Odderon in Gauge/String Duality

Marko Djurić

University of Porto

work with Richard Brower and Chung-I Tan

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Outline

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- Recently, there has been a renewed interest in both the Pomeron and the Odderon. These are the leading Regge trajectories with the quantum numbers of the vacuum.

- We can study these objects at both weak (using pQCD) and strong coupling (using gauge/string duality).

- The recent focus has been mostly on the question of what the intercept of these objects is. By calculating the intercepts to higher order from both the weak and the strong coupling side we can try to interpolate to the non-perturbative region from both sides. This can be a very important test of the gauge/string duality.

- Using both methods, the Odderon has two solutions, one fixed at 1 and one slightly below 1. An additional question for the Odderon is whether the first solution is exactly equal to one to all order.

- We will give an introduction to what the Pomeron and Odderon are, and show how they arise in string theory on AdS, focusing on the Odderon, and discuss possible applications and extensions.

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- Consider $2 \rightarrow 2$ scattering.
- Work in the Regge limit

$$s \gg t$$

- We can expand the amplitude into partial waves

$$A(s, t) = 16\pi \sum_{j=0}^{\infty} (2j + 1) A_j(t) P_j(\cos \theta_t),$$

- In the Regge limit,

$$P_j(1 + \frac{2s}{t}) \rightarrow \frac{\Gamma(2j+1)}{\Gamma^2(j+1)} (\frac{s}{2t})^j \sim f(t) s^j.$$

- If exchanged particle has spin j

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- Optical theorem:

$$\sigma_{tot} = \frac{1}{s} \Im A(s, 0)$$

- Experimentally

$$\sigma_{tot} \sim s^{0.08}$$

- The amplitude will depend on an infinite number of exchanged particles.

- We can continue the amplitude into the complex plane

$$A^{\pm}(j, t) = \begin{cases} A_j^+(t) & j\,even \\ A_j^-(t) & j\,odd \end{cases}$$

- $A(j, t)$ will have as singularities poles at integer j for fixed t. As we change t, the position of the pole will change, leading to a trajectory

$$j = \alpha(t)$$

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- We can write $A(s, t)$ as a contour integral in the complex plane

$$A^\pm(s, t) = 8\pi \sum_{j=0}^\infty (2j + 1) A_j^\pm(t)(P_j(z_t) \pm P_j(-z_t))$$

$$= 8\pi i \int_C dj(2j + 1) A^\pm(j, t) \frac{P(j, -z_t) \pm P(j, z_t)}{\sin(\pi j)}$$

- We next deform the contour C to a contour C' parallel to imaginary axis and real part $-1/2$

$$A^\pm(s, t) = -16\pi^2 \sum_i \frac{(2\alpha_i^\pm(t) + 1)\beta_i^\pm(t)}{\sin(\pi\alpha_i^\pm(t))}(P(\alpha_i^\pm(t), -z_t) \pm P(\alpha_i^\pm(t), z_t))$$

- $\alpha_i^\pm(t)$ is the position of the pole in the j plane.
- Take advantage of the asymptotic form of the Legendre polynomials

$$\sqrt{\pi} P(j, z) \sim \frac{\Gamma(j + 1/2)}{\Gamma(j + 1)}(2z)^j \quad \Re j \geq -1/2$$

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- This will give us a sum in powers of s. At high energy, we can keep just the leading term

$$A^{\pm}(s,t) \sim (1 \pm e^{-i\pi\alpha^{\pm}(t)})\beta(t)(\frac{s}{s_0})^{\alpha^{\pm}(t)}.$$

- $\alpha(t)$ is the term with the largest value of $\Re\alpha_i(t)$
- Amplitude corresponds to an exchange of a whole trajectory of particles $\alpha^{\pm}(t)$.
- Equivalently, we are exchanging a 'Reggeon' - object with spin $\alpha^{\pm}(t)$.

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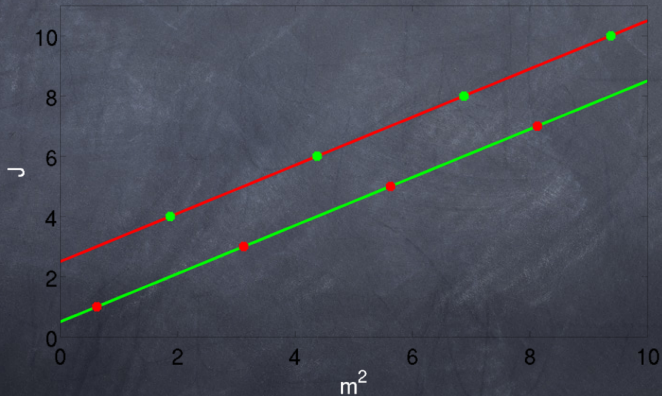

Figure: Regge trajectories.

Pomeron

- Look again at the factor

$$1 \pm e^{-i\pi\alpha^{\pm}(t)}$$

- When $\alpha^+(t)$ is odd, $1 + e^{-i\pi\alpha^+(t)} = 0$, and similarly when $\alpha^-(t)$ is even, $1 - e^{-i\pi\alpha^-(t)} = 0$.

- Two sets of trajectories, one with only particles with even non-negative spin, and one with particles with odd positive spin.

- For trajectories with that don't have the quantum numbers of the vacuum, $\alpha(0) < 1$, leading to vanishing σ_{tot}

- The leading Reggeon which has the quantum numbers of the vacuum, $C = +1$ and $I = 0$, is known as the Pomeron.

- The intercept $\alpha(0) > 1$ leading to non-vanishing

$$\sigma_{tot} \sim s^{\alpha(0)-1}$$

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Odderon

- Similarly, the Odderon is the leading negative signature trajectory
- Its quantum numbers are

$$I = 0, \quad C = -1$$

- Its intercept is either

$$\alpha(0) = 1 \ \text{ or } \ \alpha(0) < 1$$

- It is more elusive experimentally.
- We will revisit the Pomeron and Odderon from string theory.
- The Pomeron is very important - the leading exchange in total cross sections.

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We will now turn to using the *AdS/CFT* correspondence to study strong coupling. The correspondence relates operators in $\mathcal{N} = 4\,SYM$ to states in string theory on $AdS_5 \times S^5$. It is valid for large 't Hooft coupling λ.

- We will work with the metric

$$ds^2 = \frac{R^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + R^2 d\Omega_5$$

- In the hardwall model, we have a cut-off

$$0 < z < z_0$$

- The cutoff position will roughly correspond to

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- Let us write the scattering amplitude

$$A_{W_L W_R} = \int d^2 w \langle W_R w^{L_0 - 2} \bar{w}^{\tilde{L}_0 - 2} W_L \rangle$$

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$$A_{W_L W_R} = \langle W_R \mathcal{V}(T) \rangle \langle \mathcal{V}(T) W_L \rangle$$

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$$\langle \mathcal{W}_R \, e^{ikX} \eta_{MN} \partial X^M \overline{\partial} X^N \rangle \simeq O(1) \,.$$

- For $t \neq 0$ the leading term picks up a common factor of $(\partial X^- \overline{\partial} X^-)^{\frac{\alpha' t}{4}}$ on the left

$$[(h_{MN} + B_{MN}) \partial X^M \overline{\partial} X^N]^{1 + \frac{\alpha' t}{4}}$$
$$\simeq \left[h_{--}(\partial X^- \overline{\partial} X^-) + B_{-\perp}(\partial X^- \overline{\partial} X^\perp - \partial X^\perp \overline{\partial} X^-) \right]$$
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- These two terms will give us the Pomeron and Odderon vertex operators.

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- Note that $\mathcal{V}_P$ and $\mathcal{V}_O$ have opposite symmetry under $\partial \longleftrightarrow \bar{\partial}$, even and odd for $\mathcal{V}_P$ and $\mathcal{V}_O$ respectively.

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Odderon in AdS

- For vertex operators in *AdS* we replace them by the flat space vertex operators multiplied by $\phi(r)$. For the Odderon

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$$\mathcal{V}_O(j, \pm) = (\partial X^\pm \bar{\partial} X^\perp - \partial X^\perp \bar{\partial} X^\pm)(\partial X^\pm \bar{\partial} X^\pm)^{\frac{j-1}{2}} e^{\mp ik \cdot X} \phi_{\pm j \perp}(r).$$

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- where $\Delta_{O,j} = (r/R)^{-(j-1)}(\Delta_{O,1})(r/R)^{(j-1)}$. To determine the differential operator for Odderon, $\Delta_{O,1}$, we can match the EOM at $j = 1$, appropriate in the infinite λ limit.

- In the case of the Odderon, in the supergravity limit we have two equations

$$(\Box_{Maxwell} - (k+4)^2)B^{(1)}_{IJ} = 0 \, , \quad (\Box_{Maxwell} - k^2)B^{(2)}_{IJ} = 0$$

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$$[j - 1 - \frac{\alpha' t}{2} e^{-2u} - \frac{1}{2\sqrt{\lambda}}(\partial_u^2 - m_{AdS}^2)]\phi_{\pm\perp}(u) = 0$$

- This can be solved

$$\mathcal{V}_O(j, \nu, k, \pm) \sim$$
$$(\partial X^{\pm} \bar{\partial} X^{\perp} - \partial X^{\perp} \bar{\partial} X^{\pm})(\partial X^{\pm} \bar{\partial} X^{\pm})^{\frac{j-1}{2}} e^{\mp ik \cdot X} e^{(j-1)u} K_{\pm 2i\nu}(|t|^{1/2} e^{-u})$$

- and for the amplitude we would have

$$\mathcal{T}^{(-)} \sim \int \frac{dj}{2\pi i} \int \frac{d\nu\, \nu \sinh 2\pi\nu}{\pi} \frac{\Pi_-(j)\, s^j}{j - j_0^{(-)} + \mathcal{D}\nu^2}$$
$$\times \langle \mathcal{W}_{R0} \mathcal{V}_O(j, \nu, k, -) \rangle \langle \mathcal{V}_O(j, \nu, k, +) \mathcal{W}_{L0} \rangle$$

- with $j_0^{(-)}$ given by

$$j_0^{(-)} = 1 - m_{AdS}^2/2\sqrt{\lambda} + O(1/\lambda)\,.$$

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Summary

We can summarize both the strong and weak coupling results.

	Weak Coupling	Strong Coupling
$C = +1$	$j_0^{(+)} = 1 + (\ln 2)\,\lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C = -1$	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\,\lambda/\pi + O(\lambda^2)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$
	$j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

Outline

- The most direct application is to calculate scattering amplitudes. In the case of $2 \to 2$ scattering, the above expressions can be simplified.
- We can write the scattering amplitude as

$$A(s,t) = 2s \int d^2 l e^{-i\mathbf{l}_\perp \cdot \mathbf{q}_\perp} \int dz d\bar{z} P_{13}(z) P_{24}(\bar{z}) \chi(s, l, z, \bar{z})$$

- P_{13} and P_{24} are the products of incoming and outgoing scattering states, and χ is the exchange kernel.
- For the Pomeron:

$$\chi(\tau, L) = (\cot(\frac{\pi\rho}{2}) + i) g_0^2 e^{(1-\rho)\tau} \frac{L}{\sinh L} \frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{3/2}}$$

- Due to conformal invariance, χ is a function of only two variables

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- We can write the scattering amplitude as

$$A(s,t) = 2s \int d^2 l e^{-i\mathbf{l}_\perp \cdot \mathbf{q}_\perp} \int dz d\bar{z} \, P_{13}(z) P_{24}(\bar{z}) \chi(s, l, z, \bar{z})$$

- P_{13} and P_{24} are the products of incoming and outgoing scattering states, and χ is the exchange kernel.
- For the Pomeron:

$$\chi(\tau, L) = (\cot(\frac{\pi\rho}{2}) + i) g_0^2 e^{(1-\rho)\tau} \frac{L}{\sinh L} \frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{3/2}}$$

- Due to conformal invariance, χ is a function of only two variables

$$L = \log(1 + v + \sqrt{v(2+v)})$$

$$\tau = \log(\frac{\rho}{2} z z' s)$$

- Obtained by placing a sharp cut-off on the radial AdS coordinate at $z = z_0$.
- First notice that at $t = 0$ χ for conformal pomeron exchange can be integrated in impact parameter

$$\chi(\tau, t = 0, z, \bar{z}) = i\pi g_0^2 \left(\cot\left(\frac{\pi\rho}{2}\right) + i \right) (z\bar{z}) e^{(1-\rho)\tau} \frac{e^{-\frac{(\ln(\bar{z}/z))^2}{\rho\tau}}}{(\rho\tau)^{1/2}}$$

- Similarly, the $t = 0$ result for the hard-wall model can also be written explicitly

$$\chi_{hw}(\tau, t = 0, z, \bar{z}) = \chi(\tau, 0, z, \bar{z}) + \mathcal{F}(\tau, z, \bar{z})\chi(\tau, 0, z, z_0^2/\bar{z}).$$

- The function

$$\mathcal{F}(\tau, z, \bar{z}) = 1 - 4\sqrt{\pi\tau}\, e^{\eta^2} \operatorname{erfc}(\eta), \qquad \eta = \frac{-\log(z\bar{z}/z_0^2) + 4\tau}{\sqrt{4\tau}}$$

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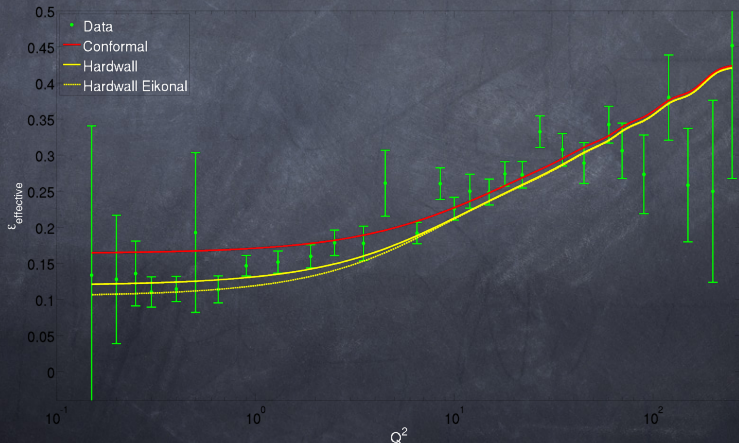
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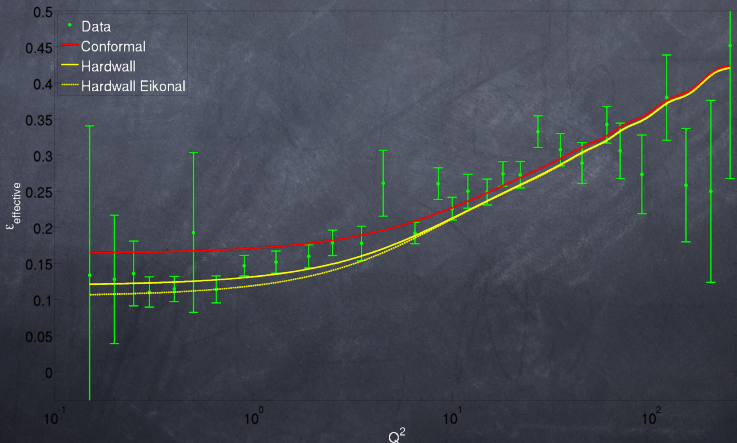
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- This has already been successfully applied to DIS [Brower, MD, Sarcevic, Tan, 2010], DVCS [Costa, MD, 2012] and VMP [Costa, MD, Evans, 2013; see talk by Evans next]. In all of those cases, when comparing our results with experimental data from HERA, we get very good results.
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- We also reproduce the 'running' of the effective pomeron intercept.

- Similiarly, we can calculate the amplitude for exclusive processes where an Odderon is exchanged.
- The odderon also accounts for the difference in the particle-particle and particle - anti-particle total cross sections. Define

$$F_{\bar{c}b \to \bar{a}d} = F^+ + F^-,$$
$$F_{ab \to cd} = F^+ - F^-.$$

- we can check that $F^\pm$ are the $C = \pm 1$ contributions to the amplitude, and that, via the optical theorem

$$\sigma_T(\bar{a}b) - \sigma_T(ab) \sim (2/s)\Im F^-$$

- We have also calculated the glueball masses [Brower, MD, Tan, 2008]

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Outline

We thus conclude today's talk. We have seen interesting methods to apply gauge/gravity duality to the Pomeron and Odderon, which are very useful indeed. Let us now look at some possible future directions of research.

- Already a lot of work has been done in studying diffractive processes using the Pomeron exchange. There are still more processes we can look at, for example *pp* total cross sections.
- It is possible to extend this beyond 2 → 2 scattering, for example for 2 → 3 scattering [Brower, MD, Tan, 2012]
- Recently the Pomeron intercept has been calculated to higher order [Costa, Goncalves, Penedones, 2012]
- Similarly, work is under way in calculating the Odderon intercept to higher order [Brower, Costa, MD, Raben, Tan, in progress, see talk by Tan tomorrow].
- From weak coupling there has been recent interest in the same question. Particularly interesting is the question if the intercept stays at 1 to all order (which all the calculations so far suggest).
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