Spatial Wilson loops and magnetic screening in heavy-ion collisions

Adrian Dumitru

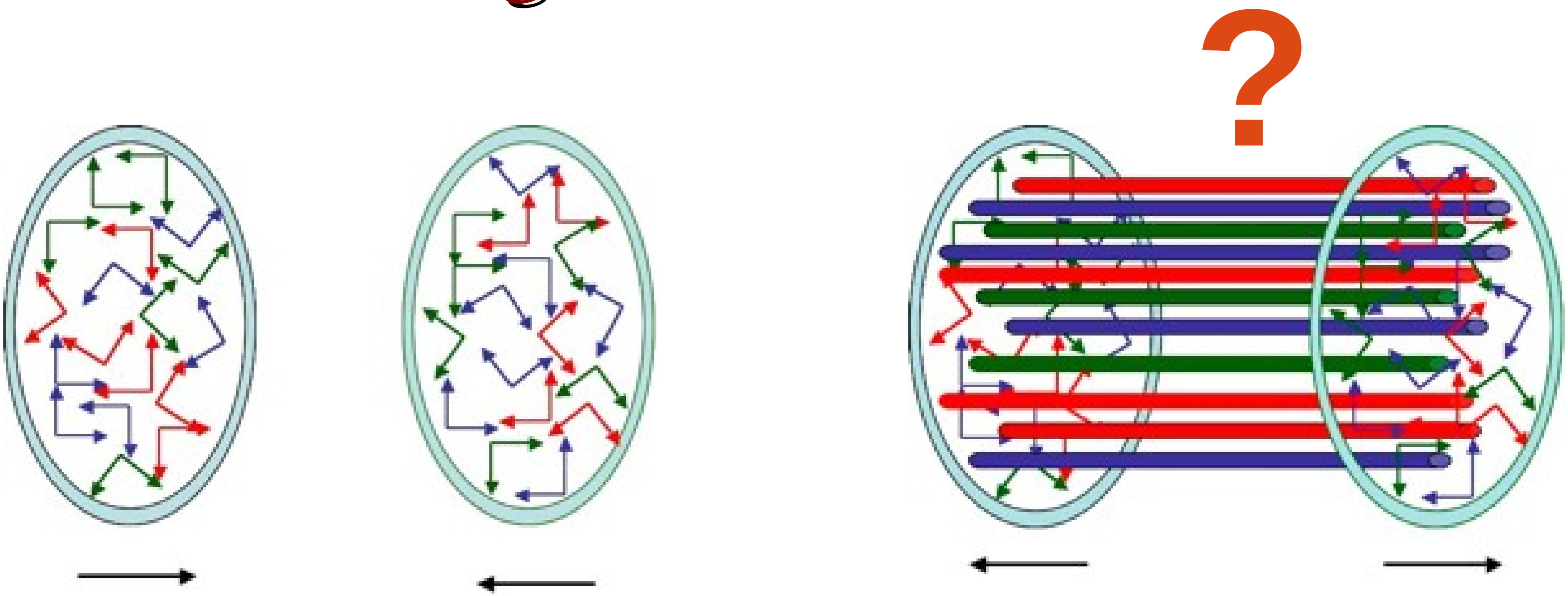
RIKEN BNL and Baruch College/CUNY

Talk based on:

A.D., Y. Nara, E. Petreska, arXiv:1302.2064

A.D., H. Fujii, Y. Nara, arXiv:1305.2780

Glasma long. initial fields



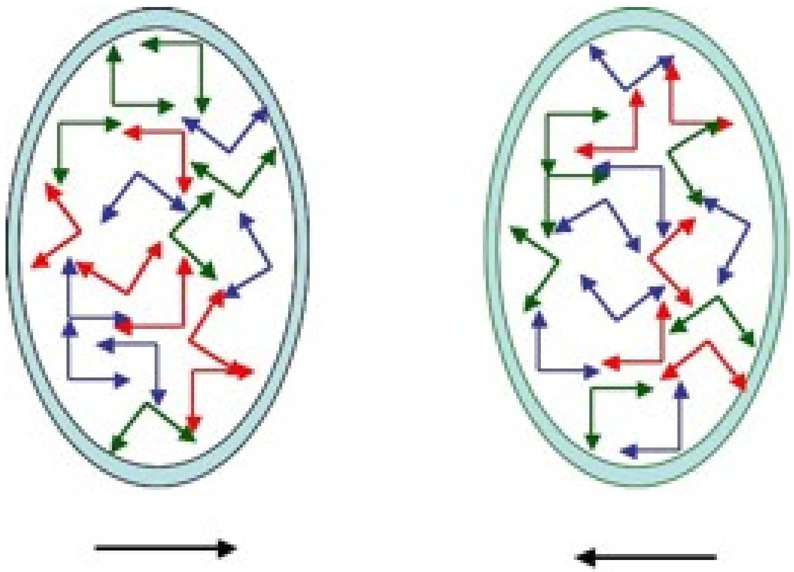
before collision   right after impact

$$E^z = ig\left[\alpha_1^i, \alpha_2^i\right] \quad , \quad B^z = ig\epsilon^{ij}\left[\alpha_1^i, \alpha_2^j\right]$$

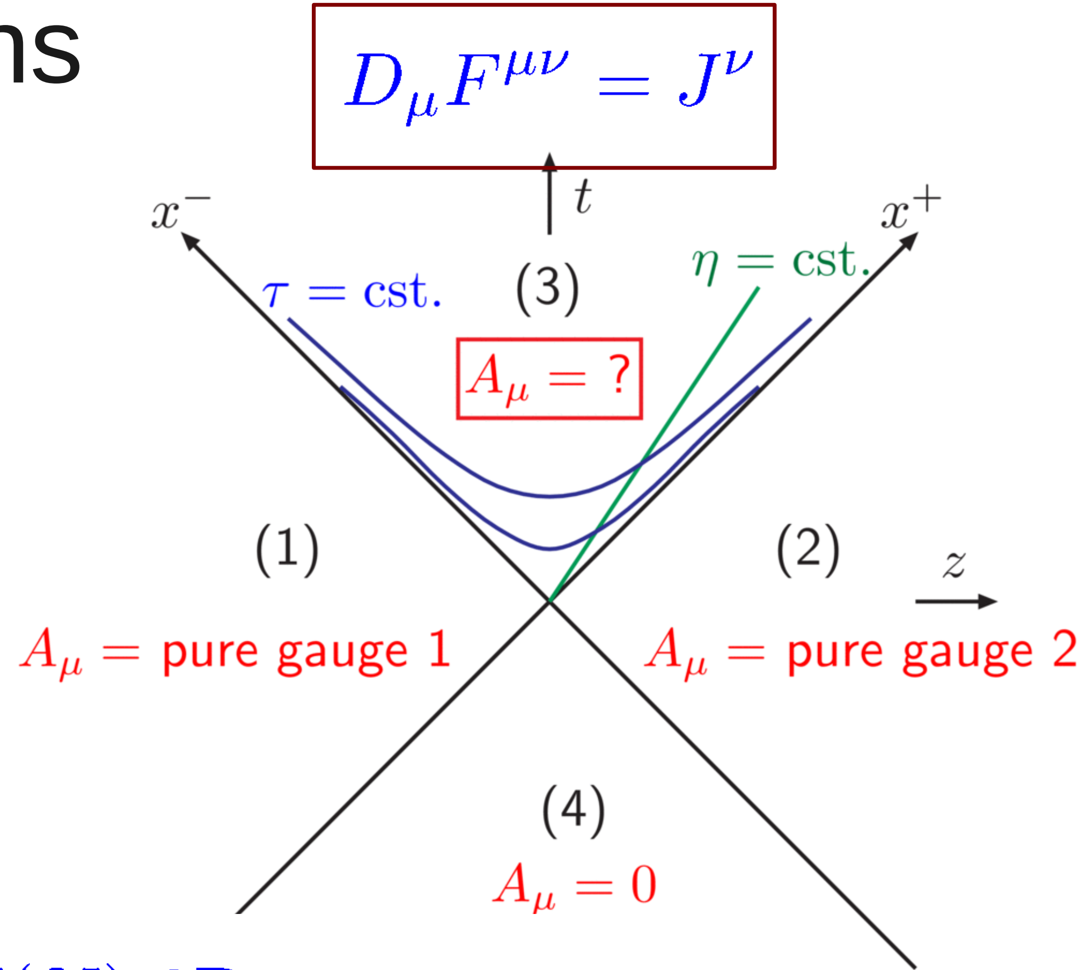
$$\nabla \cdot \mathbf{B} = ig\left[A^i, B^i\right]$$

R. Fries, J. Kapusta, Y. Li: nucl-th/0604054
Lappi + McLerran NPA 2006

Heavy-ion collisions



before collision



$$\alpha^i_m = \frac{i}{g} U_m \partial^i U^\dagger_m \qquad SU(N) \text{ 2D pure gauge}$$

$$\partial^i \alpha^i_m = g\rho_m \qquad (m = 1, 2 \ \ \text{Proj/Targ})$$

$$A^i = \alpha^i_1 + \alpha^i_2 \qquad (\tau = +0)$$

$$S_{\text{MV}} = \int d^2x_\perp \frac{1}{2\mu^2} \rho^a \rho^a$$

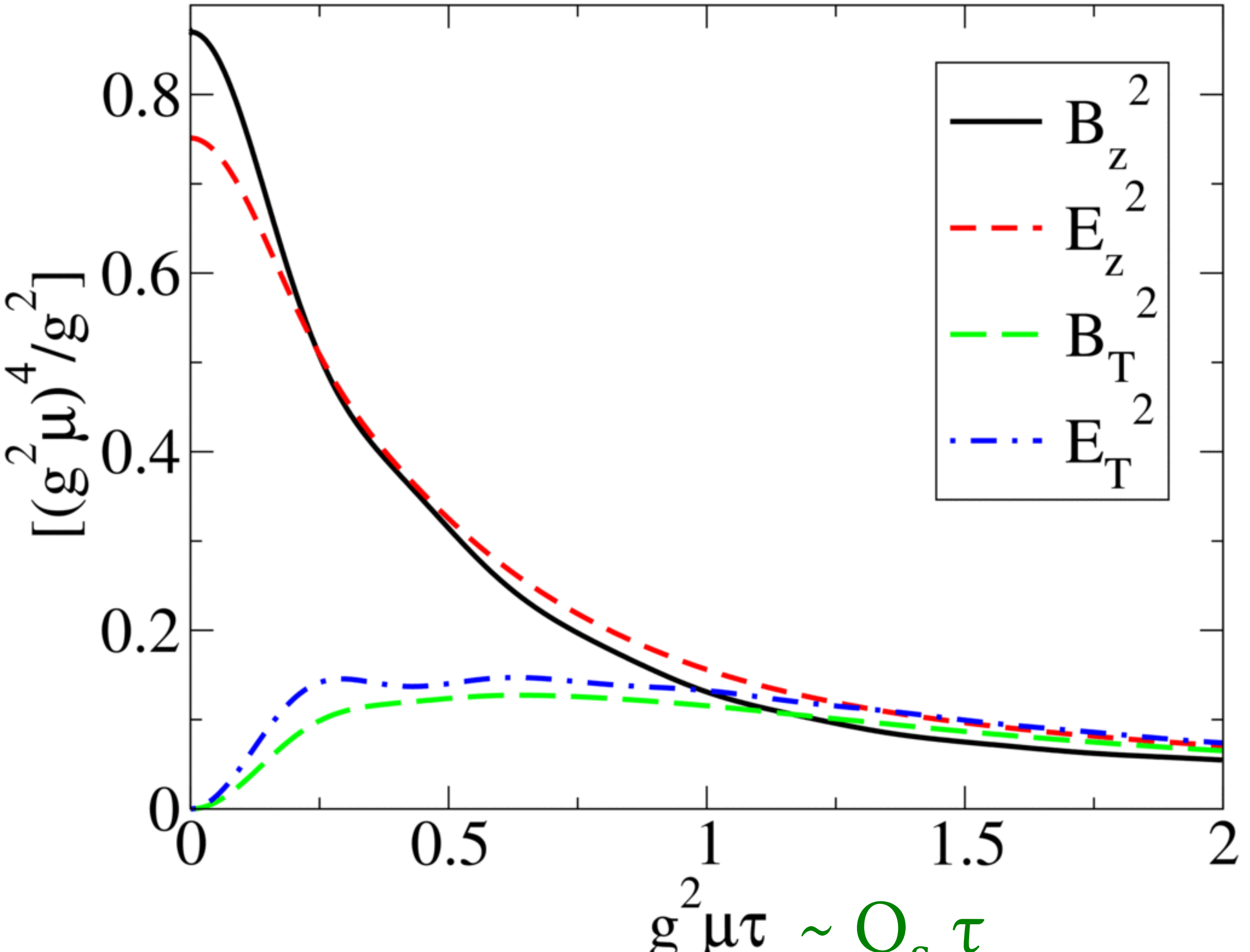
McLerran-Venugopalan action (for $A^{1/3} \to \infty$)
$\mu^2 \sim A^{1/3}$

Notes:

- $\rho(x)$ random at each point, correlation length =0
- *not* so for A^i !
  (see below)

Non-perturbative solution (using a lattice)

Lappi + McLerran NPA 2006



$\sim Q_s\,\tau$

Analyze classical field configurations at midrapidity: η=0, 2D

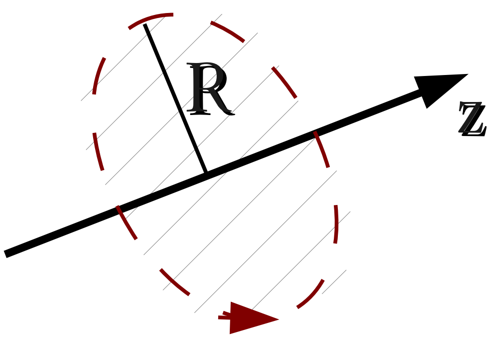
what is structure of B_z field ?

magnetic flux loop in x-y plane:

$$M(R) = \mathcal{P} \exp\left(ig \oint dx^i A^i \right)$$

$$W_M(R) = \frac{1}{N_c} \langle \mathrm{tr}\ M(R) \rangle$$

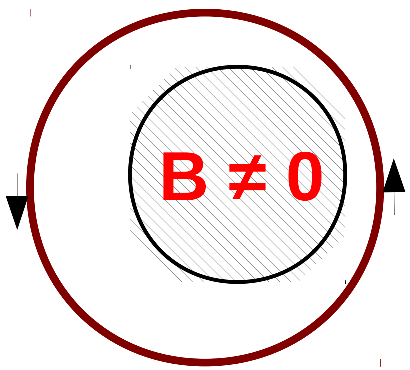
$$W_M^{Z(2)}(R) = \langle \mathrm{sgn}\ \mathrm{tr}\ M(R) \rangle$$



Magnetic flux loop: *Abelian* fields

$$\oint \vec{A}\cdot d\vec{\ell} = \int \vec{B}\cdot d\vec{a} = \Phi$$

$$\exp\ ig \oint \vec{A}\cdot d\vec{\ell} = e^{ig\Phi}$$



single-valued A field:

$$\exp\ ig \oint \vec{A}\cdot d\vec{\ell} \quad = \quad 1$$

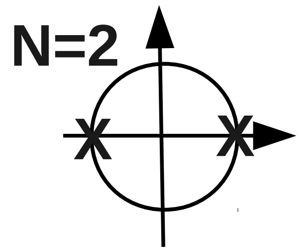
$$g \oint \vec{A}\cdot d\vec{\ell} \quad = \quad 2\pi\, n$$

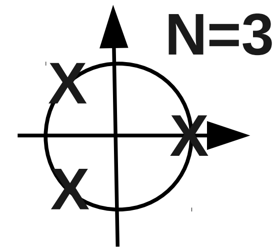
→ magnetic flux integer multiple of $\dfrac{2\pi}{g}$

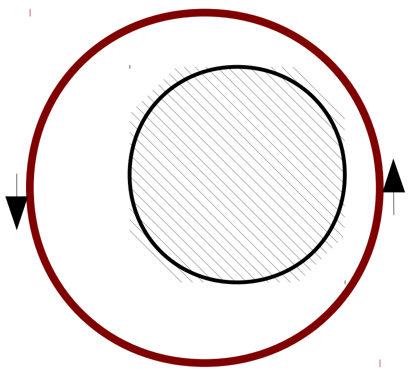
Magnetic flux loop: non-*Abelian* fields

transform under SU(N) / Z(N)

$$Z(N) = \left\{ e^{2\pi i\, n/N}\, \mathbb{1},\ n = 0 \cdots N-1 \right\}$$





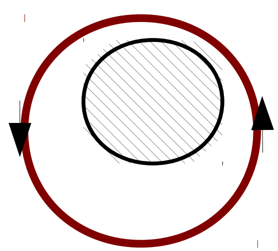


$$\mathcal{P} \exp\ ig \oint \vec{A} \cdot d\vec{\ell} \;=\; e^{2\pi i\, n/N}\, \mathbb{1}_{N\times N}$$

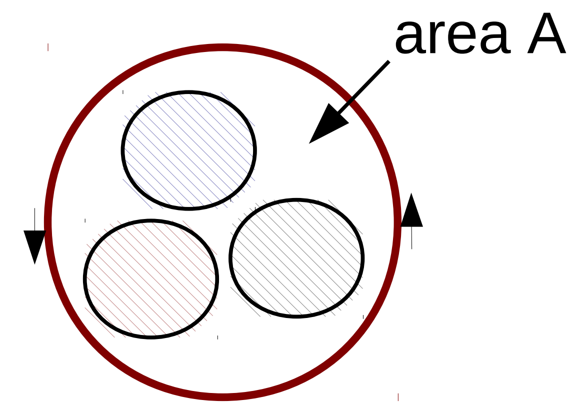
i.e., gauge transformation <u>can</u> be multi-valued by an element of Z(N)

n = Z(N) charge in shaded region

Magnetic Z(N) "vortices":

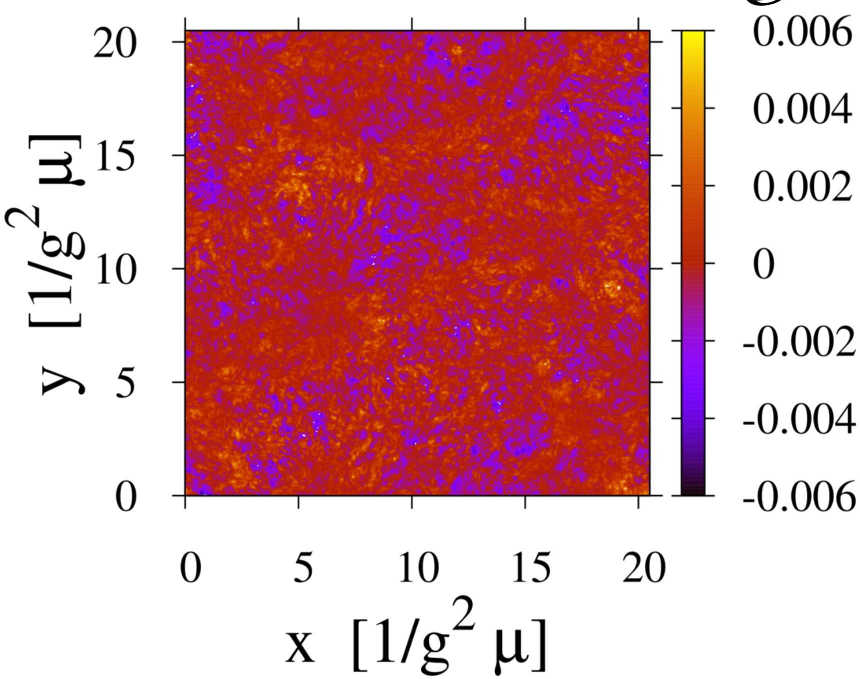


$$e^{2\pi i\, n/N}$$



$$e^{2\pi i\,(n_1+n_2+n_3)/N}$$

actual field configuration



do we find :

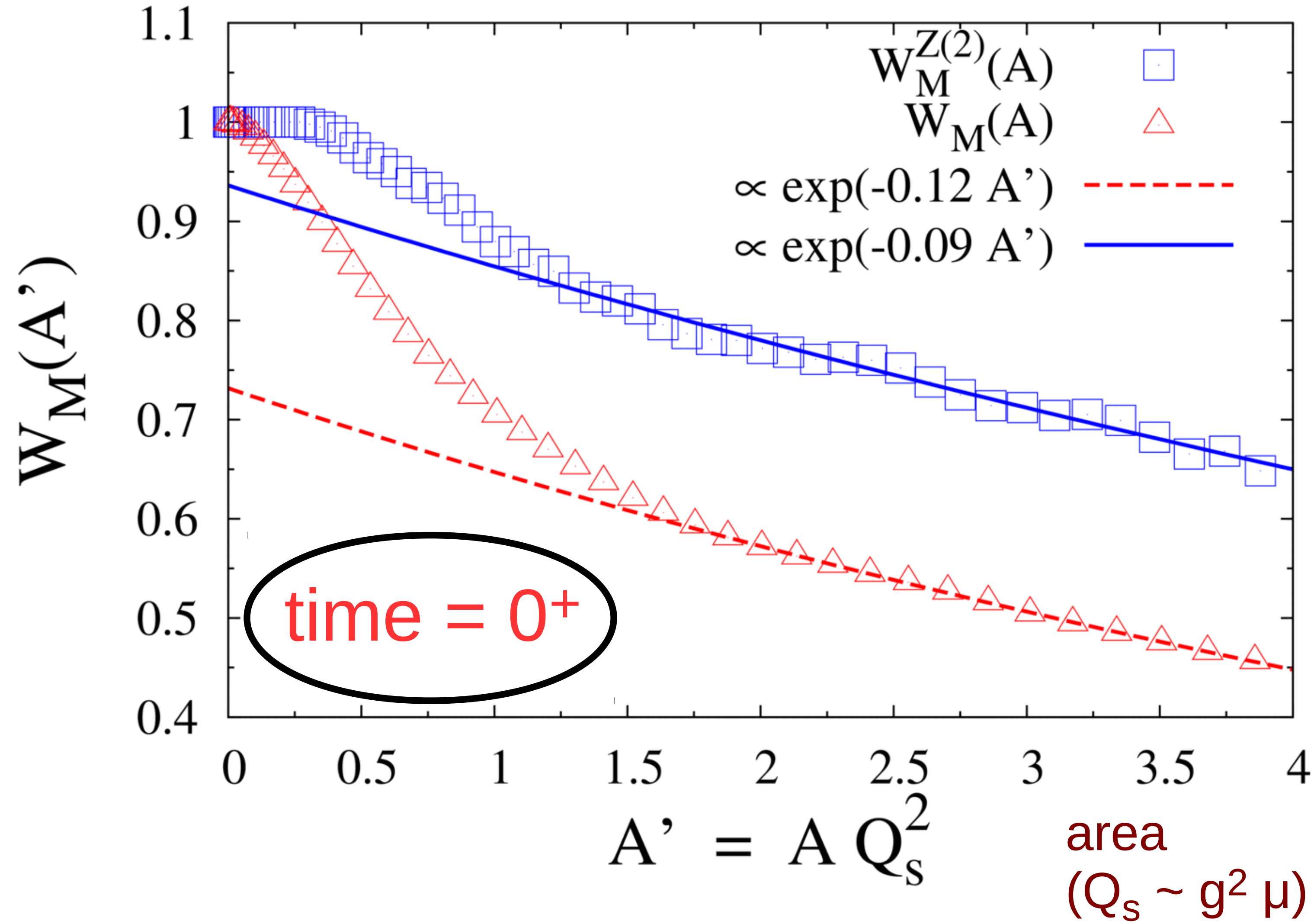
- area law ?

$$W_M(R) \sim e^{-\sigma A}$$

- loop $\in$ Z(N) ?

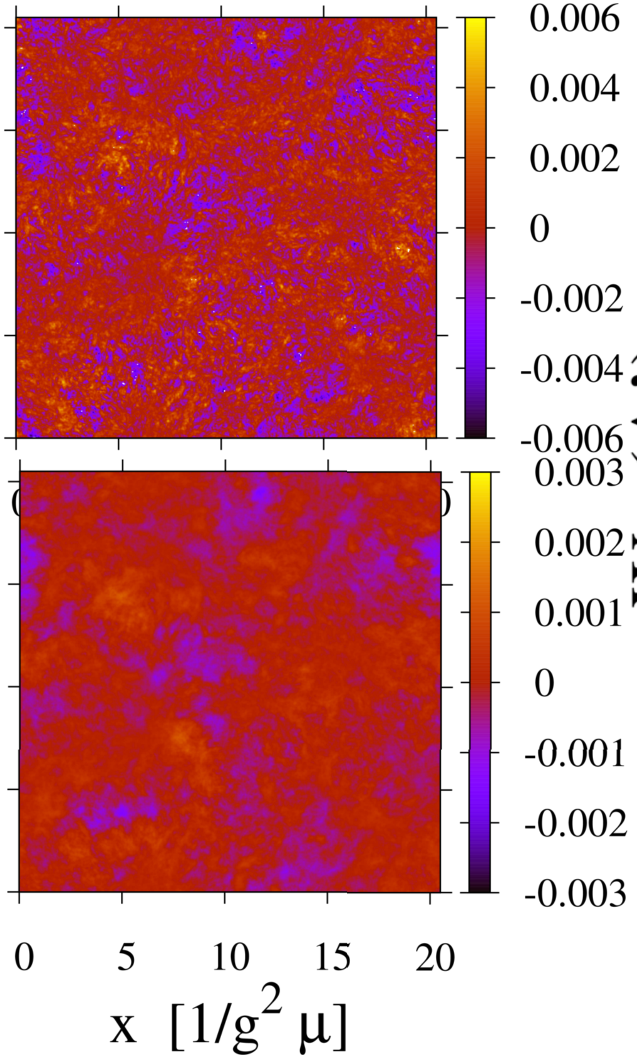
$$\langle \mathrm{sgn}\,\mathrm{tr}\ M \rangle \sim \frac{1}{N} \langle \mathrm{tr}\ M \rangle$$

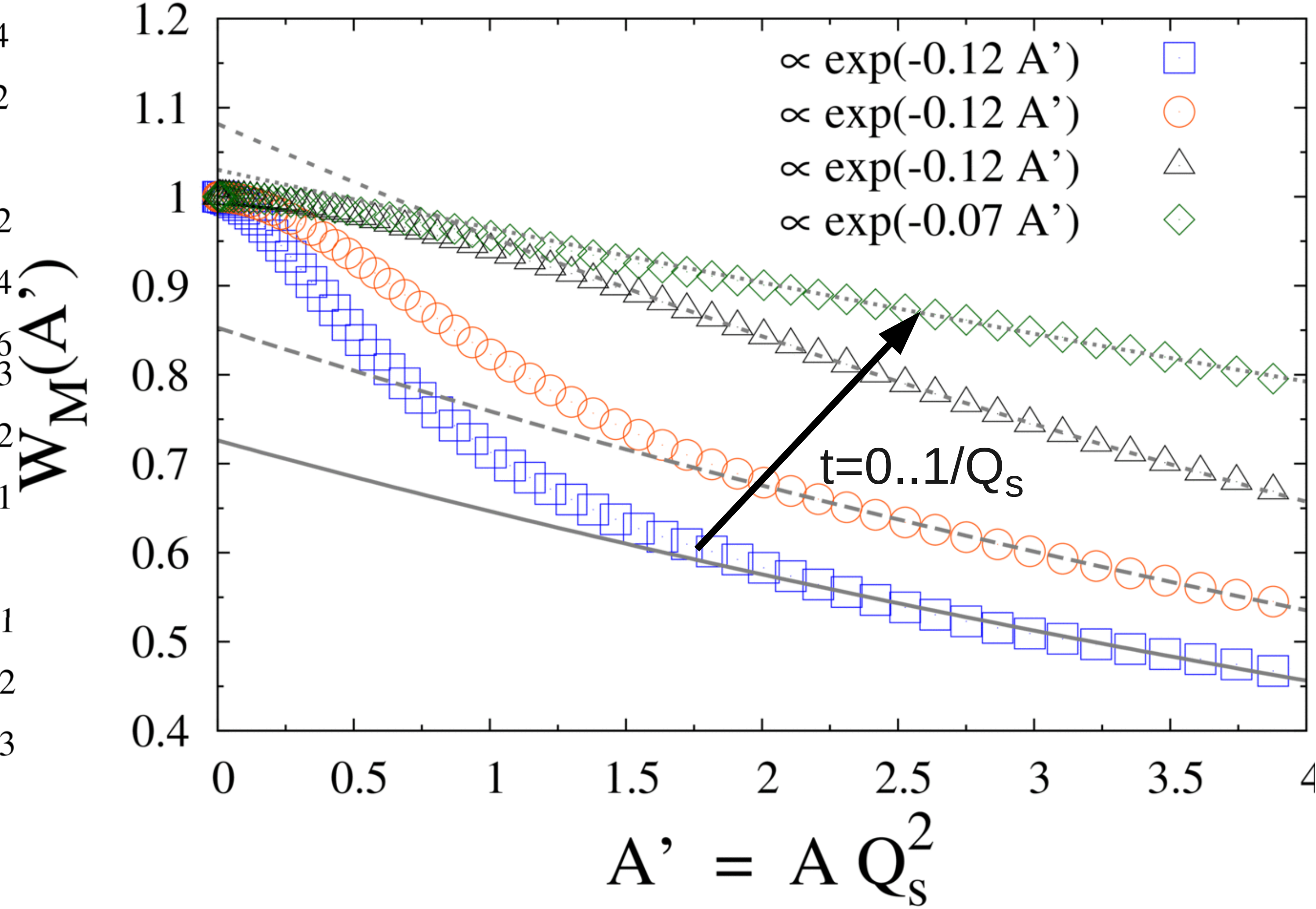
SU(2) solution :



- area law for loops with area A ≥ 1.5 – 2
- $\sigma_M \sim 0.12\ Q_s^2$; thermal SU(N): $\sigma_M \sim g_{3D}^2 \sim (g^2T)^2$
- small loops ∉ Z(2) but roughly ok for large ones!
- structure of B_z ~ uncorrelated vortices ?!
- $R_{vtx} \sim 1/Q_s$ from onset of area law

from t=0 to ~1/Q_s





- earlier onset of area law
- vortex radius decreases, almost point-like at t~1/Q_s
- condensation phenomenon ?   Or collapse of vortices?

perturbative expansion of loop / “naive” Gaussian approximation

expand $\left\langle \frac{1}{N} \mathrm{tr}\, \mathcal{P} \exp \; ig \oint \vec{A} \cdot d\vec{\ell} \right\rangle$ to 4th order in $A^i = \alpha_1^i + \alpha_2^i$

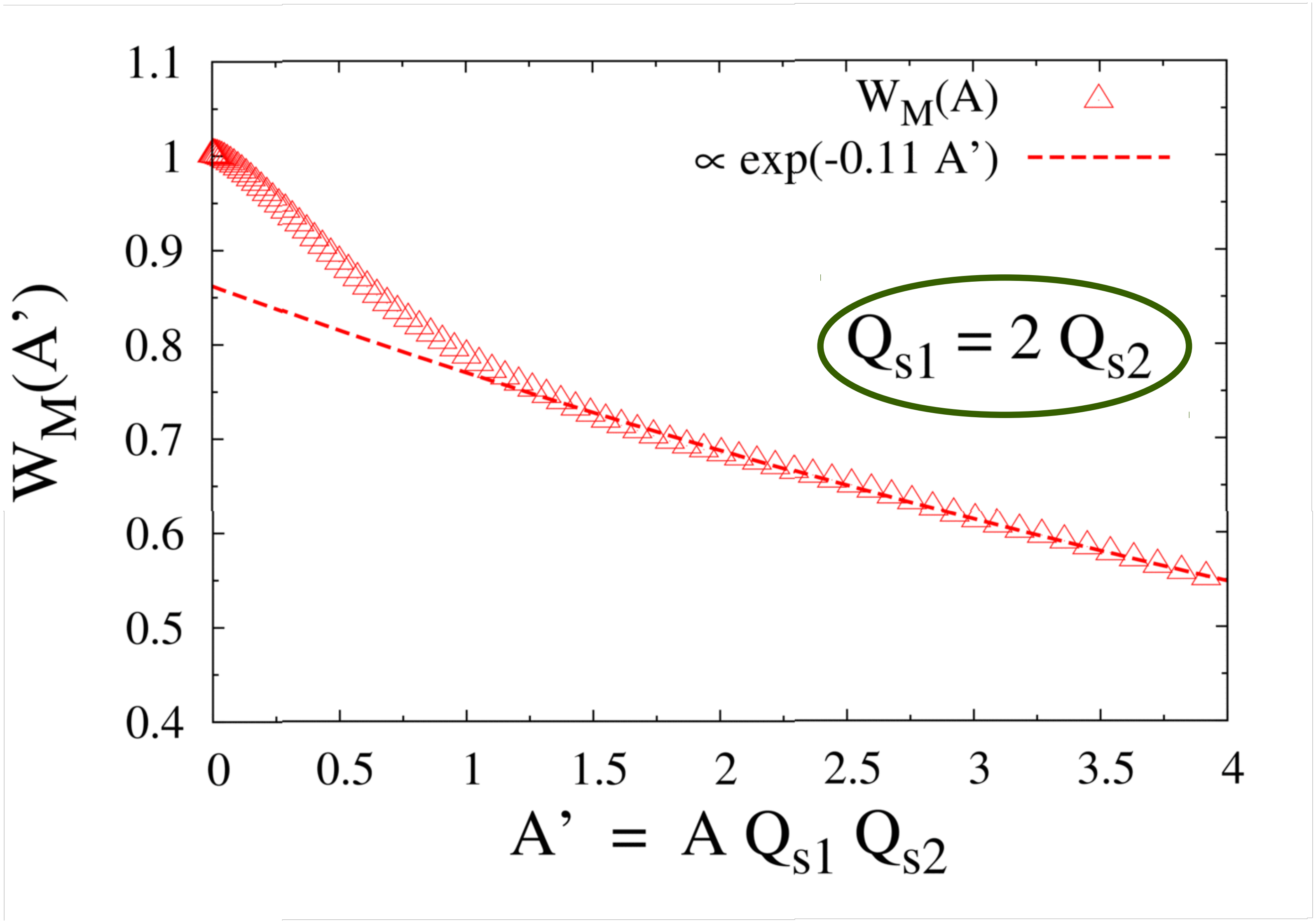
and subtract disconnected piece (remember: α_1, α_2 are pure gauges)

$$1 = \left\langle \frac{1}{N} \mathrm{tr}\, \mathcal{P} \exp \; ig \oint \vec{\alpha}_1 \cdot d\vec{\ell} \right\rangle \left\langle \frac{1}{N} \mathrm{tr}\, \mathcal{P} \exp \; ig \oint \vec{\alpha}_2 \cdot d\vec{\ell} \right\rangle$$

Result: $W_M(\alpha_1^i + \alpha_2^i) - W_M^{\mathrm{sngl}}(\alpha_1^i)\, W_M^{\mathrm{sngl}}(\alpha_2^i) \quad = \quad \frac{\pi^2}{3} \frac{2N^2 - 3}{N^2 - 1} A^2 Q_{s1}^2 Q_{s2}^2$

- ~A^2, not ~A, higher multipole moments gone

lattice solution for ***asymmetric*** collision



- ~ same string tension $\sigma_M = 0.11\ Q_{s1}\ Q_{s2}$

perturbative expansion cont'd :

- connected contribution ~area must be ~ A Q_{s1} Q_{s2}
  by dimensional analysis...
- “naive” perturbative expansion of loop is built from two-point functions
  $<\rho\rho> \sim \mu^2 \sim Q_s^2$

magnetic screening !

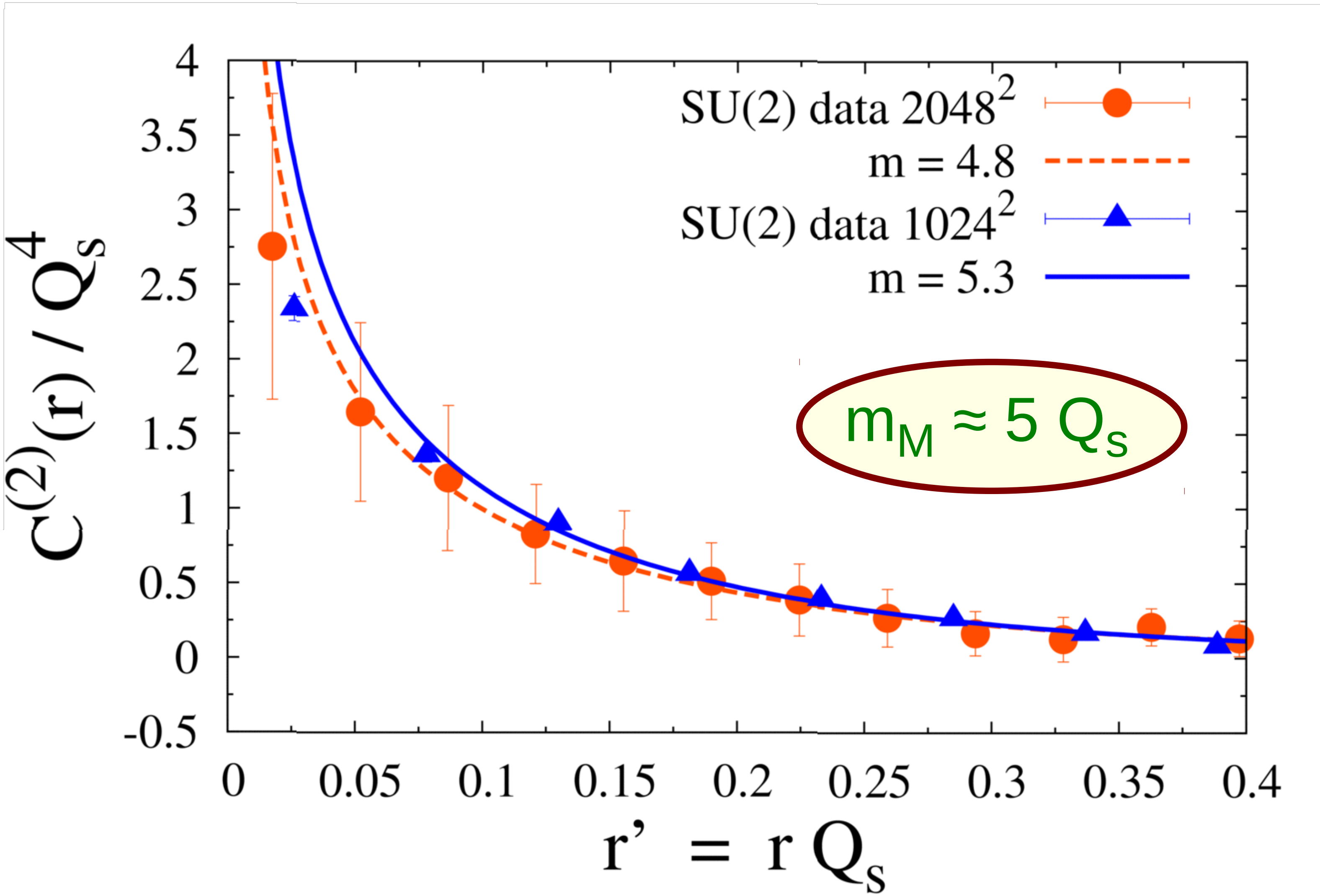
$$C^{(2)}(r) = \langle \mathrm{tr}\, G(\mathbf{0})\, G(\mathbf{x}) \rangle$$

$$G(\mathbf{x}) = g\, U(\mathbf{0} \to \mathbf{x})\, F_{xy}(\mathbf{x})\, U(\mathbf{x} \to \mathbf{0})$$

expectation: $$\int d^d p \, \frac{1}{p^2 + m^2} \sim \frac{1}{r^{(d-1)/2}} \; \exp(-m\, r)$$

Notes:

- in thermal equilibrium $f_{cl}(k) \sim 1/k$; screening masses UV divergent (Bödeker, McLerran, Smilga '95) here $\sim 1/k_T^4$
- gauge links: interactions of external legs with produced gluons $A^i = \alpha_1^i + \alpha_2^i$

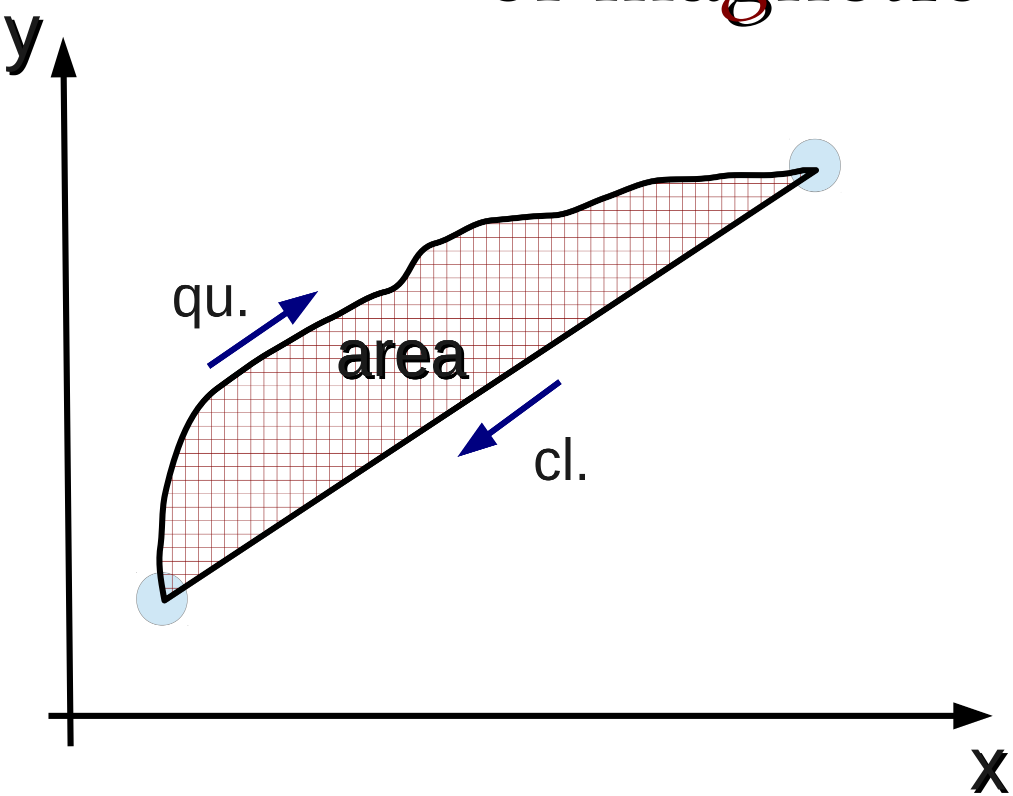


$$\sigma_M = \frac{1}{2} \int d^2 r\ C^{(2)}(r)$$

(Yu. Simonov et al)

satisfied to good approx

Propagation of hard particles in background of magnetic Z(N) vortices



classical trajectory ?

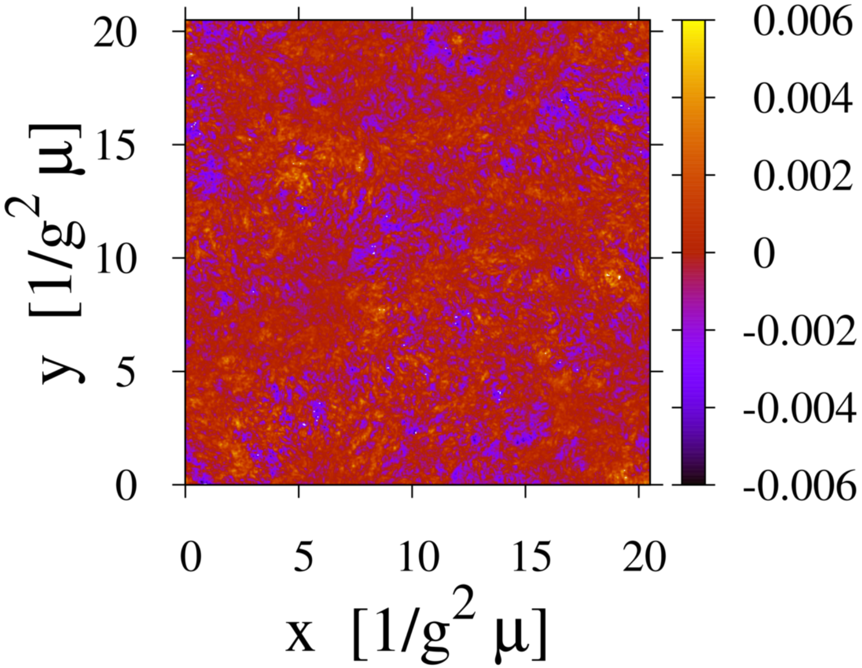
- only if paths within one de Broglie length ($1/p_T$) have *same Aharonov-Bohm phase*
- destructive interference leads to Anderson localization

$$\int_0^\infty ds \int \mathcal{D}x^\mu \left\langle \exp i \int_0^s d\tau \left(m\dot{x}^2 + g A_\mu \dot{x}^\mu \right) \right\rangle \sim$$

$$\int_0^\infty ds \int \mathcal{D}x^\mu \exp \left(i \int_0^s d\tau \, m\dot{x}^2 \right) \exp(-\sigma_M A) = \frac{i}{p^2 + i\sigma_M \frac{m}{p_T}}$$

Summary

(structure of long. fields in initial stage)



- "Clumping" of magnetic field
- area law: $W_M(A) \sim \exp(-\sigma_M A)$ for loop radius $R \sim 1/Q_s$
- Z(2) projected loop gives similar σ_M
- magnetic screening at scale $m_M \cong 5\, Q_s$
- hard particles: classical trajectories only for p_T sufficiently far above Q_s

Backup Slides

Perturbative evaluation of magnetic Wilson loop:

$$W_M = 1 + \frac{1}{N_c} \mathrm{tr} \left\langle -g^2 \int_{-\pi}^{\pi} ds \int_{-\pi}^{s} ds' \frac{\partial x^i}{\partial s} \frac{\partial z^j}{\partial s'} A^{ai}(\tau, s) A^{bj}(\tau, s') t^a t^b \right\rangle$$
$$+ \frac{g^4}{N_c} \int_{-\pi}^{\pi} ds \int_{-\pi}^{s} ds' \int_{-\pi}^{s'} d\bar{s} \int_{-\pi}^{\bar{s}} d\bar{s}' \frac{\partial x^i}{\partial s} \frac{\partial z^j}{\partial s'} \frac{\partial u^k}{\partial \bar{s}} \frac{\partial v^l}{\partial \bar{s}'}$$
$$\left\langle A^{ai}(\tau, s) A^{bj}(\tau, s') A^{ck}(\tau, \bar{s}) A^{dl}(\tau, \bar{s}') \right\rangle \; \mathrm{tr}\, t^a t^b t^c t^d$$

Single nucleus: $A^i = \alpha^i$

$$W_M = 1 - \pi A Q_s^2 + \cdots$$

Forward light cone: $A^i = \alpha_1^i + \alpha_2^i$

$$W_M = 1 - \pi A \left(Q_{s1}^2 + Q_{s2}^2\right) + \frac{\pi^2}{6} \frac{2N_c^2 - 3}{N_c^2 - 1} A^2 \left(Q_{s1}^4 + Q_{s2}^4\right)$$
$$+ \frac{\pi^2}{3} \frac{2N_c^2 - 3}{N_c^2 - 1} A^2 Q_{s1}^2 Q_{s2}^2$$