Scale hierarchy in high-temperature QCD

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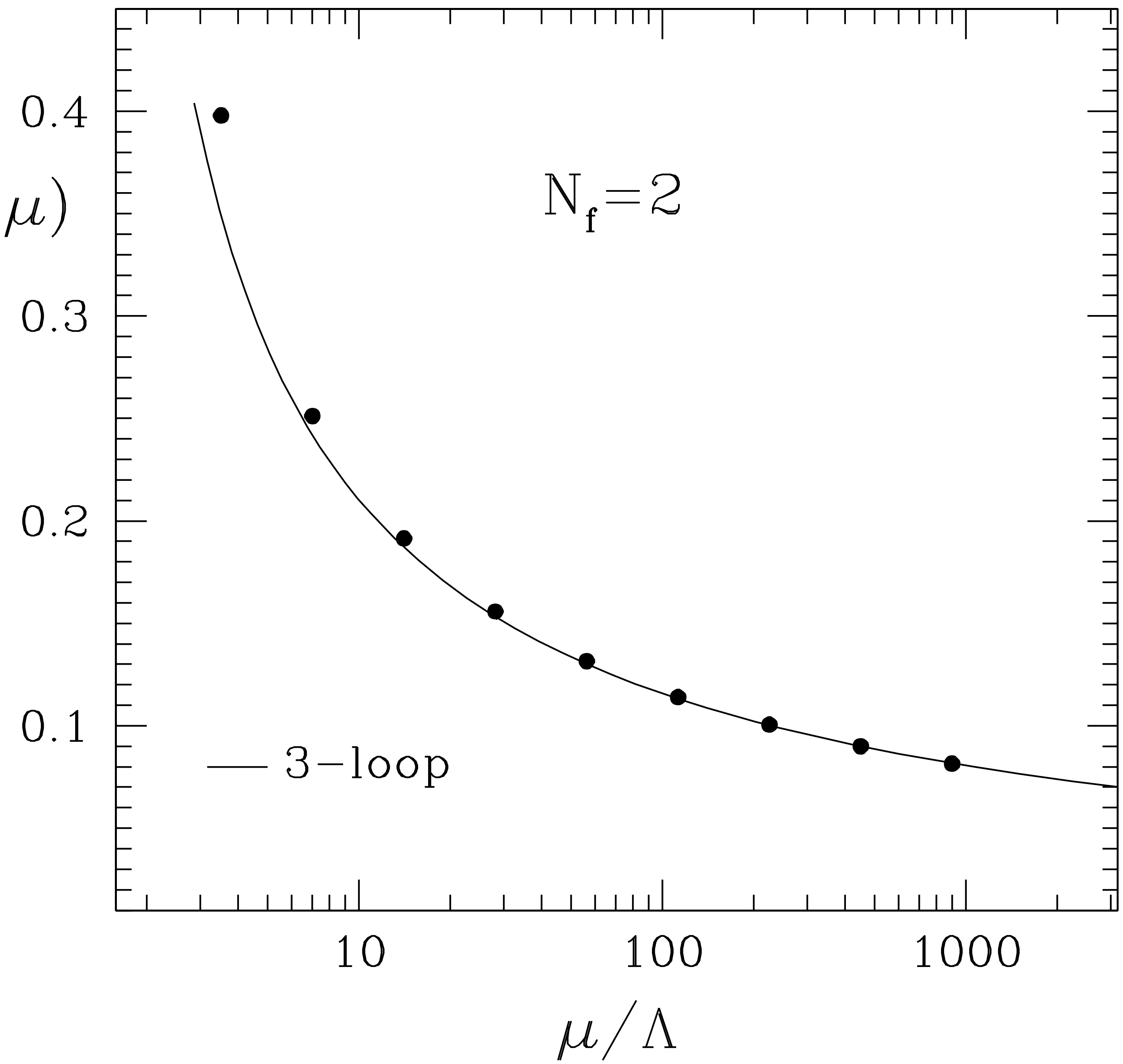
QCD is asymptotically free

$$\frac{g^2}{4\pi} = \alpha(\mu)$$



- High temperature: $g(T) \to 0$, deconfinement for $T > T_c$

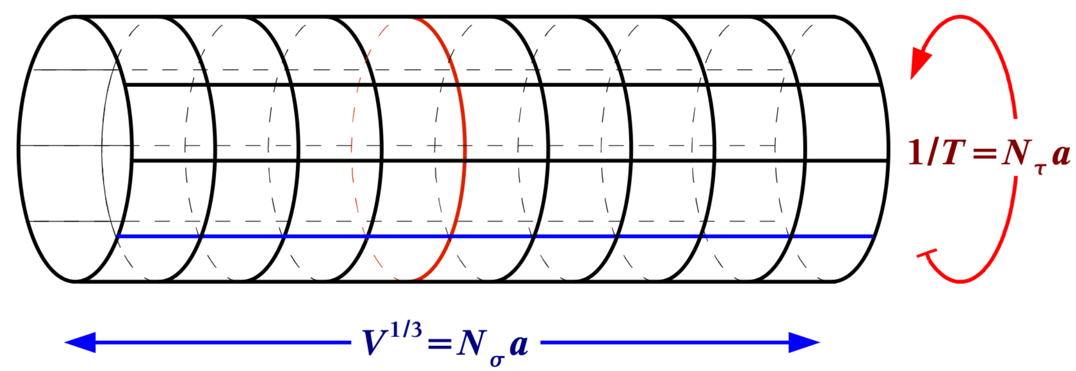
$\lim_{T\to\infty} g(T) = 0 \implies$ perturbative treatment OK

at "sufficiently large" T

- Dimensional reduction: $4d \to 3d$

- Thermal boundary conditions:

$\phi(x + 1/T) = \pm\phi(x)$ bosons / fermions



- Fourier decomposition: $\tilde{\phi}_n(x) = \int_0^{1/T} dt\ e^{i2\pi(n+\mathbf{q})t}\phi(x,t) \qquad q = \{0, 1/2\}$

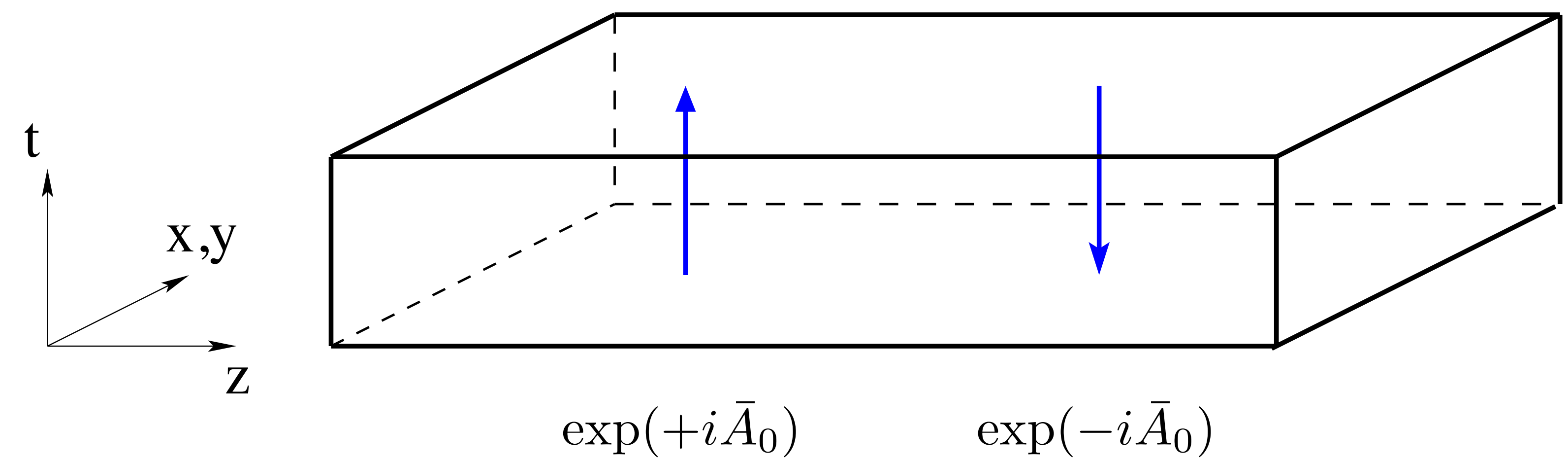
- Tower of states: $E_n^2 = |\vec{k}|^2 + [(2\pi T(n+\mathbf{q}))^2 + m^2] = |\vec{k}|^2 + (m_{\rm eff}^{3d})^2$

- $|\vec{k}| << T \longrightarrow$ static $(n = 0)$ modes for bosons; fermions decouple

Same for gauge fields

- Effective d.o.f.: $\bar{A}_i \equiv A_{i,n=0}$ and $\bar{A}_0 \equiv A_{0,n=0}$ or Polyakov loop

$$\bar{A}_0 \equiv A_{0,n=0}(\vec{x}) = \int_0^{1/T} dt\; A_0^a(\vec{x},t)\tau_a$$



- Effective action:

$$S^{4d} = \int d^3x\, dt\; \mathrm{Tr}F_{\mu\nu}^2 \qquad \longrightarrow \quad S_{\rm eff}^{3d} = \int d^3x\; [\mathrm{Tr}\bar{F}_{ij}^2 + m^2\bar{A}_0^2 + (D_i\bar{A}_0)^2 + \lambda\bar{A}_0^4 + \cdots]$$

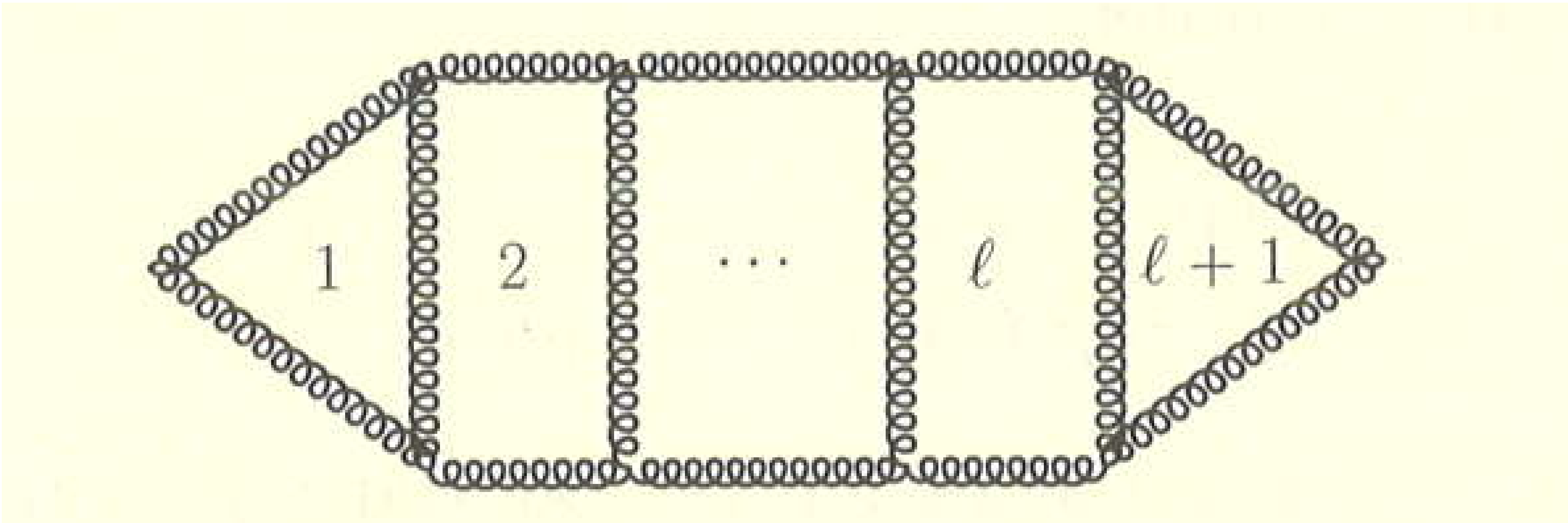
ie. 3d Yang-Mills + adjoint Higgs

3d couplings by integrating out non-static modes. Tree-level: $(g_{\rm eff}^{3d})^2 = g(T)T$

Note: 3d theory is confining, ie. non-perturbative in IR

Non-perturbative $\mathcal{O}(g^2T)$: Linde problem

- Consider perturbative expansion of free energy (pressure)



$(l+1)$ loops, $2l$ vertices, $3l$ propagators: $$g^{2l}\left(T\int d^3k\right)^{l+1} k^{2l}(k^2+m^2)^{-3l}$$

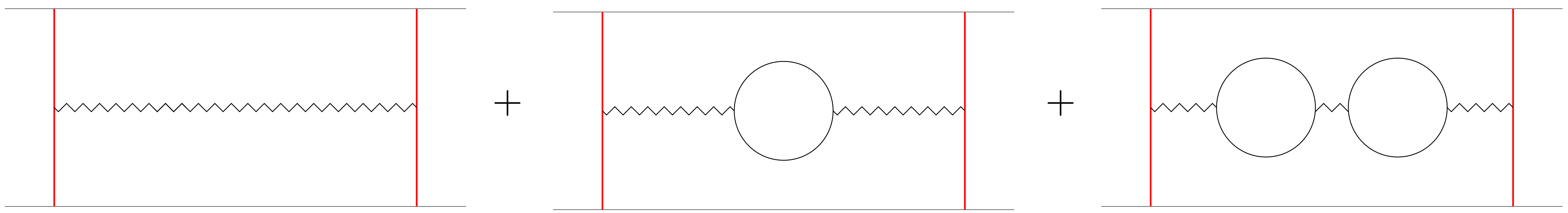
$m=0 \ \rightarrow \ \int dk\ k^{3l+2-4l}$ IR-divergent if $l \geq 3$ ie. non-perturbative at $\mathcal{O}(g^6)$

Divergence cured by non-perturbative mass $m_G \sim \mathcal{O}(g^2T)$ mass gap of 3d theory

(3d glueball)

Debye screening: QED

- QED: e^+e^- thermal pair creation screens static charges



$$+\cdots$$

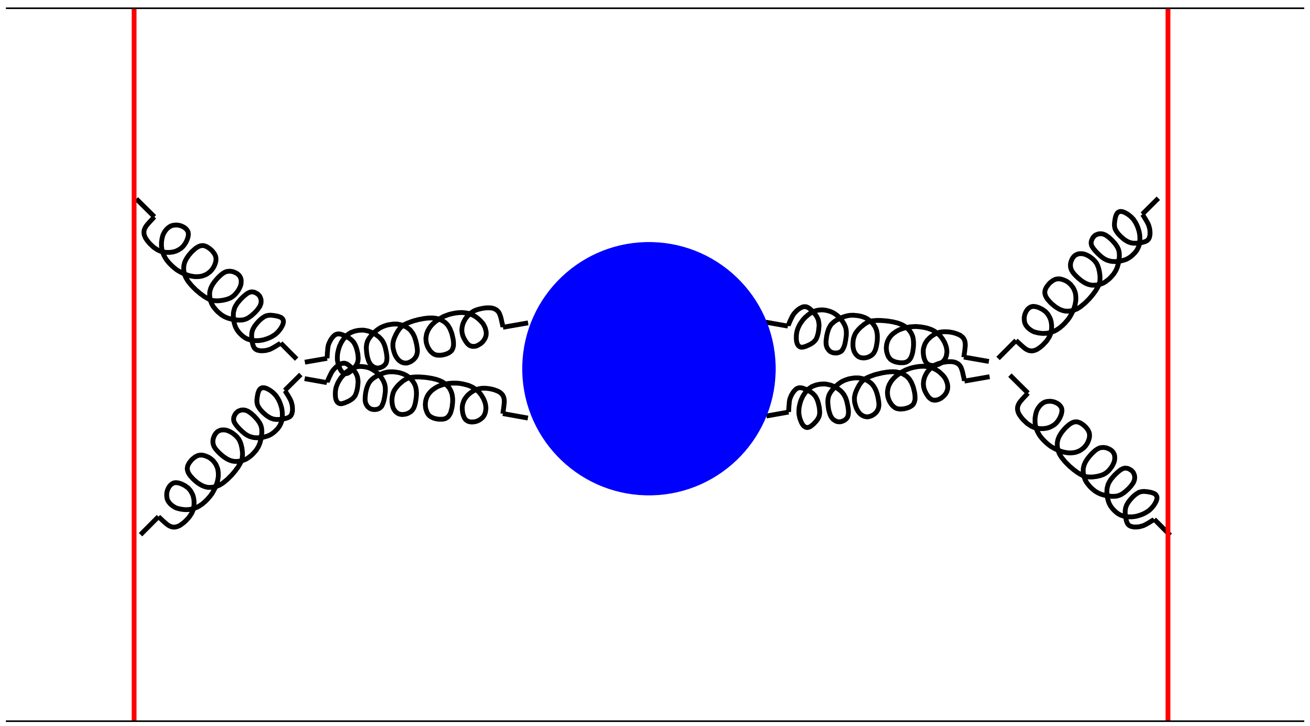
Resum all orders: $\sum_k \mathcal{O}(e^2)^k \to \mathcal{O}(e)$

Coulomb potential $\propto \frac{1}{r}$ at $T = 0$ $\longrightarrow$ Yukawa $\propto \frac{\exp(-m_D r)}{r}$ at $T > 0$

$$m_D = \frac{eT}{\sqrt{3}}(1 + \mathcal{O}(e^2))$$

Debye screening: QCD

- QCD: $\mathrm{Tr(Polyakov\ loop)}$ is color singlet, so at least 2 gluons emitted
- to lowest order, similar to QED: $m_D = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$
- higher order: 3- and 4-gluon vertex couples to $3d$ glueball: $m_G \sim \mathcal{O}(g^2 T)$



Confining potential $\propto \sigma r$ at $T = 0$ $\longrightarrow$ $\propto \frac{\exp(-m_D r)}{r}$ then $r \nearrow$ $\propto \frac{\exp(-m_G r)}{r}$

Recap: 3 scales in high-T QCD

- "hard" scale $2\pi T$ non-static modes
- "soft" scale $g(T)T$ Debye mass ("electric")
- "ultrasoft" scale $g^2(T)T$ 3d glueball mass ("magnetic")

Hierarchy as $T \to \infty, \quad g(T) \to 0$

- integrate out "hard" scale $\longrightarrow$ effective theory EQCD (electric)

  3d Yang-Mills + adjoint Higgs

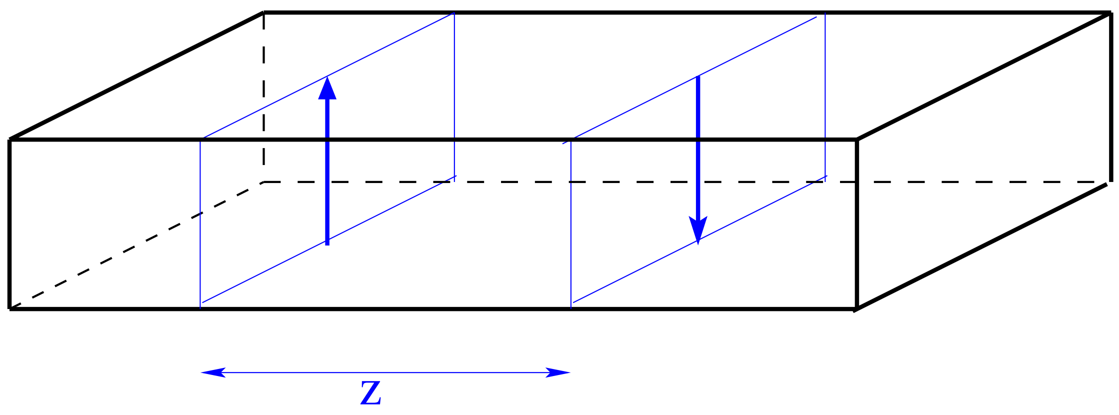
- integrate out "soft" scale $\longrightarrow$ effective theory MQCD (magnetic)

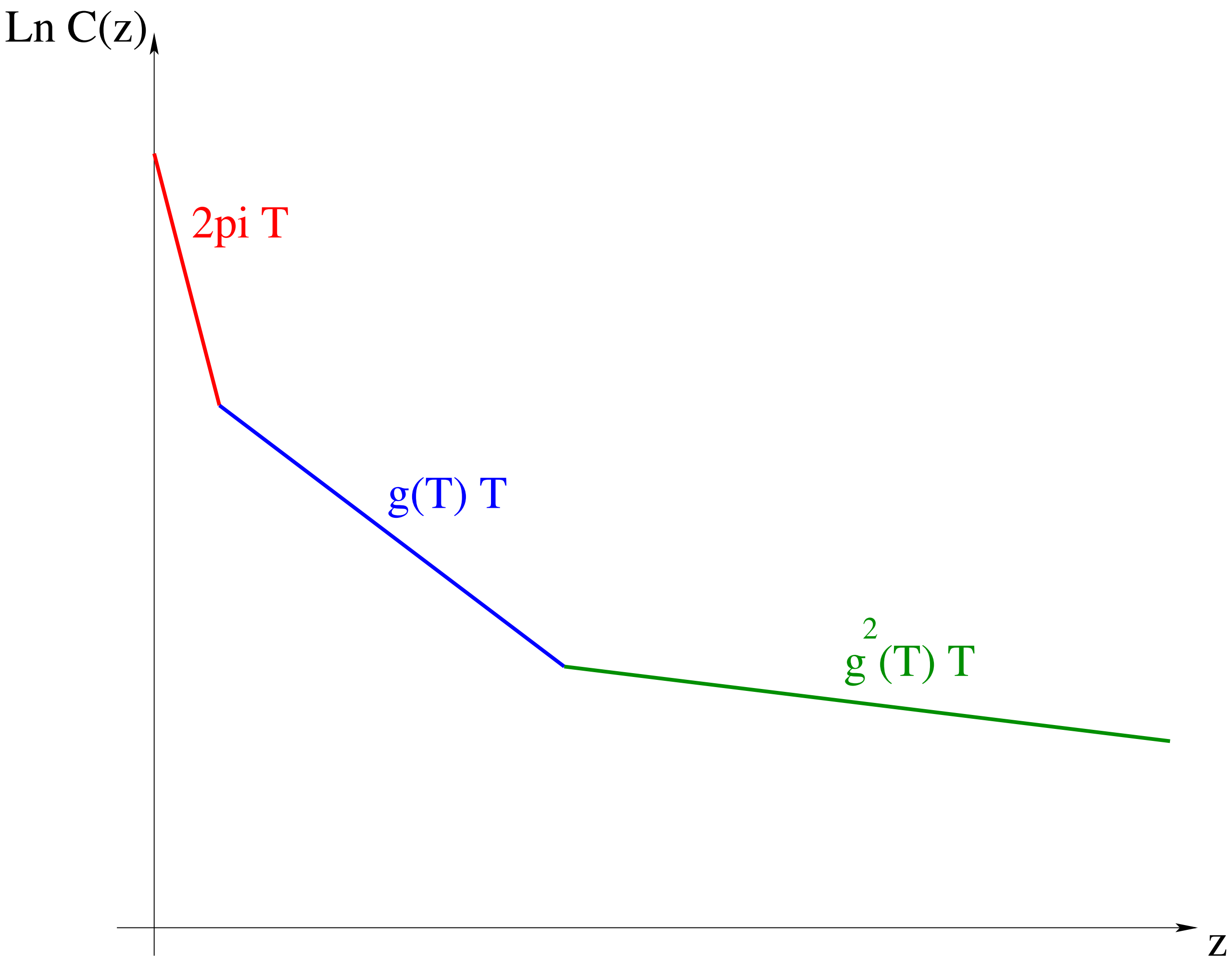
  3d Yang-Mills

How high should T be ?

$$g^2 T \ll gT \ll 2\pi T$$

Look for 3 scales in decay rate of correlator of Polyakov loop:

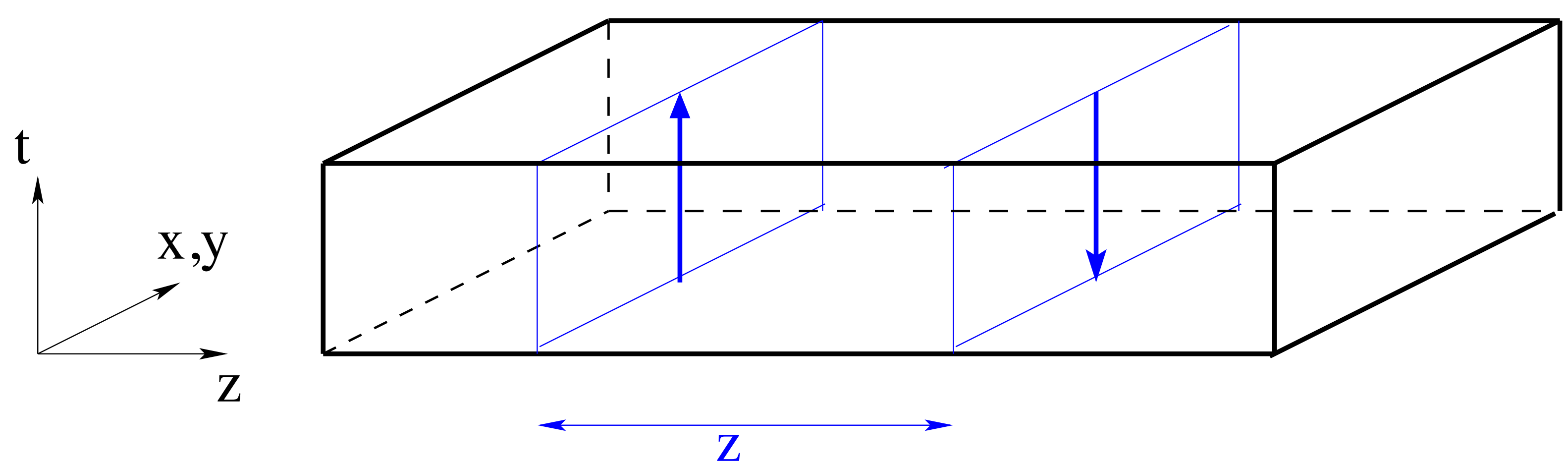




Symmetries

- Reversal of Euclidean time "R": $t \to -t, \quad A_0 \to -A_0, \; \mathrm{TrPol} \to \mathrm{TrPol}^\dagger$

- MQCD (3d effective theory) is "R"-even $\quad\to\quad$ scale g^2T in "R"-even observables only



$$\langle [\sum_{xy} \mathrm{ImTrPol}(x,y,0)][\sum_{xy} \mathrm{ImTrPol}(x,y,z)] \rangle$$ projects on $\{2\pi T, gT\}$

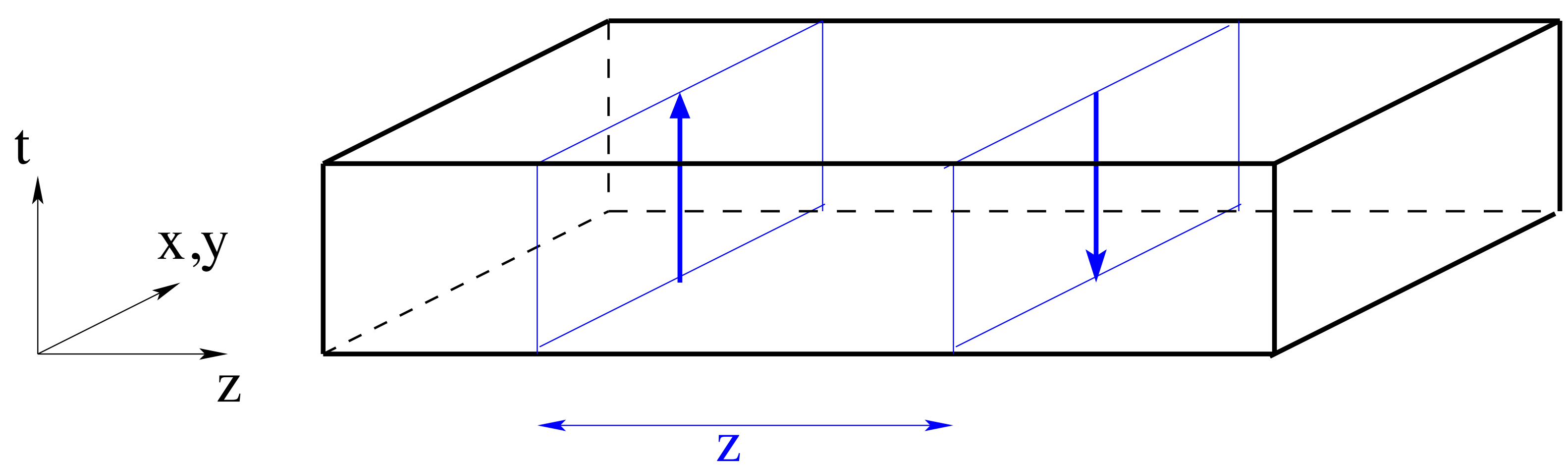
$$\langle [\sum_{xy} \mathrm{ReTrPol}(x,y,0)][\sum_{xy} \mathrm{ReTrPol}(x,y,z)] \rangle_c$$ projects on $\{2\pi T, gT, g^2T\}$

Expectations

- Polyakov loop: $\mathrm{Pol} = \exp(i\bar{A}_0) \underset{\bar{A}_0 \ll 1}{\approx} 1 + i\bar{A}_0 - \frac{1}{2}\bar{A}_0^2 - \frac{i}{6}\bar{A}_0^3 + \cdots$ with $\mathrm{Tr}\bar{A}_0 = 0$

- At $\mathcal{O}(gT)$: $\mathrm{ImTrPol} \sim \bar{A}_0^3 \longrightarrow \text{mass } 3m_0$

  $\mathrm{ReTrPol} \sim \bar{A}_0^2 \longrightarrow \text{mass } 2m_0$



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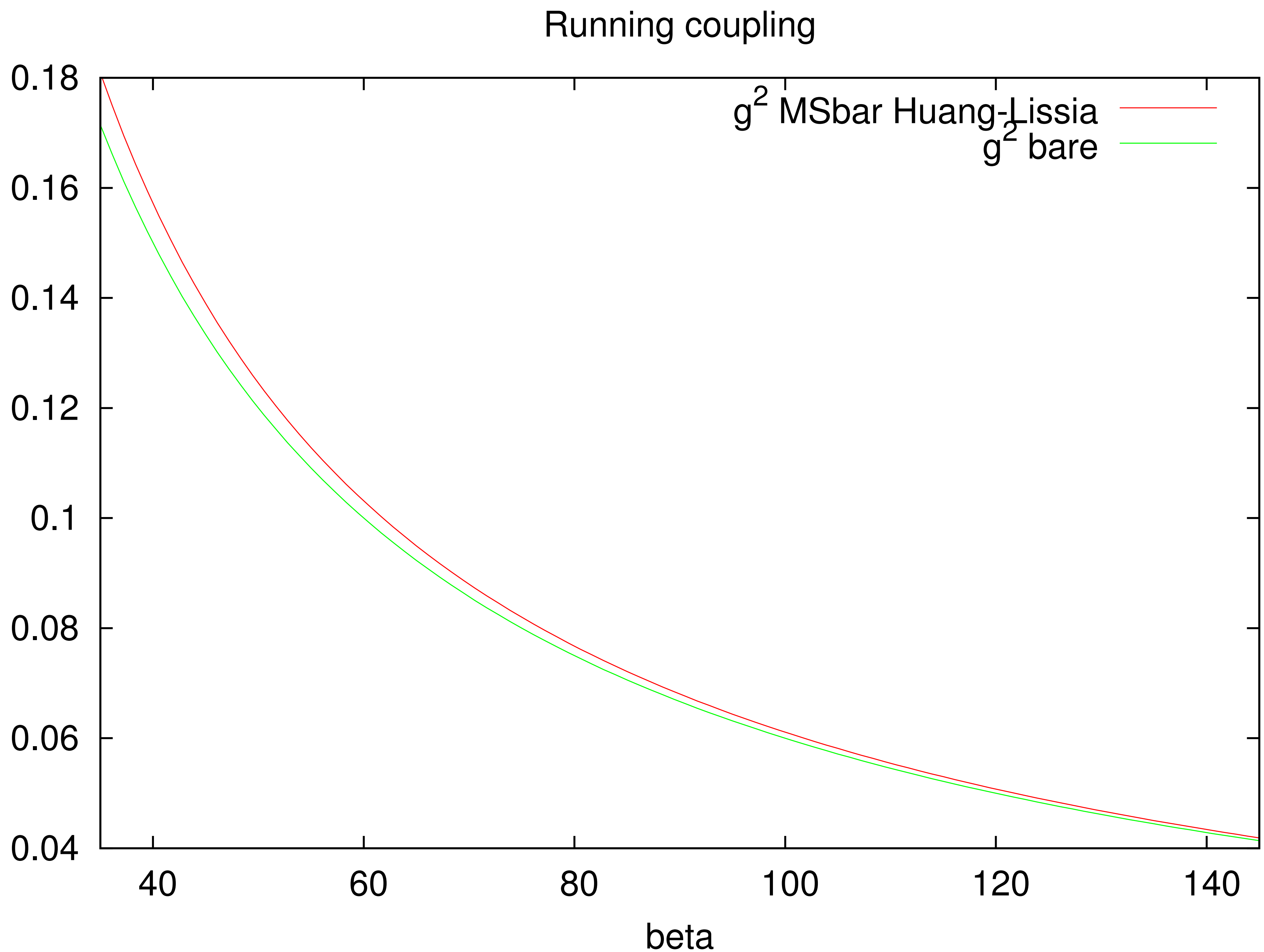
Results: $SU(3), 24^2 \times (24)80 \times 2 \text{ lattice}, \beta = \frac{6}{g^2} \in [40..140]$



g chosen suitably small (0.2 - 0.4) for a clear scale hierarchy

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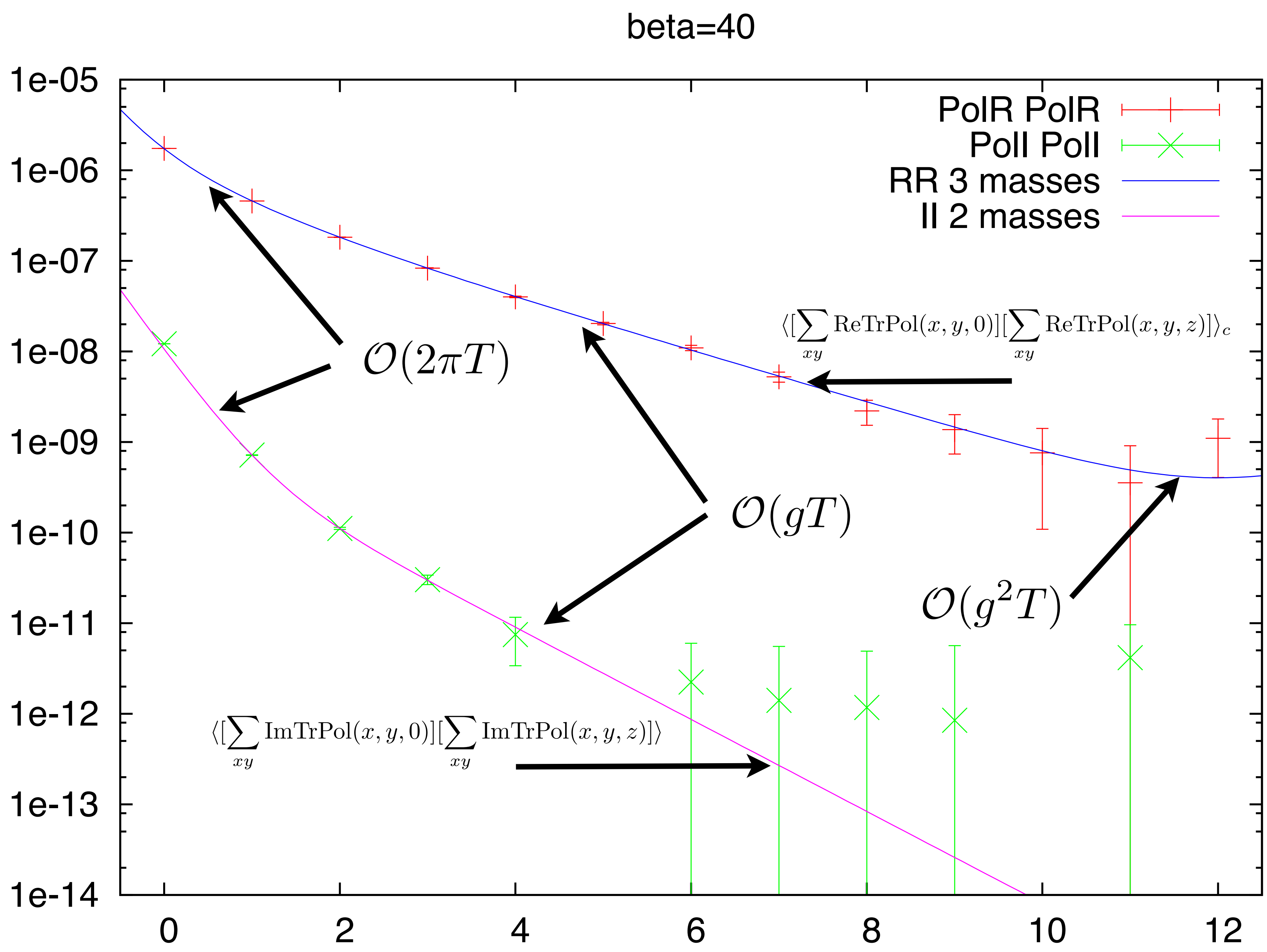

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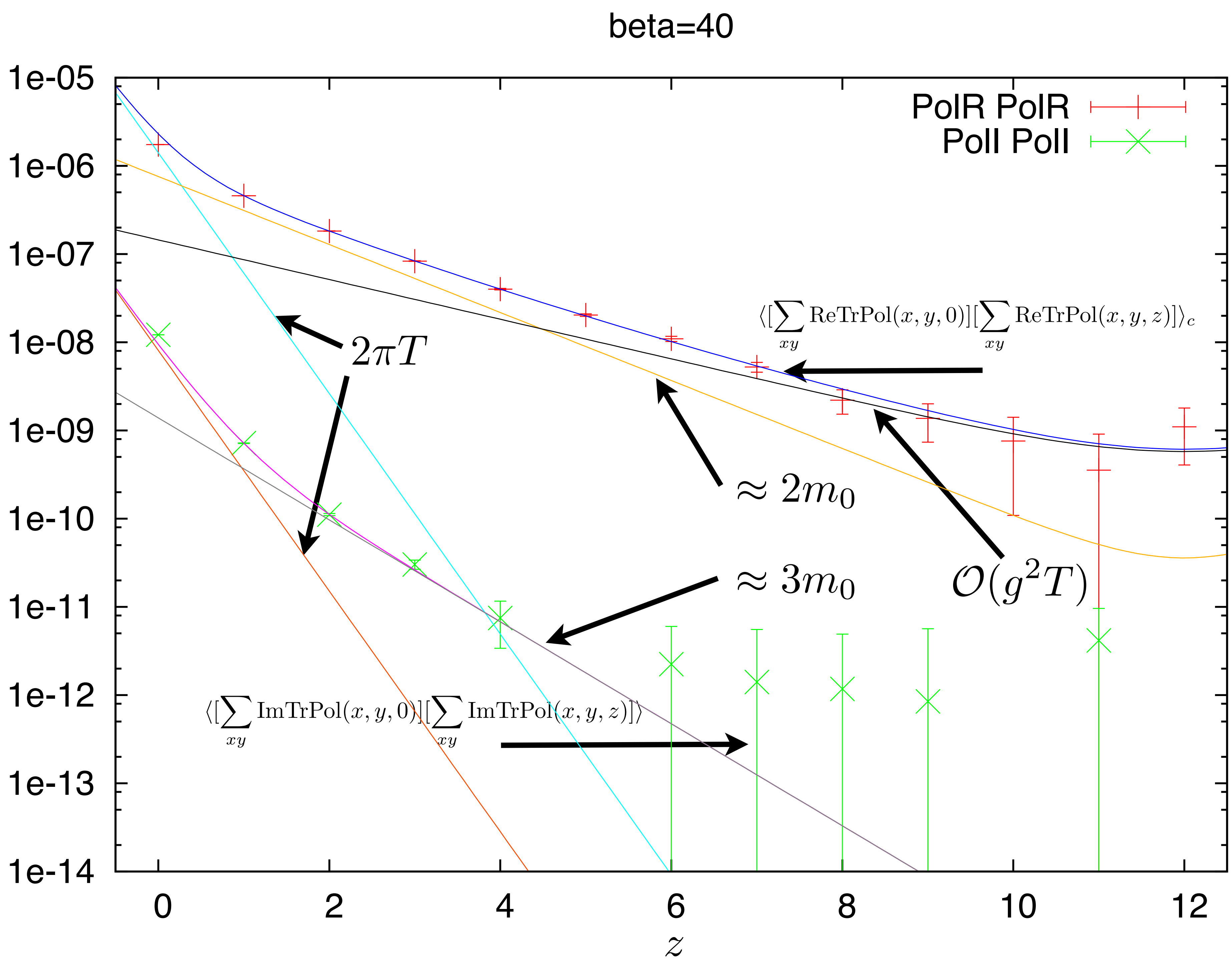

Fit starts at $z = 2$ but ansatz describes $z = 0, 1$ data very well

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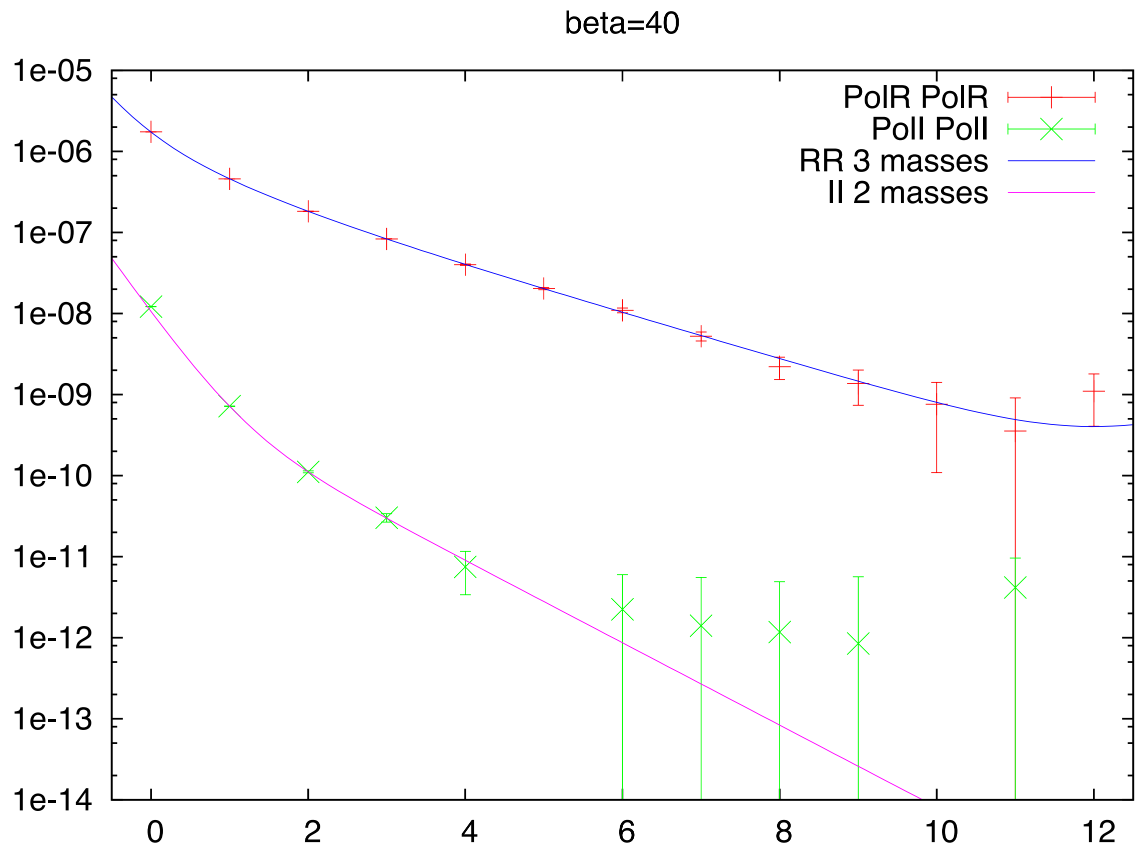
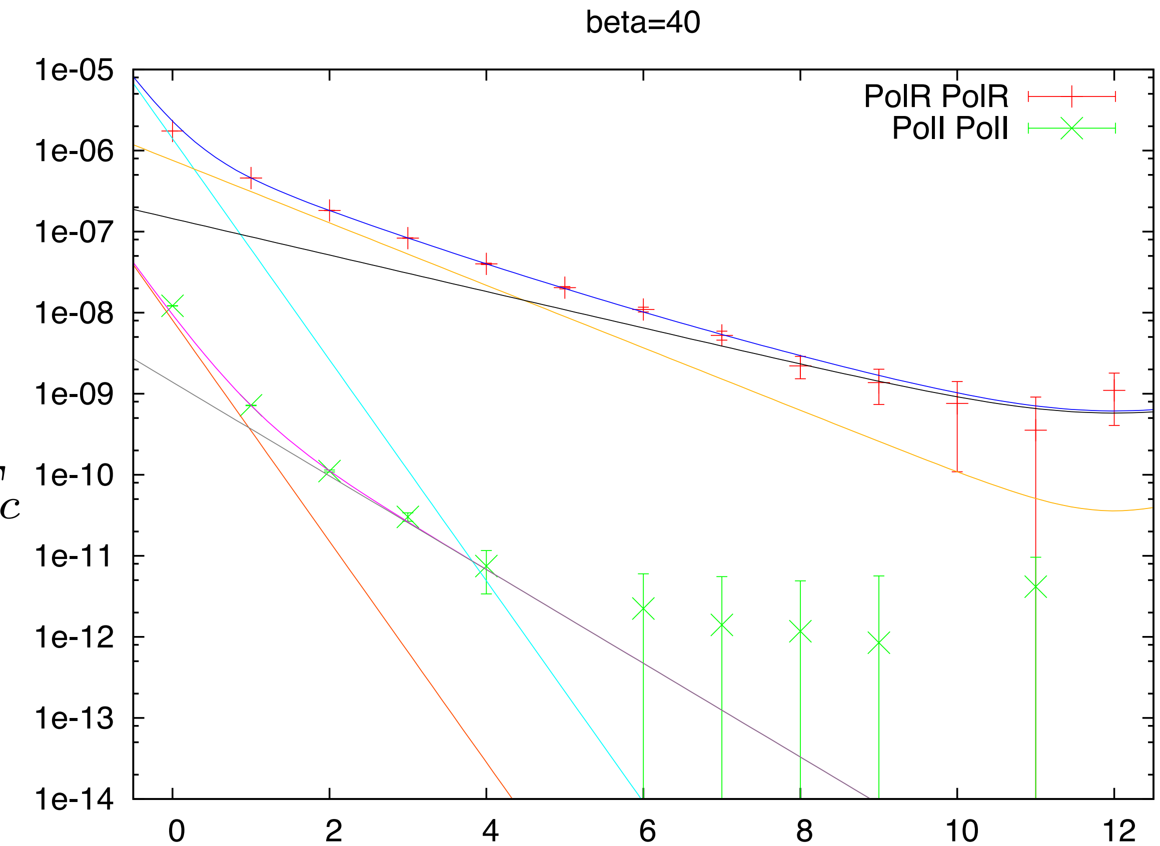
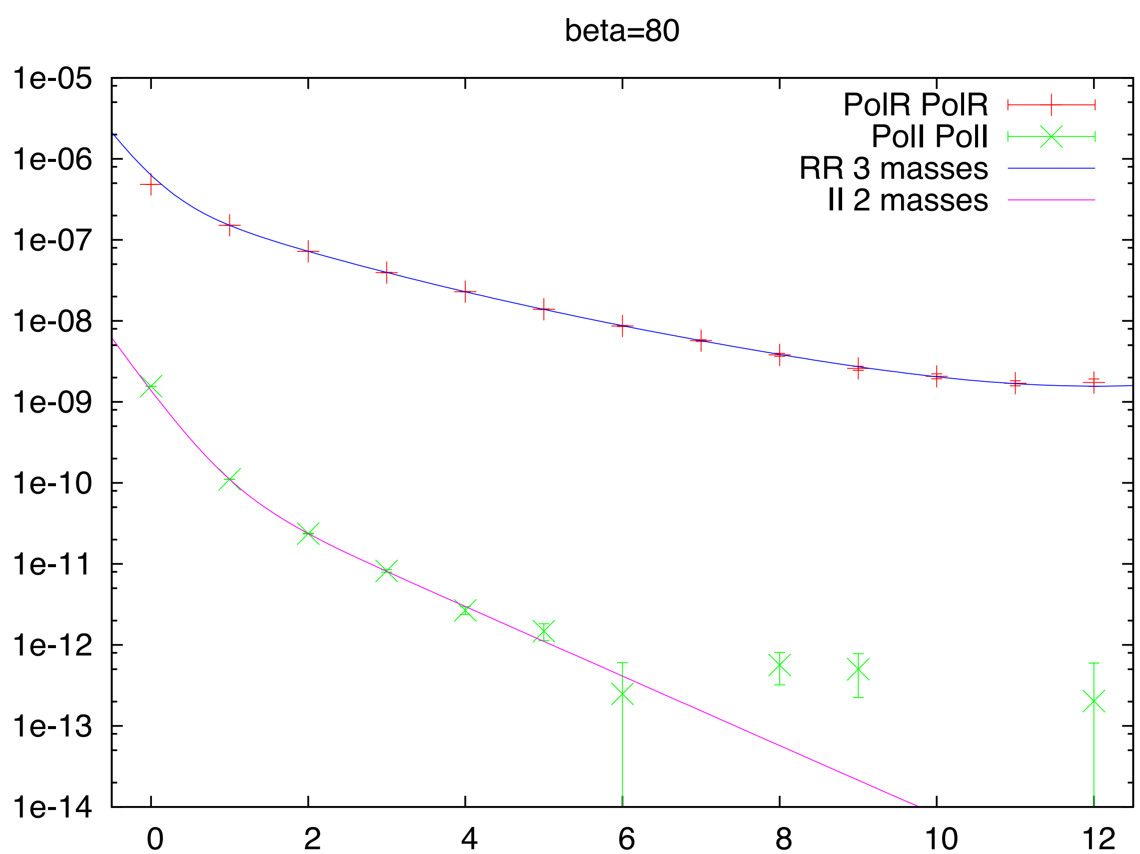


$\beta = 40$

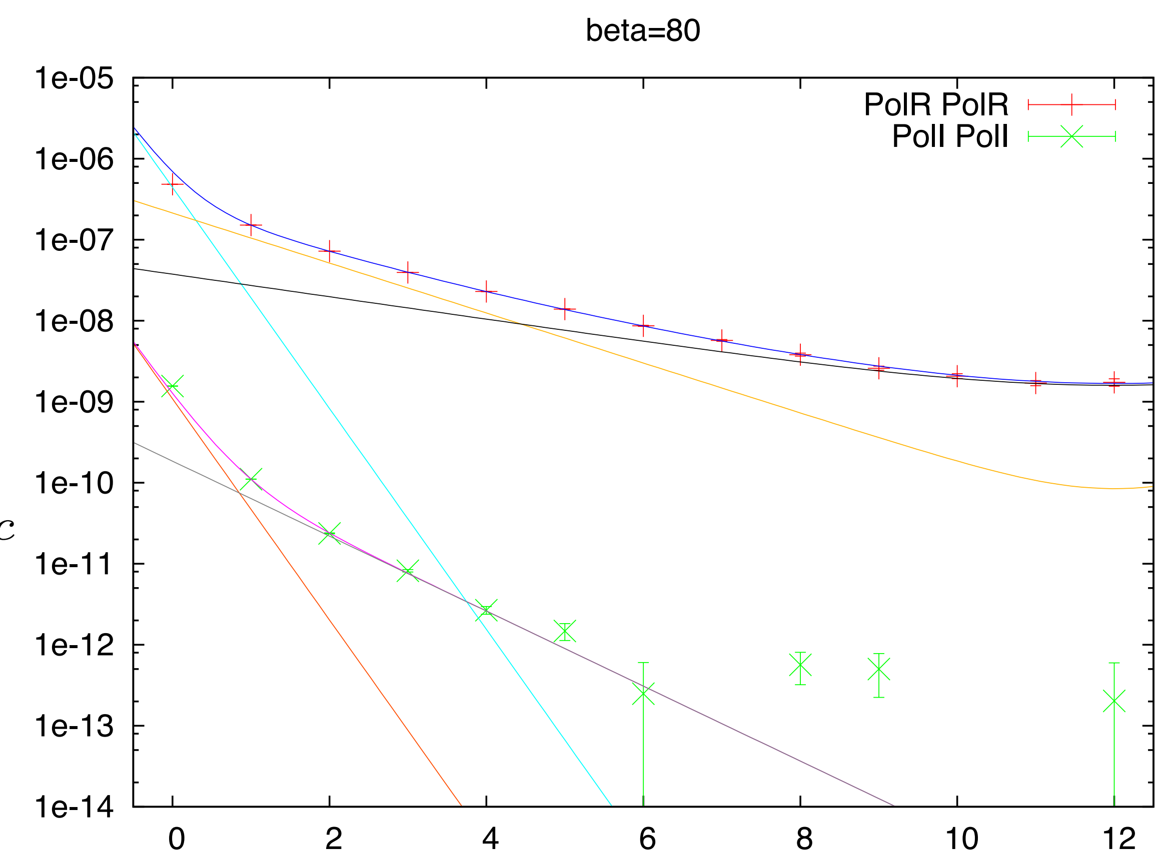
$T \sim 10^{19} T_c$





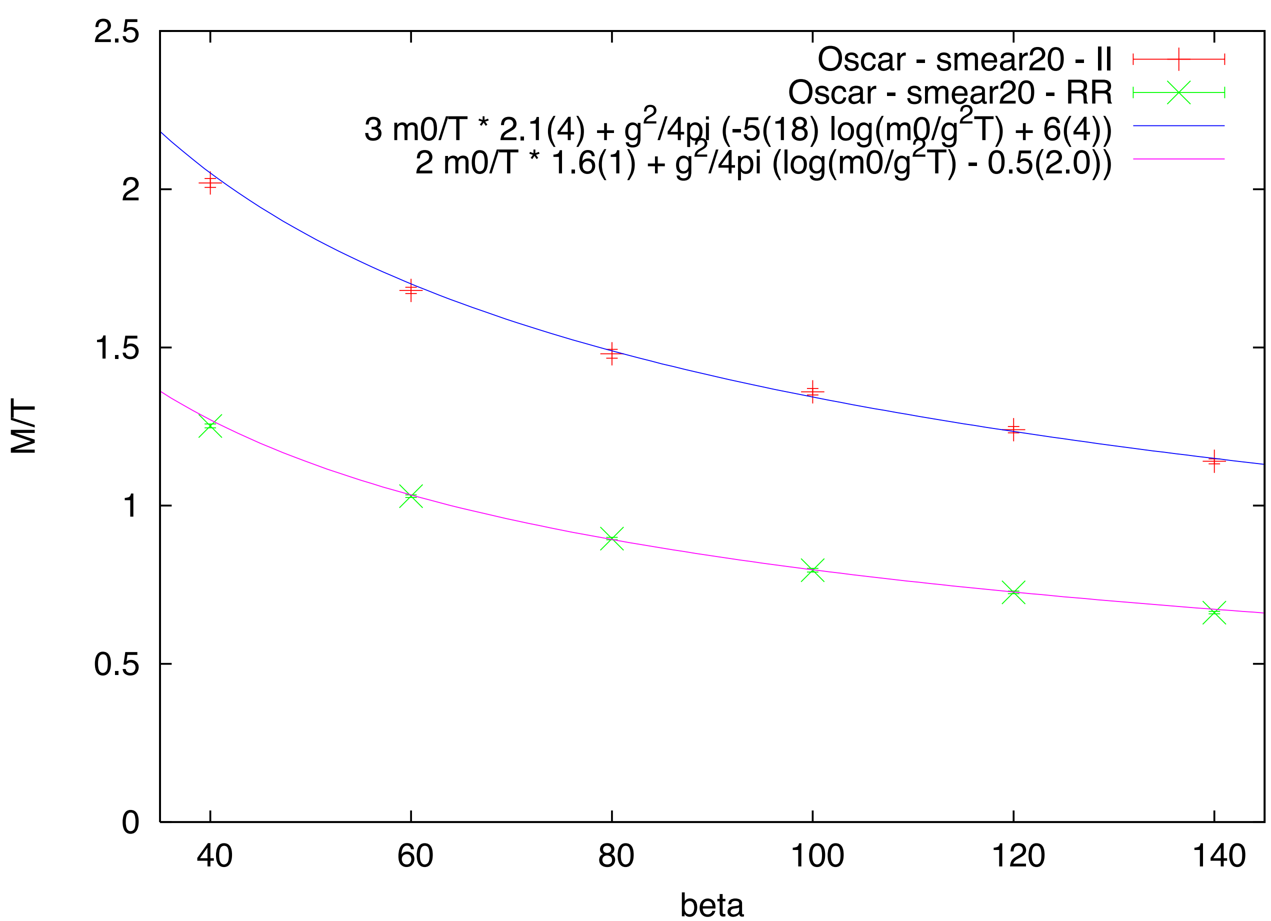
$\beta = 80$

$T \sim 10^{39} T_c$



Measured masses

- Expected (II): $\frac{M(\mathrm{Tr}[A_0^3])}{T} = 3\frac{m_0}{T} + \frac{g^2 N_c}{4\pi}\left(b_3 \log\frac{m_0}{g^2 T} + c_3\right),$ b_3, c_3 non-perturbative (large)
- And (RR): $\frac{M(\mathrm{Tr}[A_0^2])}{T} = 2\frac{m_0}{T} + \frac{g^2 N_c}{4\pi}\left(\log\frac{m_0}{g^2 T} + c_2\right),$ c_2 non-perturbative (large)
- Fitted:
  - ★ b_3, c_3, c_2 all $\mathcal{O}(1)$
  - ★ Need lattice correction $\mathcal{O}(\frac{1}{N_t^2})$ on m_0 (coeff. 1.6 -- 2.1)



Note that $\frac{M}{T} \gtrsim 1$

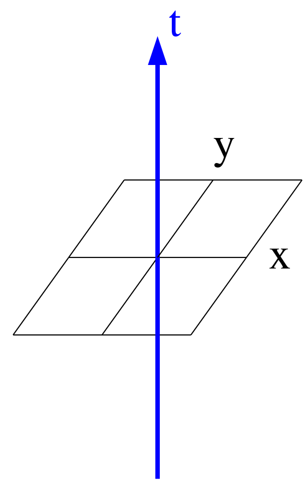
No clear scale hierarchy even at $T \sim 10^{30} T_c$

$\mathcal{O}(g^2 T)$ masses not shown (too noisy)

To do:

- Measure glueball mass $\mathcal{O}(g^2 T)$

- Measure mass from $\mathrm{ReTr}[A_0 F_{12}] \sim m_0$



$$\frac{M(\mathrm{Tr}[A_0 F_{12}])}{T} = \frac{m_0}{T} + \frac{g^2 N_c}{4\pi}\left(\log\frac{m_0}{g^2 T} + c_1\right)$$

- Extrapolate to continuum: $N_t = 2, 3, 4, \cdots \longrightarrow$ check m_0

- Then, obtain non-perturbative coeffs $\{b_3, c_1, c_2, c_3\}$

Lesson: success of effective 3d description at $T \gtrsim 3 - 10\ T_c$

does not necessarily imply scale hierarchy