Inflation as the Precursor of Dark Energy

H. M. Fried, Brown University

Co-author: Y. Gabellini,

INLN, Universite de Nice

- A QED-based Model of a new version of Vacuum Energy is suggested, leading to a simple, finite, one-parameter representation of Dark Energy.
- An elementary, obvious, but somewhat radical generalization is then able to describe both Dark Energy and Inflation in the same Vacuum Energy framework…
- …as well as the origins of Dark Matter, Ultra-High-Energy Cosmic Rays, including GeV Gammas, with and without X-Ray and Optical Radiation tails.
- One further generalization then leads to a relation between Inflation and the Big Bang, and to a possible understanding of the Birth and Death of a Universe.

- Here is an Idea, simple and compelling; why has it been overlooked?
- Conventional QED: Electromagnetic fields are either “quantized” or “external” = classical fields, which can be switched on and off.
- Simultaneously, one speaks of the “Quantum Vacuum”, in which quantized fields of arbitrary complexity are fluctuating, and whose effects can be indirectly inferred (as in the Lamb Shift of H).
- Imagine a virtual photon in the QV, suddenly transformed into a bubble = virtual e+ e- loop. While this loop exists, there is (thinking classically) an electric field across that loop, an emag. fluctuation containing energy which disappears when the bubble collapses.
- But then another loop appears, with its normal vector pointing in a different direction, and then disappears; and then another, and another… so that, on the <u>average</u>, there is a certain amount of energy associated with such continuously random fluctuations.

- We phrase the question in terms of an A(x) effective c-number vector field, just like a “classical” field, except that it <u>cannot</u> be turned off.
- Because of the random orientations of its 3-dim. polarization vector, A must lie along the time-like direction; and this is true for every Lorentz observer who imagines such an A in his/her own Frame.
- Imagine an energy present, in every LF, an average potential energy of all the fluctuating loops, which can have significant effects on a large classical scale…
- If it is <u>this</u> potential energy that represents the Vacuum Energy which was responsible for Inflation, and is now responsible for the Dark Energy accelerating our Universe outwards.
- <u>NB</u>: This has absolutely nothing to do with Zero-point energy!!

The conventional mathematical apparatus used to define a vacuum state makes no reference to the scales on which vacuum properties are observed; and we here suggest that this description is incomplete.

On distance scales larger than the electron's Compton wavelength, $\sim$ 10^(-10) cm, one can imagine that the average separation of the virtual loop e+ e- is not distinguishable; but on much smaller scales - where our A_μ^{vac} (x) oscillations will be so rapid that they cannot affect the motion of ordinary charged particles - it is possible to have effective, average, vacuum fields which exist and contain electromagnetic energy.

We therefore postulate that, in the absence of ordinary, external, classical fields, the vev of the QED current operator

$$J_\mu(x) = ie\bar{\psi}(x)\gamma_\mu\psi(x)$$

need be neither x-independent nor zero, and is given by

$$A_\mu^{vac}(x) = \int d^4y\, D_{S,\mu\nu}(x-y)\langle 0|J_\nu(y)|0\rangle$$

where $D_{c,\mu\nu}$ is the usual, free-field, photon propagator, for convenience defined in the Lorentz gauge, $0 = \partial_\mu A_\mu^{vac} = \partial_\mu D_{c,\mu\nu} = \partial_\nu D_{c,\mu\nu}$.

In classical E&M, vector [potentials can always be written in terms of classical currents J,

$$A_\mu(x) = \int D_{c,\mu\nu}(x-y) J_\nu(y) ,$$

while the transition to operator QED, in the absence of external, classical currents, involves the replacement of the classical potential and currents by the operators A_μ (x) and j_μ(x),

$$A_\mu = \int D_{c,\mu\nu} j_\nu + \hat{A}_\mu$$

where $\hat{A}_\mu$ denotes a free-field photon operator satisfying $(-\partial^2)\hat{A}_\mu = 0$; and its vev is zero.

Calculating the vev of the above equation yields

$$\langle 0| A_\mu(x) |0\rangle = \int d^4y\, D_{c,\mu\nu}(x-y) \langle 0| j_\nu(y) |0\rangle , \qquad (2)$$

and conventionally, in the absence of the usual, large-scale external fields, both sides of this equation are to vanish. But if we assume that non-zero $\langle 0| j_\mu(x) |0\rangle$ can exist on ultra-short scales, a comparison of (1) with (2) suggests that the A_μ^{vac}(x) produced by such small-scale currents are to be identified with $\langle 0| A_\mu(x) |0\rangle$ found in conventional QED in the presence of the same, c-number A_μ^{vac}(x). In other words,

$$A_\mu^{\rm vac}(x) \; = <0\,|\,A_\mu(x)|\,0> = \; \frac{1}{i}\,\frac{\delta}{\delta J_\mu(x)}\,\mathcal{Z}[\eta,\bar\eta,J]\Big|_{\eta=\bar\eta=J=0}$$

$$= \frac{1}{i}\,\int d^4y\,D_{c,\mu\nu}(x-y)\,\frac{\delta}{\delta A_\nu(y)}\;e^{\displaystyle -\frac{i}{2}\int\frac{\delta}{\delta A}D_c\frac{\delta}{\delta A}} \tag{2.5}$$

$$\times <0\,|\,S[A^{\rm vac}]|\,0>^{-1}\,\exp[L[A+A^{\rm vac}]]\Big|_{A=0}$$

which provides a bootstrap equation with which to determine such short scale $A_\mu^{\rm vac}(x)$, if any exist. In (2.5), which can be transformed into a functional integral relation, the vacuum to vacuum amplitude is given by [2] :

$$<0\,|\,S[A^{\rm vac}]|\,0> = \; e^{\displaystyle -\frac{i}{2}\int\frac{\delta}{\delta A}D_c\frac{\delta}{\delta A}}\,\exp[L[A+A^{\rm vac}]]\Big|_{A=0} \tag{2.6}$$

How does one go about finding a solution to (2.5) ? The first requirement is a representation for $L[A + A^{\rm vac}]$ which is sufficiently transparent to allow the functional operation of (2.5) to be performed. Use of the Fradkin functional representation [3] for $L[A]$ is one such possibility, because that representation is gaussian in A, and (2.5) can be performed immediately. But one is then left with the task of evaluating the remaining functional operations, which is not a trivial affair. What shall be done here is to use the well known, gauge independent, second order perturbative approximation to L :

$$L[A + A^{\rm vac}] \longrightarrow \frac{i}{2} \int d^4x \, d^4y \, [A^\mu + A^{\mu {\rm vac}}](x) \, K_{\mu\nu}(x-y) \, [A^\nu + A^{\nu {\rm vac}}](y) \tag{3.1}$$

which is clearly quadratic in A, and corresponds to the simplest Feynman diagram of a virtual lepton–antilepton bubble. The strictly gauge invariant form of that bubble may be represented as :

$$\tilde{K}_{\mu\nu}(k) = (\delta_{\mu\nu} \, k^2 - k_\mu k_\nu) \, \Pi(k^2) \ , \qquad k^2 = \vec{k}^2 - k_0^2$$

and the relevant form of $\Pi(k^2)$ will follow immediately :

$$\Pi(k^2) = -\frac{2\alpha}{\pi}\int_0^1 dy\, y(1-y)\int_0^\infty \frac{ds}{s}\, e^{-is[m^2 + y(1-y)k^2]} \tag{3.2}$$

where m denotes the mass of the charged particle whose " bubble " is the source of $A_\mu^{\rm vac}$. The obvious UV logarithmic divergence of (3.2) will require the usual regularisation and substraction procedure, and the renormalized $\Pi_R(k^2)$ may easily be put into the form [4] :

$$\Pi_R(k^2) = \frac{2\alpha}{\pi}\int_0^1 dy\, y(1-y)\ln[1 + y(1-y)\frac{k^2}{m^2}] \tag{3.3}$$

Higher order perturbative terms should each yield a less important contribution to the final answer, although the latter could be qualitatively changed by their sum; we shall assume that this is not the case, and that the perturbative approximation (properly unitarized by the functional calculation which automatically sums over all such loops) gives a qualitatively reasonable approximation.

Using the approximation (3.1), the functional operation of (2.5), now equivalent to a gaussian functional integration, is immediate [2] and yields the approximate relation for this $A_\mu^{\rm vac}(x)$:

$$A_\mu^{\rm vac}(x) = \int d^4y \left(D_c K \, \frac{1}{1 - D_c K} \right)_{\mu\nu} (x-y) A_\nu^{\rm vac}(y) \tag{3.4}$$

or, using its Fourier transform $\tilde{A}_\mu^{\rm vac}(k)$:

$$\tilde{A}_\mu^{\rm vac}(k) = \left(\Pi_R(k^2) \frac{1}{1 - \Pi_R(k^2)} \right) \tilde{A}_\mu^{\rm vac}(k) \tag{3.5}$$

which can be written :

$$\left(\frac{1 - 2\Pi_R(k^2)}{1 - \Pi_R(k^2)} \right) \tilde{A}_\mu^{\rm vac}(k) = 0 \tag{3.6}$$

Non zero solutions to (3.6) may be found in the “ tachyonic ” form [5] :

$$\tilde{A}_\mu^{\rm vac}(k) = C_\mu(k) \, \delta(k^2 - M^2) = C_\mu(k) \, \delta(\vec{k}^2 - k_0^2 - M^2) \tag{3.7}$$

with M^2 such that :

$$\Pi_R(M^2) = \frac{1}{2} \tag{3.8}$$

and which serves to determine M. Note that a solution of form $C_\mu \, \delta(k^2 + \mu^2)$ would not be possible, since the log of $\Pi_R(-\mu^2)$ picks up an imaginary contribution for time–like $k^2 = -\mu^2$, for $\mu > 2m$. $\Pi_R(M^2)$ must be a real quantity to satisfy (3.8).

In order to describe this vacuum field, one must define C_μ. Remembering that we are using the Lorentz gauge, one choice of C_μ might be

$$C_\mu(k) = v_\mu - k_\mu (k \cdot v)/k^2 ,$$

where v_μ is a constant to be determined. But the part proportional to k_μ is a pure gauge term, which cannot contribute to any emag. field; while the replacement of C_μ by v_μ generates fields that diverge in the region of the light cone, and require an ad hoc regularization.

This is the form of solution first used in 2003 for Dark Energy… but a far better solution follows by enforcing the Lorentz gauge condition by the choice

$$C_\mu(k) = Q\, v_\mu\, \delta(k \cdot v),$$

where Q is a constant to be determined; this choice produces a field both simple and everywhere finite.

But there are certain questions, and requirements which must be satisfied:

a) What is v_μ ? Physically, this vector should represent an electric field polarization, corresponding to the electric field in the plane of the fluctuating loops. But the QED vacuum will have loops fluctuating in all possible planes, and no single $\vec{v}$ direction can be relevant. It is then intuitively clear that v_μ should have only a fourth component.

b) But in which Lorentz frame ? If it is to represent a field generated by the same vacuum processes in every frame, it should have the same value to each observer in his own Lorentz frame.

We now show that this choice of solution for the vacuum field does satisfy both of these requirements, and has just the correct behavior to suggest a mechanism for inflation, while producing a present day energy density that can be associated with dark energy.

Insert a representation for both delta functions of :

$$\tilde{A}_\mu^{\rm vac}(k) = \kappa\, v_\mu\, \delta(k\cdot v)\, \delta(k^2 - M^2) \tag{4.1}$$

and calculate the inverse Fourier transform of $\tilde{A}_\mu^{\rm vac}$. It will be represented by the parametric, " proper time " integral :

$$A_\mu^{\rm vac}(x) = \frac{\kappa}{(2\pi)^4}\sqrt{\frac{i\pi}{4}}\Big(\frac{-iv_\mu}{\sqrt{v^2}}\Big)\int_{-\infty}^{+\infty} ds\, s^{-3/2}\,\epsilon(s)\, e^{\,isM^2 + iX^2/4s} \tag{4.2}$$

with $\epsilon(s) = \dfrac{s}{|s|}$, $x\cdot v = \vec{r}\cdot\vec{v} - x_0 v_0$, and where $X^2 = x^2 - (x\cdot v)^2/v^2$, and is a Lorentz invariant quantity. The corresponding solution in another Lorentz frame, represented by a prime on x and a prime on v, will have exactly the same form.

Now, consider the solution (4.2) in our frame, and ask what value should be assigned to the spatial components. From the argument of a) above, the only sensible value for this field $A_\mu^{\rm vac}$ is the choice $\vec{v}=0$. Then, the quantity X^2 reduces to r^2, and (4.2) can be evaluated trivially :

$$A_\mu^{\rm vac}(x) \longrightarrow A_4^{\rm vac}(x) = i\,\frac{\kappa}{(2\pi)^3}\,\epsilon(v_0)\,\frac{\sin(Mr)}{r} \tag{4.3}$$

and depends only on r and on the sign of v_0.

Now switch to another Lorentz frame, where the result is given by (4.2) using prime variables, related to the unprimed variables by standard Lorentz transformation. An observer in that frame asks what value he should assign to the spatial components of his v'; and for his vacuum field, he comes to exactly the same conclusion as did we : the only sensible choice for a vacuum field as seen by him must require $\vec{v}\,' = 0$. Note that his square root variable $\sqrt{X^2}$ containing all the x' dependence is a Lorentz invariant quantity, and is equal to our square root variable. In our frame, when we set $\vec{v} = 0$, that variable reduces to r; and in his frame, when he sets $\vec{v}\,' = 0$ it reduces to r'. But both must be equal, since they were derived from the same invariant; when the observer in the primed frame sets his $\vec{v}\,' = 0$, as he must to describe his vacuum field, he is using the same functional form as (4.3) in terms of his r'.

The only possible difference between the two expressions of (4.3), primed or unprimed, is the sign of v_0 and that of v_0'. But, as always when dealing with physical entities, we restrict all admissible Lorentz transformations to those which are orthochronous, keeping the same sense of time, or of energy, or in this case of v_0; and hence $\epsilon(v_0) = \epsilon(v_0')$ and the two versions of (4.3) are the same. In this way, observers in every Lorentz frame see the same vacuum field. For simplicity, we shall choose $\epsilon(v_0) = +1$, although this choice of sign has no bearing on the vacuum energy densities to be calculated.

Following the arguments above, any observer in any frame will see a " vacuum electrostatic " potential of form :

$$\phi^{\text{vac}}(r) = \frac{\kappa}{(2\pi)^3} \frac{\sin(Mr)}{r}$$

where κ is a constant to be determined.

The resulting electric field has a rapid spatial variation, as does its energy density :

$$\rho = \vec{\mathcal{E}}^{\,2}/8\pi = \xi\, \frac{M^2}{r^2}\Big(\cos(Mr) - \frac{\sin(Mr)}{Mr}\Big)^2 = \xi\, M^4 f(x) \tag{4.4}$$

where $\xi = (\frac{1}{8\pi})\frac{\kappa^2}{(2\pi)^6}$, $x = Mr$, and $f(x) = \frac{1}{x^2}\Big(\cos x - \frac{\sin x}{x}\Big)^2$. A plot of $f(x)$ has the form indicated in Fig.1, which suggests the basic idea of this approach : the first pulse serves to kick start Inflation, which is supposed to begin at $t \sim 10^{-42}$ s, and have an average energy density ρ such that $\rho^{1/4} \sim 10^{18}$ GeV. These numbers, and the time when Inflation stops, $t \sim 10^{-32\pm6}$ s, with an average $\rho^{1/4} \sim 10^{13\pm3}$ GeV, are from Table 2.1 of Liddle and Lyth [6]. However, the initial $\rho^{1/4}$ has simply been specified as the Planck mass, with no uncertainties attached, for the Planck mass just symbolizes the beginning of Inflation; in reality, several orders of magnitude of uncertainties should be associated with that number, or with any such number for which a model exists. Again, the relation of our model to Inflation – in particular, to the first peak of Fig.1 – will be discussed elsewhere, in detail.

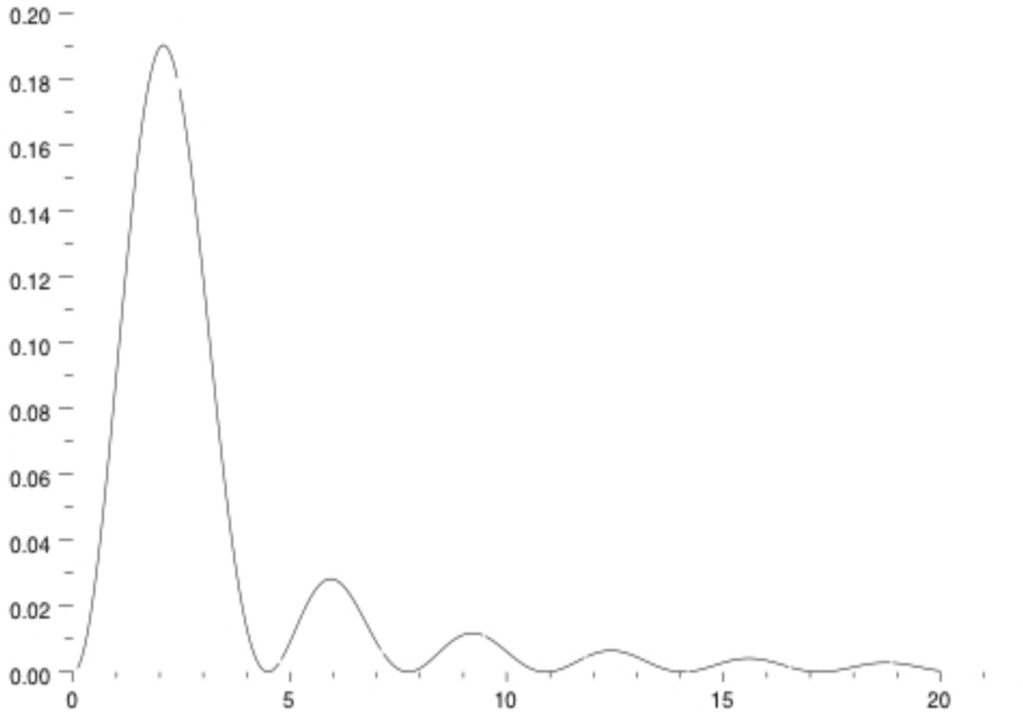

Figure 1: A plot of $f(x) = \dfrac{1}{x^2}\left(\cos x - \dfrac{\sin x}{x}\right)^2$ vs. x

The relevant equation determining M is then :

$$\frac{1}{2} = \Pi_R(M^2) = \frac{2\alpha}{3\pi}\Big[3\ln\Big(\frac{M}{\bar{m}_l}\Big) + 3\times 3\times(\frac{4}{9}+\frac{1}{9})\ln\Big(\frac{M}{\bar{m}_q}\Big)\Big] \tag{5.1}$$

With $\bar{m}_l = \bar{m}_q = m_0 = 1\,\mathrm{GeV}$, $\alpha = 1/137$ and a color factor of 3 being included, this equation reduces to :

$$\frac{1}{2} = \Pi_R(M^2) = \frac{2\alpha}{3\pi}\Big[8\ln\Big(\frac{M}{m_0}\Big)\Big]$$

and we find $M = 10^{18}\,\mathrm{GeV}$. It is interesting to note that this perfectly finite calculation, in the context of QED, is able to produce a " mass term " on the order of the Planck mass.

With this value of M, we are then able to compute the order of magnitude of present day vacuum energy. If we compute that energy density by integrating the energy density of this vacuum field, as expressed in (4.4) – or more simply, by extracting the dominant, non oscillatory behavior of that integral – and dividing by the present day volume of the Universe, $(4\pi/3)R^3_{\text{now}}$, one finds :

$$\rho_{\text{now}} \sim 6\,\xi M^2/R^2_{\text{now}} = 6\,\xi \left(\frac{M}{m}\right)^2\left(\frac{mc}{\hbar}\right)\left(\frac{m}{R^2_{\text{now}}}\right) \tag{5.2}$$

where $m \simeq 10^{-27}\,\text{gm}$ is the electron mass, $(\hbar/mc) \simeq 10^{-10}\,\text{cm}$, $M/m \simeq 5.10^{-24}$, and $(4\pi/3)R^3_{\text{now}} \simeq 10^{85}\,\text{cm}^3$, or $R^2_{\text{now}} \simeq 10^{57}\,\text{cm}^2$. The result is :

$$\rho_{\text{now}} \sim \xi\,10^{-28}\,\text{gm}/\text{cm}^3 \tag{5.3}$$

which, choosing $\xi \sim O(.1)$ is precisely the order of magnitude needed to fit the acceleration data.

This model of a QED vacuum energy, defined by a particular choice of $C_\mu(k)$, is, of course, not the only possibility; but it is everywhere finite, with a simple Lorentz invariance form.

- We now come to Inflation… and immediately, there is is one crucial difference between the VE obtained for present-day Dark Energy - built from renormalized loop fluctuations - and that for Inflation.
- Inflation begins when our Universe was infinitesimally small. QED charge renormalization requires viewing a "bare charge" at distances larger than the Compton wavelength of the charged particle. But those distances do not yet exist… and and such fluctuations must be viewed in terms of the bare charge, e_0 .
- Is this possible? In all QED perturbative approximations e_0 diverges… but in the non-perturbative summations (of Fried and Gabellini, Annals of Physics 327 (2012) 1645)), all the divergences cancel, so that $(Z_3)^{-1}$ and e_0 are finite, and $\alpha_0 = e_0^2/4\pi = \pi/2$.
- For Inflation, we can therefore use α_0 , rather than α , but we must provide a cut-off to the log divergence of the integral

$$\Pi(k^2) = -\frac{2\alpha_0}{\pi}\int_0^1 dy \cdot y \cdot (1-y) \int_\epsilon^\infty \frac{ds}{s}\, e^{-is[m^2 + y(1-y)k^2]}$$

- at its lower limit, where $\epsilon = \Lambda^{-2} \rightarrow M_P^{-2}$, where the (Planck mass)^(-1) represents the smallest distance describable in terms of relativistic QM and Gravity.

But one immediately sees that the previous integral may be re-written as

$$\Pi(M^2) = \frac{i\pi^2}{12} - \int_0^1 dy \cdot y(1-y) \int_\epsilon^\infty \frac{d\tau}{\tau} e^{-\tau[m^2 + y(1-y)M^2]},$$

and has an imaginary part, conflicting with the Model's demand that $\Pi(M^2)$ must be real. And each lepton or quark loop contributes another and equal imaginary part. What to do? Is there any other type of "particle" whose loop could contribute an imaginary part of opposite sign?

Let's try a daring assumption! Suppose that, in the QV, there exist loops built from massive, electrically-charged, fermionic tachyons, particles whose p > E, v > c, and which couple to photons as do the leptons. Immediately, one sees that their imaginary contributions exactly cancel out those of the leptons and quarks, one tachyon loop for every lepton and/or quark loop.

Does such an assumption violate any experimental result? Not at all! What it does violate is the long-standing prejudice against allowing any a-causal Physics to enter our description of the physical world. And the grand irony is that, following this picture to its conclusion, one obtains a coherent picture of Dark Energy, Inflation, Dark Matter, Ultra-High Energy Cosmic Rays that can violate the GZK Limit, GeV Gammas with and without Optical and X-Ray tails, and of the relation of Inflation to the Big Bang, and to the Birth and Death of a Universe.

- Can one imagine a situation in which such massive, electrically-charged Ts and Tbars would be able to move from the QV to the Real Vacuum?
- Yes: e.g., in the immediate vicinity of a supernova explosion, where huge electric fields, acting for a short time, could provide a Schwinger mechanism, and tear such T-Tbar pairs out of the QV. (Or, perhaps there were Ts and Tbars in the original Big Bang.)
- But there can be no experimental objection to bound T-Tbar pairs in the QV. Distaste, and prejudice, Yes; but experimental objection, No.
- Incidentally, one brief word: A high-energy tachyon is a perfect candidate for a Dark Matter particle…

- To summarize the results which follow from including T-Tbar loops together with lepton loops:
- a) Such Ts can have masses 10^(10   6) M_P, while the M parameter is 10^(8   6) GeV.
- b) These numbers are compatible with the initial (10^(-42) sec.) and final (10^(-32   6) sec.) times accepted for the beginning and end of Inflation, as well as for the energy density at the end of Inflation.
- c) And the same proportionality constant for the energy density as used for Dark Energy, can be used here, for Inflation.

Finally, A Cosmological Speculation:

Suppose that there exist stray tachyons moving through the Real Vacuum, and suppose that at some distant point in the Universe, a T of energy >> Planck mass meets and annihilates with a similar Tbar.

Suppose that the space-time continuum cannot absorb such a huge energy density at that point. What will happen if there is a rupture, or a breakdown of the space-time continuum at that spot?

If wherever there is a T_mu,nu there must be a g_mu,nu, a new coordinate system, of a New Universe, forms at that point, while just at that spot the separation of the QV and the Real Vacuum breaks down. One can imagine that the immense amount of the Old Universe's VE, its stored Potential Energy, begins to force its way through that spot, and into the Real Vacuum, where it can then act upon the loop fluctuations of the Old Universe, driving them – in a spectacular Schwinger mechanism – into the Real Vacuum of the New Universe.

And this is the Big Bang, the Birth of a New Universe, as the energy- and charge-conserving loop pairs are torn apart, and explode into the New Universe. As this happens, the QV of the New U. starts to build up with its own loop fluctuations, as the loop fluctuations of the Old U. disappear. Because the potential energy of the loop fluctuations is what balances and exceeds gravitational attraction, as the Old U. loses its loop fluctuations and their associated potential energy, it must eventually collapse into a monstrous black hole, one whose radiation might well be seen by an observer (of the New U.).

Please remember that this is Speculation… but coherent speculation, all following from a simple, QED-based Model of Vacuum Energy.

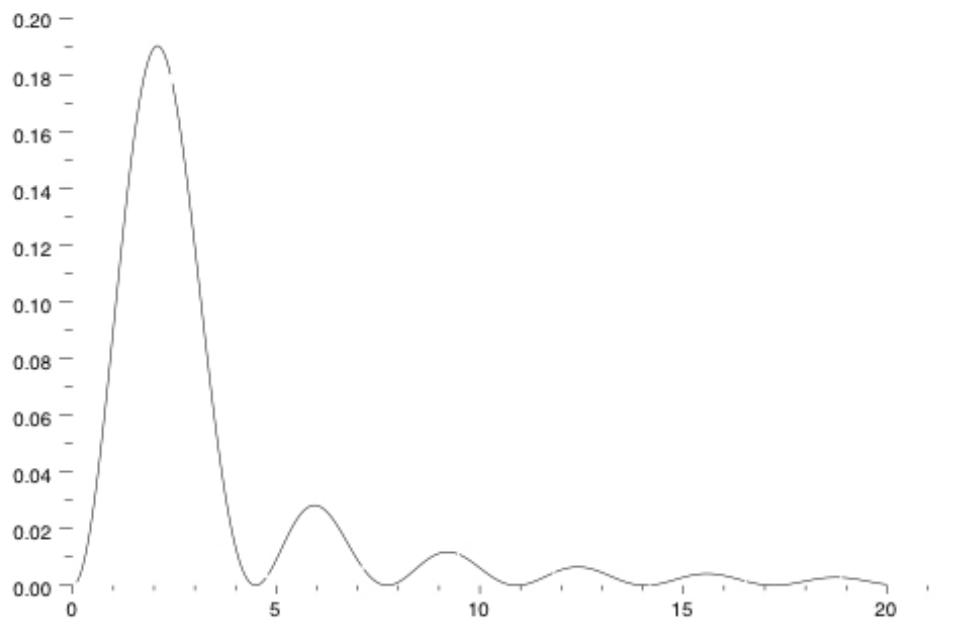

Figure 1: A plot of $f(x) = \frac{1}{x^2}\left(\cos x - \frac{\sin x}{x}\right)^2$ vs. x

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