Anisotropic flow in ALICE at the LHC

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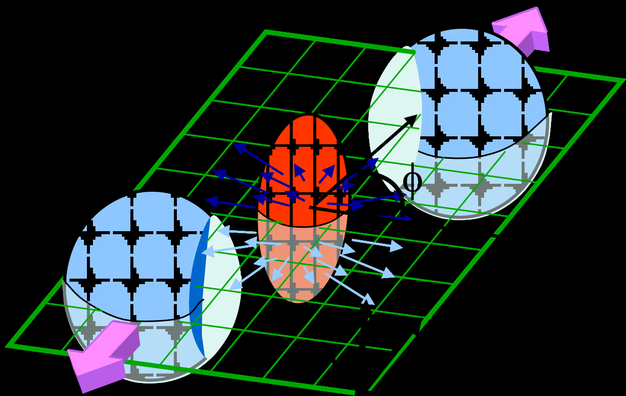
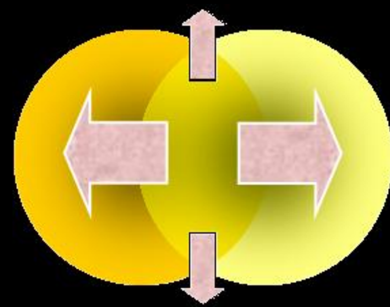
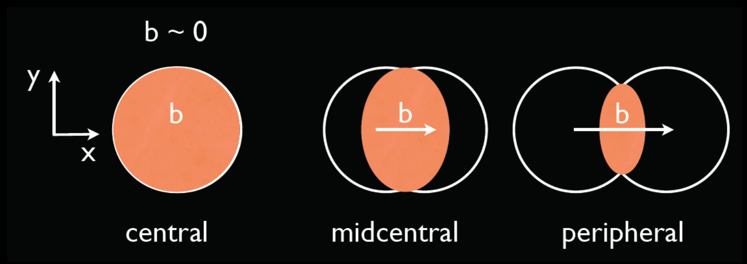
Introduction

Anisotropic flow







- Anisotropies in momentum space S. Voloshin and Y. Zhang (1996) =>

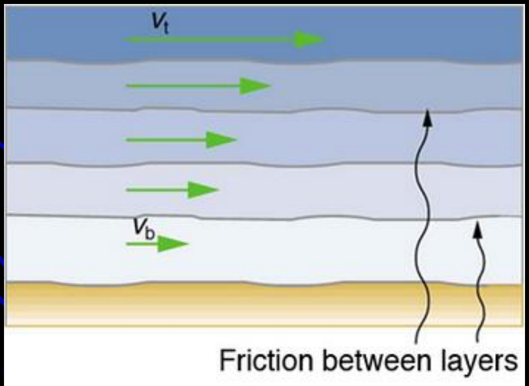
$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{RP}\right)\right)\right)$$

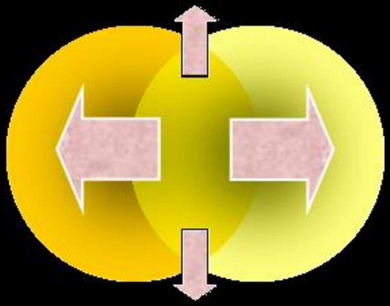
$$v_n = \langle\cos(n(\phi-\Psi_{RP}))\rangle$$

- Coordinate => momentum space
- In case of fluctuations $\Psi_{RP} \Rightarrow \Psi_n$
    - Reaction plane is a plane spanned by the impact parameter and beam axis
- Harmonics v_n quantify anisotropic flow
    - v_1 is directed flow, v_2 is elliptic flow, v_3 is triangular flow, etc.

Why flow?

- Goal: expect to see/study the QGP (deconfined quark-gluon matter)
- Test: whether hydrodynamic (liquid) description can describe the data
  - If yes, can we extract the transport properties of this matter (e.g. viscosity)?
  - Other items of interest: Equation of state (EoS), energy loss...
- For detailed description: need realistic time evolution => initial conditions (fluctuations), deconfined phase (hydro), hadronization
- Anisotropic flow is sensitive to the system evolution (in particular it probes the properties of the created matter such as shear viscosity)
  - Perfect liquid ⇔ shear viscosity zero

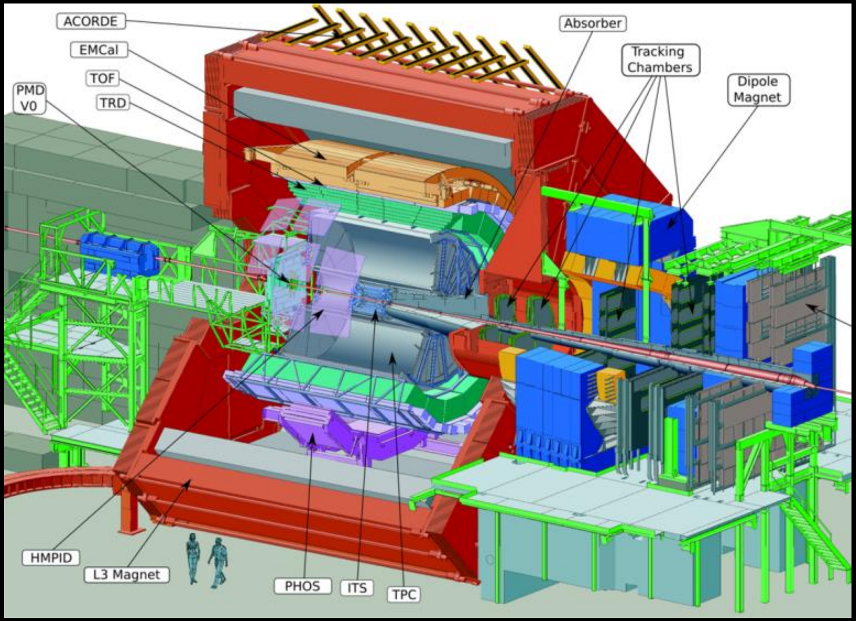




- Shear viscosity characterizes quantitatively the resistance of the liquid to displacement of its layers

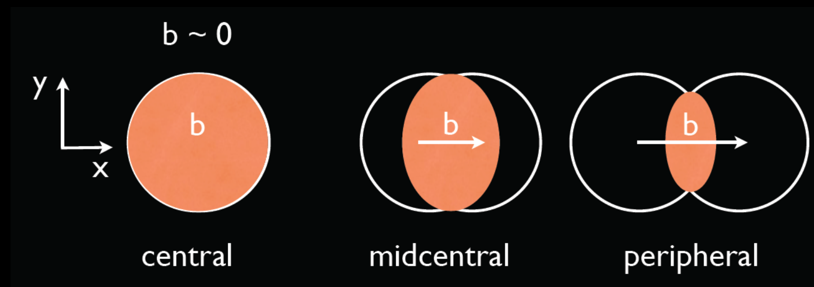
Analysis outline

- Data:
  - 2010 + 2011 => Pb-Pb events at 2.76 TeV
  - Acceptance => $|\eta| < 0.8$ (Time Projection Chamber)
    $|\eta| < 5.1$ (Forward Multiplicity Detector)

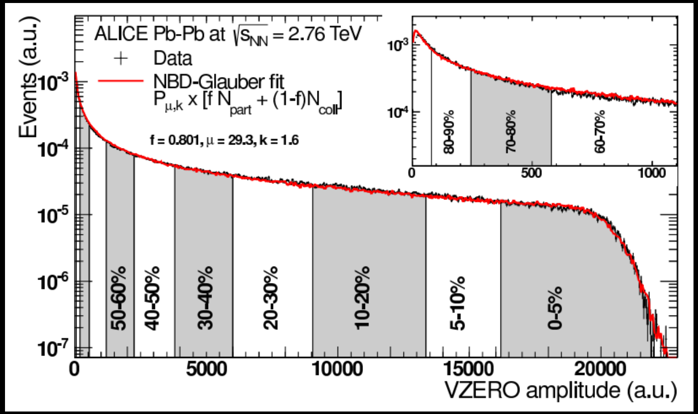


- Charged particle tracking:
  - Time Projection Chamber
  - Inner Tracking System
- Centrality determination
  - VZERO detectors
    - VZERO-A => $2.8 < \eta < 5.1$
    - VZERO-C => $-3.7 < \eta < -1.7$
- Systematic uncertainties:
  - Nonflow
  - Centrality determination
  - Inefficiencies in detectors azimuthal acceptance
  - Variation of track quality cuts
  - Secondaries in the material

Centrality



- Elliptic flow is geometrical quantity => need to classify all events in terms of initial geometry
- Another geometrical quantity available: Multiplicity
  - In central collisions more nucleons within nuclei interact than in the peripheral collisions => more particles are produced in the central collisions than in the peripheral
- Glauber model: Quantitative description of multiplicity distribution, centrality classes of events
  - Most central => Centrality class 0-5%
  - Peripheral => Centrality class 70-80%



How do we measure anisotropic flow?

Azimuthal correlations

- Definition of flow harmonics :

$$v_n = \langle \cos(n(\varphi - \Psi_{\mathrm{RP}})) \rangle$$

- Two particle azimuthal correlations (double brackets indicate an event sample and particles in each event averages):

$$\left\langle\!\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\!\right\rangle = \left\langle\!\!\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}}-(\phi_2-\Psi_{\mathrm{RP}}))} \right\rangle\!\!\right\rangle$$

$$= \left\langle\!\!\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}})} \right\rangle \left\langle e^{-in(\phi_2-\Psi_{\mathrm{RP}})} \right\rangle\!\!\right\rangle = \langle v_n^2 \rangle$$

- Systematic biases:
  - Few-particle correlations unrelated to the initial geometry (nonflow)
  - Multiplicity fluctuations
- Suppress nonflow with multi-particle cumulants

N. Borghini, P. M. Dinh and J.-Y. Ollitrault, "Flow analysis from multiparticle azimuthal correlations," PRC 64 (2001) 054901
Ryogo Kubo, "Generalized Cumulant Expansion Method"

2-particle cumulant

- The most general decomposition of 2-particle correlation

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

X_1 and X_2 are two observables (e.g. particle density)

- 2-particle cumulant (non-factorisable 2-particle correlation):

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

3-particle cumulant

- The most general decomposition of 3-particle correlation:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c \end{aligned}$$

- Using expression for 2-particle cumulants obtain:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \end{aligned}$$

=> Recursively one can obtain cumulants for any number of observables

Cumulants in flow analysis

- Observables in the context of anisotropic flow analysis (Ollitrault *et al*)

$$X_1 \equiv e^{in\phi_1}, \qquad X_2 \equiv e^{in\phi_2}$$
$$X_3 \equiv e^{-in\phi_3}, \qquad X_4 \equiv e^{-in\phi_4}$$

- An event average of single particle observable vanish:

$$\langle\langle X_i \rangle\rangle = \langle\langle e^{\pm in\varphi} \rangle\rangle = 0 \qquad \text{for all } n$$

- Correlate only distinct particles (exclude self correlation):

$$\langle 2 \rangle \equiv \langle \cos n(\phi_1 - \phi_2) \rangle \,, \qquad \phi_1 \neq \phi_2$$

$$\langle 4 \rangle \equiv \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle \,, \qquad \phi_1 \neq \phi_2 \neq \phi_3 \neq \phi_4$$

Flow vector and cumulants

- Q_n-vector (or flow vector) evaluated for harmonic n, and for event with multiplicity M:

$$Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$

- Analytical expressions for multi-particle azimuthal correlations in terms of Q-vectors

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2 \cdot \mathbf{Re}\left[Q_{2n} Q_n^* Q_n^*\right] - 4(M-2) \cdot |Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

Flow cumulants

- Cumulants expressed in terms of azimuthal correlations, defined in terms of Q-vectors:

$$
\begin{aligned}
c_n\{2\} &= \langle\langle 2\rangle\rangle \\
c_n\{4\} &= \langle\langle 4\rangle\rangle - 2\,\langle\langle 2\rangle\rangle^2 \\
c_n\{6\} &= \langle\langle 6\rangle\rangle - 9\,\langle\langle 2\rangle\rangle\,\langle\langle 4\rangle\rangle + 12\,\langle\langle 2\rangle\rangle^3 \\
c_n\{8\} &= \langle\langle 8\rangle\rangle - 16\,\langle\langle 6\rangle\rangle\,\langle\langle 2\rangle\rangle - 18\,\langle\langle 4\rangle\rangle^2 \\
&+ 144\,\langle\langle 4\rangle\rangle\,\langle\langle 2\rangle\rangle^2 - 144\,\langle\langle 2\rangle\rangle^4
\end{aligned}
$$

- If all correlations are expressed analytically in terms of Q-vectors => Q-cumulants (QC)

R. Snellings, S. Voloshin, AB: *"Flow analysis with cumulants: Direct calculations"*, PRC 83, 044913 (2011)

Flow contribution to cumulants

- In the absence of flow fluctuations, flow harmonics enter in a power of the cumulant order:

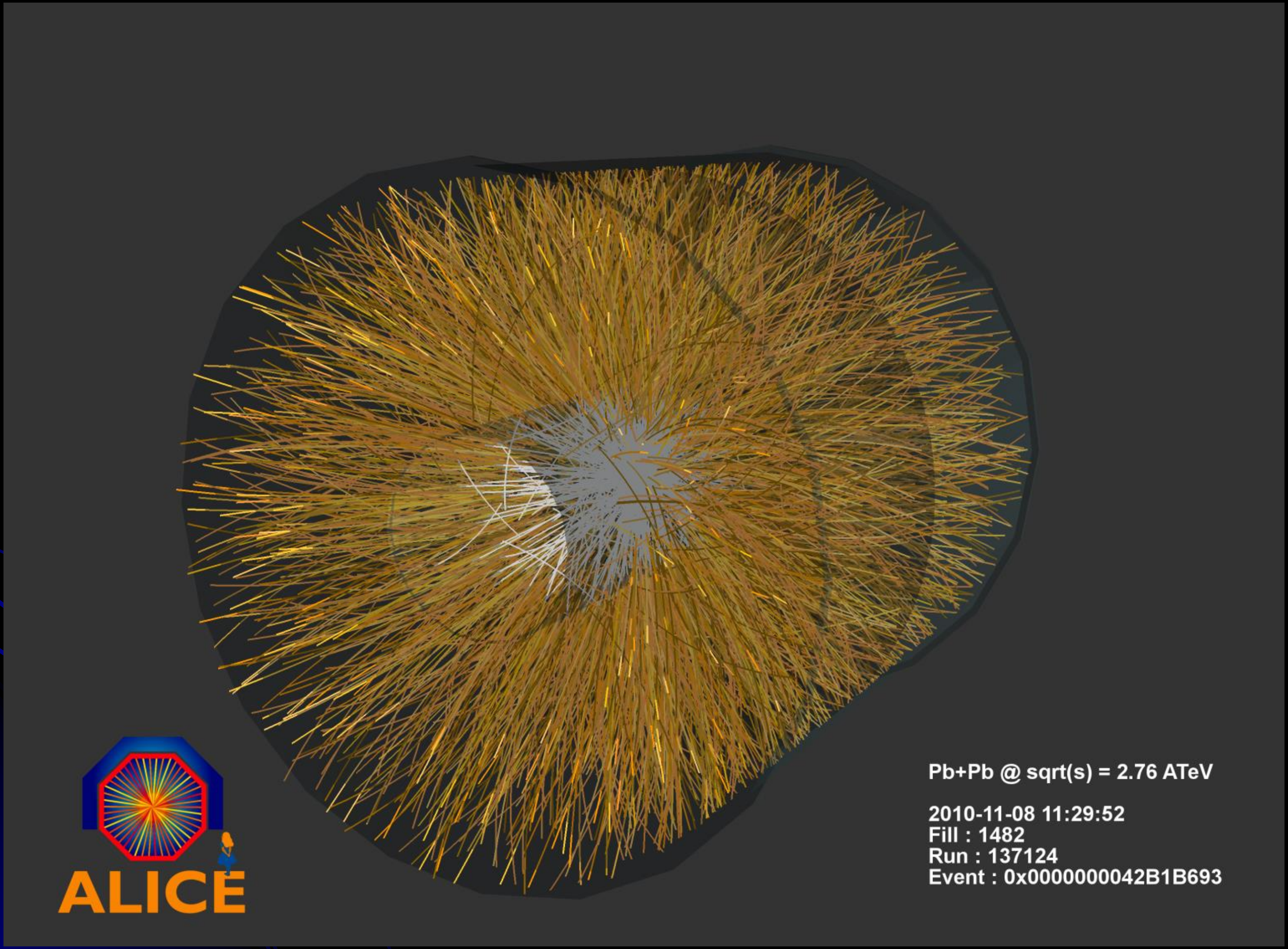
$$
\begin{aligned}
c_n\{2\} &= v_n^2 \\
c_n\{4\} &= -v_n^4 \\
c_n\{6\} &= 4v_n^6 \\
c_n\{8\} &= -33v_n^8
\end{aligned}
$$

These relations hold for any harmonic

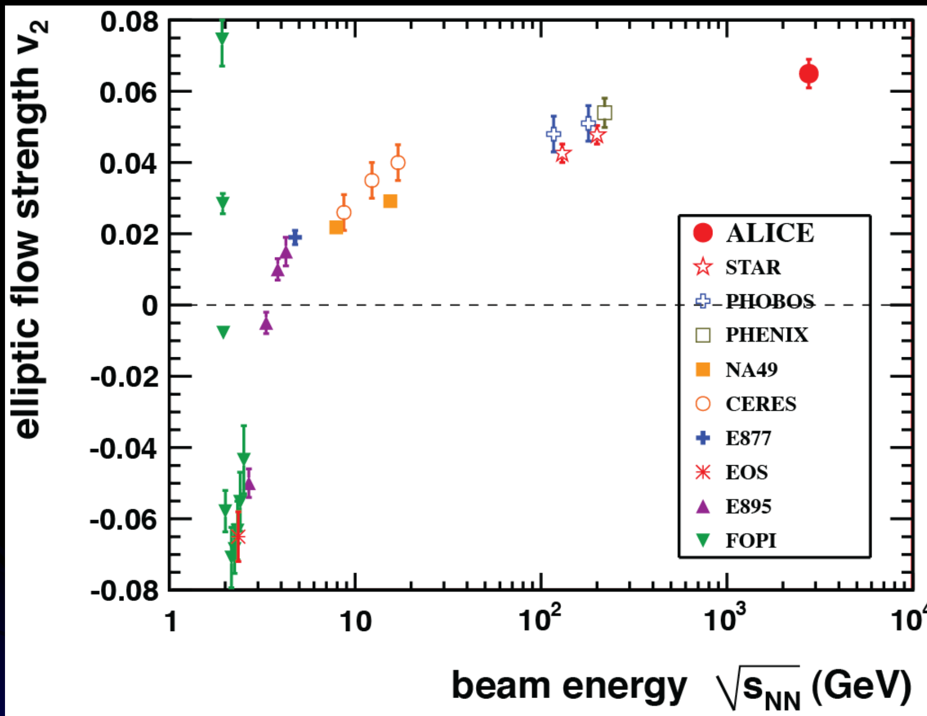
- Each of the equations above gives an independent observable for v_n => $v_n\{2\}$, $v_n\{4\}$, $v_n\{6\}$, etc.
  - Important for flow fluctuations studies (discussed later)

Results

A central heavy-ion collision as seen by ALICE

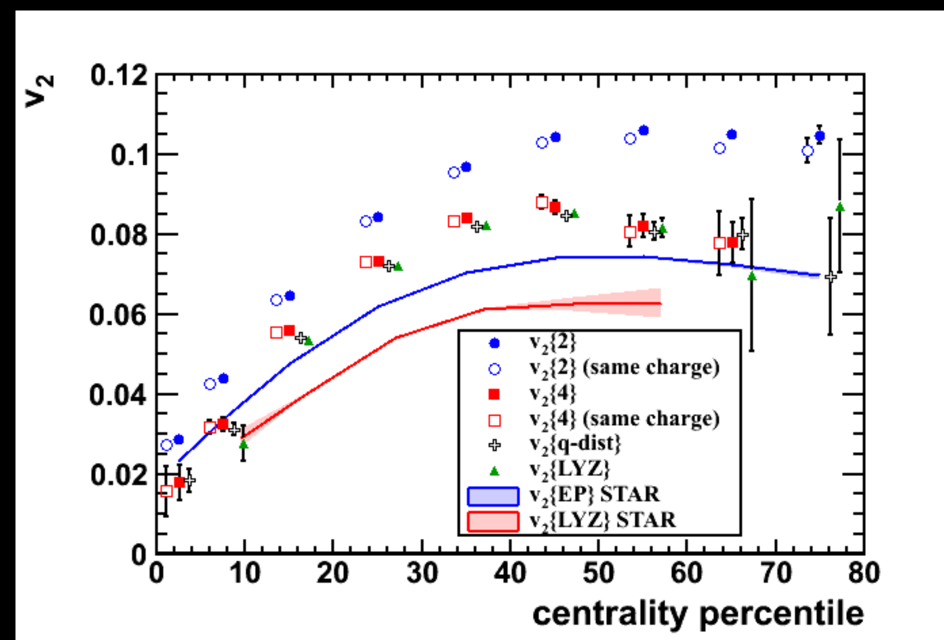


First results on the elliptic flow at the LHC



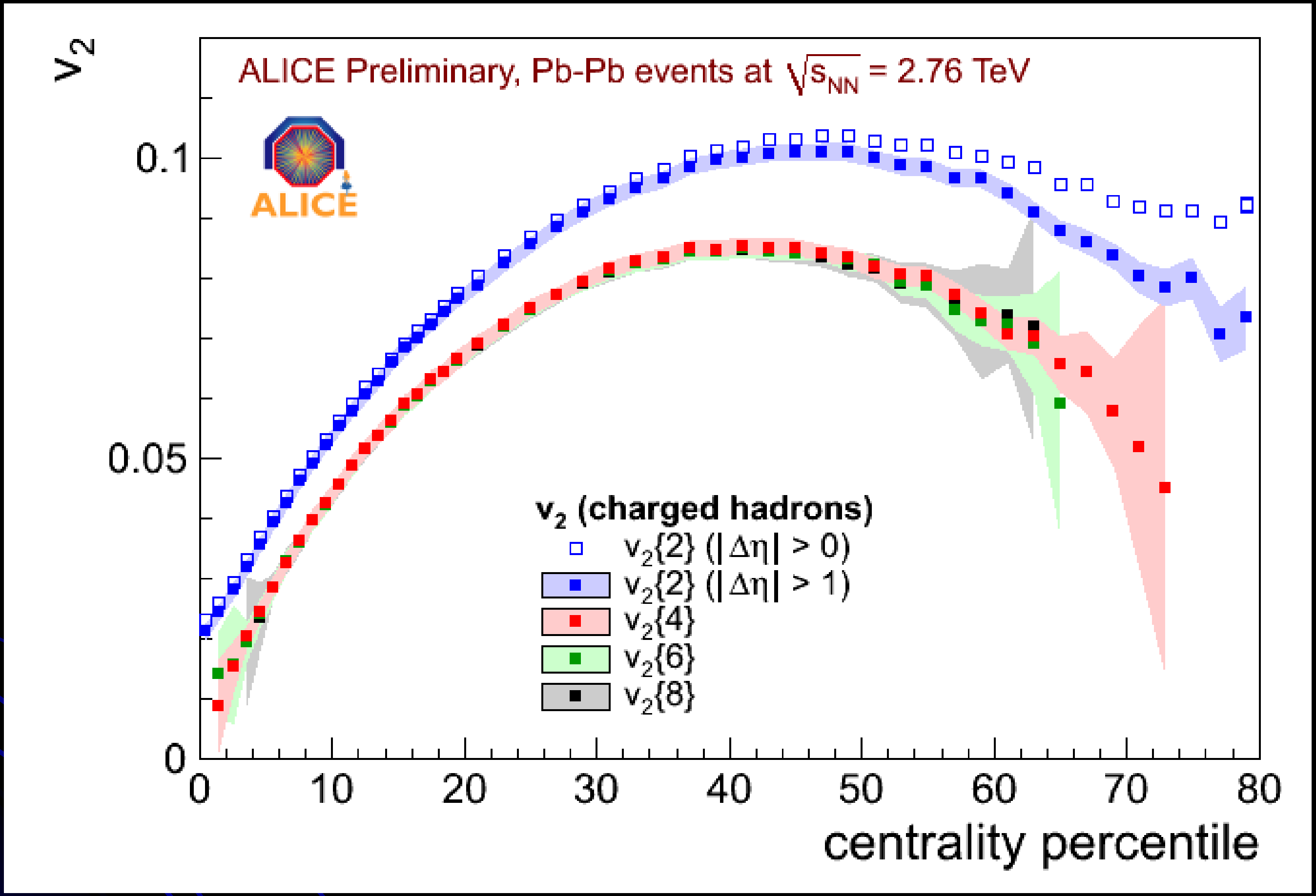
Phys. Rev. Lett. 105, 252302 (2010)

Cited by now ~ 300 times!
(most cited LHC heavy-ion paper according to Spires)



=> Elliptic flow increases by ~ 30% when compared to RHIC energies

Two and multi-particle cumulants in ALICE



=> The difference between $v_2\{2\}$ with and without eta gap is driven by the contribution from nonflow

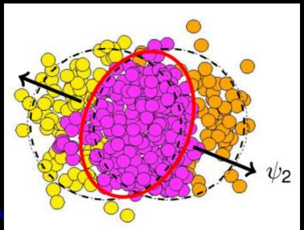
=> The difference between 2- and multi-particle estimates is due to fluctuations in the initial geometry

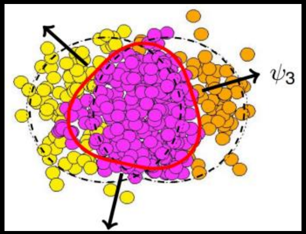
Flow fluctuations

- Event-by-event fluctuations in the positions of participating nucleons => non-zero odd harmonics:

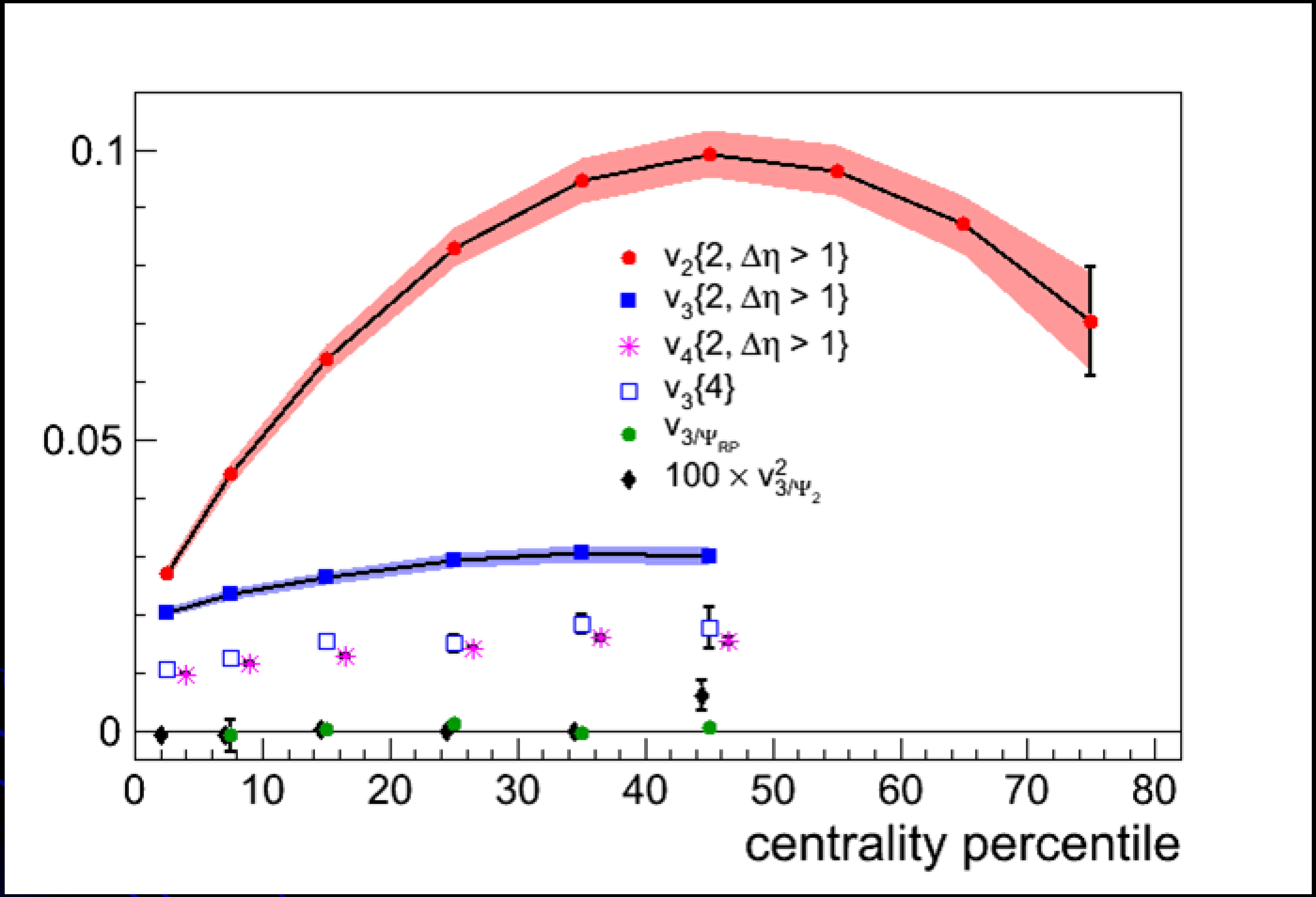
$$\Psi_{\mathrm{RP}} \Rightarrow \Psi_n$$
$$v_n = \langle \cos(n(\varphi - \Psi_n)) \rangle$$

- Each harmonic v_n has its own symmetry plane Ψ_n
- Experimental consequences of e-b-e flow fluctuations:
  - $\langle v_n^k \rangle$ is not the same as $\langle v_n \rangle^k$





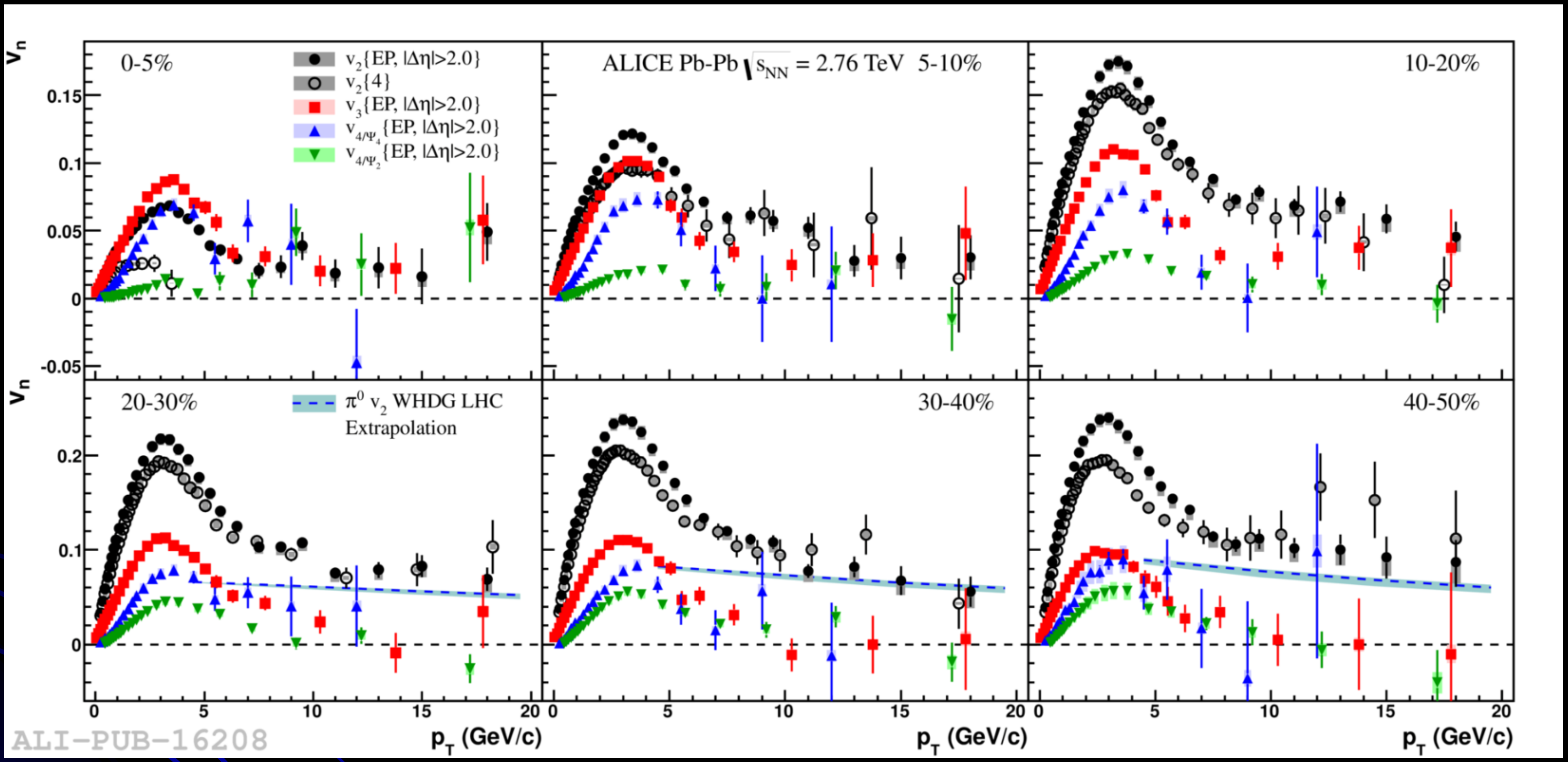
Charged particle v_3



- **Phys.Rev.Lett. 107 (2011) 032301**
- Nonzero v_3 develops along its own symmetry plane
- Symmetry plane of v_2 shows no correlation with the plane of v_3

v_n vs. transverse momentum

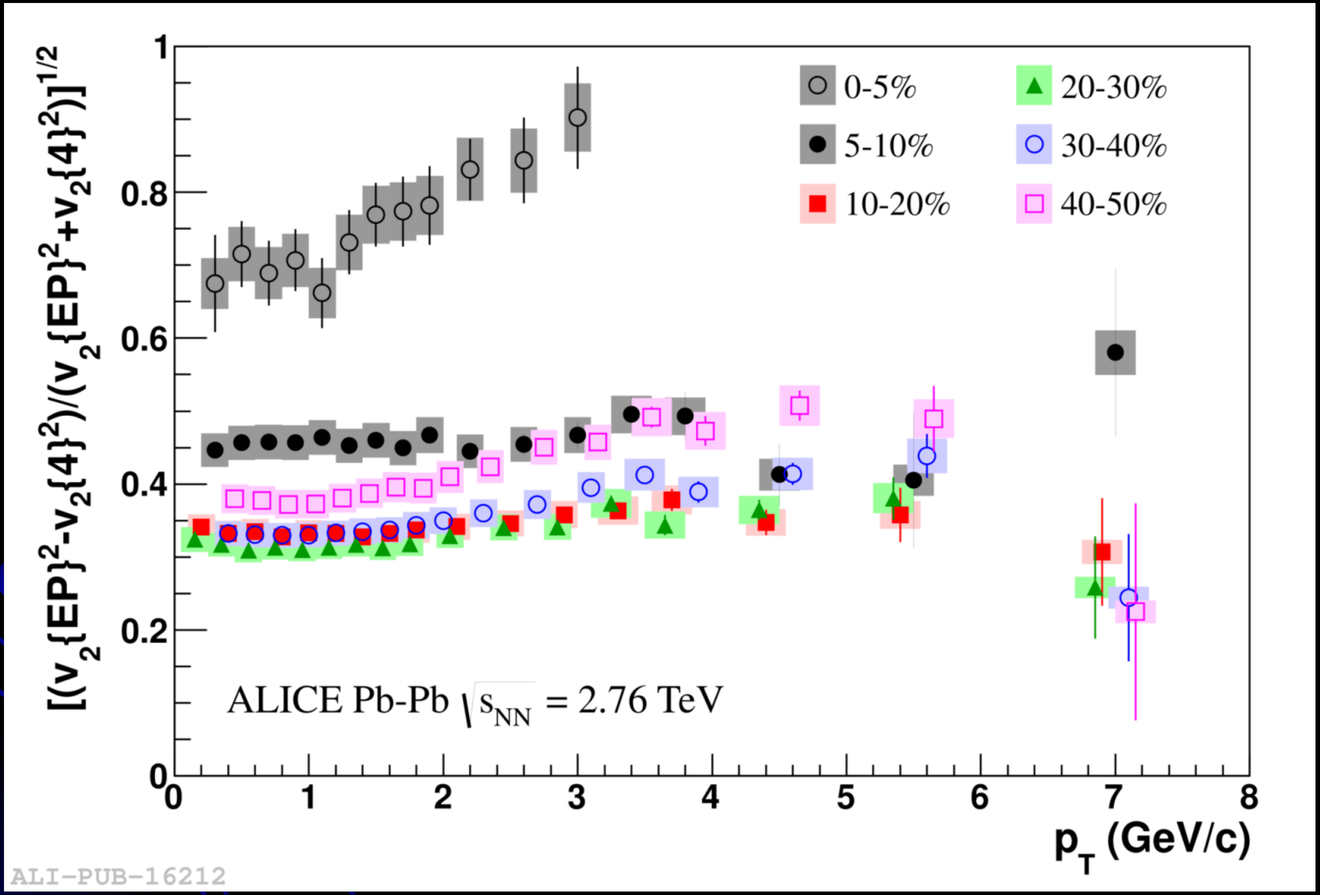
Phys.Lett.B 719 (2013) 18-28



* Central collisions: all harmonics are similar
* v_3 is almost independent of centrality, dominant harmonic for more central events very soon i.e. at moderate p_T values.
* $v_4\{\Psi_4\} > v_4\{\Psi_2\}$ => flow fluctuations
* Non-vanishing v_2 at high-p_T (p_T > 8 GeV/c) => path length dependence of energy loss of high-p_T partons traversing the non-isotropic medium

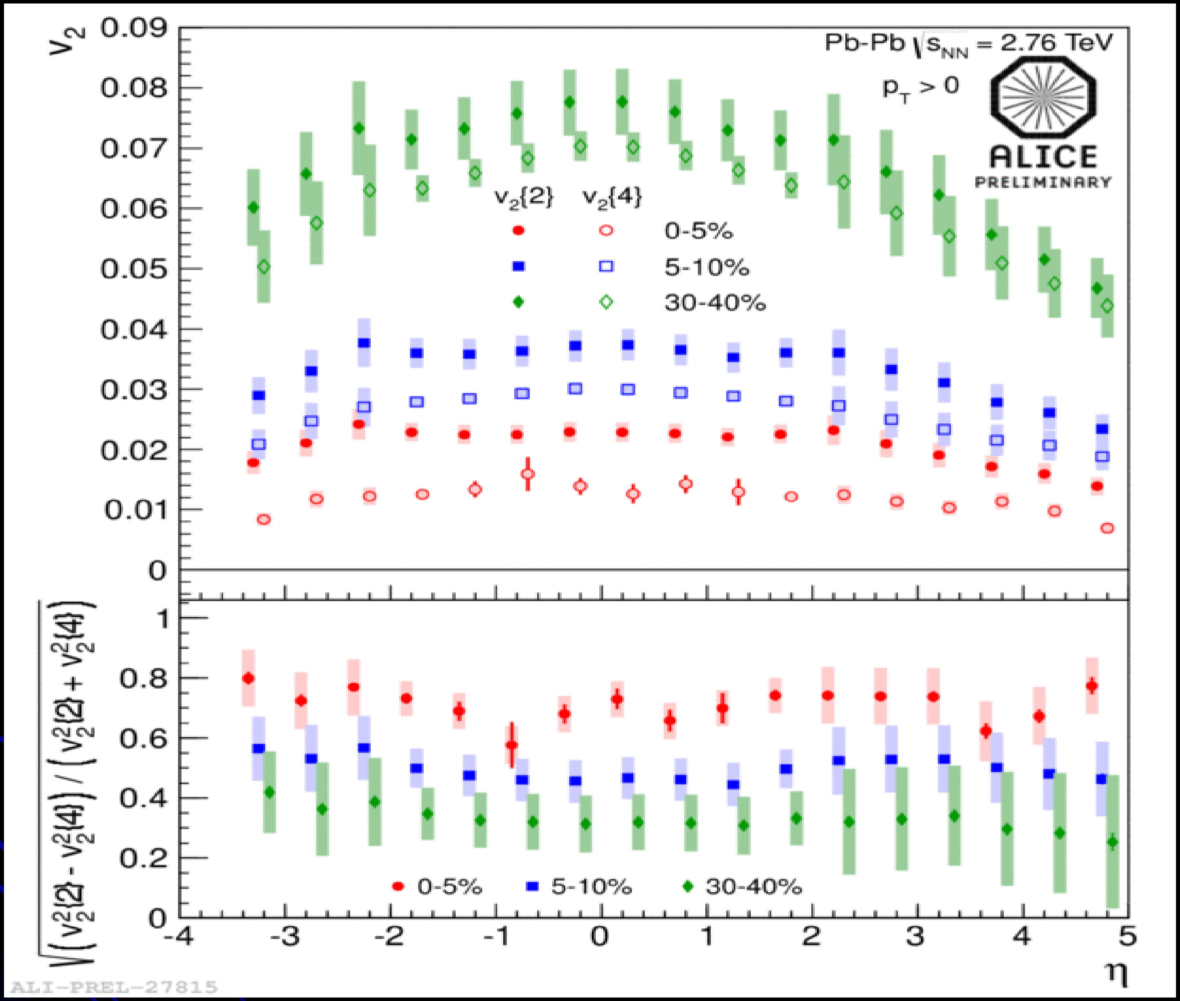
Estimate of v_2 fluctuations

Phys.Lett.B 719 (2013) 18-28



=> Fluctuations are similar up to p_T ~ 6 GeV, with the exception of most central events

v_2 in wide pseudorapidity range



=> Fluctuations do not change significantly with rapidity
(A. Hansen QM12, arXiv:1210.7095)

p.d.f. of flow fluctuations

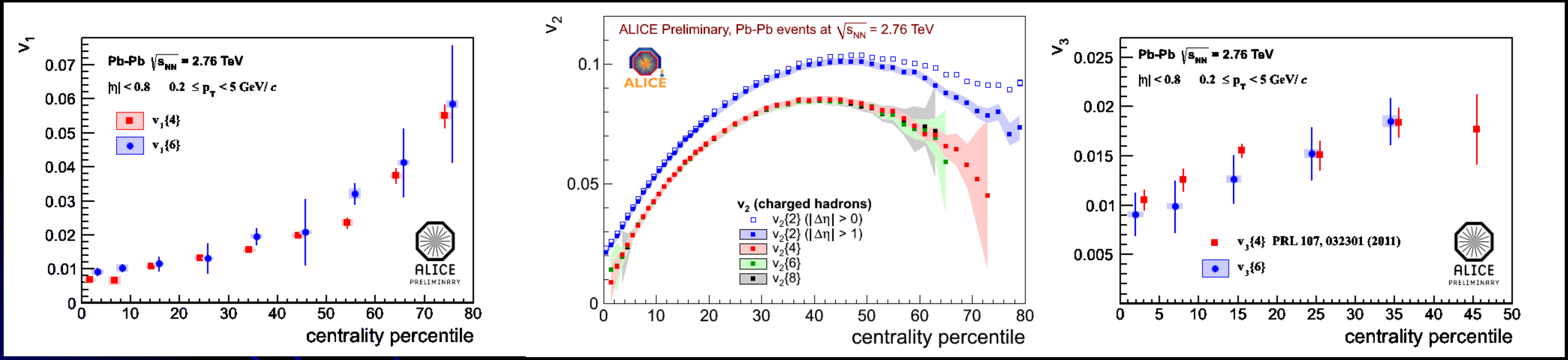
- Equivalence: p.d.f. ⇔ moments ⇔ cumulants
  - If for the 1st ($\langle v \rangle$) and 2nd (σ_v) moments $(\sigma_v/\langle v \rangle)^2 \ll 1$ is satisfied, then all multi-particle cumulants for any p.d.f. are the same
  - Odd harmonics originate from fluctuations => $(\sigma_v/\langle v \rangle)^2 \ll 1$ is never satisfied
- Bessel-Gaussian p.d.f: All higher moments degenerated $v_n\{4\} = v_n\{6\} = v_n\{8\} = \ldots.$

$$
\begin{aligned}
f(v) &= \frac{v}{b^2} \exp\left(-\frac{v^2+a^2}{2b^2}\right) I_0\left(\frac{va}{b^2}\right) \\
v\{2\} &= \sqrt{a^2+2b^2} \\
v\{4,6,\ldots\} &= a
\end{aligned}
$$

Voloshin *et al*: PLB 659, 537 (2008)

v_1, v_2, and v_3 from multi-particle cumulants

- Established experimentally that $v_n\{4\} \sim v_n\{6\}$ => p.d.f. of e-b-e flow fluctuations must have non-negligible 3rd/higher moments (when compared to the 1st/2nd moment)



=> Bessel-Gaussian function is an example of p.d.f. with $v_n\{4\} = v_n\{6\}$

Symmetry plane correlation

- Observable to determine correlation between different symmetry planes:

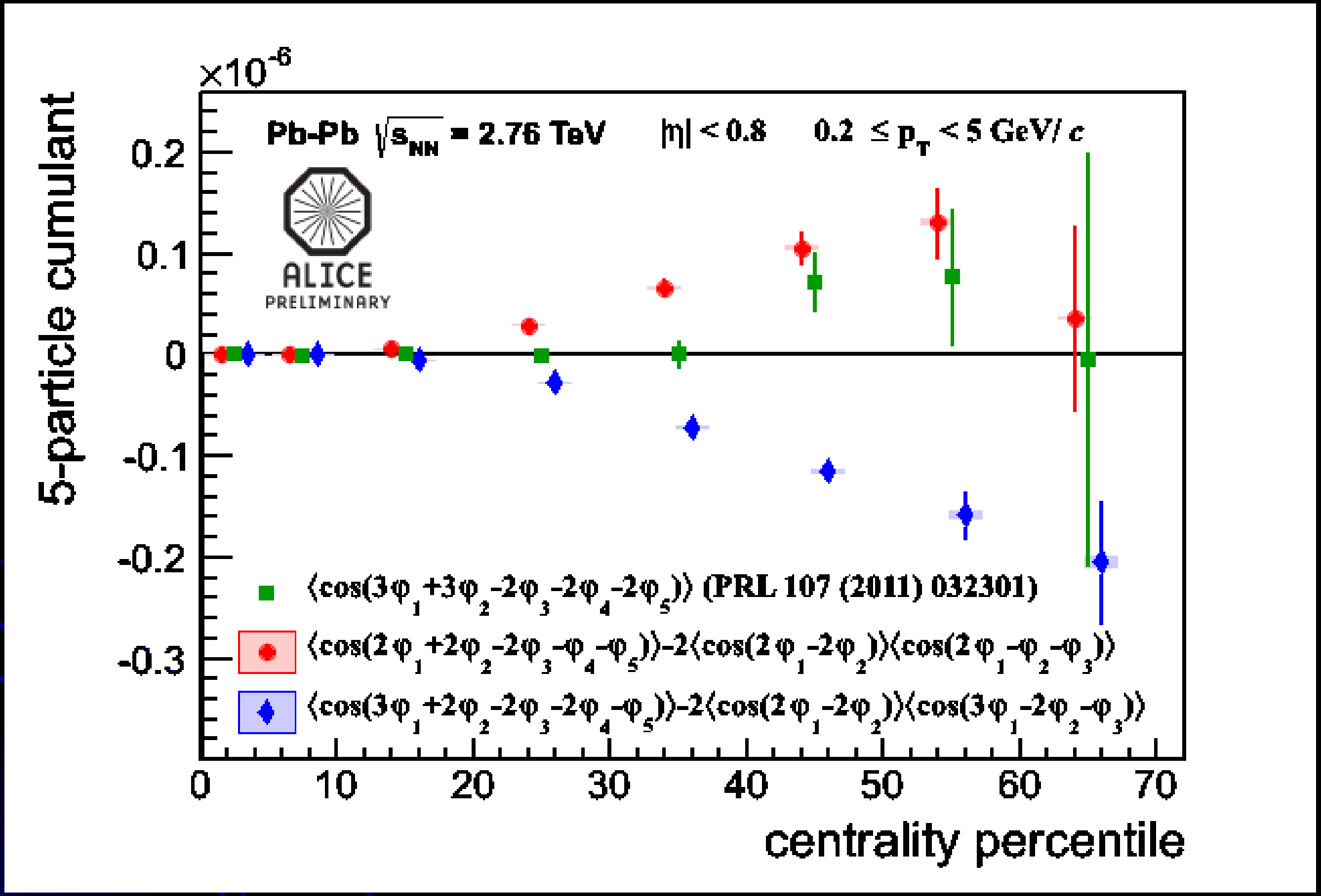
$$\langle \cos(n_1\phi_1 + \cdots + n_k\phi_k) \rangle = v_{n_1} \cdots v_{n_k} \cos(n_1\Psi_1 + \cdots + n_k\Psi_k)$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, 'Determining initial-state fluctuations from flow measurements in heavy-ion collisions,' PRC **84** 034910 (2011)

- Example:

$$\langle \cos(3\varphi_1 + 3\varphi_2 - 2\varphi_3 - 2\varphi_4 - 2\varphi_5) \rangle = v_3^2 v_2^3 \cos[6(\Psi_3 - \Psi_2)]$$

What is the relation between symmetry planes Ψ_n?

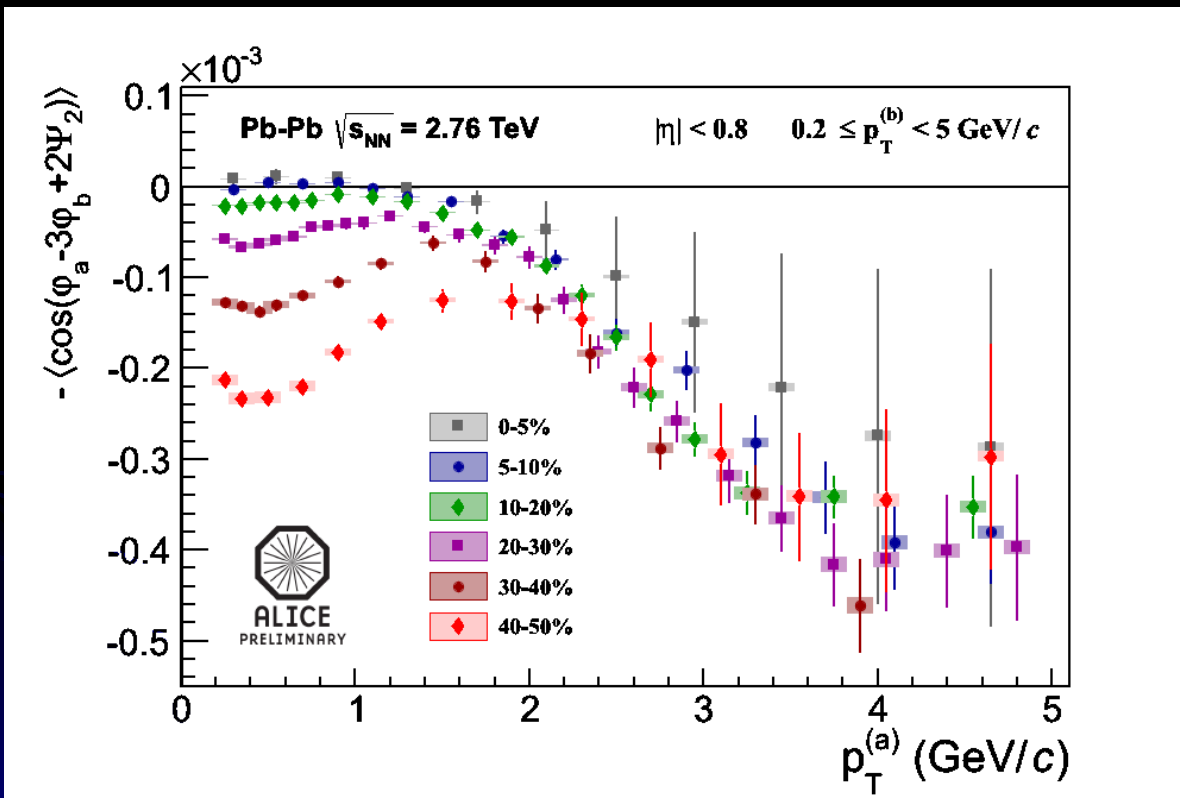


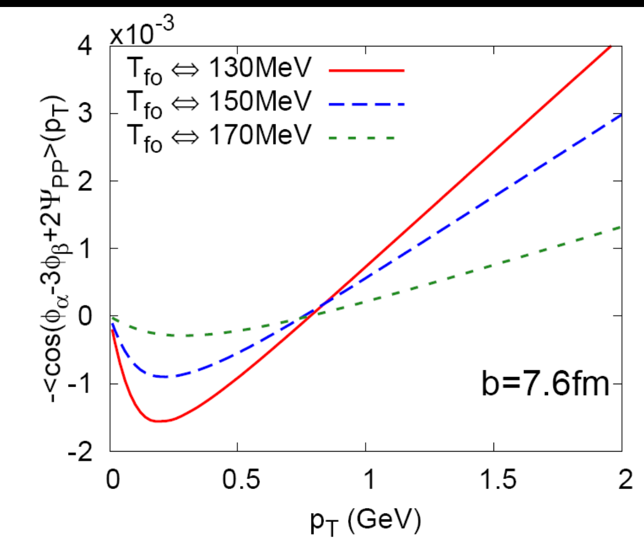
=> Observe non-zero genuine 5-particle correlation

=> Correlation strength is related to three-plane correlations

Three plane correlation

$$\langle\cos(\varphi_a - 3\varphi_b + 2\varphi_c)\rangle = \langle\cos(\varphi_a - 3\varphi_b + 2\Psi_2)\rangle \times v_2$$





Teaney, Yan PRC **83**, 064904 (2011)

* Observe non-zero 3-plane correlation

* The shape at low p_T is similar to that expected from the hydrodynamic model calculations, but differs at higher p_T

Summary

- Elliptic flow at LHC energies is 30% larger than at RHIC, compatible with expectations from hydrodynamic model calculations
- Flow fluctuations
  - Observed significant triangular flow
  - Do not change significanly neither with p_T nor with eta
  - Underlying p.d.f. is consistent with Bessel-Gaussian function
- Studied the correlation between symmetry planes along which flow develops
  - Symmetry plane of v_2 is not the same as the symmetry plane of v_3
- Observed non-zero mixed harmonic 3-particle correlation => indication of the three plane correlation
- Wealth of experimental results which demands a theoretical interpretation

Thanks!

Backup slides

Anisotropic flow

- Why we do not care about 'sinus terms'?
  - It is equally probable for a particle to be produced in directions phi and -phi:

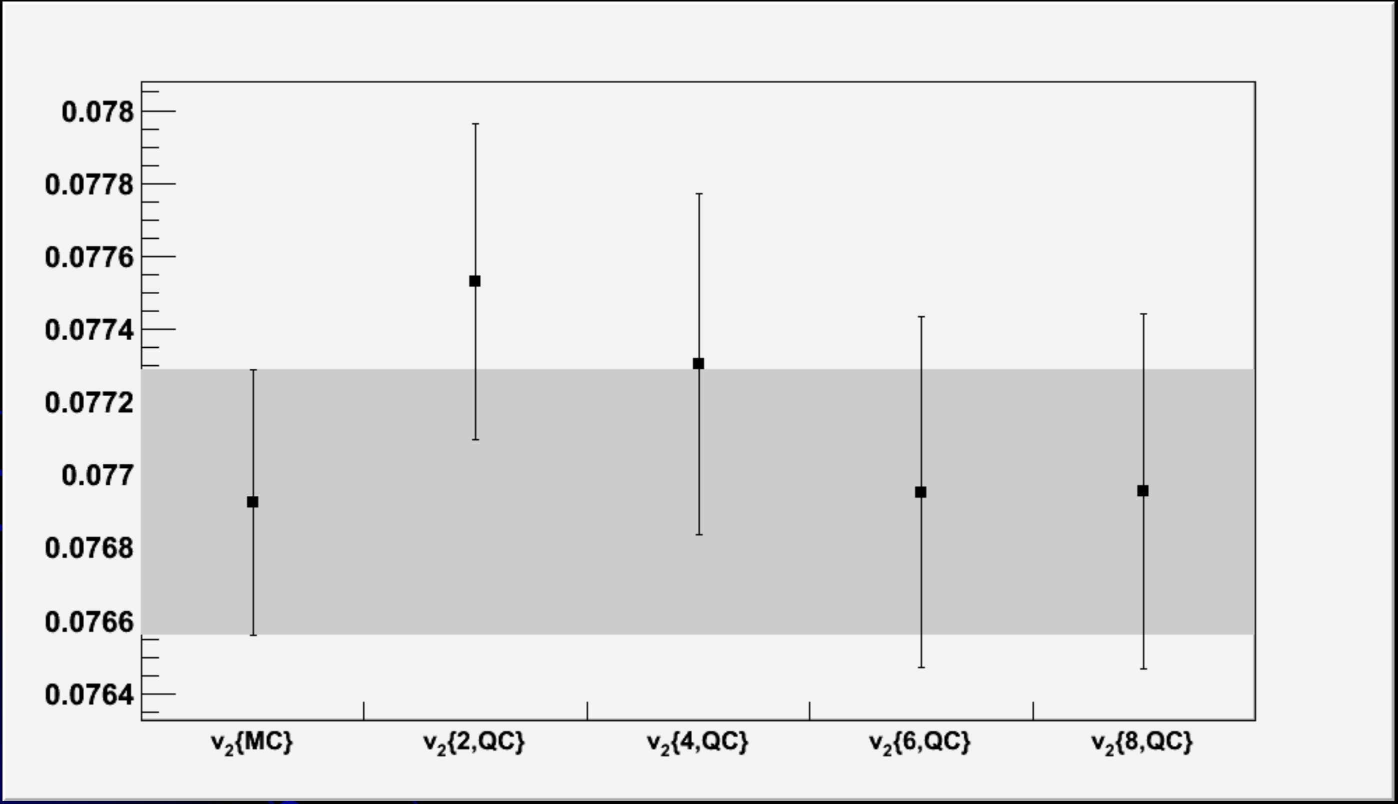
$$\sin(n\varphi) + \sin[n(-\varphi)] = \sin(n\varphi) - \sin(n\varphi) = 0$$

- Can 'odd cosine terms' be non-zero for ideal geometry?
  - It is equally probable for a particle to be produced in directions phi and phi + Pi:

$$\begin{aligned}
\cos(n\phi) + \cos[n(\phi + \pi)] &= \cos(n\phi) + \cos(n\phi)\cos(n\pi) - \sin(n\phi)\sin(n\pi) \\
&= \cos(n\phi) + \cos(n\phi)(-1)^n - \sin(n\phi)\cdot 0 \\
&= \cos(n\phi)\cdot(1 + (-1)^n) = 0 \text{ for odd } n
\end{aligned}$$

Q-cumulants

- Proof of the principle => Using Therminator events (realistic Monte Carlo generator of heavy-ion events, has both anisotropic flow and all resonances in the standard model)



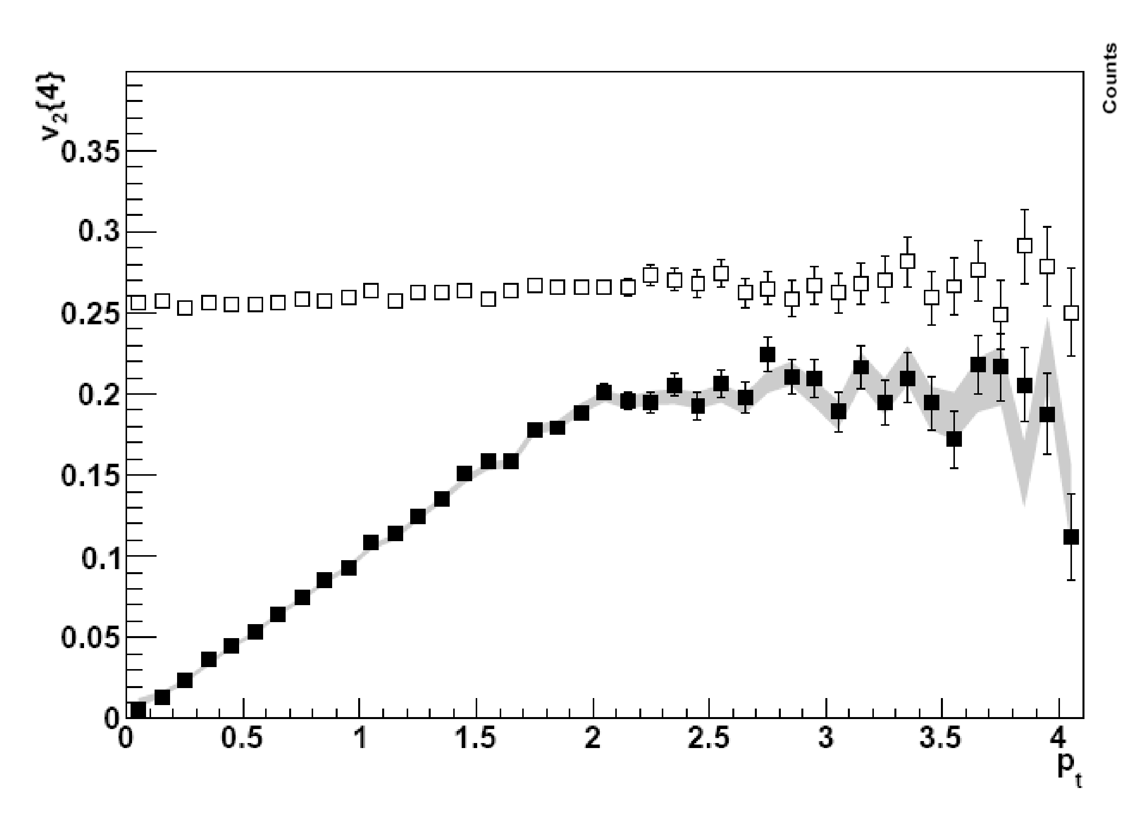
In this regime multi-particle QCs are precision method

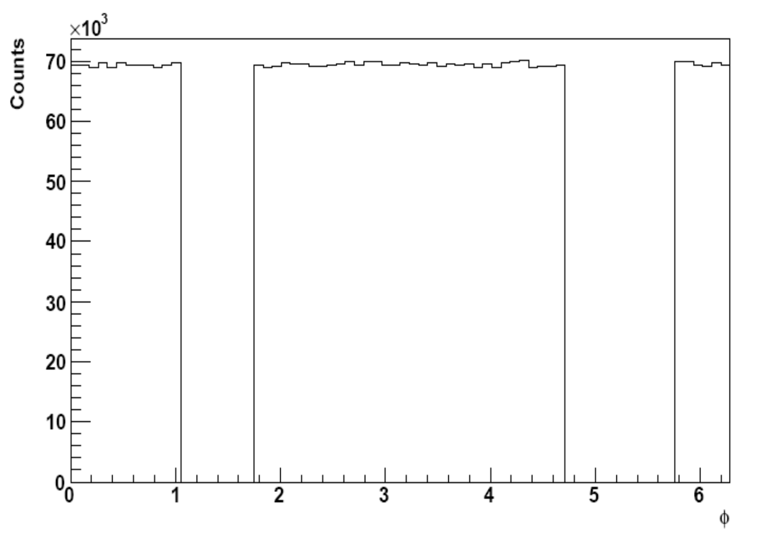
Non-uniform acceptance (1/2)

- If a detector has non-uniform acceptance in azimuthal angle, than in each event we have trivial anisotropies in momentum distributions of detected particles => this has nothing to do with anisotropic flow!
  - Can we disentangle one anisotropy from another?

Non-uniform acceptance (2/2)

- **Generalized *Q*-cumulants** can correct for non-uniform acceptance very well





Technical details => Appendix C in R. Snellings, S. Voloshin, A.B. *"Flow analysis with cumulants: Direct calculations"*, PRC 83, 044913 (2011)

Grey band => $v_2\{MC\}$; open markers => $v_2\{4\}$ from isotropic *Q*-cumulants; filled markers => $v_2\{4\}$ from generalized *Q*-cumulants

Is it really that trivial?

- Anisotropic flow measurement => ‘recipe’:
  - Step 1: Measure/estimate reaction (symmetry) plane in an event
  - Step 2: Take azimuthal angles of all reconstructed particles in an event
  - Step 3: Evaluate anisotropic flow harmonics via the average

$$v_n = \langle\cos(n(\phi - \Psi_{\mathrm{RP}}))\rangle$$

- In experimental practice the above prescription will not work
  - We cannot neither measure directly nor estimate reaction (symmetry) plane reliably event-by-event
- Can we estimate anisotropic flow harmonics v_n without requiring the reaction (symmetry) plane?

Autocorrelations

- We have to correlate only distinct particles, because autocorrelations are dominant and useless (really!) contribution in all averages. So:

$$\langle 2\rangle \equiv \langle \cos n(\phi_1-\phi_2)\rangle\ , \qquad \phi_1 \neq \phi_2$$

$$\langle 4\rangle \equiv \langle \cos n(\phi_1+\phi_2-\phi_3-\phi_4)\rangle\ , \qquad \phi_1 \neq \phi_2 \neq \phi_3 \neq \phi_4$$

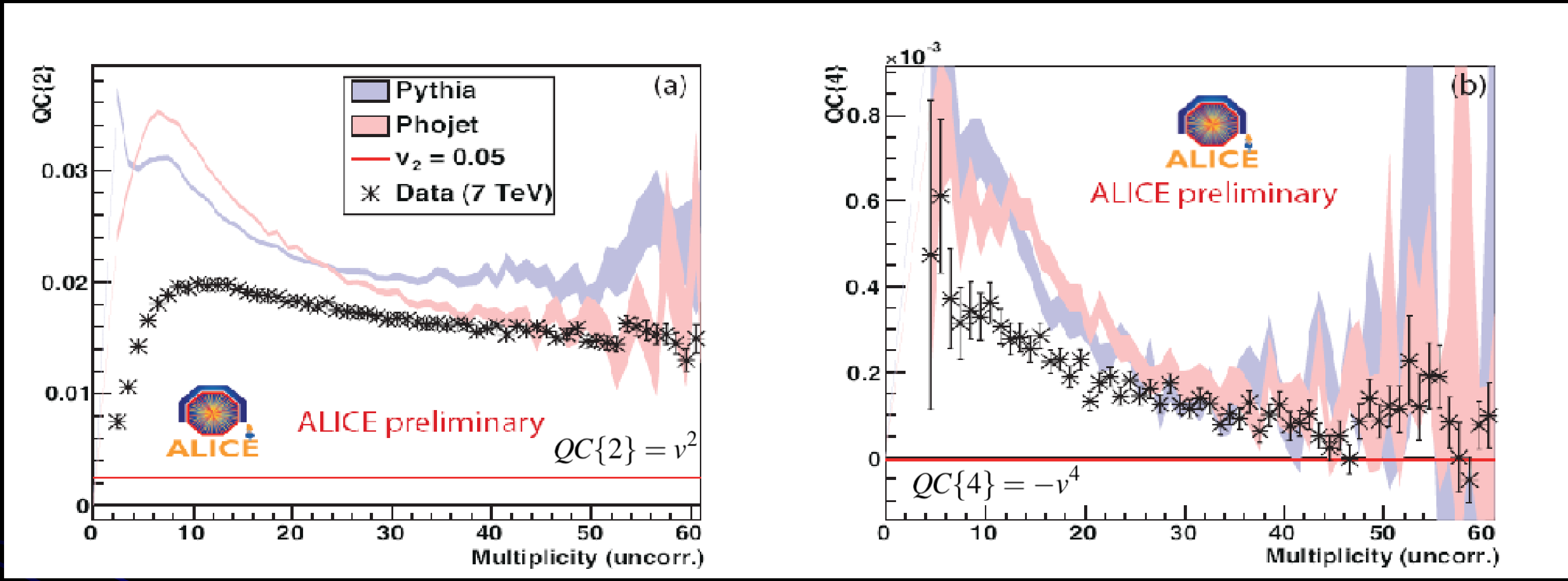
- How to enforce above constrains in practice?
  - Brute force evaluation via nested loops? => not feasible
  - Formalism of generating functions? => only approximate

$$G_n(z) \equiv \prod_{j=1}^{M}\left(1+\frac{z^* e^{in\phi_j}+ze^{-in\phi_j}}{M}\right)$$

$$\langle G_n(z)\rangle = \sum_{k=0}^{M/2}\frac{|z|^{2k}}{M^{2k}}\binom{M}{k}\binom{M-k}{k}\left\langle e^{in(\phi_1+\cdots+\phi_k-\phi_{k+1}-\cdots-\phi_{2k})}\right\rangle$$

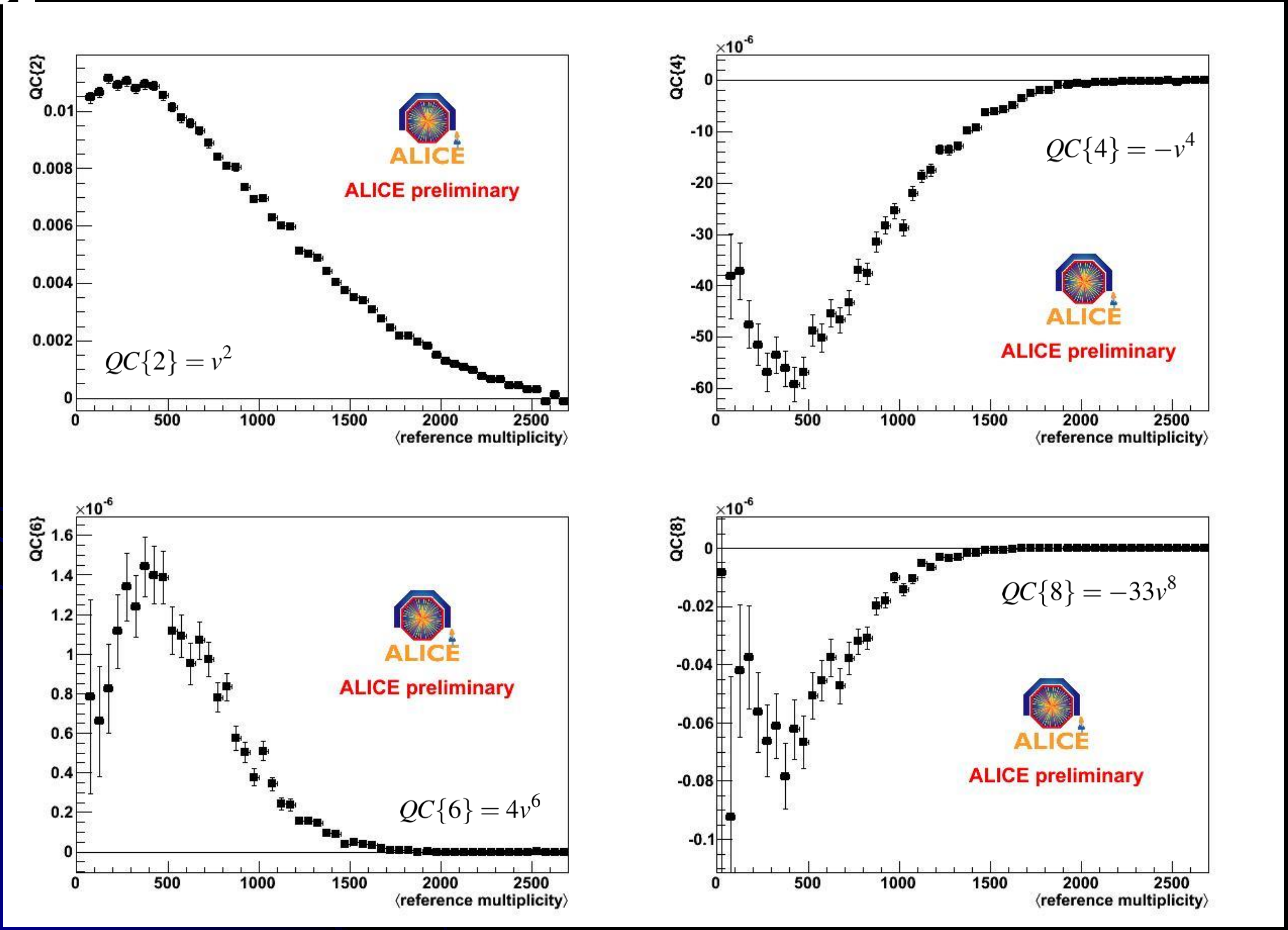
N. Borghini, P. M. Dinh and J.-Y. Ollitrault, "Flow analysis from multiparticle azimuthal correlations," PRC 64 (2001) 054901

Elliptic flow in *pp*?

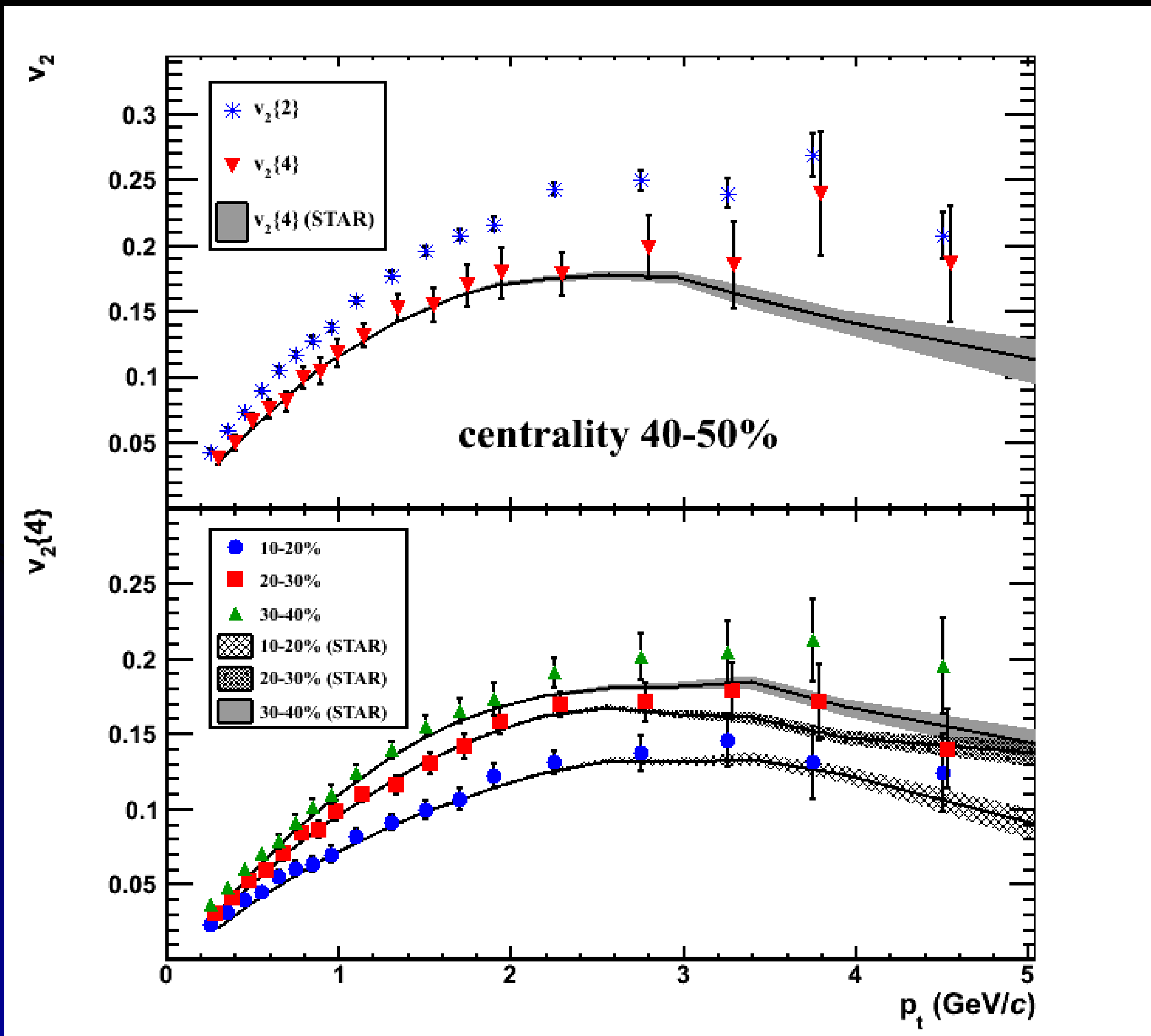


- Both 2- and 4-particle correlations decrease with multiplicity: Typical for non-collective behavior
- Pythia and Phojet are overestimating the strength of the correlations (and these two generators are dominated by jets and resonances)
- 4-p cumulant comes with a “wrong sign” => its dominant contribution is not coming from flow
- Current status – **We do not see elliptic flow in *pp***

Flow at first sight!



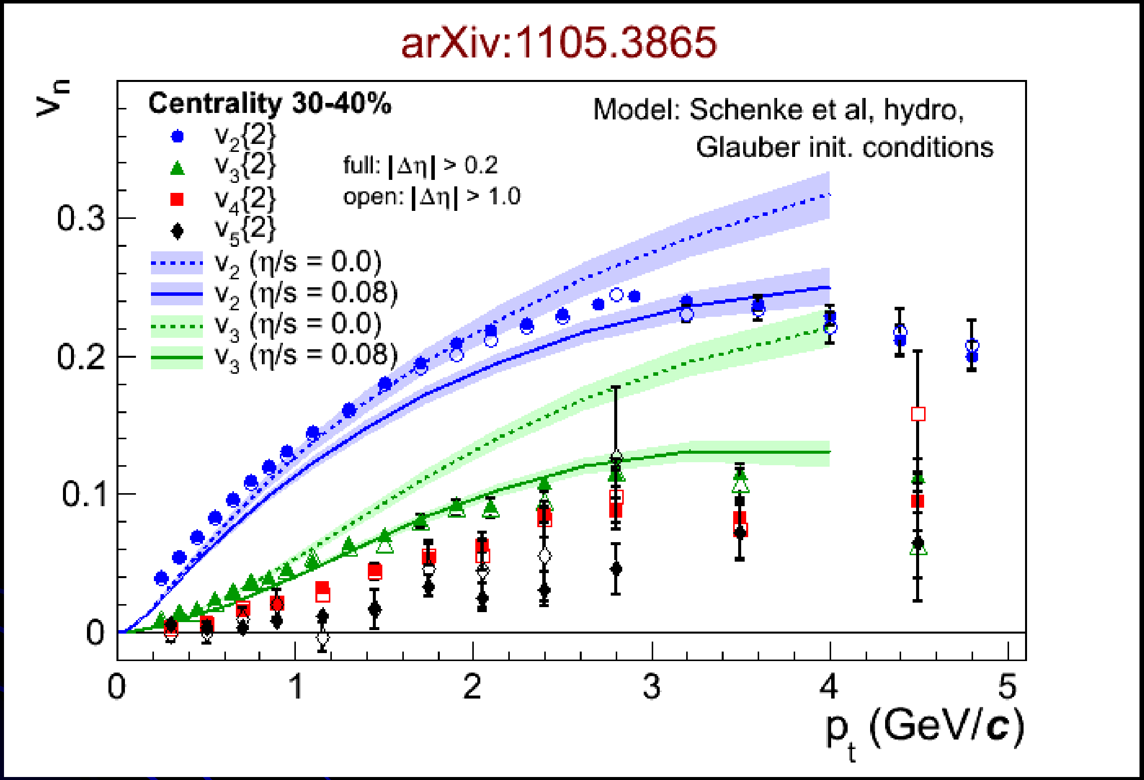
Early results



Phys. Rev. Lett. 105, 252302 (2010)

p_T dependence of elliptic flow at LHC close to the one at RHIC!

Comparison to models

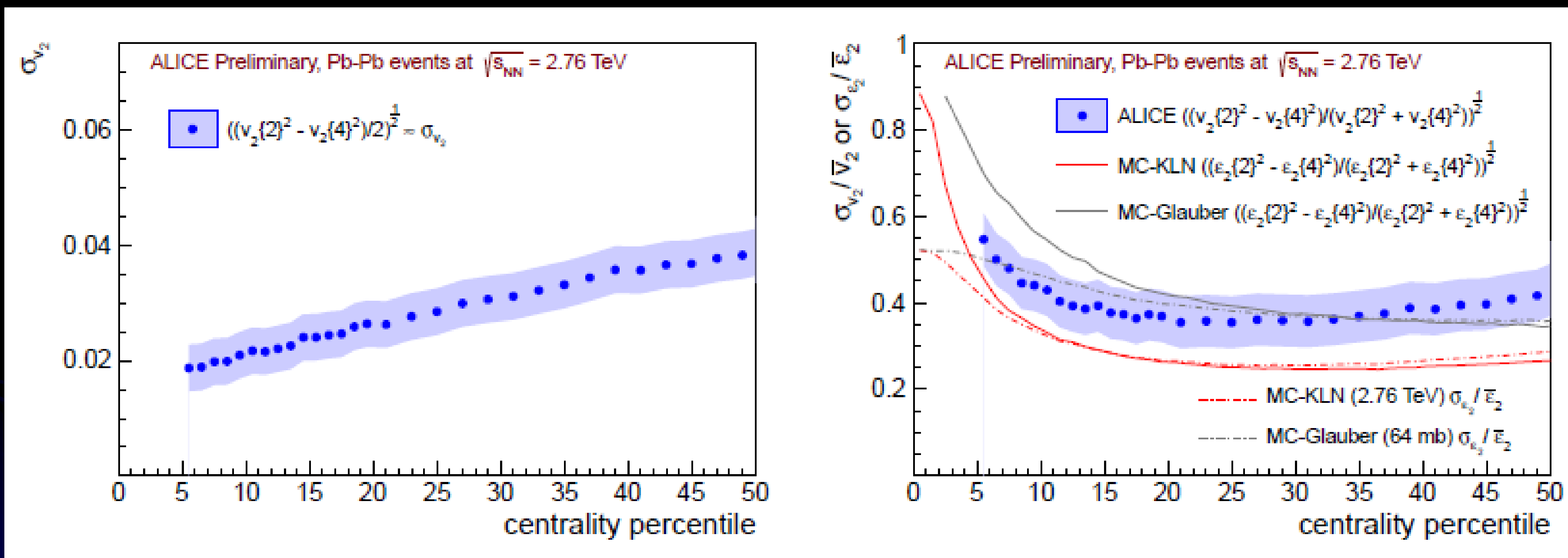


Within this model overall magnitude of v_2 and v_3 seems to be fine, but the details of p_t dependence are not well described

- More quantitative statement: The magnitude of $v_2(p_t)$ is described better with eta/s = 0, while for v_3(pt) eta/s = 0.08 provides a better description
- This model fails to describe well v_2 and v_3 simultaneously
- Produced matter in Pb-Pb collisions at LHC continues to behave as a nearly perfect liquid

ALICE fluctuations

http://arxiv.org/abs/arXiv:1106.6284



Similar values for relative flow fluctuations at LHC and at RHIC

QC{5}

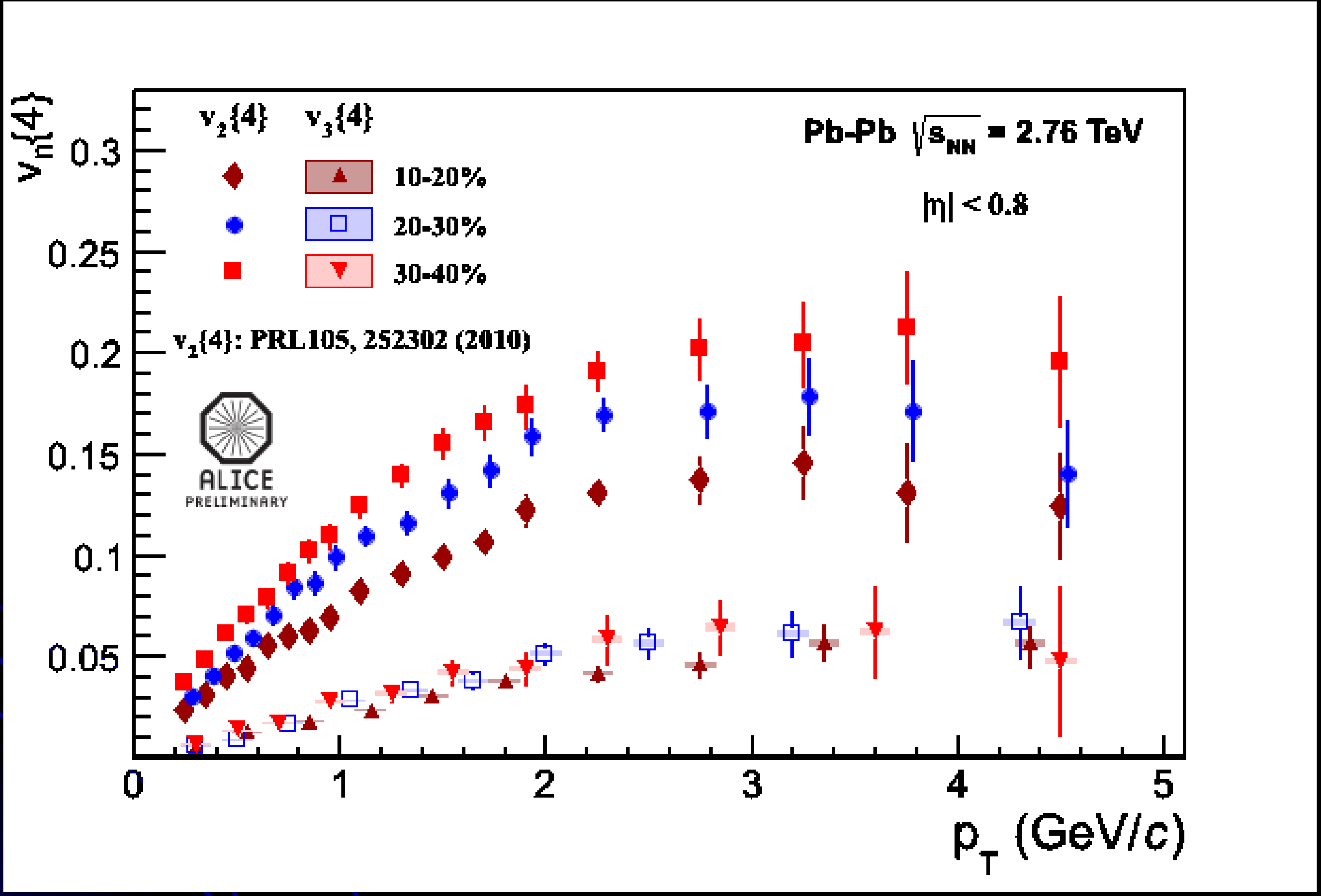
- Isolating the corresponding cumulants and quantifying the theoretical contributions:

$$
\begin{aligned}
QC\{5\} &= \langle\cos(2\phi_1+2\phi_2-2\phi_3-\phi_4-\phi_5)\rangle \\
&\quad -2\cdot\langle\cos(2\phi_1-\phi_2-\phi_3)\rangle\langle\cos(2\phi_1-2\phi_2)\rangle \\
&\overset{\text{in theory}}{=} -v_2^3 v_1^2\cos[2(\Psi_2-\Psi_1)]
\end{aligned}
$$

and:

$$
\begin{aligned}
QC\{5\} &= \langle\cos(3\phi_1+2\phi_2-2\phi_3-2\phi_4-\phi_5)\rangle \\
&\quad -2\cdot\langle\cos(3\phi_1-2\phi_2-\phi_3)\rangle\langle\cos(2\phi_1-2\phi_2)\rangle \\
&\overset{\text{in theory}}{=} -v_3 v_2^3 v_1\cos[3\Psi_3-2\Psi_2-\Psi_1)]
\end{aligned}
$$

$v_n\{4\}$ *vs* transverse momentum



- Strong centrality dependence of $v_2\{4\}$ => contribution from v_2 wrt. Ψ_{RP}
- Weak centrality dependence of $v_3\{4\}$ typical for pure flow fluctuations