positive metric??
- Unitarity time evolution??

# The Hamiltonian determines its own adjoint!

**Must find the secret symmetry:**

$$[\mathcal{C}, \mathcal{PT}] = 0,$$

$$[\mathcal{C}^2 = 1],$$

$$[\mathcal{C}, H] = 0$$

Replace $\dagger$ by $\mathcal{CPT}$

# Unitarity

With respect to the ***CPT*** adjoint the theory has **UNITARY** time evolution.

Norms are strictly positive!
Probability is conserved!

# Example: 2 x 2 Non-Hermitian matrix *PT*-symmetric Hamiltonian

$$H = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad (r,\ s,\ \theta \text{ real})$$

$\mathcal{T}$ is complex conjugation and $\mathcal{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$E_{\pm} = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta} \qquad \text{real if } s^2 > r^2\sin^2\theta$$

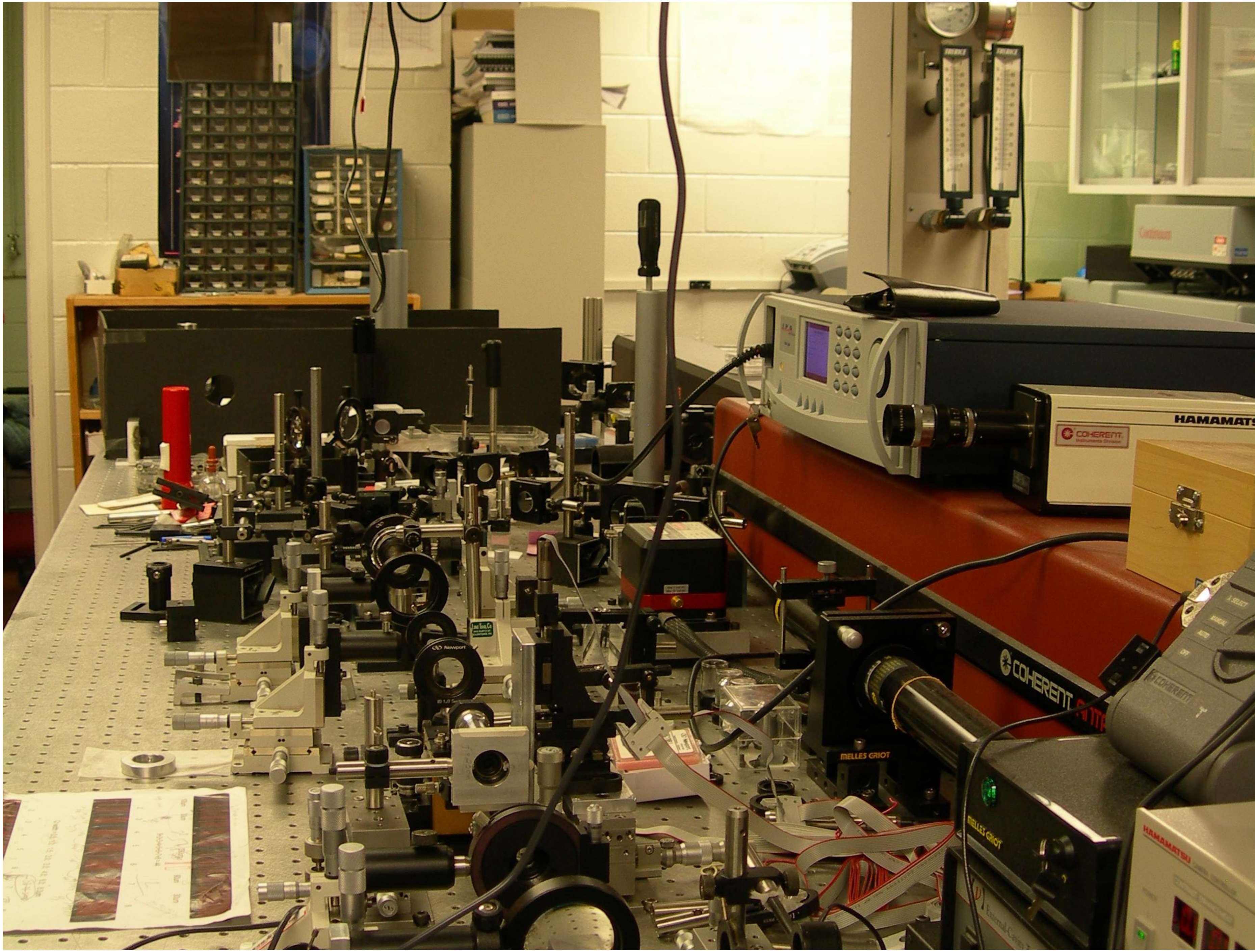
$$\mathcal{C} = \frac{1}{\cos\alpha}\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$

where $\sin\alpha = (r/s)\sin\theta.$

# ***PT*–symmetric systems are being observed experimentally!**

# First laboratory observation of *PT* transition using optical wave guides

A. Guo, G. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. Siviloglou, and D. Christodoulides, *Physical Review Letters* **103**, 093902 (2009)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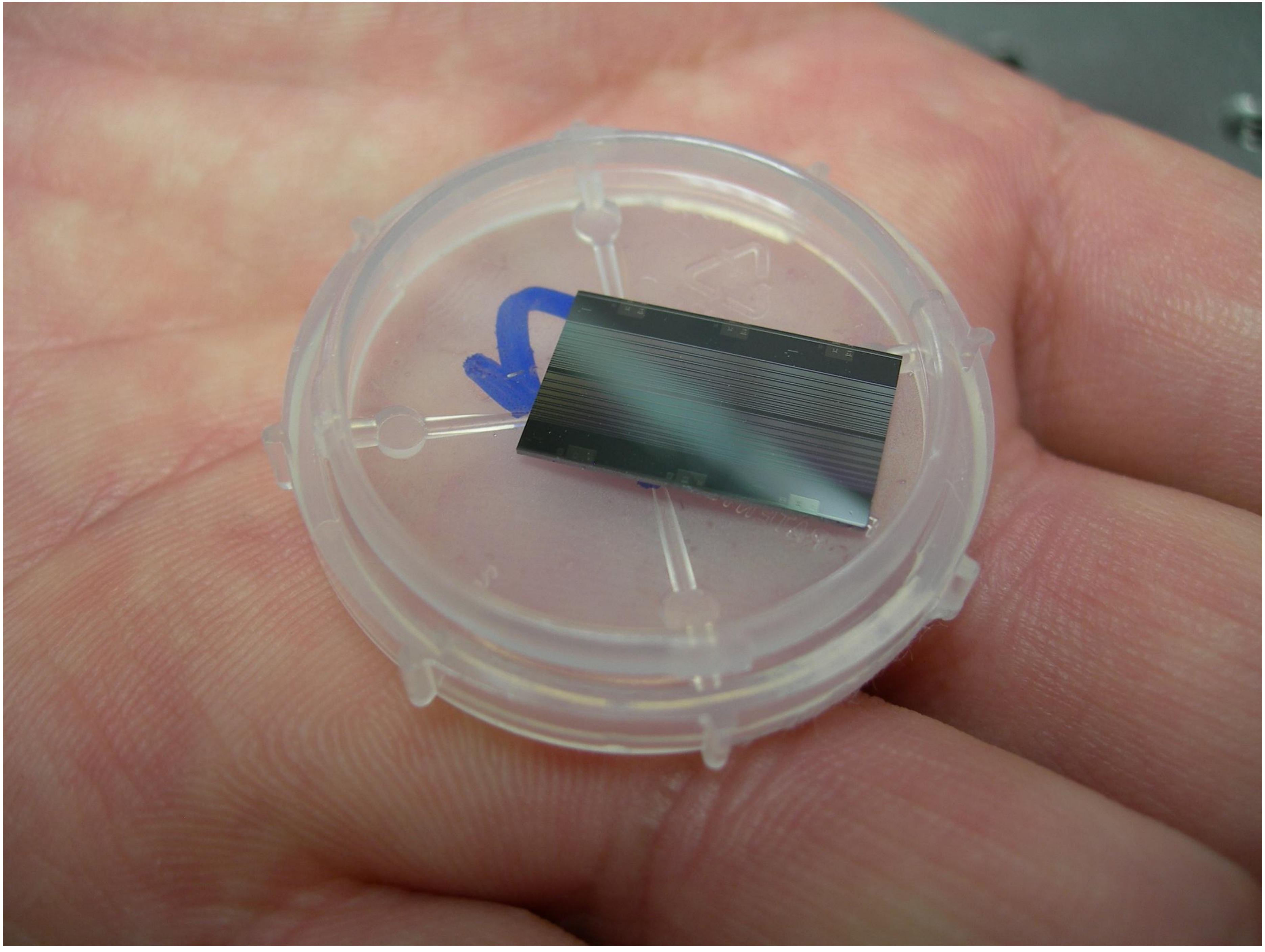
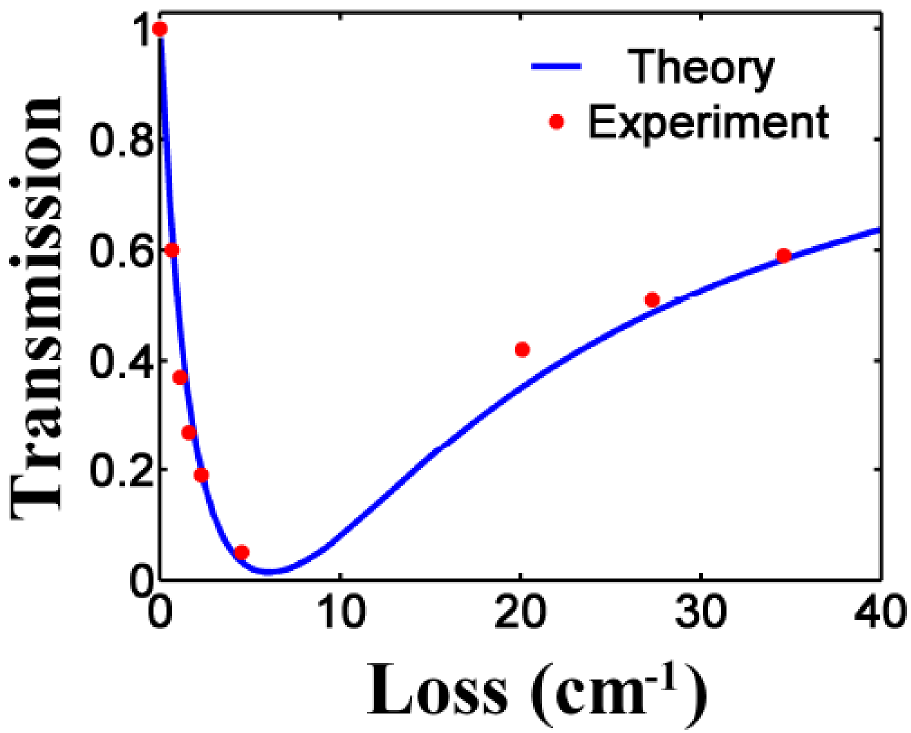


# The observed *PT* transition

**Figure 4:** Experimental observation of spontaneous passive $\mathcal{PT}$-symmetry breaking. Output transmission of a passive $\mathcal{PT}$ complex system as the loss in the lossy waveguide arm is increased. The transmission attains a minimum at 6 cm$^{-1}$.



nature physics

LETTERS

PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515

# Observation of parity–time symmetry in optics

Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]*

**One of the fundamental axioms of quantum mechanics is associated with the Hermiticity of physical observables[1]. In the case of the Hamiltonian operator, this requirement not only implies real eigenenergies but also guarantees probability conservation. Interestingly, a wide class of non-Hermitian Hamiltonians can still show entirely real spectra. Among these are Hamiltonians respecting parity–time (*PT*) symmetry[2–7]. Even though the Hermiticity of quantum observables was never in doubt, such concepts have motivated discussions on several fronts in physics, including quantum field theories[8], non-Hermitian Anderson models[9] and open quantum systems[10,11], to mention a few. Although the impact of *PT* symmetry in these fields is still debated, it has been recently realized that optics can provide a fertile ground where *PT*-related notions can be implemented and experimentally investigated[12–15]. In this letter we report the first observation of the behaviour of a *PT* optical coupled system that judiciously involves a complex index potential. We observe both spontaneous *PT* symmetry breaking and power oscillations violating left–right symmetry. Our results may pave the way towards a new class of *PT*-synthetic materials with intriguing and unexpected properties that rely on non-reciprocal light propagation and tailored transverse energy flow.**

($\varepsilon > \varepsilon_{th}$), the spectrum ceases to be real and starts to involve imaginary eigenvalues. This signifies the onset of a spontaneous *PT* symmetry-breaking, that is, a 'phase transition' from the exact to broken-*PT* phase[7,20].

In optics, several physical processes are known to obey equations that are formally equivalent to that of Schrödinger in quantum mechanics. Spatial diffraction and temporal dispersion are perhaps the most prominent examples. In this work we focus our attention on the spatial domain, for example optical beam propagation in *PT*-symmetric complex potentials. In fact, such *PT* 'optical potentials' can be realized through a judicious inclusion of index guiding and gain/loss regions[7,12–14]. Given that the complex refractive-index distribution $n(x) = n_R(x) + in_I(x)$ plays the role of an optical potential, we can then design a *PT*-symmetric system by satisfying the conditions $n_R(x) = n_R(-x)$ and $n_I(x) = -n_I(-x)$.

In other words, the refractive-index profile must be an even function of position $x$ whereas the gain/loss distribution should be odd. Under these conditions, the electric-field envelope $E$ of the optical beam is governed by the paraxial equation of diffraction[13]:

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\frac{\partial^2 E}{\partial x^2} + k_0[n_R(x) + in_I(x)]E = 0$$

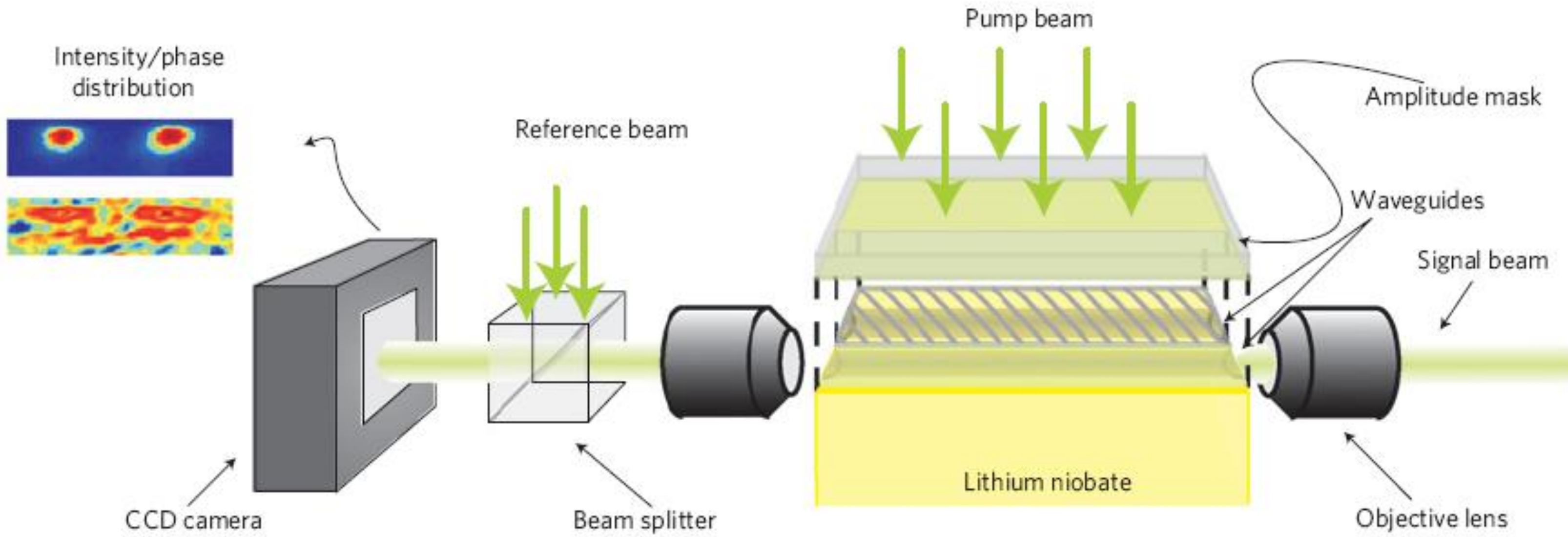



**Figure 2 | Experimental set-up.** An $Ar^+$ laser beam (wavelength 514.5 nm) is coupled into the arms of the structure fabricated on a photorefractive $LiNbO_3$ substrate. An amplitude mask blocks the pump beam from entering channel 2, thus enabling two-wave mixing gain only in channel 1. A CCD camera is used to monitor both the intensity and phases at the output.

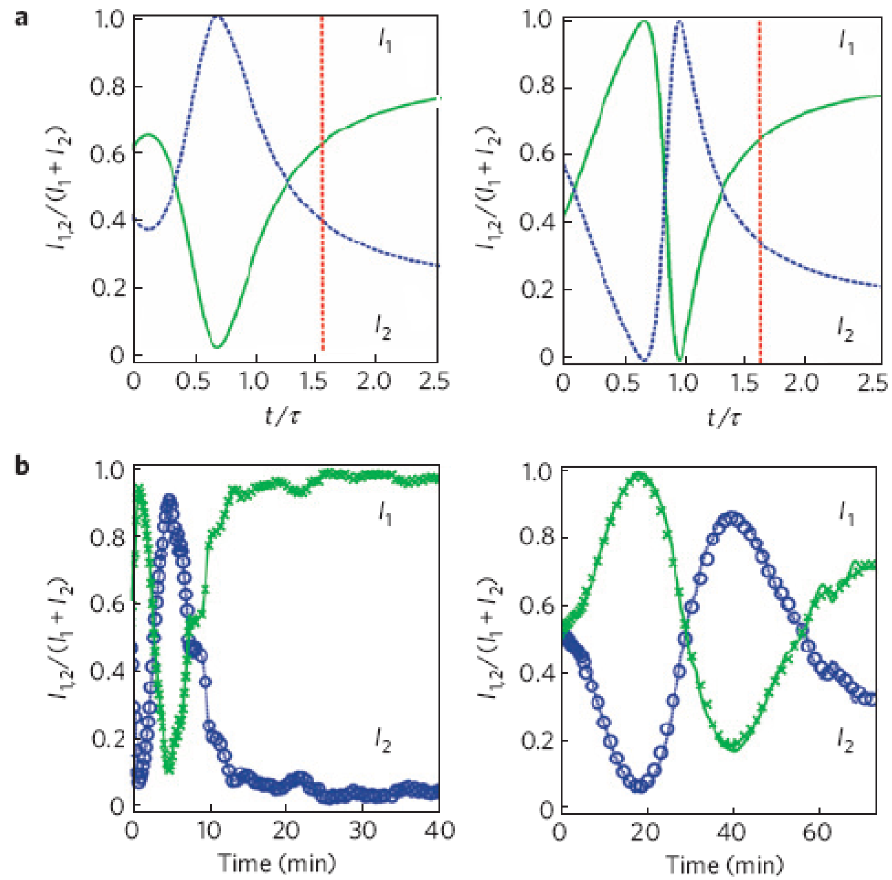



**Figure 3 | Computed and experimentally measured response of a *PT*-symmetric coupled system. a**, Numerical solution of the coupled equations (1) describing the *PT*-symmetric system. The left (right) panel shows the situation when light is coupled into channel 1 (2). Red dashed lines mark the symmetry-breaking threshold. Above threshold, light is predominantly guided in channel 1 experiencing gain, and the intensity of channels 1 and 2 depends solely on the magnitude of the gain. **b**, Experimentally measured (normalized) intensities at the output facet during the gain build-up as a function of time.

# *PT*-symmetric diffusion – Shanghai/Rutgers

PHYSICAL REVIEW A **81**, 042903 (2010)

## Enhanced magnetic resonance signal of spin-polarized Rb atoms near surfaces of coated cells

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[2]*Department of Physics, Rutgers University, Newark, New Jersey 07102, USA*
(Received 12 November 2009; published 21 April 2010)

We present a detailed experimental and theoretical study of edge enhancement in optically pumped Rb vapor in coated cylindrical pyrex glass cells. The Zeeman polarization of Rb atoms is produced and probed in the vicinity ($\sim 10^{-4}$ cm) of the cell surface by evanescent pump and probe beams. Spin-polarized Rb atoms diffuse throughout the cell in the presence of magnetic field gradients. In the present experiment the edge enhanced signal from the back surface of the cell is suppressed compared to that from the front surface, due to the fact that polarization is probed by the evanescent wave at the front surface only. The observed magnetic resonance line shape is reproduced quantitatively by a theoretical model and yields information about the dwell time and relaxation probability of Rb atoms on Pyrex glass surfaces coated with antirelaxation coatings.

DOI: 10.1103/PhysRevA.81.042903 PACS number(s): 34.35.+a, 75.40.Gb, 76.70.Hb, 87.57.nt

# *PT*-symmetric optics -- Caltech

SCIENCE VOL 333 5 AUGUST 2011

## Nonreciprocal Light Propagation in a Silicon Photonic Circuit

Liang Feng,[1,2,4]*† Maurice Ayache,[3]* Jingqing Huang,[1,4]* Ye-Long Xu,[2] Ming-Hui Lu,[2] Yan-Feng Chen,[2]† Yeshaiahu Fainman,[3] Axel Scherer[1,4]†

Optical communications and computing require on-chip nonreciprocal light propagation to isolate and stabilize different chip-scale optical components. We have designed and fabricated a metallic-silicon waveguide system in which the optical potential is modulated along the length of the waveguide such that nonreciprocal light propagation is obtained on a silicon photonic chip. Nonreciprocal light transport and one-way photonic mode conversion are demonstrated at the wavelength of 1.55 micrometers in both simulations and experiments. Our system is compatible with conventional complementary metal-oxide-semiconductor processing, providing a way to chip-scale optical isolators for optical communications and computing.

[1]Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125, USA. [2]Nanjing National Laboratory of Microstructures, Nanjing University, Nanjing, Jiangsu 210093, China. [3]Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, USA. [4]Kavli Nanoscience Institute, California Institute of Technology, Pasadena, CA 91125, USA.

# *PT*-symmetric superconducting wires -- Indiana

PRL **99,** 167003 (2007) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2007

## Bifurcation Diagram and Pattern Formation of Phase Slip Centers in Superconducting Wires Driven with Electric Currents

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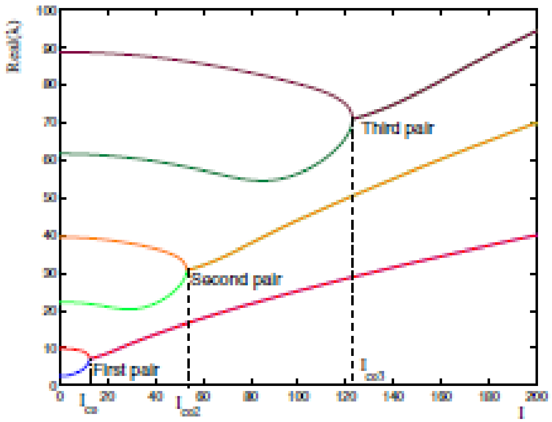
(Received 14 February 2007; published 18 October 2007)

We provide here new insights into the classical problem of a one-dimensional superconducting wire exposed to an applied electric current using the time-dependent Ginzburg-Landau model. The most striking feature of this system is the well-known appearance of oscillatory solutions exhibiting phase slip centers (PSC's) where the order parameter vanishes. Retaining temperature and applied current as parameters, we present a simple yet definitive explanation of the mechanism within this nonlinear model that leads to the PSC phenomenon and we establish where in parameter space these oscillatory solutions can be found. One of the most interesting features of the analysis is the evident collision of real eigenvalues of the associated *PT*-symmetric linearization, leading as it does to the emergence of complex elements of the spectrum.



# *PT*-symmetric microwave cavities -- Germany

PRL **108,** 024101 (2012) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2012

## $\mathcal{PT}$ Symmetry and Spontaneous Symmetry Breaking in a Microwave Billiard

S. Bittner,[1] B. Dietz,[1,*] U. Günther,[2] H. L. Harney,[3] M. Miski-Oglu,[1] A. Richter,[1,4,†] and F. Schäfer[1,5]

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(Received 21 July 2011; published 10 January 2012)

We demonstrate the presence of parity-time ($\mathcal{PT}$) symmetry for the non-Hermitian two-state Hamiltonian of a dissipative microwave billiard in the vicinity of an exceptional point (EP). The shape of the billiard depends on two parameters. The Hamiltonian is determined from the measured resonance spectrum on a fine grid in the parameter plane. After applying a purely imaginary diagonal shift to the Hamiltonian, its eigenvalues are either real or complex conjugate on a curve, which passes through the EP. An appropriate basis choice reveals its $\mathcal{PT}$ symmetry. Spontaneous symmetry breaking occurs at the EP.

DOI: 10.1103/PhysRevLett.108.024101 PACS numbers: 05.45.Mt, 02.10.Yn, 11.30.Er

# *PT*-symmetric cavity lasers -- Yale

PRL **106**, 093902 (2011) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2011

## $\mathcal{PT}$-Symmetry Breaking and Laser-Absorber Modes in Optical Scattering Systems

Y. D. Chong,* Li Ge,† and A. Douglas Stone

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(Received 30 August 2010; revised manuscript received 27 January 2011; published 2 March 2011)

Using a scattering matrix formalism, we derive the general scattering properties of optical structures that are symmetric under a combination of parity and time reversal ($\mathcal{PT}$). We demonstrate the existence of a transition between $\mathcal{PT}$-symmetric scattering eigenstates, which are norm preserving, and symmetry-broken pairs of eigenstates exhibiting net amplification and loss. The system proposed by Longhi [Phys. Rev. A **82**, 031801 (2010).], which can act simultaneously as a laser and coherent perfect absorber, occurs at discrete points in the broken-symmetry phase, when a pole and zero of the $S$ matrix coincide.

DOI: 10.1103/PhysRevLett.106.093902 PACS numbers: 42.25.Bs, 42.25.Hz, 42.55.Ah

# *PT*-symmetric superconducting wires -- Argonne

PRL 109, 150405 (2012) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2012

## Stimulation of the Fluctuation Superconductivity by $\mathcal{PT}$ Symmetry

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(Received 6 May 2012; published 9 October 2012)

We discuss fluctuations near the second-order phase transition where the free energy has an additional non-Hermitian term. The spectrum of the fluctuations changes when the odd-parity potential amplitude exceeds the critical value corresponding to the $\mathcal{PT}$-symmetry breakdown in the topological structure of the Hilbert space of the effective non-Hermitian Hamiltonian. We calculate the fluctuation contribution to the differential resistance of a superconducting weak link and find the manifestation of the $\mathcal{PT}$-symmetry breaking in its temperature evolution. We successfully validate our theory by carrying out measurements of far from equilibrium transport in mesoscale-patterned superconducting wires.

DOI: 10.1103/PhysRevLett.109.150405 PACS numbers: 11.30.Er, 03.65.Ge, 73.63.−b

# *PT*-symmetric photonic graphene -- Technion

**RAPID COMMUNICATIONS**

PHYSICAL REVIEW A **84**, 021806(R) (2011)

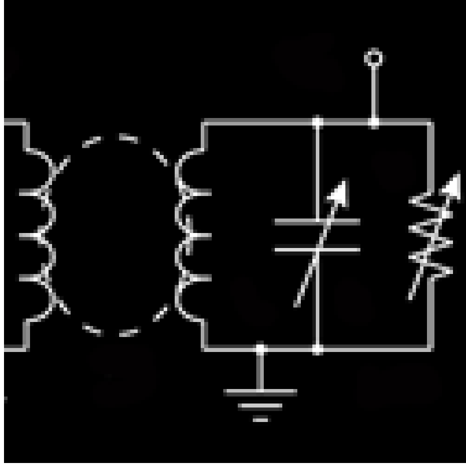
## $\mathcal{PT}$-symmetry in honeycomb photonic lattices

Alexander Szameit, Mikael C. Rechtsman, Omri Bahat-Treidel, and Mordechai Segev
*Physics Department and Solid State Institute, Technion, 32000 Haifa, Israel*
(Received 21 April 2011; published 19 August 2011)

We apply gain and loss to honeycomb photonic lattices and show that the dispersion relation is identical to tachyons—particles with imaginary mass that travel faster than the speed of light. This is accompanied by $\mathcal{PT}$-symmetry breaking in this structure. We further show that the $\mathcal{PT}$-symmetry can be restored by deforming the lattice.

DOI: 10.1103/PhysRevA.84.021806 PACS number(s): 42.25.−p, 42.82.Et

# *PT*-symmetric LRC circuits -- Wesleyan



*APS: Spotlighting exceptional research*

J. Schindler *et al.*, Phys. Rev. A (2011)

**Experimental study of active *LRC* circuits with *PT* symmetries**

Joseph Schindler, Ang Li, Mei C. Zheng, F. M. Ellis, and Tsampikos Kottos

Phys. Rev. A **84**, 040101 (2011)

Published October 13, 2011

Everyone learns in a first course on quantum mechanics that the result of a measurement cannot be a complex number, so the quantum mechanical operator that corresponds to a measurement must be Hermitian. However, certain classes of complex Hamiltonians that are not Hermitian can still have real eigenvalues. The key property of these Hamiltonians is that they are parity-time (*PT*) symmetric, that is, they are invariant under a mirror reflection and complex conjugation (which is equivalent to time reversal).

Hamiltonians that have *PT* symmetry have been used to describe the depinning of vortex flux lines in type-II superconductors and optical effects that involve a complex index of refraction, but there has never been a simple physical system where the effects of *PT* symmetry can be clearly understood and explored. Now, Joseph Schindler and colleagues at Wesleyan University in Connecticut have devised a simple *LRC* electrical circuit that displays directly the effects of *PT* symmetry. The key components are a pair of coupled resonant circuits, one with active gain and the other with an equivalent amount of loss. Schindler *et al.* explore the eigenfrequencies of this system as a function of the "gain/loss" parameter that controls the degree of amplification and attenuation of the system. For a critical value of this parameter, the eigenfrequencies undergo a spontaneous phase transition from real to complex values, while the eigenstates coalesce and acquire a definite chirality (handedness). This simple electronic analog to a quantum Hamiltonian could be a useful reference point for studying more complex applications.

– *Gordon W. F. Drake*

# *PT*-symmetric mechanical system

CMB, B. Berntson, D. Parker, E. Samuel, *American Journal of Physics* (in press) [arXiv: math-ph/1206.4972]

Why science teachers should not be given playground duty.

*PT* quantum mechanics is fun! You can re-visit things you already know about traditional Hermitian quantum theory

# Three examples:

*1*. "Ghost Busting: ***PT***-Symmetric Interpretation of the Lee Model"
CMB, S. Brandt, J.-H. Chen, and Q. Wang
*Phys. Rev. D* **71**, 025014 (2005) [arXiv: hep-th/0411064]

*2*. "No-ghost Theorem for the Fourth-Order Derivative
Pais-Uhlenbeck Oscillator Model"
CMB and P. Mannheim
*Phys. Rev. Lett.* **100**, 110402 (2008) [arXiv: hep-th/0706.0207]

*3*. "Resolution of Ambiguity in the Double-Scaling Limit"
CMB, M. Moshe, and S. Sarkar
*Journal of Physics A* **46**, 102002 (2013) [*IOP select*]

# Double scaling (correlated) limits

Correlated limits arise frequently in physical problems when there are two parameters, say $\varepsilon$ and $\alpha$, and $\varepsilon$ is treated as small ($\varepsilon \ll 1$) so that it plays the role of a perturbation parameter. If we treat $\alpha$ as fixed, the solution $\mathcal{S}(\varepsilon, \alpha)$ to the problem is a formal perturbation series in powers of $\varepsilon$: $\mathcal{S}(\varepsilon, \alpha) \sim \sum_{n=0}^{\infty} a_n(\alpha)\varepsilon^n$.

A correlated limit occurs when we do *not* treat $\alpha$ as fixed, but rather allow it to tend to a critical value $\alpha \to \alpha_{\text{crit}}$ as $\varepsilon \to 0$

# Correlated limits:

A nontrivial correlated limit arises if we choose the functional dependence so that *all* terms in the perturbation series become comparable as $\varepsilon \to 0$. When this happens, the series undergoes a transmutation in which it depends on just *one parameter*, which we call $\gamma$. In this correlated limit the perturbation series still diverges, but we sum the series for $\mathcal{S}(\gamma)$ by using Borel summation. Correlated limits are remarkable in that $\mathcal{S}(\gamma)$ is a *universal* function that reveals the essential features of the problem while being insensitive to specific details. Often, $\mathcal{S}(\gamma)$ is *entire* (analytic for all $\gamma$).

# *Example*: Nonuniformly convergent Fourier sine series near the edge of the interval of convergence

$$f(x) = \sum_{1}^{\infty} a_n \sin(nx)$$

$$\sum_{1}^{N} a_n \sin(nx)$$

$$N \to \infty, \qquad x \to 0, \qquad \gamma \equiv Nx$$

This limit is described by the Gibbs function $G(\gamma) = \mathrm{Si}(2\gamma)$

# *Example*: Transition in a QM wave function between a classically allowed and a classically forbidden region (one-turning-point problem) as a correlated limit

$$\hbar^2 \phi''(x) = Q(x)\phi(x) \qquad\qquad Q(x) \sim ax$$

$$\phi_{\mathrm{WKB}}(x) = \exp\left[\frac{1}{\hbar}\int_0^x ds \sum_{n=0}^{\infty} \hbar^n S_n(s)\right] \quad (\hbar \to 0)$$

$$\hbar \to 0 \qquad x \to 0 \qquad \gamma = a^{1/2}x^{3/2}/\hbar$$

$$\boxed{\phi(\gamma) = c\mathrm{Ai}(\gamma)}$$

The solution to the famous one-turning-point problem is a correlated limit. The Airy function is an *entire* function of $\gamma$ and it is universal because it is valid for all potentials $Q(x)$ that vanish linearly at the turning point.

# *Example*: Laplace's method for the asymptotic expansion of an integral

Large-$N$ behavior of the Laplace integral

$$Z(N) = \int_0^\infty dr\, e^{-NS(r)}$$

Assume that $S'(r) > 0$ for all $r \geq 0$ and use repeated integration by parts to obtain the complete asymptotic expansion of $Z(N)$ as $N \to \infty$:

$$Z(N) \sim e^{-NS(0)} \sum_{k=1}^{\infty} N^{-k} \left[\frac{1}{S'(r)} \frac{d}{dr}\right]^{k-1} \frac{1}{S'(r)}\bigg|_{r=0}$$

This is an uncorrelated large-$N$ expansion.

Suppose that $S'(0)$ is small but that higher derivatives of $S(r)$ are not small at $r = 0$

As $S'(0) \to 0$, $k$th term in series is approximated by

$$N^{-k}[-2S''(0)]^{k-1}[S'(0)]^{1-2k}\Gamma(k-1/2)/\Gamma(1/2)$$

because this has the greatest number of powers of $S'(0)$ in the denominator.

Correlated limit:

$N \to \infty$, $S'(0) \to 0$, $\gamma^2 \equiv N[S'(0)]^2/S''(0)$ is fixed

Assume that $S''(0) > 0$ so that $\gamma^2 > 0$

$$Z(\gamma) \sim \frac{e^{-NS(0)}}{\sqrt{NS''(0)}} \sum_{k=0}^{\infty} (-2)^k \gamma^{-2k-1} \frac{\Gamma(k+1/2)}{\Gamma(1/2)}$$

Series diverges but its Borel sum is a parabolic cylinder function

$$\boxed{Z(\gamma) \sim e^{-NS(0)} \exp\left(\gamma^2/4\right) \mathrm{D}_{-1}(\gamma)/\sqrt{NS''(0)}}$$

$Z(\gamma)$ is entire. It is universal because it depends only on the two numbers $S(0)$ and $S''(0)$, and thus it applies *universally* to all functions $S(r)$ with these particular values.

[The uncorrelated series depends on *all* derivatives of $S(r)$ at $r = 0$.]

For the special value $\gamma = 0$, $\mathrm{D}_{-1}(0) = \sqrt{\pi/2}$ gives the famous result known as Laplace's method

$$\boxed{Z(N) \sim e^{-NS(0)} \sqrt{\pi/[2NS''(0)]} \quad (N \to \infty)}$$

Laplace's method is a limiting case of the correlated limit for which $S'(0) = 0$ and $S''(0) > 0$. The correlated limit describes the approach of $Z(\gamma)$ to Laplace's asymptotic formula.

The correlated limit describes in a smooth and *universal* fashion what happens as the derivative of $S(r)$ approaches 0 at the Laplace point, just as the Gibbs function describes in a smooth and universal fashion how a nonuniformly convergent Fourier series for $f(x)$ behaves as $x$ approaches the boundary of the interval.

# Uncorrelated large-$N$ expansion for an O($N$) QFT in 0 dimensions

Partition function

$$Z = \int d^{N+1}x \exp\left[-\frac{1}{2}\sum_{n=1}^{N+1} x_n^2 - \frac{\lambda}{4}\left(\sum_{n=1}^{N+1} x_n^2\right)^2\right]$$

Rotational symmetry:

$$\sum_{n=1}^{N+1} x_n^2 = Nr^2$$

$$\lambda = g/N$$

$$Z = \mathcal{A}_{N+1}\int_0^\infty dr\, e^{-NL(r)} \qquad L(r) = r^2/2 + gr^4/4 - \log r$$

We must assume that $g$ is positive so that the integral representation for $Z$ converges!

Laplace's method: Locate the Laplace points – zeros of

$$L'(s) = r + gr^3 - 1/r$$

One Laplace point lies in the range of integration $0 \leq r < \infty$:

$$r_0 = \sqrt{(G-1)/(2g)}$$

$$G \equiv \sqrt{1+4g}$$

$$Z \sim \frac{\mathcal{A}_{N+1} e^{-NL(r_0)}}{\sqrt{NG/\pi}} \sum_{k=0}^{\infty} a_k N^{-k} \quad (N \to \infty)$$

$$a_0 = 1$$

$$a_1 = \frac{5 - 6G^2 - G^3}{24G^3}$$

$$a_2 = \frac{385 - 924G^2 - 10G^3 + 684G^4 + 12G^5 - 143G^6}{1152G^6}$$

This is the full asymptotic expansion of $Z$ and it is the *uncorrelated* large-$N$ expansion of the partition function.

## Correlated limit of the large-$N$ expansion

For all terms in the expansion to have the same order of magnitude, the correlated limit must be

$$N \to \infty$$

and

$$g \to g_{\rm crit} = -1/4$$

(that is, $G \to 0$)

with

$$\gamma \equiv NG^3/2$$

$$Z \sim \frac{\mathcal{A}_{N+1} e^{-NL(r_0)}}{\sqrt{NG/\pi}} \left( 1 + \frac{5}{48\gamma} + \frac{385}{4608\gamma^2} + \ldots \right)$$

This correlated limit is invalid because it requires that $g < 0$. Furthermore, the series is a *nonalternating* divergent series; such a series is not Borel summable.

## $\mathcal{PT}$-symmetric reformulation of the theory

New O$(N+1)$-symmetric partition function

$$Z = \mathrm{Re} \int d^{N+1}x \, e^{-L}$$

Take $N$ to be an *even* integer.

The Lagrangian $L$ is

$$L = \frac{1}{2}\sum_{j=1}^{N+1} x_j^2 + \frac{\lambda i^{\varepsilon}}{2+\varepsilon}\Big(\sum_{j=1}^{N+1} x_j^2\Big)^{1+\varepsilon/2}$$

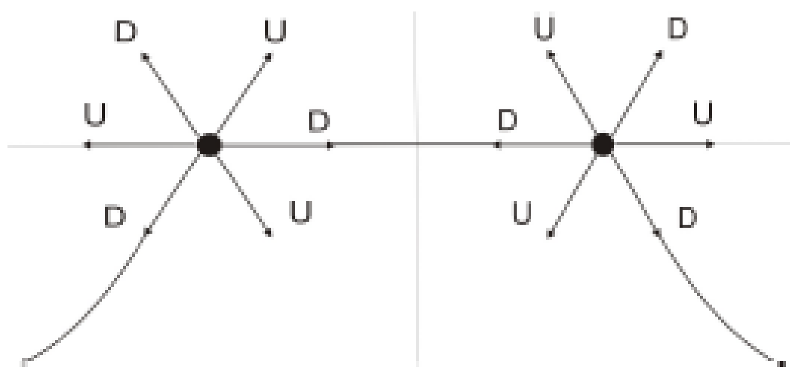
The integral is taken on the real axis and it converges if $\varepsilon < 1$.

We let $\lambda = gN^{-\varepsilon/2}$ and again introduce the radial variable $r$ by $\sum_{n=1}^{N+1} x_n^2 = Nr^2$. The crucial assumption that $N$ is *even* allows us to extend the radial integral to the entire real-$r$ axis:

$$Z = \frac{1}{2}\mathcal{A}_{N+1}\int_{-\infty}^{\infty} dr \, e^{-N\mathcal{L}(r)}$$

$$\mathcal{L} = r^2/2 + gr^2(ir)^{\varepsilon}/(2+\varepsilon) - \log r$$

Boundary conditions on integral: Path of integration lies in a pair of $\mathcal{PT}$-symmetric Stokes wedges centered about $-\pi\varepsilon/(4+2\varepsilon)$ and $-(4\pi+\pi\varepsilon)/(4+2\varepsilon)$. The wedges have angular opening $\pi/(2+\varepsilon)$ and contain the real-$r$ axis if $\varepsilon<1$. As $\varepsilon$ increases above 1, the wedges rotate downward into the complex plane and become narrower. At $\varepsilon=2$ the wedges are centered about $-\pi/4$ and $-3\pi/4$ and have angular opening $\pi/4$.



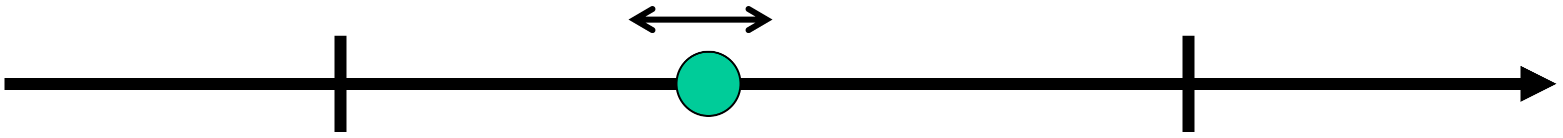

$$\boxed{Z \sim \mathcal{A}_{N+1} e^{NL(\sqrt{2})} 2^{-1/6} \pi N^{-1/3} \mathrm{Bi}(\gamma^{2/3}) e^{-2\gamma/3}}$$

# THE END!

# Intuitive explanation of *PT* transition …

# Classical harmonic oscillator

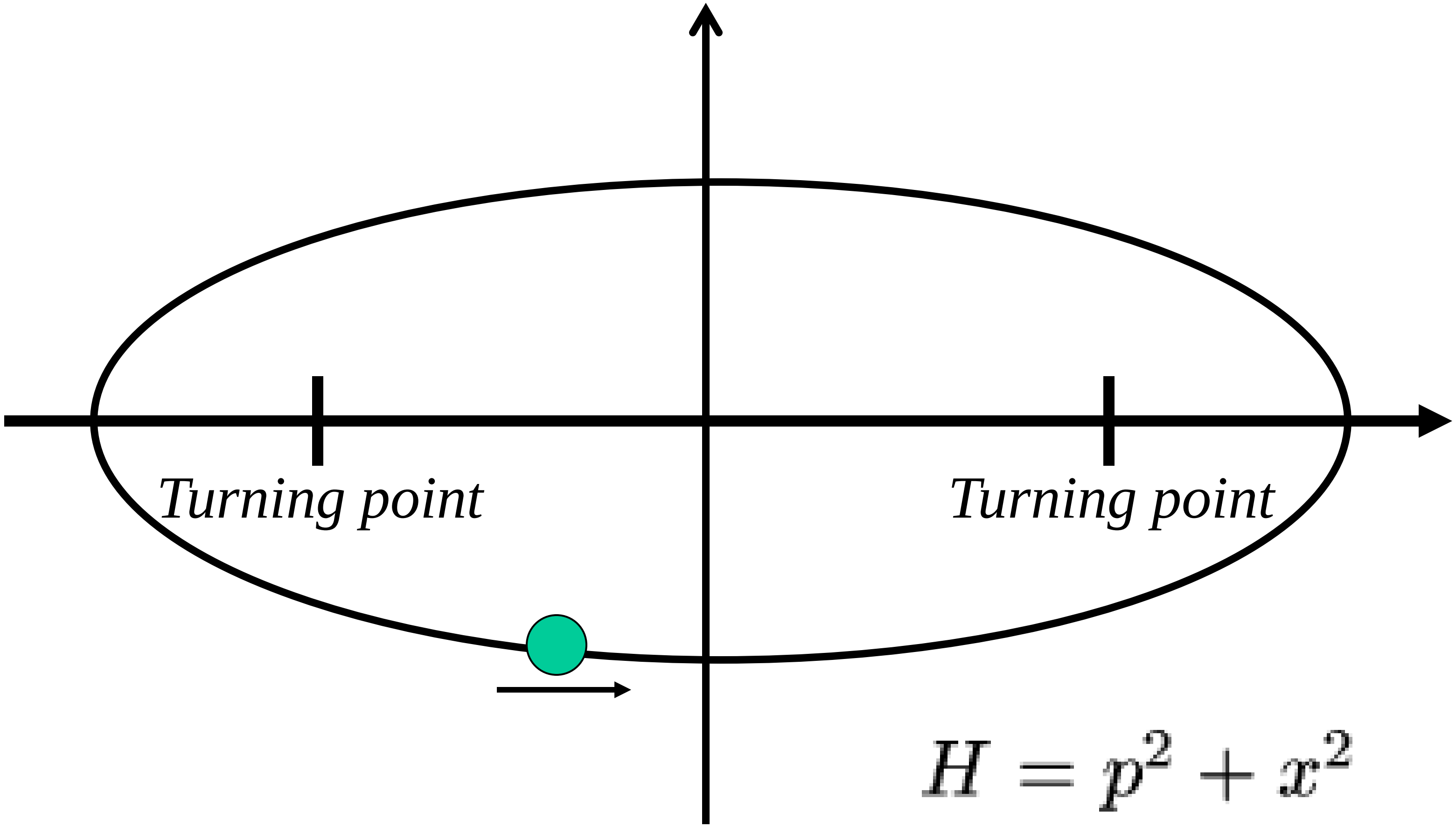
Back and forth motion on the real axis:



*Turning point* *Turning point*

$$H = p^2 + x^2 \qquad (\epsilon = 0)$$

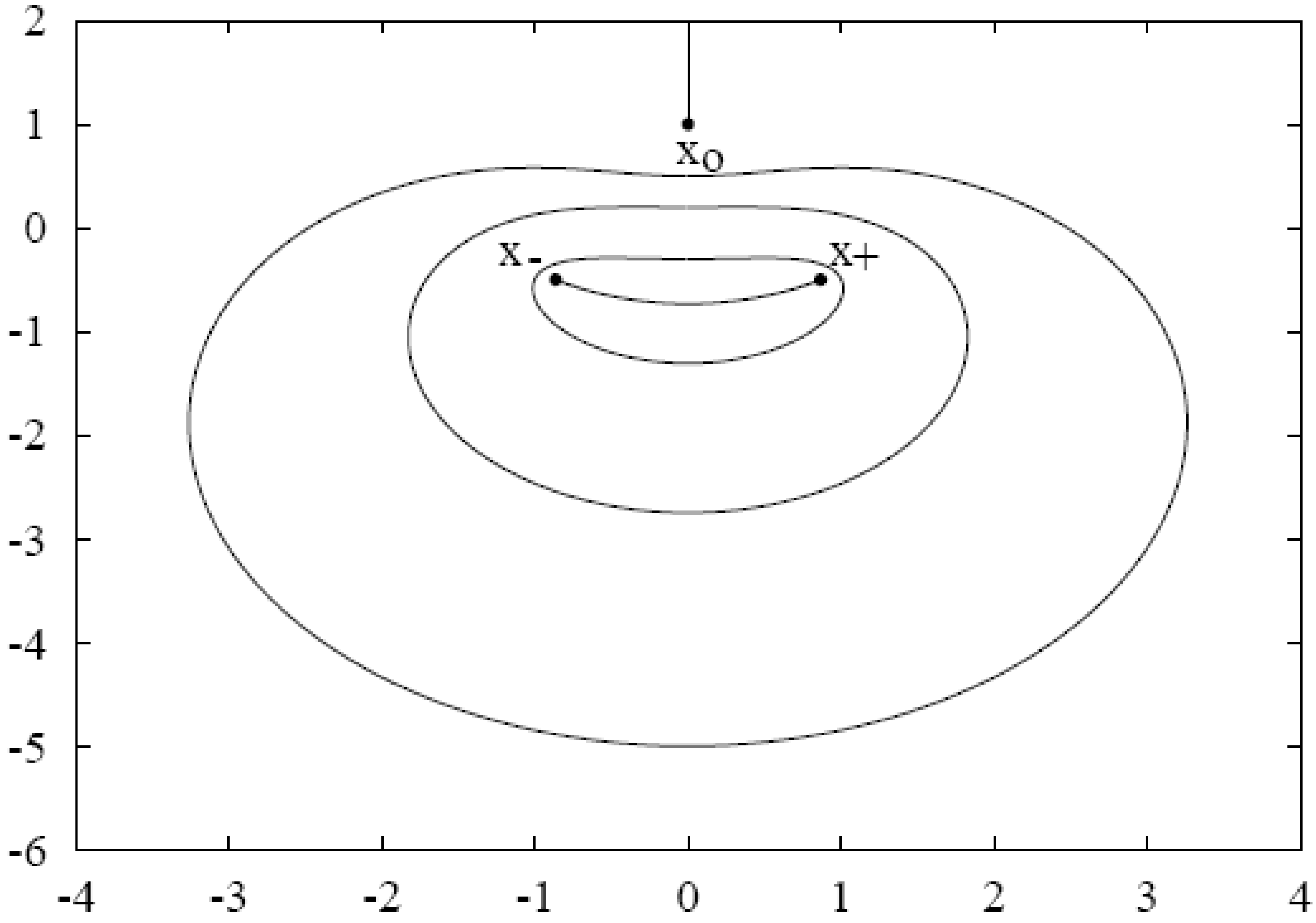
# Harmonic oscillator in complex plane



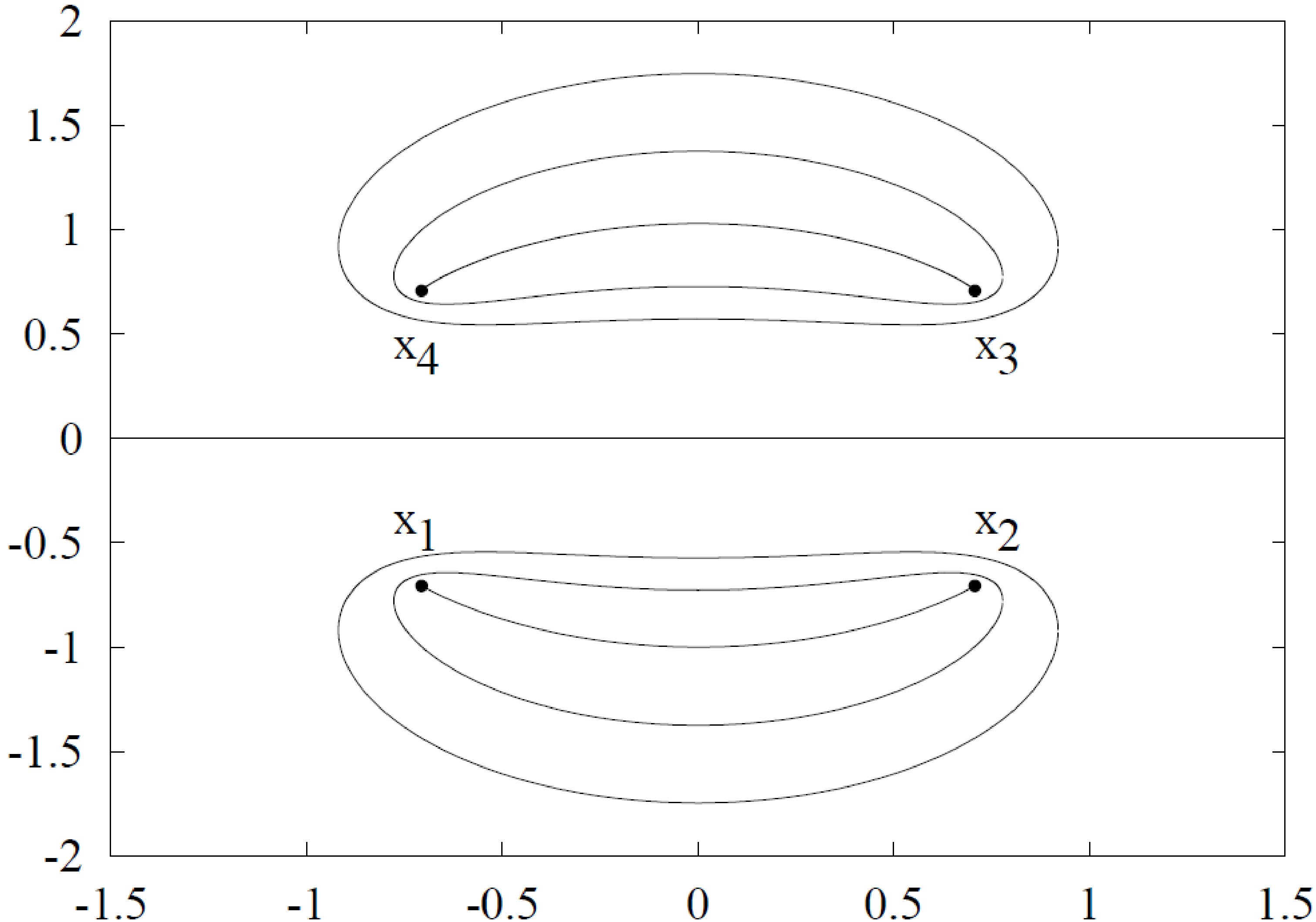

$$H = p^2 + x^2$$

$(\epsilon = 0)$

$$H = p^2 + ix^3 \qquad (\varepsilon = 1)$$



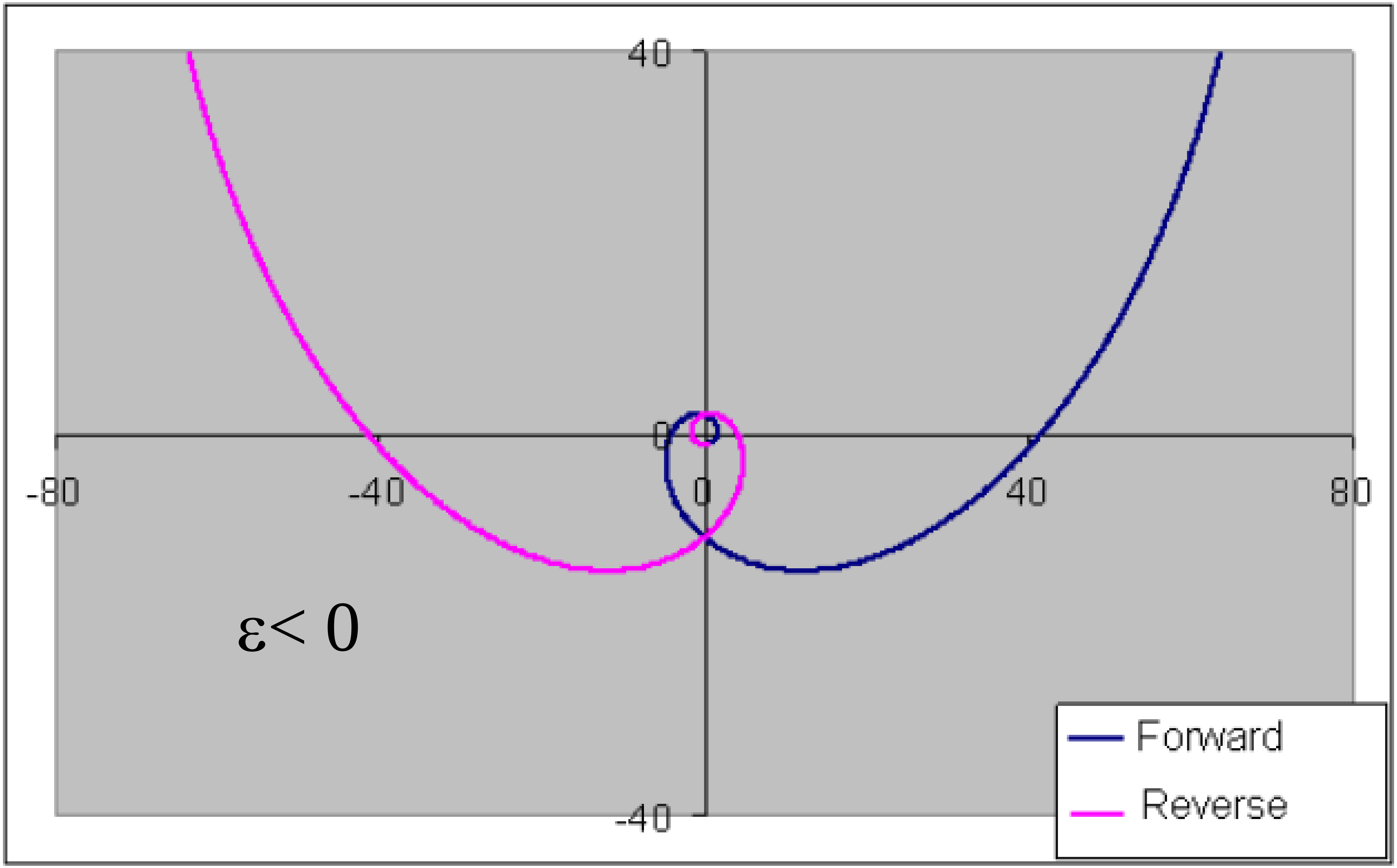
$$H = p^2 - x^4 \qquad (\varepsilon = 2)$$



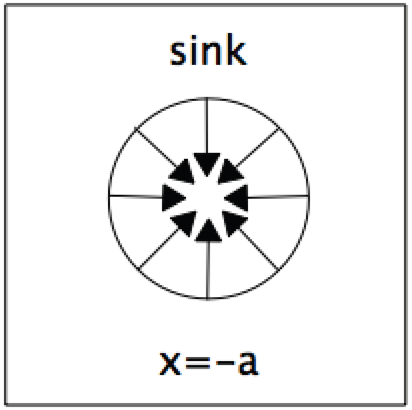
# Bohr-Sommerfeld Quantization of a complex atom

$$\oint dx\, p = \left(n + \frac{1}{2}\right)\pi$$

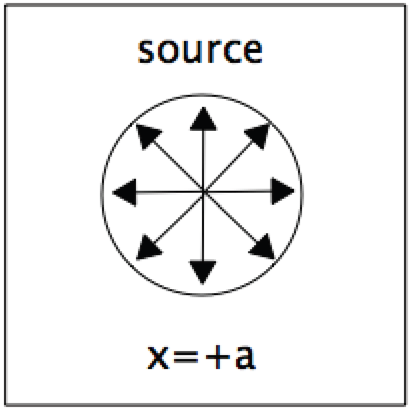
# Broken *PT* symmetry – orbit not closed



## Box 1: Loss




## Box 2: Gain




$$-i\frac{d}{dt}\phi(t) = H\phi(t)$$

$$H = [E_1] = \left[re^{i\theta}\right]$$

$$H = [E_2] = \left[re^{-i\theta}\right]$$

$$\psi(t) = \psi(0)e^{iE_1 t}$$

$$\psi(t) = \psi(0)e^{iE_2 t}$$

**Two boxes together as a single system:**

$$H = \begin{bmatrix} re^{i\theta} & 0 \\ 0 & re^{-i\theta} \end{bmatrix}$$

This Hamiltonian is ***PT*** symmetric,

where ***T*** is complex conjugation and

$$\mathcal{P} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

## Couple boxes together with coupling strength *s*

$$H = \begin{bmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{bmatrix}$$

Eigenvalues become real if *s* is sufficiently large.
Critical value given by:

$$s_{\text{crit}}^2 = r^2 \sin^2 \theta$$

**Examining CLASSICAL limit of *PT* quantum mechanics provides intuitive explanation of the *PT* transition:**

$$H = p^2 + ix^3$$

Source antenna becomes infinitely strong as

$$x \longrightarrow -\infty$$

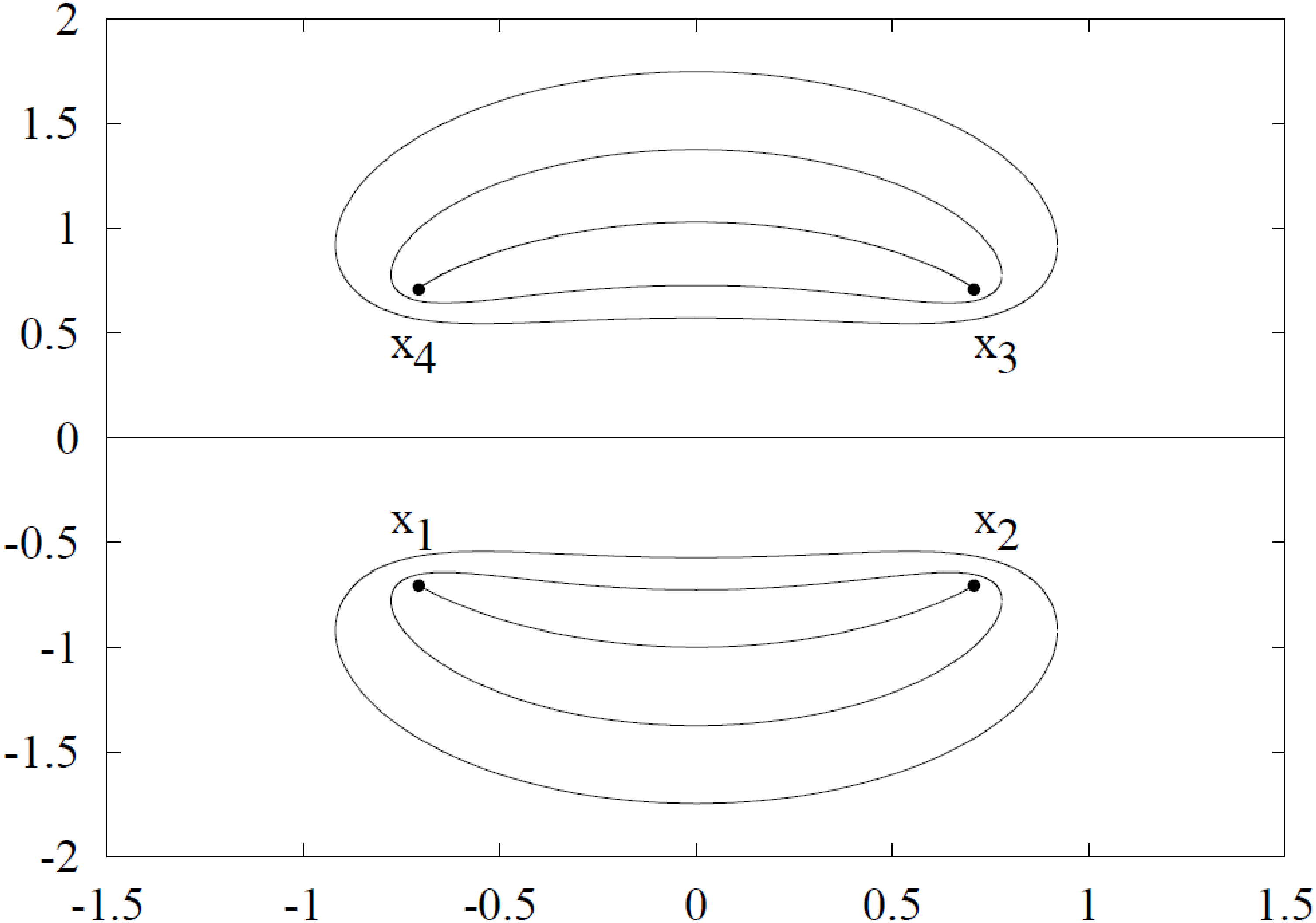
Sink antenna becomes infinitely strong as

$$x \longrightarrow +\infty$$

Time for classical particle to travel from source to sink:

$$T = \int dt = \int \frac{dx}{p} = \int_{x=-\infty}^{\infty} \frac{dx}{\sqrt{E - ix^3}}$$

$H = p^2 - x^4$ **Source and sink localized at + and - infinity**



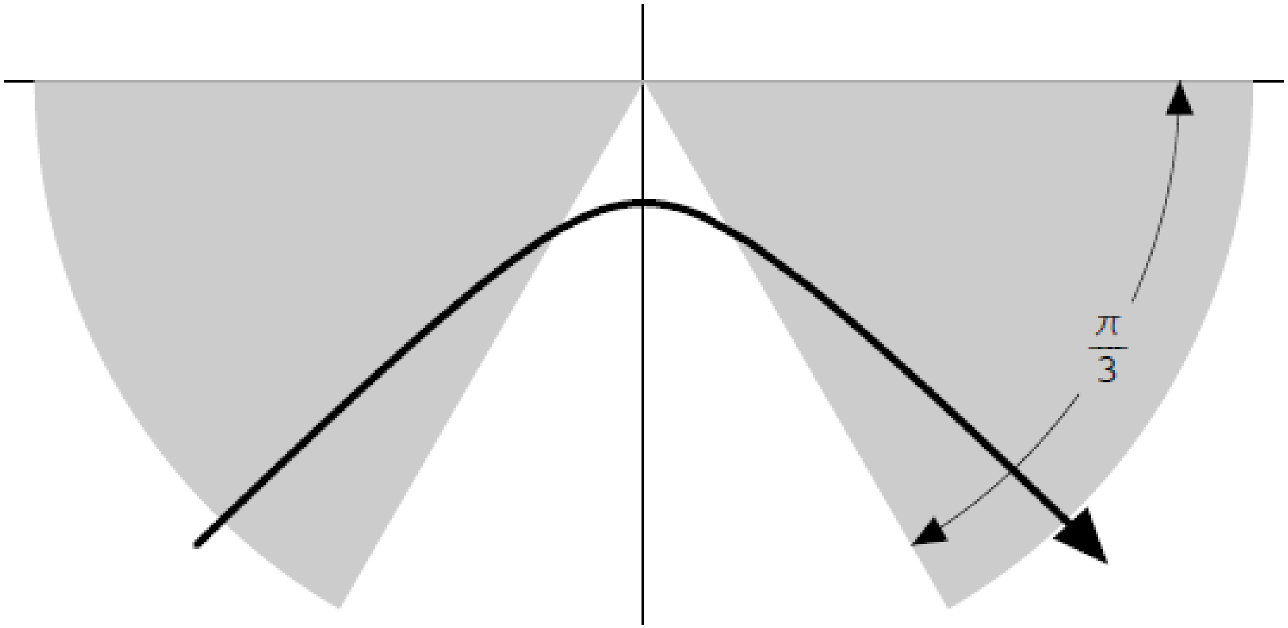
# Complex eigenvalue problems and Stokes wedges…

**At the quantum level:** $H = p^2 - x^4$

# Upside down potential

$$H = \frac{1}{2m}p^2 - gx^4$$

$$-\frac{\hbar^2}{2m}\psi''(x) - gx^4\psi(x) = E\psi(x)$$



# Step 1: Change path of integration

$$x = -2iL\sqrt{1 + iy/L}$$

fundamental unit of length is $[\hbar^2/(mg)]^{1/6}$

$$L = \lambda \left(\frac{\hbar^2}{mg}\right)^{1/6}$$

$\lambda$ is an arbitrary positive dimensionless constant

$$-\frac{\hbar^2}{2m}\left(1 + \frac{iy}{L}\right)\phi''(y) - \frac{i\hbar^2}{4Lm}\phi'(y) - 16gL^4\left(1 + \frac{iy}{L}\right)^2\phi(y) = E\phi(y)$$

# Step 2: Fourier transform

$$\tilde{f}(p) \equiv \int_{-\infty}^{\infty} dy\, e^{-iyp/\hbar} f(y)$$

$$\frac{1}{2m}\left(1-\frac{\hbar}{L}\frac{d}{dp}\right)p^2\tilde{\phi}(p)+\frac{\hbar}{4Lm}p\tilde{\phi}(p)-16gL^4\left(1-\frac{\hbar}{L}\frac{d}{dp}\right)^2\tilde{\phi}(p)=E\tilde{\phi}(p)$$

$$-16gL^2\hbar^2\tilde{\phi}''(p)+\left(-\frac{\hbar p^2}{2mL}+32gL^3\hbar\right)\tilde{\phi}'(p)+\left(\frac{p^2}{2m}-\frac{3p\hbar}{4mL}-16gL^4\right)\tilde{\phi}(p)=E\tilde{\phi}(p)$$

# Step 3: Change dependent variable

$$\tilde{\phi}(p) = e^{Q(p)/2}\Phi(p)$$

$$Q(p) = \frac{2L}{\hbar}p - \frac{1}{96gmL^3\hbar}p^3$$

$$-16gL^2\hbar^2\Phi''(p) + \left(-\frac{\hbar p}{4mL} + \frac{p^4}{256gm^2L^4}\right)\Phi(p) = E\Phi(p)$$

# Step 4: Rescale *p*

$$p = zL\sqrt{32mg}$$

$$-\frac{\hbar^2}{2m}\Phi''(z) + \left(-\hbar\sqrt{\frac{2g}{m}}z + 4gz^4\right)\Phi(z) = E\Phi(z)$$

# Result: A pair of exactly isospectral Hamiltonians

$$H = \frac{1}{2m}p^2 - gx^4$$

$$\tilde{H} = \frac{\tilde{p}^2}{2m} - \hbar\sqrt{\frac{2g}{m}}\,z + 4gz^4$$

CMB, D. C. Brody, J.-H. Chen, H. F. Jones , K. A. Milton, and M. C. Ogilvie
*Physical Review D* **74**, 025016 (2006) [arXiv: hep-th/0605066]

# Reflectionless potentials!

Z. Ahmed, CMB, and M. V. Berry,
J. Phys. A: Math. Gen. **38**, L627 (2005) [arXiv: quant-ph/0508117]