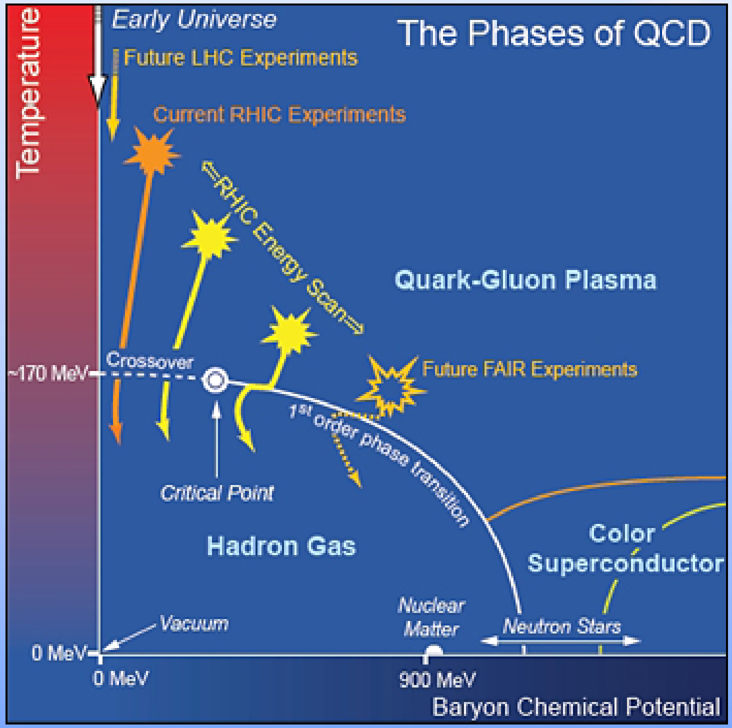
High density phase of QCD in the PNJL model

How does deconfinement occurs at high density

Hubert Hansen <hansen@ipnl.in2p3.fr>



Institut de Physique Nucléaire de Lyon, CNRS/IN2P3 and université Claude Bernard de Lyon

Collaborators: G. Goessens, A. Gatti, P. Costa, M. Ruivo, C. Da Sousa

Motivation: phase transition in RHIC at finite T and density

Confinement and asymptotic freedom are difficult questions analytically ; can be studied in simulation (Lattice QCD). Besides finite density increases the difficulty (no lattice result at large density).

- QCD at high density in the non-perturbative sector $\Rightarrow$ effective model.
- Phase transition study: need chiral symmetry + confinement (associated to $\mathbb{Z}_3$ symmetry) $\Rightarrow$ we use the PNJL model (relation between chiral restoration and deconfinement accessible).
- Only with two flavors: indeed we want to shed light on internal mecanisms occuring at high density (driven by the Fermi momentum) so we do not want to add other effects yet (strangeness, vector, etc.) that could hide those.
- We will study mean field predictions and then use mesonic correlations as a probe of the phase diagram.

The PNJL model is surprinsingly good considering it is "easy" to treat and to understand ; it gives a good phenomenological picture where it can be compared with LQCD, even if it only implements a statistical confinement.

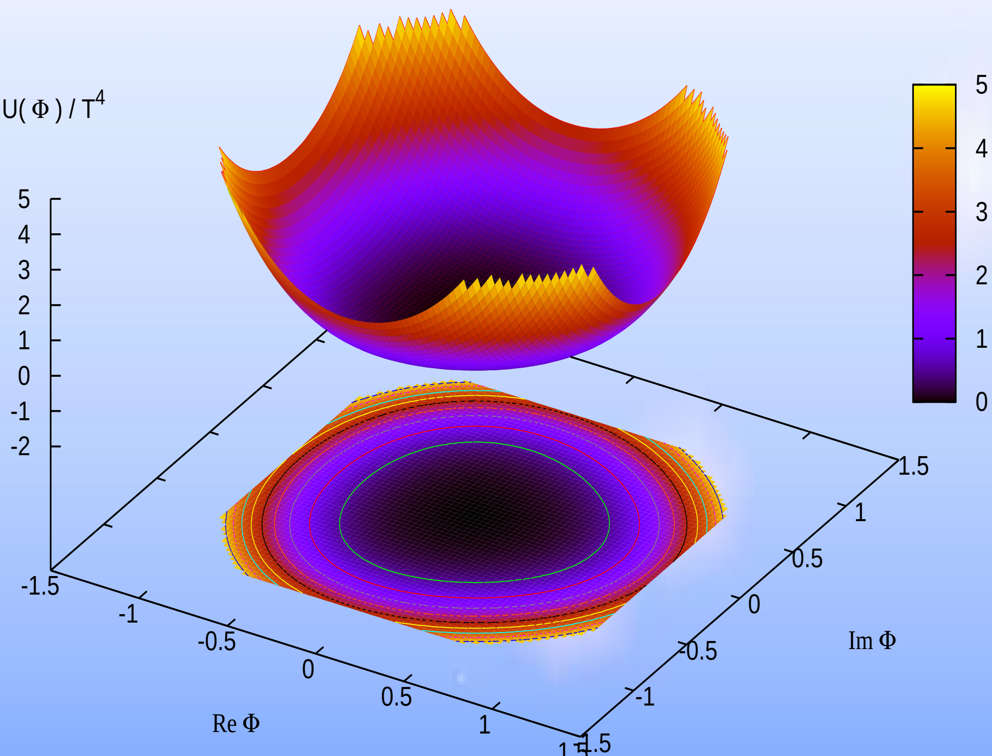
$\Rightarrow$ Understand physically some mechanisms by varying parameters one by one ($e.g.$ changing the coupling between the chiral sector and the deconfinement one ; changing the pure gauge deconfinement temperature, study the influence of the 't Hooft coupling constant on the CEP, etc.).

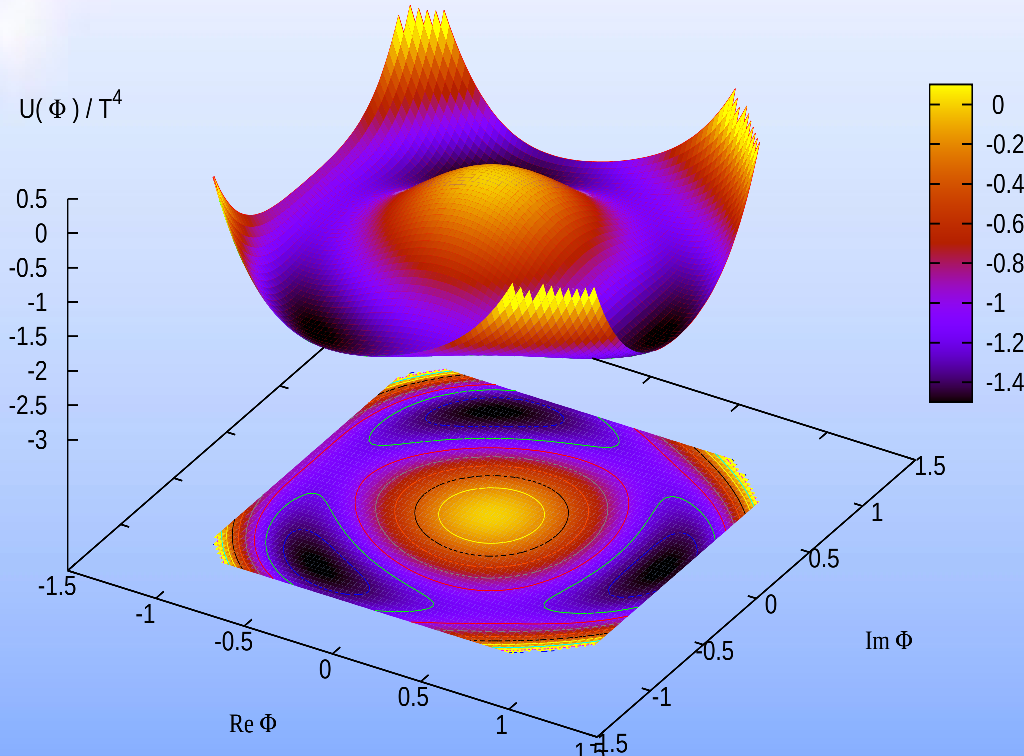
Pure gauge sector: the effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$

$\mathbb{Z}_3$ ("confinement symmetry") spontaneously broken with temperature. To reproduce this phenomenologically, one can choose a potential ($\simeq$ static gluon pressure term) with this form:

$T < T_0$, Color "confinement",
$\langle\Phi\rangle = 0 \longrightarrow$ no $\mathbb{Z}_3$ breaking

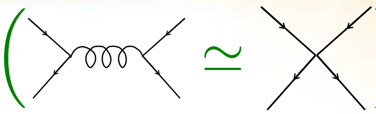
$T > T_0$, Color "deconfinement",
$\langle\Phi\rangle \neq 0 \longrightarrow$ $\mathbb{Z}_3$ broken



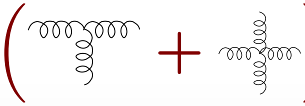


The PNJL model (Polyakov – Nambu – Jona-Lasinio) in a nutshell

$$\mathcal{L}_{PNJL} = \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)q + G_1\left[\left(\bar{q}q\right)^2 + \left(\bar{q}i\gamma_5\vec{\tau}q\right)^2\right]\left(\right.$$



$$\left. \right) - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right)\left(\right.$$

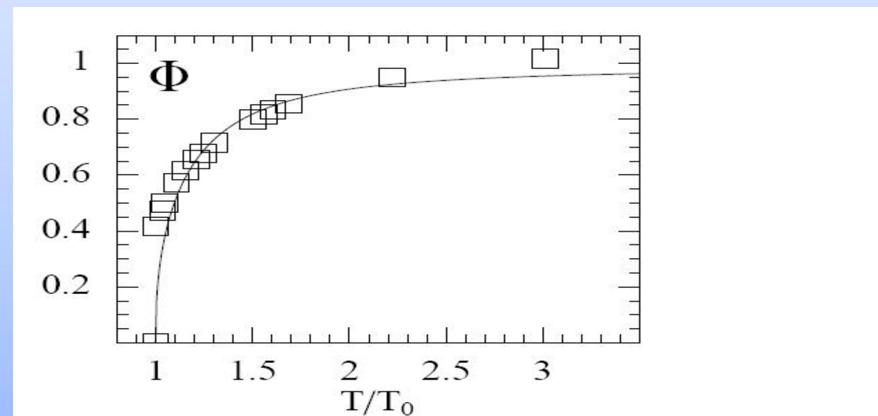


$$\left. \right) + \mu\bar{q}\gamma_0 q$$

- **NJL parameters choosen to fit hadronic input in vaccuum**

- **Polyakov loop in imaginary time and Polyakov gauge:** $L\left(\vec{x}\right) = \mathcal{P}\exp\left[i\int_0^\beta dx_4\, A_4\left(\vec{x}, x_4\right)\right] \Rightarrow$

Effective field $\Phi = \frac{1}{N_c} Tr_C\, L$; L transports the field A_μ from the point in space-time $(\vec{x}, 0)$ to $(\vec{x}, \beta)$ $\Rightarrow$ $\Phi = 0$: confinement ; $\Phi = 1$: free propagation (deconfinement)



- **Effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$ (gluon pressure):**

$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}; T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \text{ and}$$

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

(T_0 = 270 MeV)

In the following $\mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right)$ can also be choosen in its logarithmic form.

C. Ratti, M. Thaler, W.Weise, hep-ph/0604025 : lattice: O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, Phys. Lett. B **543**, 41 (2002).

Mean field grand potential

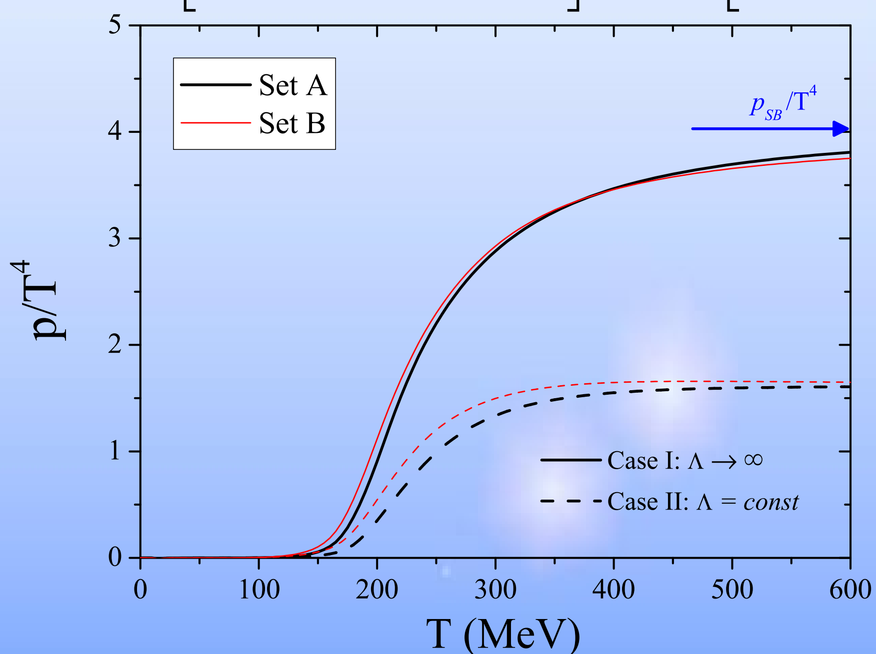
Grand potential at finite temperature and density: with $E_p = \sqrt{\vec{p}^{\,2} + m^2}$

$$\Omega = \boldsymbol{\mathcal{U}}\left(\boldsymbol{\Phi}, \bar{\boldsymbol{\Phi}}, \boldsymbol{T}\right) + \frac{(m - m_0)^2}{2G_1} - 6N_f \int_\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p -$$

$$2N_f\, T \int_{\Lambda_T} \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ \mathrm{Tr}_c \ln\left[1 + \boldsymbol{L} \mathrm{e}^{-(E_p - \mu)/T}\right] + \mathrm{Tr}_c \ln\left[1 + \boldsymbol{L}^\dagger \mathrm{e}^{-(E_p + \mu)/T}\right] \right\}$$

The propagation of the quarks into the medium filled with (background) gluon fields with pressure $\mathcal{U}$ leads to statistical suppression of 1- and 2-quarks propagation (statistical confinement) :

$$\mathrm{Tr}_c \ln\left[1 + \boldsymbol{L} \mathrm{e}^{-(E_p - \mu)/T}\right] = \ln\left[1 + 3\boldsymbol{\Phi} e^{-\beta(E_p - \mu)} + 3\bar{\boldsymbol{\Phi}} e^{-2\beta(E_p - \mu)} + e^{-3\beta(E_p - \mu)}\right]$$



Cutoff of the model:

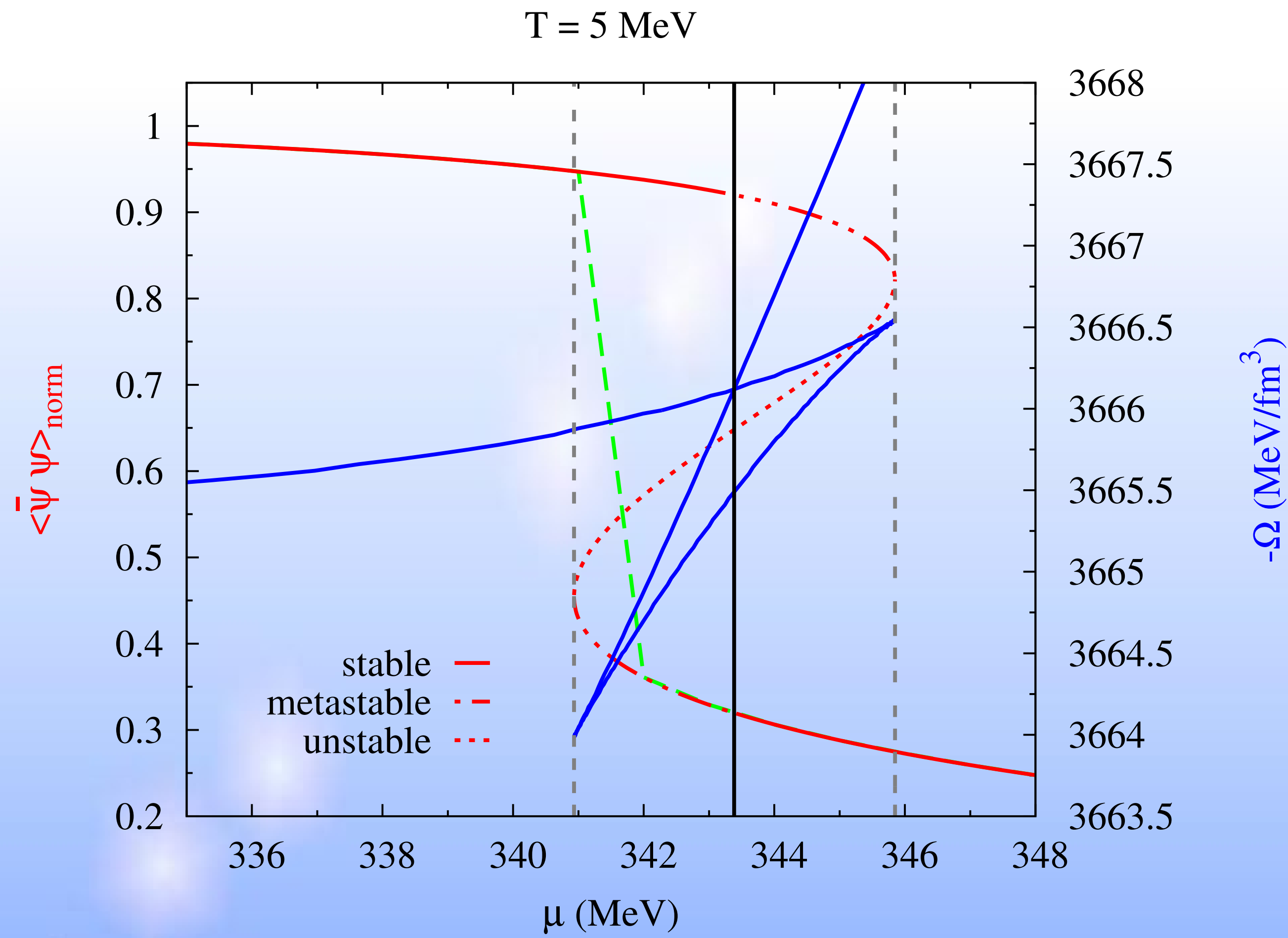
The model needs a cutoff in vaccuum: $\int_0^\Lambda$ with Λ a typical hadronic scale ($\Lambda \simeq 600$ MeV).

In medium (thermal) part of the model: Boltzmann factors are enough to regularize the integral but ...

pressure with finite and infinite Λ: lack of high momentum quarks to saturate the pressure (Stephan-Boltzmann limit) if $\Lambda_T = \Lambda$.

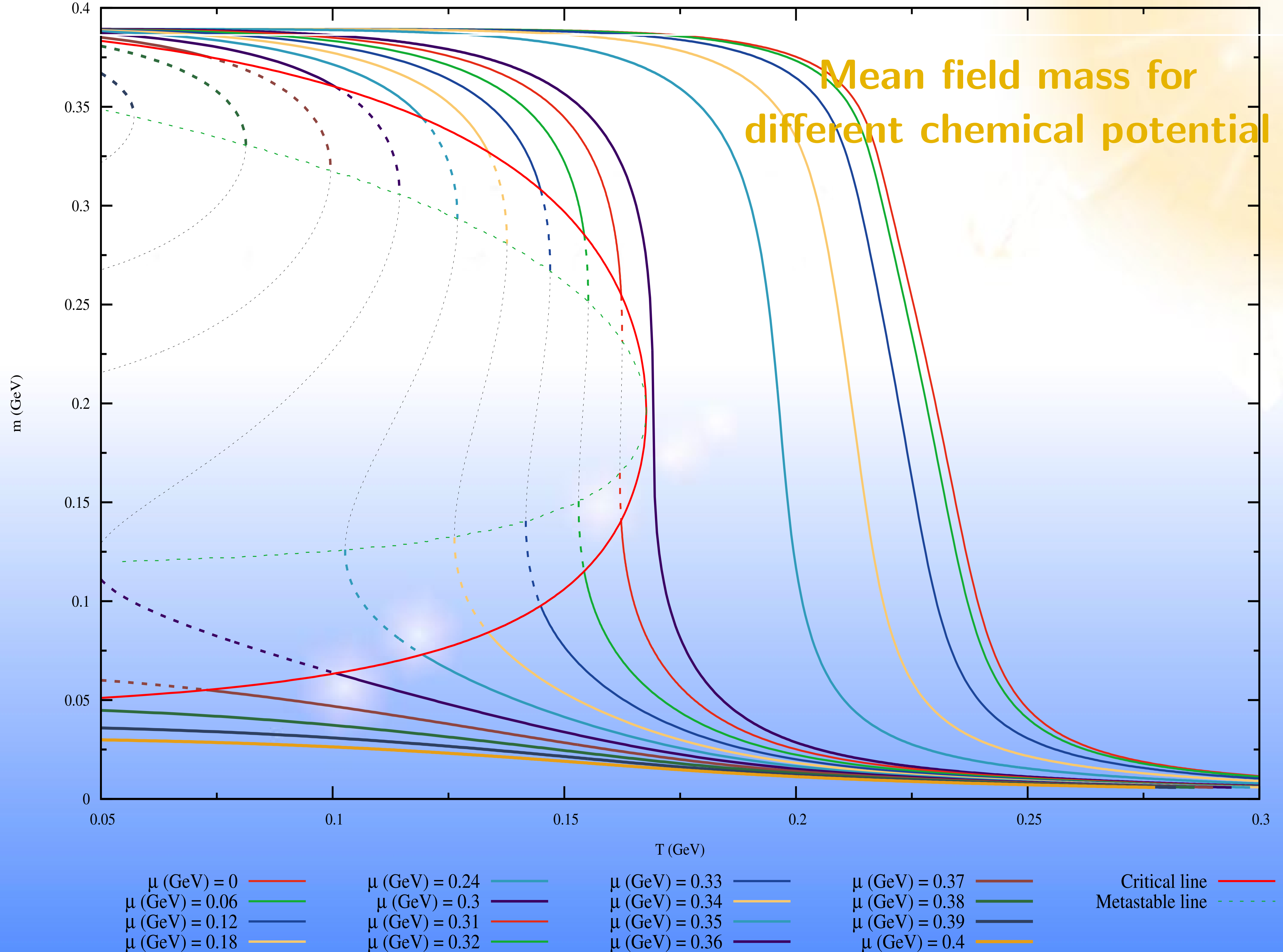
Determination of the first order transition

After solving the mean field equations (minimizing Ω):



We suppose the first order transition happens when the grand potentials are equal in the high and low mass (meta)stable phase.

Mean field mass for different chemical potential



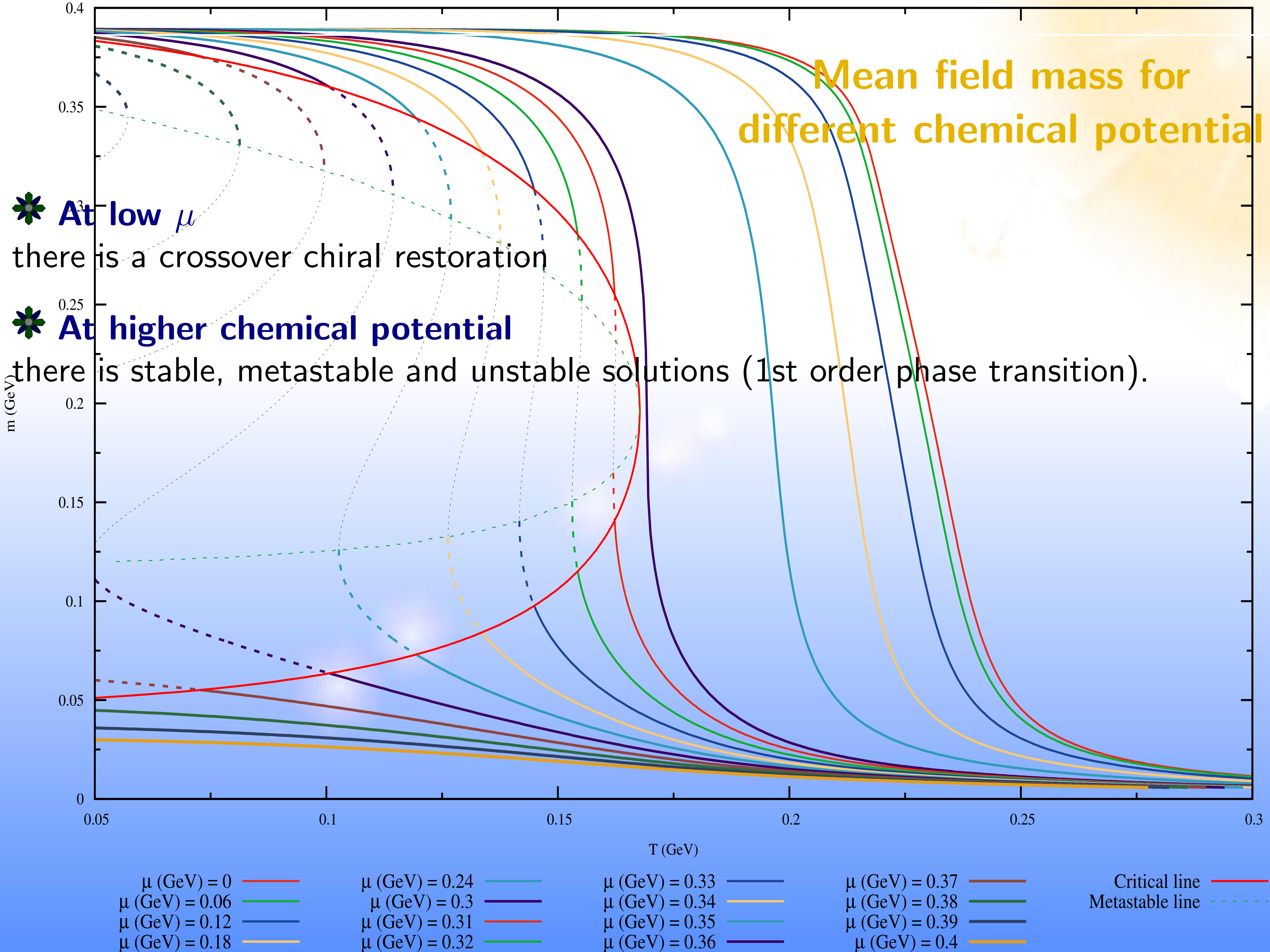
Mean field mass for different chemical potential

- **At low μ**

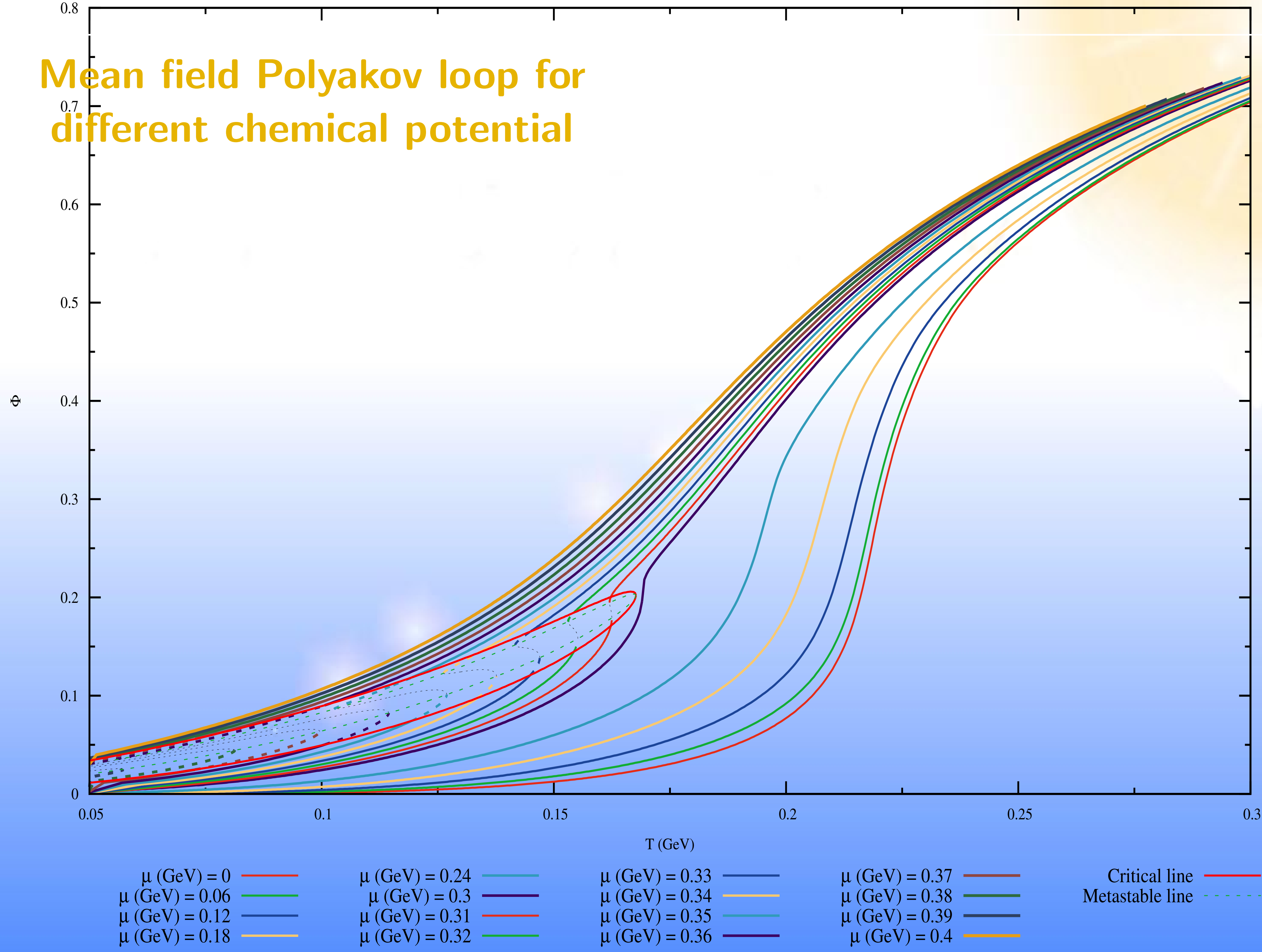
there is a crossover chiral restoration

- **At higher chemical potential**

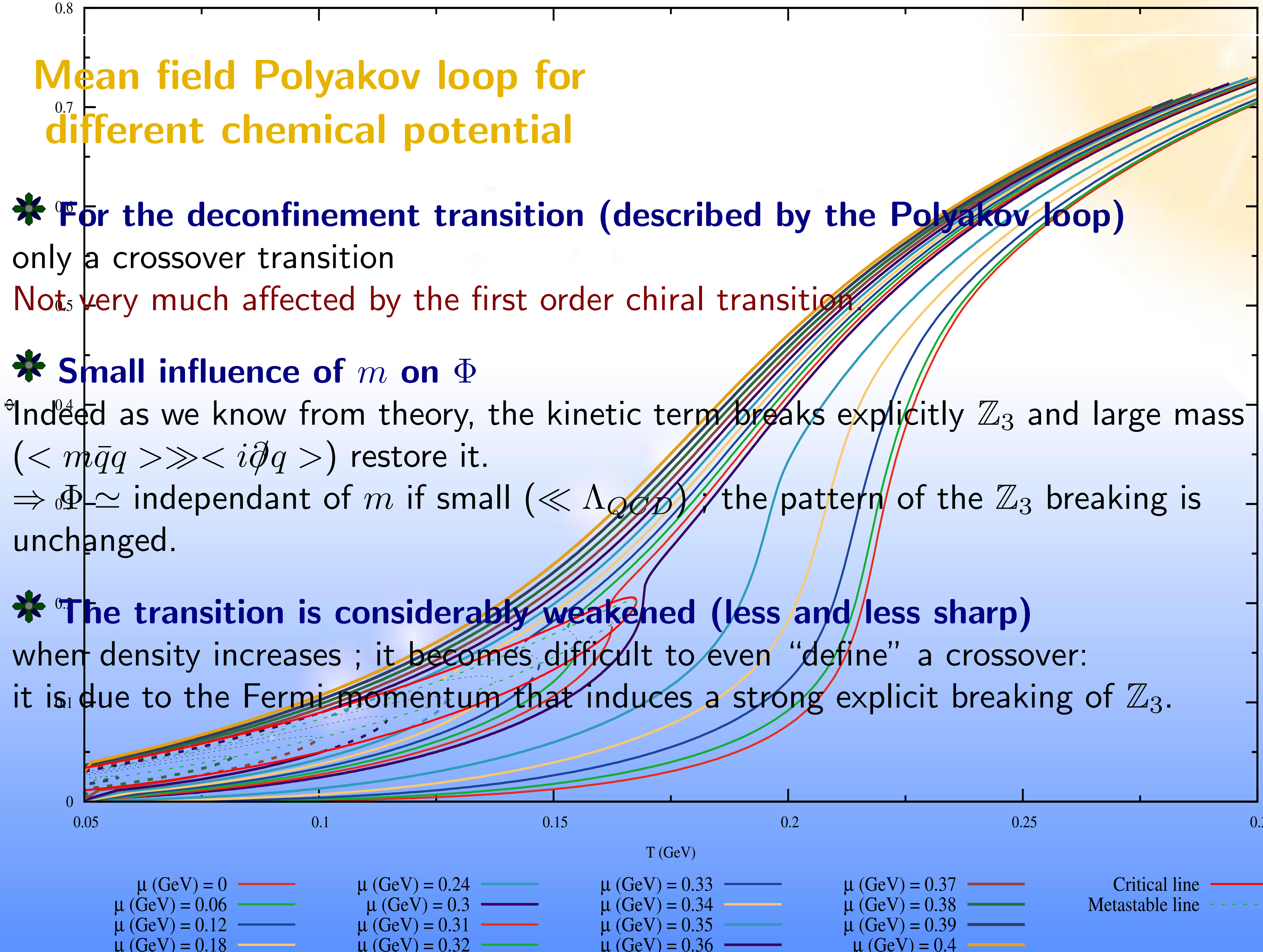
there is stable, metastable and unstable solutions (1st order phase transition).



Mean field Polyakov loop for different chemical potential



Mean field Polyakov loop for different chemical potential

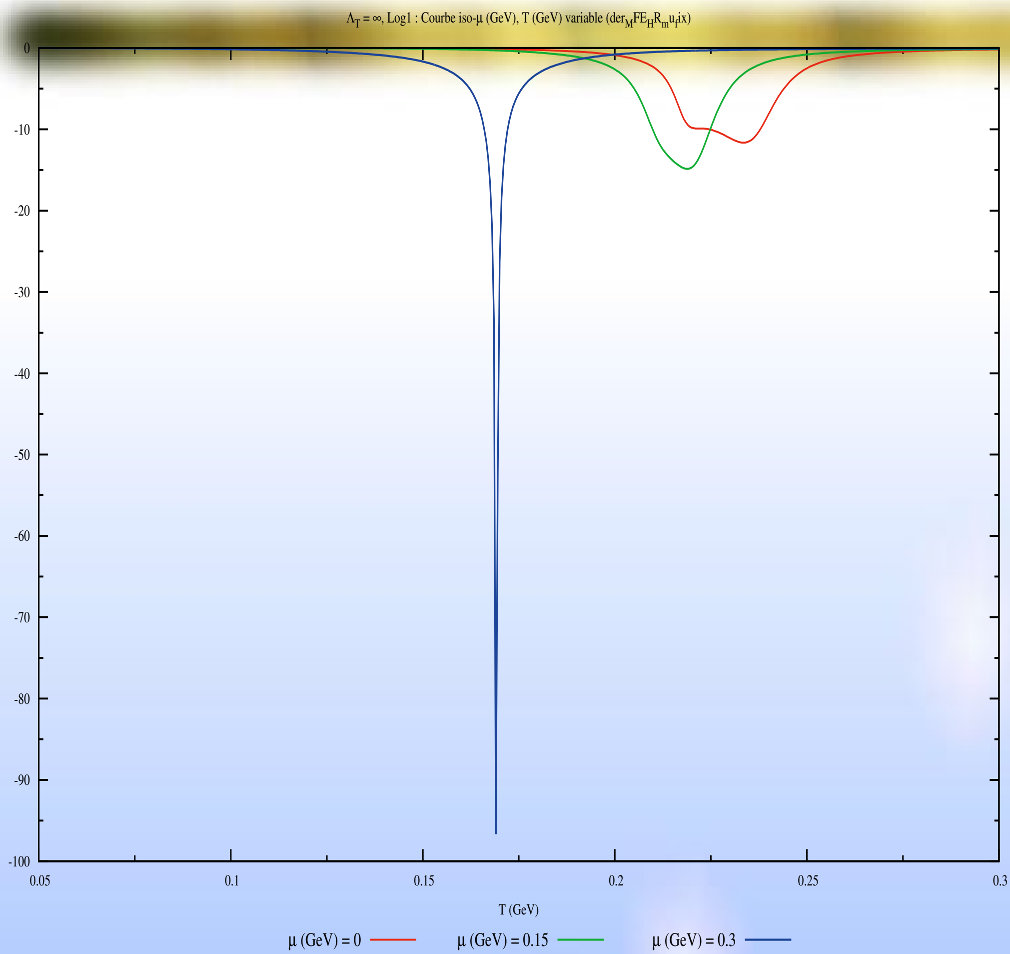


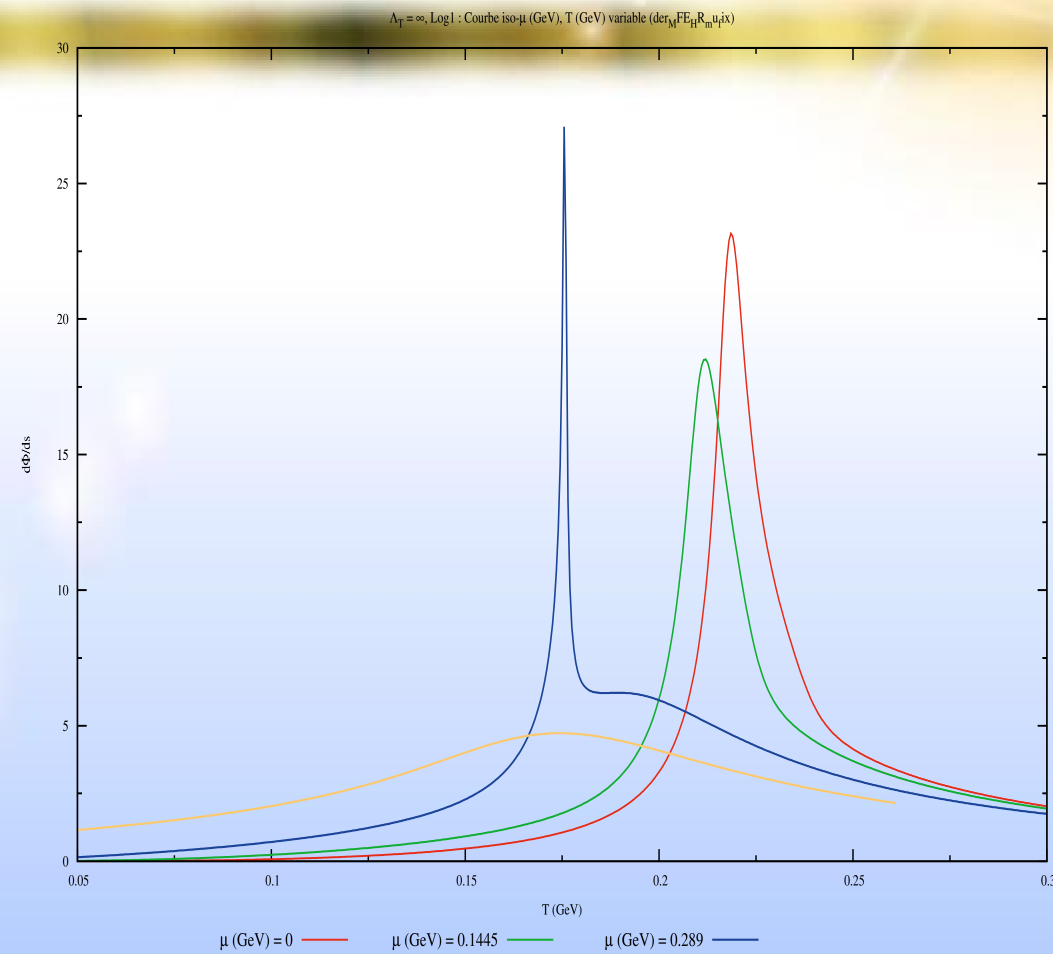
- **For the deconfinement transition (described by the Polyakov loop)**
only a crossover transition
Not very much affected by the first order chiral transition.

- **Small influence of m on Φ**
Indeed as we know from theory, the kinetic term breaks explicitly $\mathbb{Z}_3$ and large mass $(< m\bar{q}q > \gg < i\partial\!\!\!/ q >)$ restore it.
$\Rightarrow \Phi \simeq$ independant of m if small ($\ll \Lambda_{QCD}$) ; the pattern of the $\mathbb{Z}_3$ breaking is unchanged.

- **The transition is considerably weakened (less and less sharp)**
when density increases ; it becomes difficult to even "define" a crossover:
it is due to the Fermi momentum that induces a strong explicit breaking of $\mathbb{Z}_3$.

Susceptibilities at different chemical potential

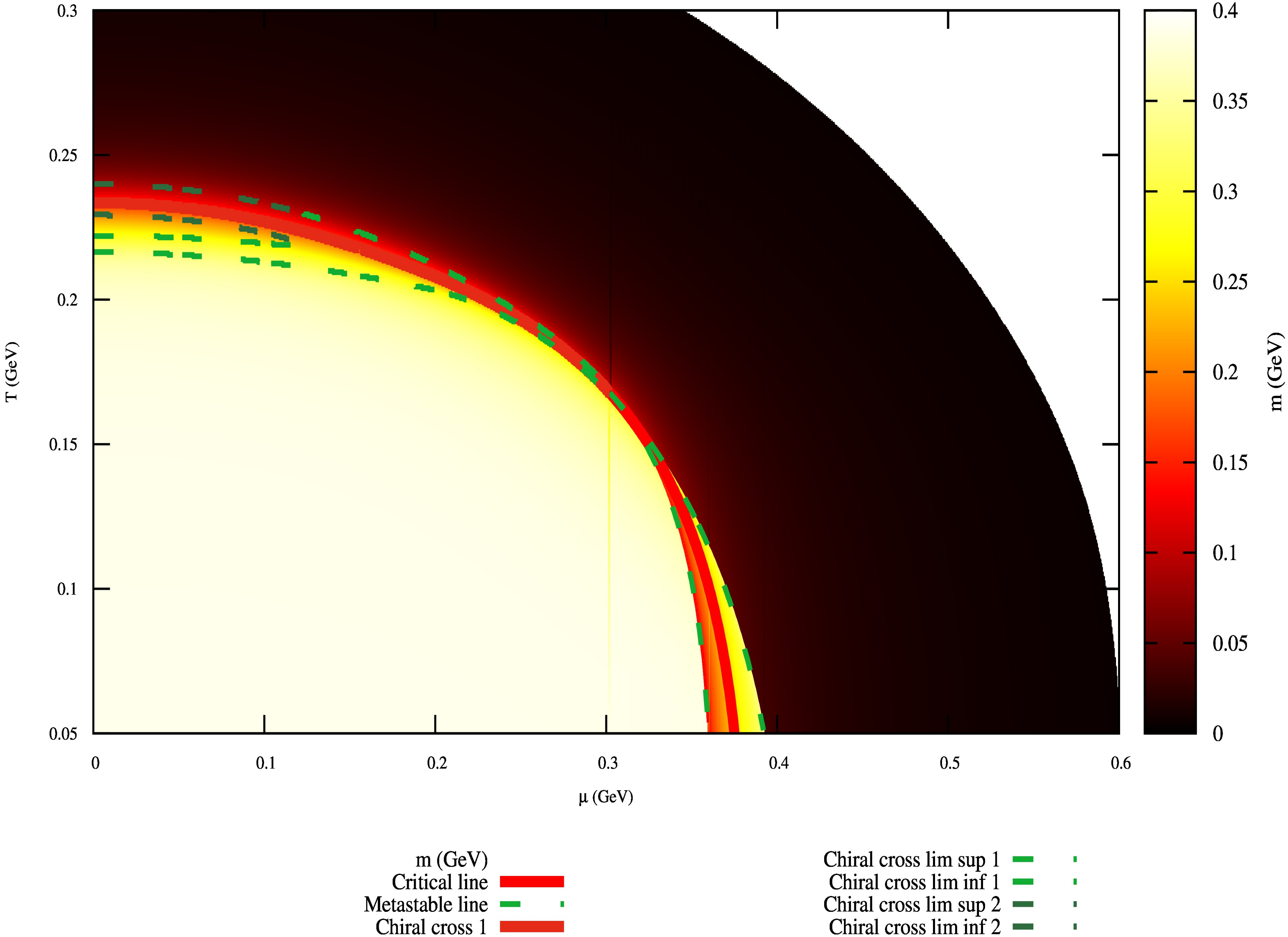


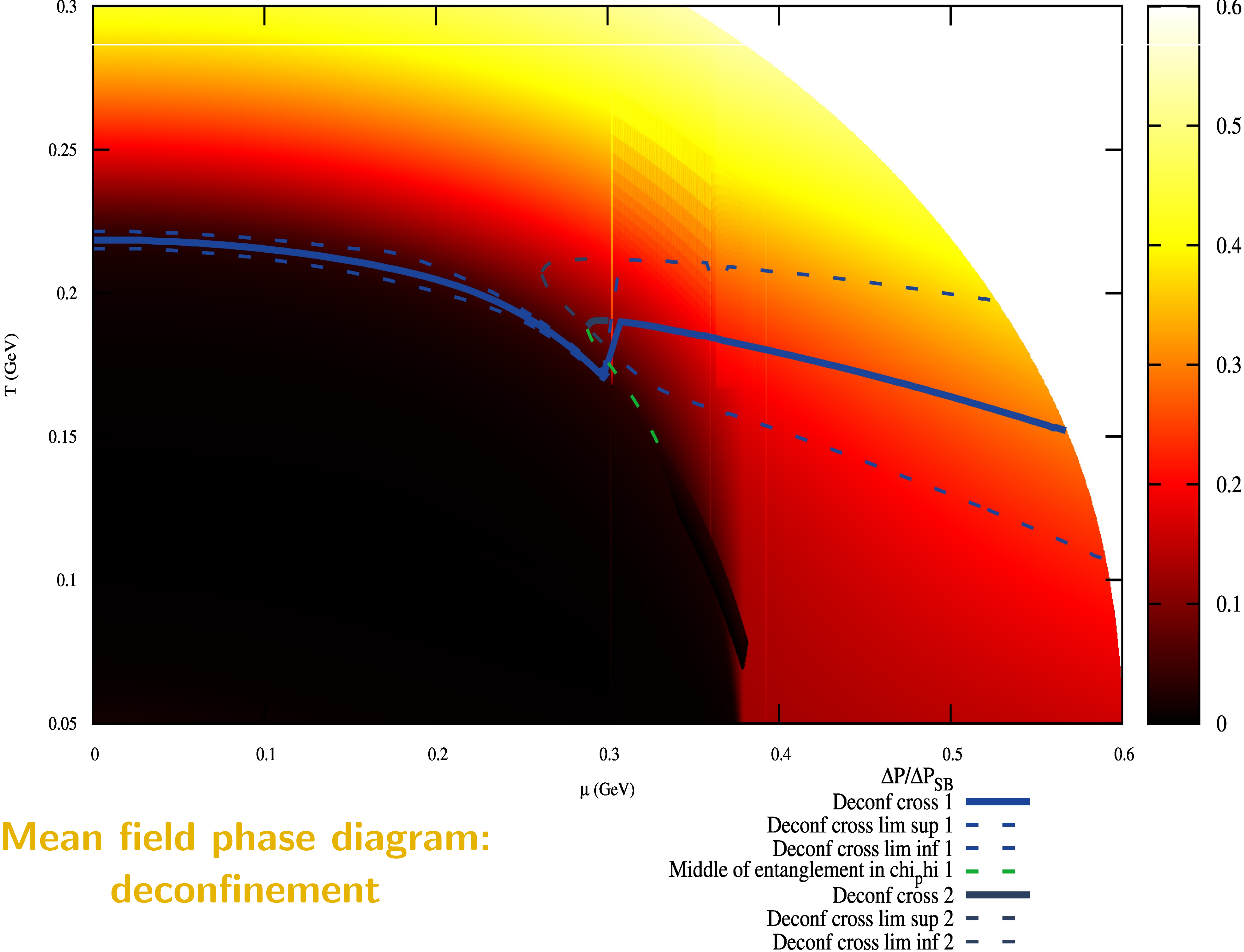


Left: Chiral susceptibilities $\chi_m = \dfrac{\partial m}{\partial T}$; Right: Deconfinement susceptibilities $\chi_\Phi = \dfrac{\partial \Phi}{\partial T}$

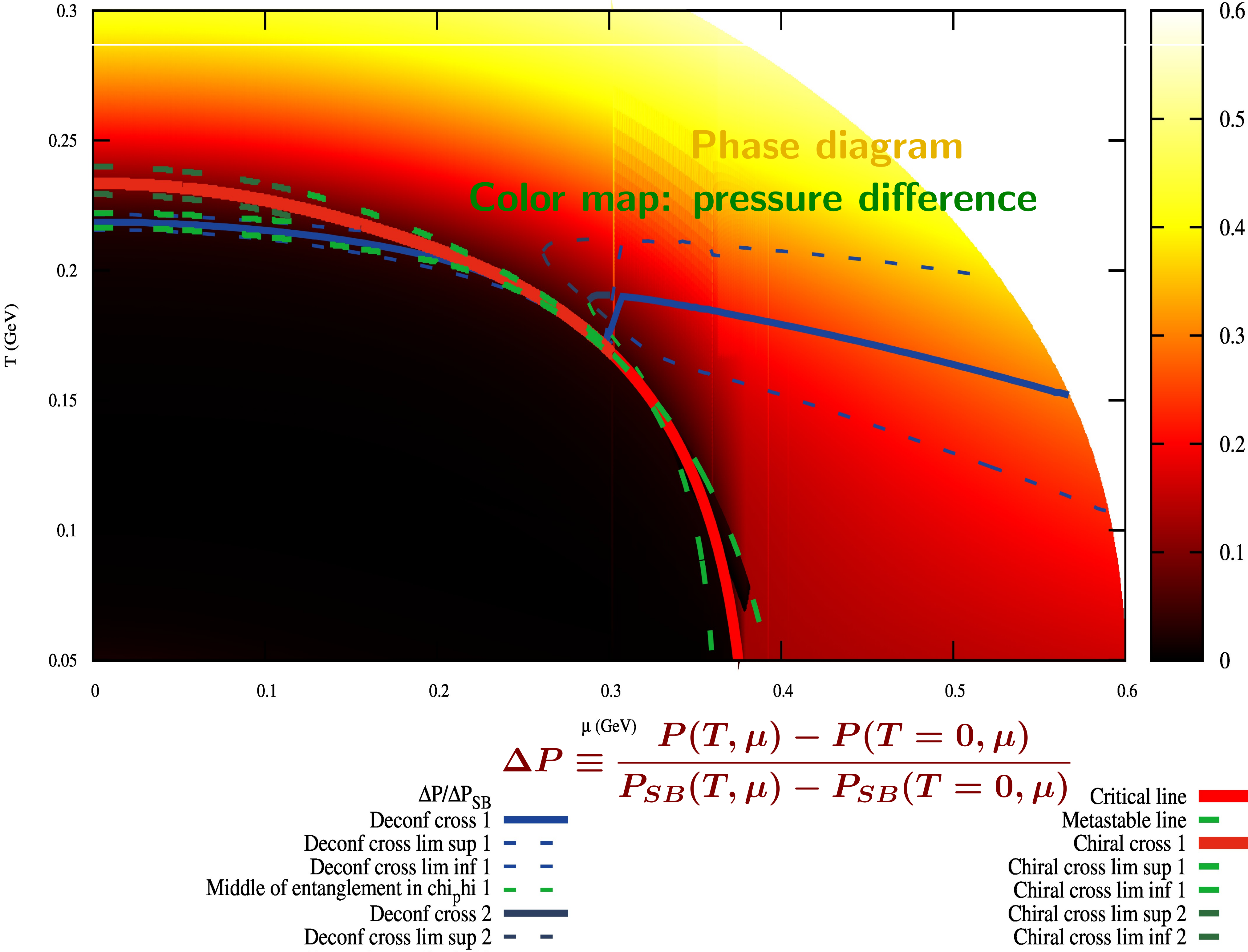
Crossover region (where most of the transition occurs) $\simeq$ delimitated by the inflexion points of χ ; depending on the parameters (μ here) a rich structure appears ($e.g.$ a maximum in χ_m, two max in χ_φ) $\Rightarrow$ modification of the cross section.

Mean field phase diagram: chiral restoration



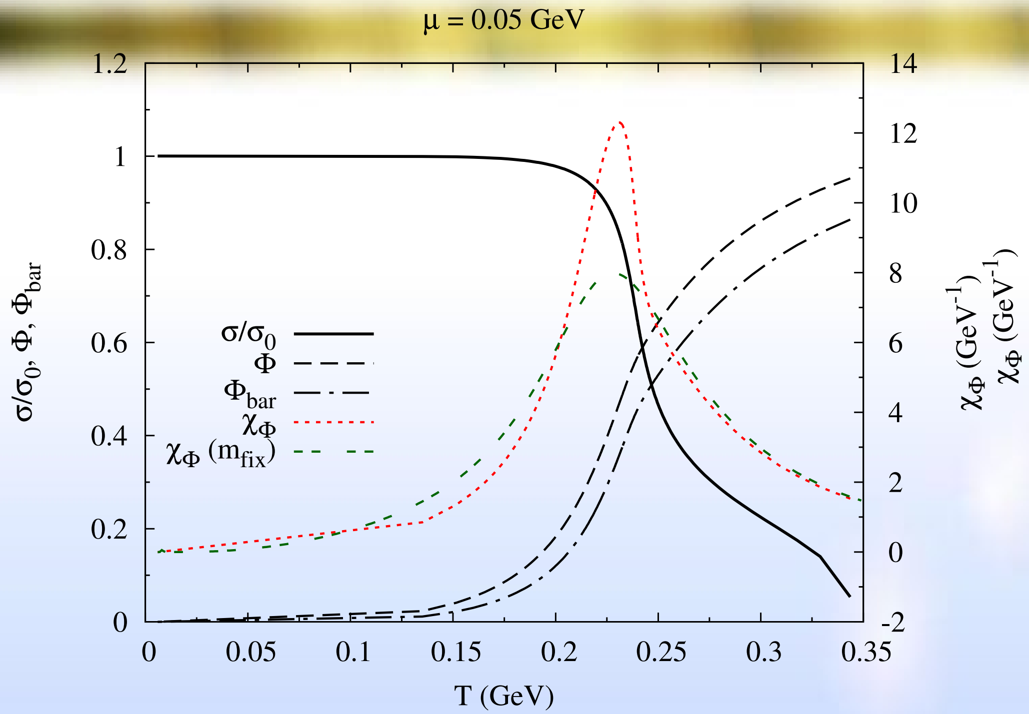


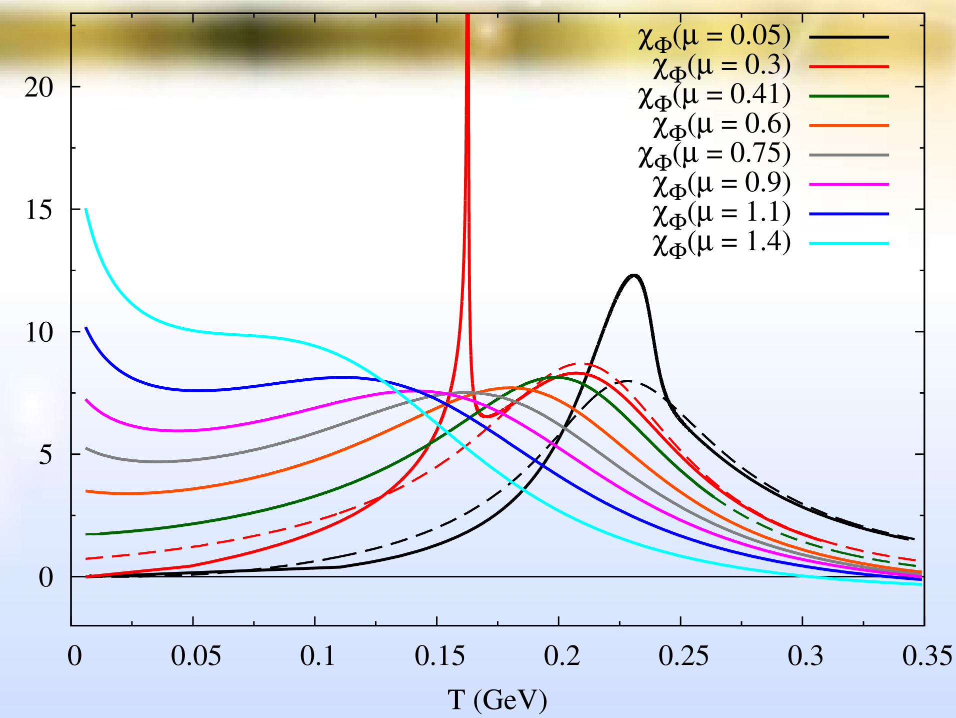
Mean field phase diagram: deconfinement



$$\Delta P \equiv \frac{P(T,\mu) - P(T=0,\mu)}{P_{SB}(T,\mu) - P_{SB}(T=0,\mu)}$$

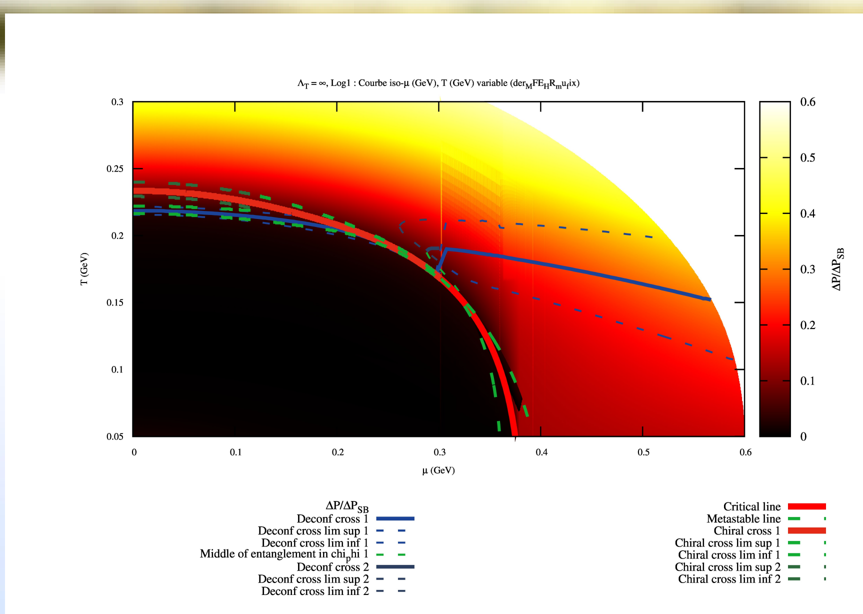
Chiral and deconfinement crossover entanglement

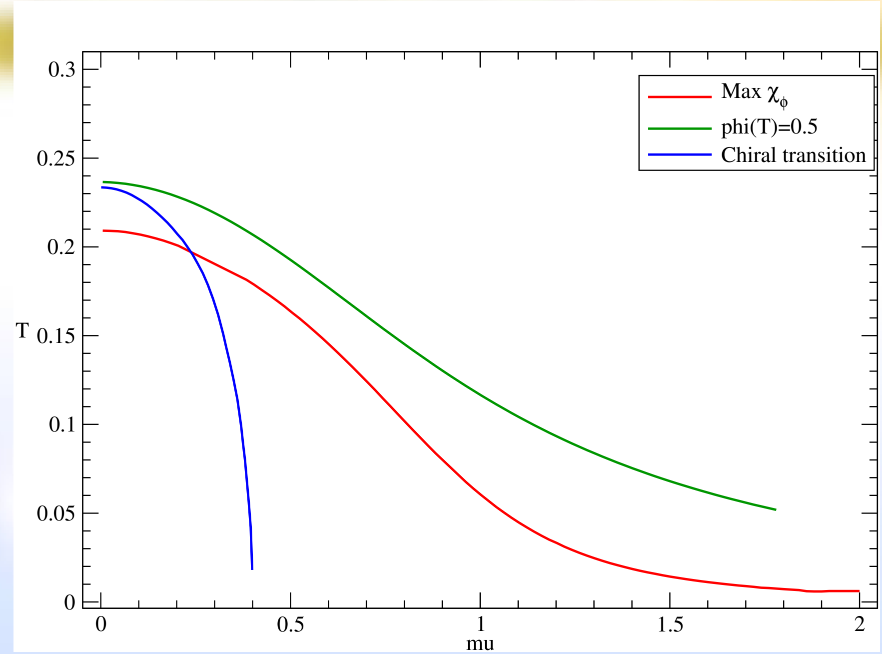




Left: Black: order parameters $< \bar{q}q >$ (chiral symmetry) and Φ (deconfinement) ; Red: deconfinement thermal susceptibility $\chi_\Phi = \frac{\partial \Phi}{\partial T}$; green: susceptibility with m_{fix} ($i.e.$ solving the mean field equations with m fixed to a given value) $\Rightarrow$ allow to remove the deconfinement and chiral transition entanglement.
Right: Φ-susceptibility for different values of μ. The solid lines represent χ_Φ calculated with the Hartree mass, the dashed lines χ_Φ calculated with the constant mass (mean value between the mass at $(T = 0,\ \mu)$ and m_0). From the green line, we cannot see the dashed lines because χ_Φ is the same if we take a constant mass or the Hartree mass.
The small influence of the mass on Φ allows to desentangle chiral and deconfinement transition.
High density: **does not look like a crossover.**

Full phase diagram





Phase diagrams with and without m_{fix}

⇒ decoupling of deconfinement and chiral restoration at the Critical End Point (CEP).

In this model, 3 phases: chirally broken and "confined" ; chirally broken and "deconfined" ; chirally restored and "deconfined" (QGP)

⇒ tendancy to deconfinement at high density (even at vanishing temperature !), an effect driven by the Fermi motion of quarks that broke $\mathbb{Z}_3$.

Note that the model is push above its limit (because of the lack of dynamical gluonic degrees of freedom): we take this results as an indication that it *may be* the same in QCD.

Relevance for phenomenology of heavy ion experimental program at high density ?

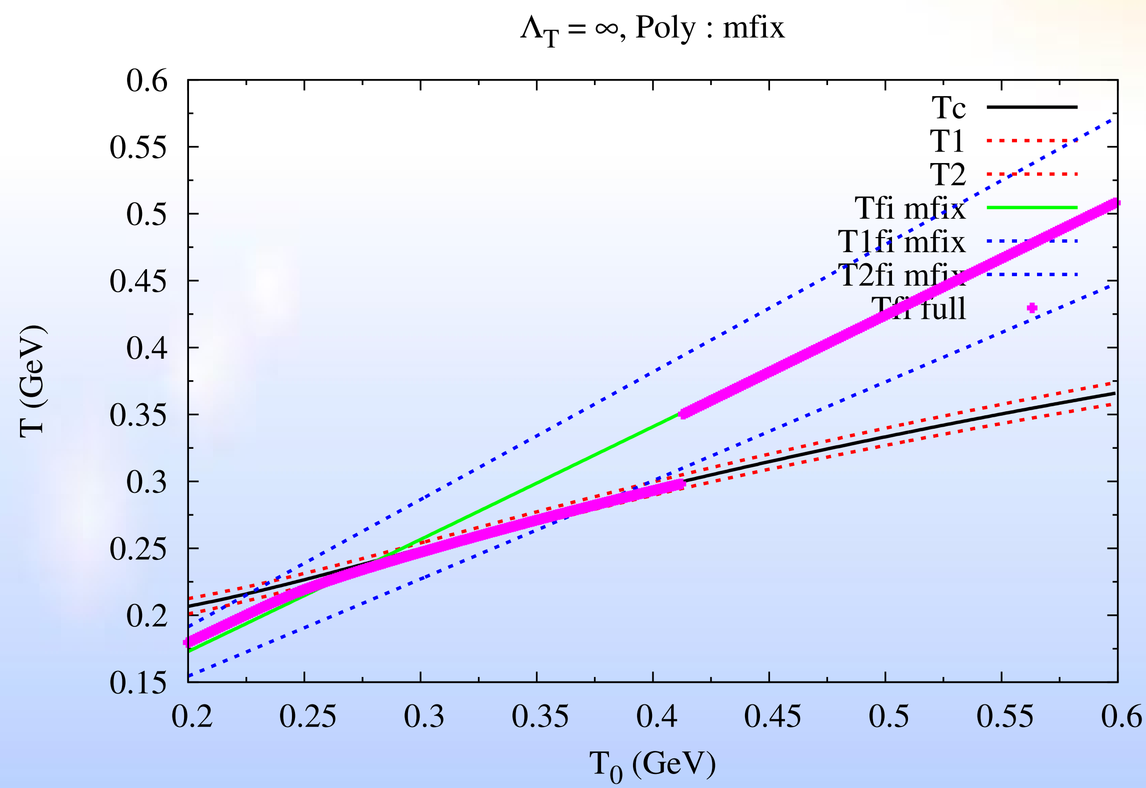
As a first step, we will take a look on a particular QGP probe, the mesonic correlations.

The Casher argument a $\mu = 0$

Casher: deconfinement induces a breaking of the chiral symmetry.

At zero density the only parameter of the PNJL model with some freedom is T_0. Changing it changes the order of the occurence of deconfinement and chiral restoration as T increases:

$\Rightarrow$ it does not seem to be a very strong coupling between the two ; the almost perfect coincidence seen in some model calculation is probably not related to a microscopic mechanism, even if as seen before (or here) when chiral and deconfinement transition are close, the deconfinement is somehow hidden by the chiral one but after a few MeV the two gets desantangled.

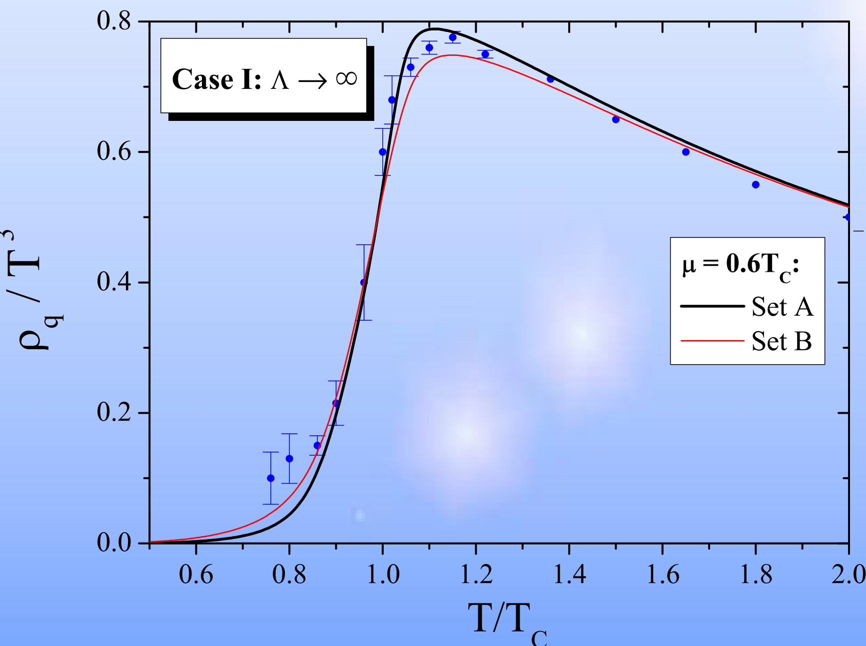


On this picture, one sees that the entanglement does not survive long when one forces with T_0 the two transitions to be at different temperature.

Spectral function $Meson \to q\bar{q}$ as a probe of the phase diagram

Calculation of the mesonic polarisation with the ring approximation (the spectral function describes only $M \to q\bar{q}$): we choose to remove high momentum quarks ($\Lambda_T = \Lambda$) from the quark loops as a reminder of the QCD asymptotic freedom (high momentum quarks have a weak, negligible interaction for what concern formation of bound states or resonances). Anyway, there is only a small quantitative difference and it is "better" for sum rules ($e.g.$ the V-A sum rule in the vector sector).

No confinement in the PNJL model (only an effect on the thermal bath via the suppression of 1- and 2-quarks Boltzmann factor when $\Phi \simeq 0$): in PNJL "deconfinement" means $\Phi \to 1$.

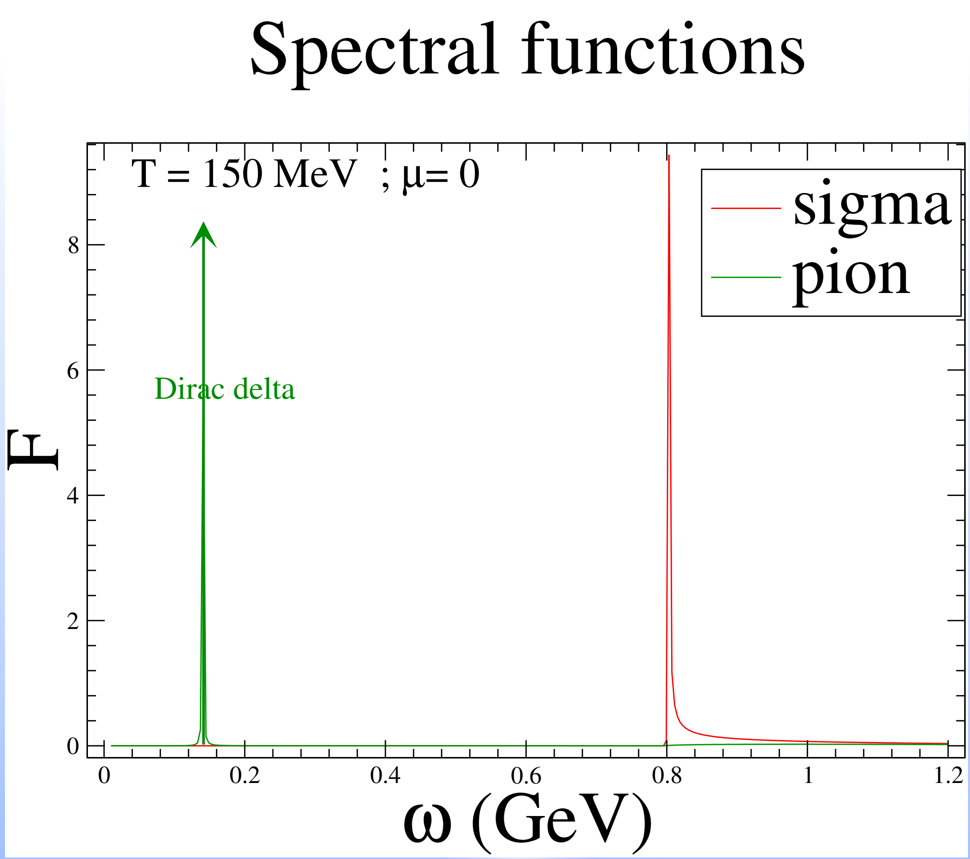


Consequence: the (mean field) quark density is vanishing in the vaccuum ; saturates SB limit in QGP $\Rightarrow$ deconfinement can be read in quark abundances.

Hence: the $M \to q\bar{q}$ spectral function will be a tool to study the consequences beyond the mean field of the deconfinement.

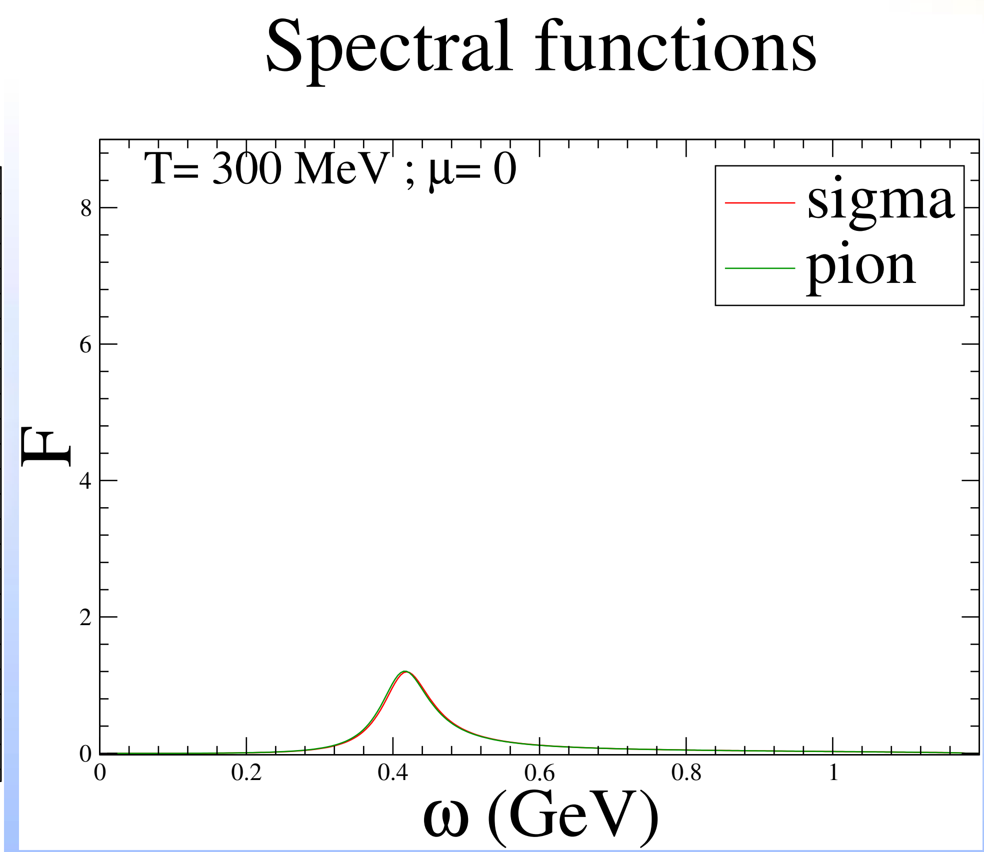
Survival of resonant state after the chiral restoration ?

Qualitative picture



Confining regime: σ is a narrow resonance ; chiral symmetry broken

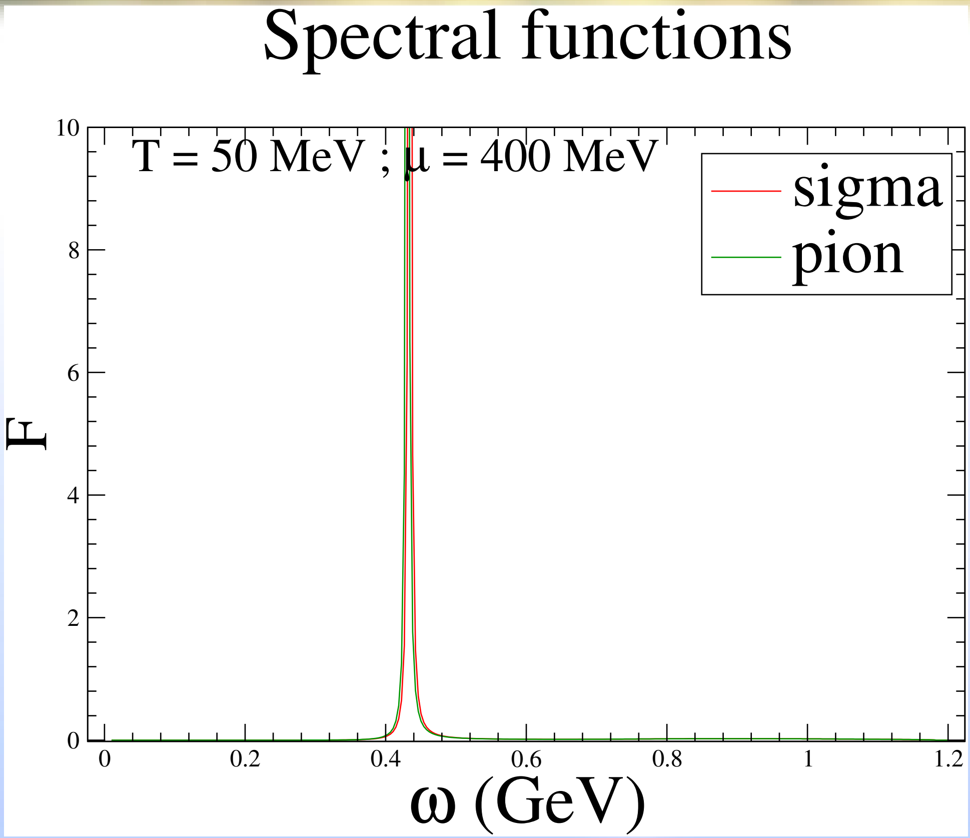
$$< \bar{q}q > \neq 0 \ ; \ \Phi \simeq 0$$



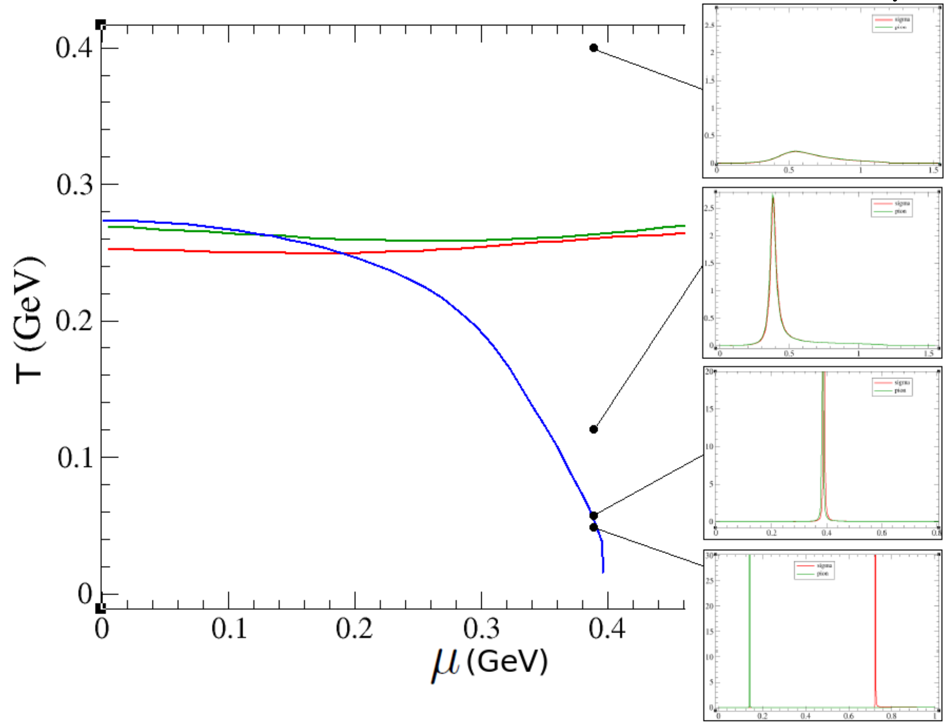
Deconfinement: σ and π spectral function are alsmost the same as an uncorrelated $\bar{q}q$ pair ; chiral symmetry restored

$$< \bar{q}q > \simeq 0 \ ; \ \Phi \neq 0$$

Survival of resonant state after the chiral restoration ?



Evolution on a line at constant μ



Slow deconfinement (*"Confining"* regime): σ and π still narrow resonances ; chiral symmetry restored

$< \bar{q}q > \simeq 0 \;;\; \Phi \simeq 0$

The qualitative conclusions

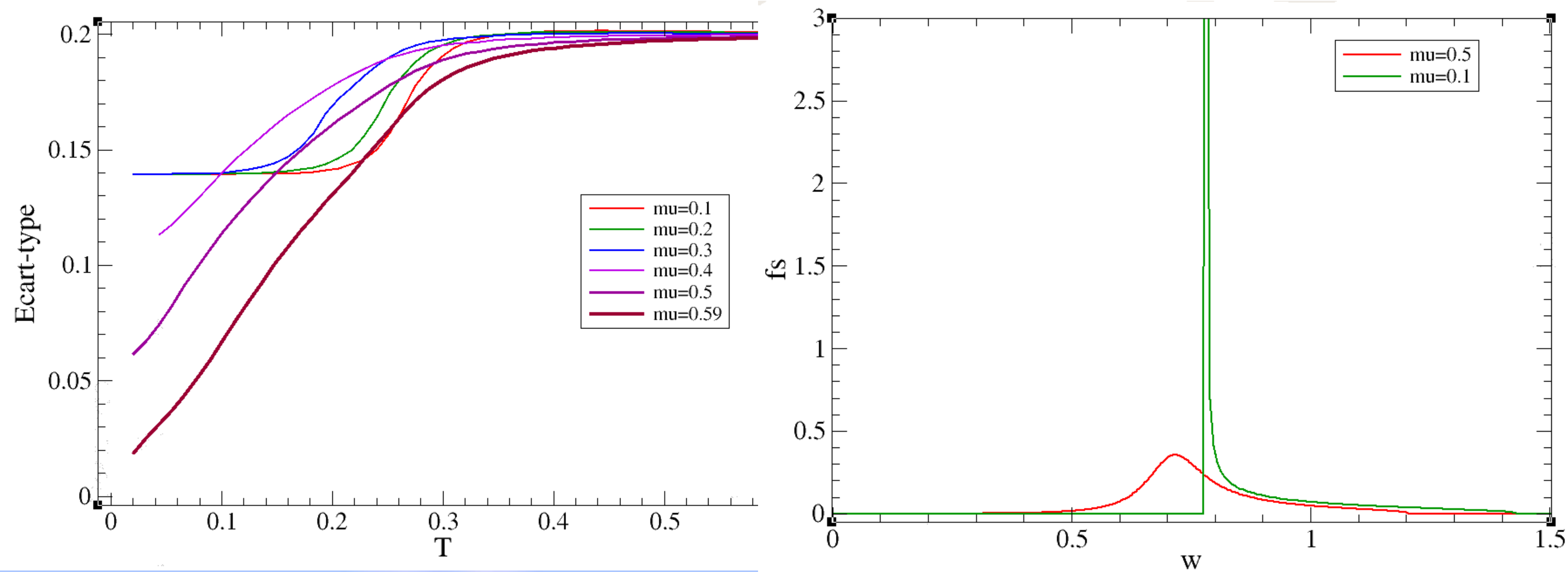
The deconfinement crossover induces very different behavior of the spectral function depending on the density:

- at low density (below the CEP) vanishing of the spectral functions very close to the chiral restoration $\Rightarrow$ "fast" deconfinement that coincides with chiral crossover
- above CEP: vanishing of the spectral function "far" from the chiral transition $\Rightarrow$ "slow" deconfinement that ends well after the 1st order transition.

In the second case: possibility of the existence of a phase where chiral symmetry is restored but the mesons are still confined (no definite answer because of the lack of confinement of the PNJL model but an indication that it may be possible in full QCD).

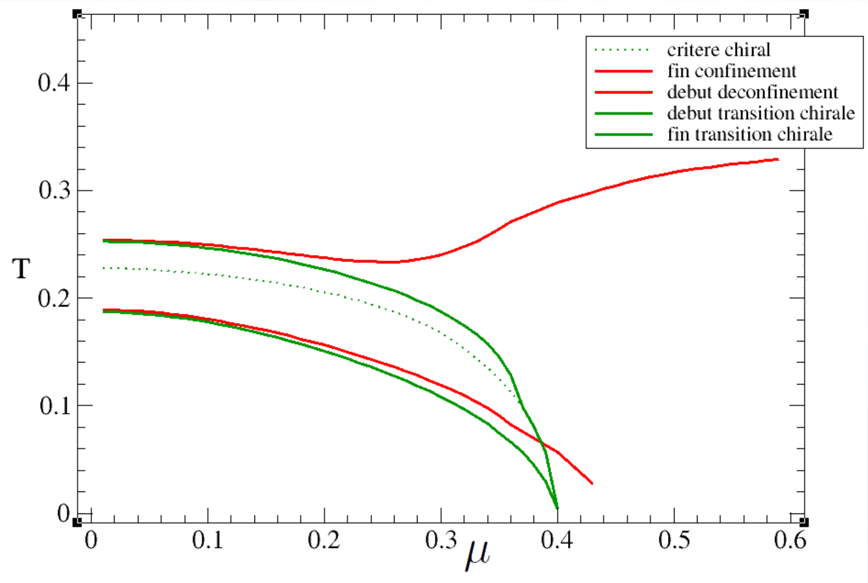
Quantitative picture

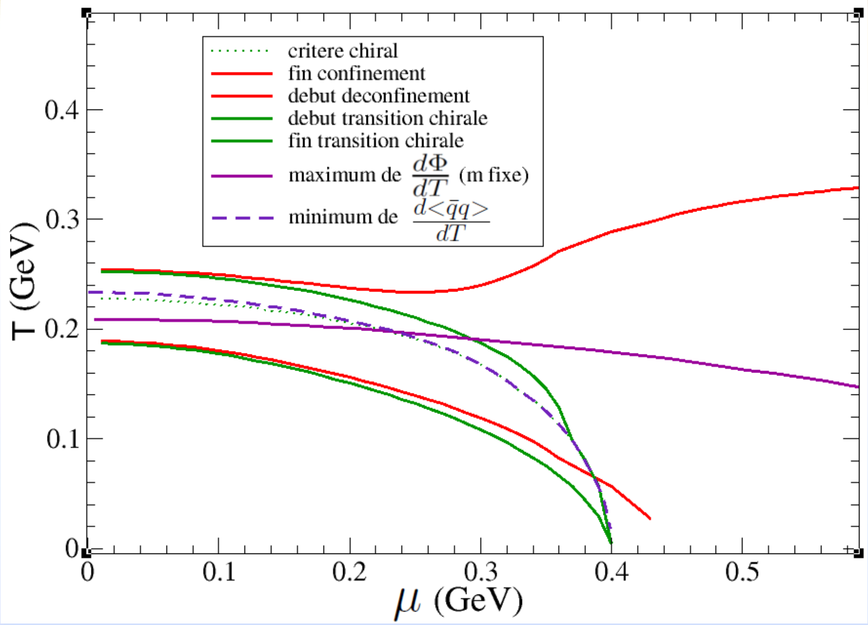
Criterion used to quantify what we can "see" qualitatively: standard deviation of the spectral function. It presents (at moderate density) two plateaux at low temperature (narrow resonance) and high temperature (spectral function $\simeq \mathrm{Im}\Pi_0$ and the standard deviation saturate).



Left: standard deviation as a function of T for different μ.
Right: σ spectral function for $T = 50$ MeV and two different μ corresponding to the crossing point of the standard deviation: $\Rightarrow$ it is not the absolute value that is used as a criterion to probe the possibility of resonant state, but the relative value with respect to the plateaux.

Comparison with mean field





Left: ending points of the plateaux in the (T, μ) plane.
Right: superposition of the mean field phase diagram and the behavior of the mesons.

$\Rightarrow$ confirms the coherence of the model: the spectral function (the strength) picture follows the order parameter Φ, not the condensate.

As a conclusion

Does this CCS phase really exists (a hadronic spectrum chirally symmetrical composed of true bound state) ? Probably no (many things omitted here, diquarks for example, lack of true confinement, etc.). But at least, this calculation indicates that the phenomenology of the phase transition at high density in future experiment could be quite different from the one at zero density:

- obviously because of the 1st order chiral transition
- but also because of this very slow breaking of $\mathbb{Z}_3$ (indicating a "slow" deconfinement)

Backup

The center symmetry of QCD at finite temperature

Finite temperature T : Equilibrium canonical ensemble $\Longrightarrow x_0 \to ix_4$ (imaginary time or euclidean metric: quantum fluctuations $\leftrightarrow$ thermal bath), $x_4 \in [0, \beta]$; $\beta = 1/T$

The Euclidean QCD Lagrangian is invariant under a gauge transformation h:
$A_\mu \longrightarrow {}^hA_\mu = hA_\mu h^\dagger - ih\partial_\mu h^\dagger$ and $q \longrightarrow {}^hq = hq$ with A_μ a gauge field, q a quark field.

From boundary condition in imaginary time:
$A(\vec{x}, x_4 + \beta) = A(\vec{x}, x_4)$ and $q(\vec{x}, x_4 + \beta) = -q(\vec{x}, x_4)$.

Hence the constraints:
${}^hA(\vec{x}, x_4 + \beta) = {}^hA(\vec{x}, x_4)$ and ${}^hq(\vec{x}, x_4 + \beta) = -{}^hq(\vec{x}, x_4)$.

Searching a solution of the constraints equations with
$h(\vec{x}, x_4 + \beta) = fh(\vec{x}, x_4)$
where $f \in$ center of $SU_c(3) \equiv \mathbb{Z}_3 = \{z_n\mathbb{I}, n = 1, 2, 3\}$ ($z_n = e^{2in\pi/3}$).
Since $[f, A_\mu] = 0$ by definition, it follows:
${}^hA(\vec{x}, x_4 + \beta) = {}^hA(\vec{x}, x_4)$ (satisfies the constraint)
and ${}^hq(\vec{x}, x_4 + \beta) = -z\,{}^hq(\vec{x}, x_4)$ (breaking of $\mathbb{Z}_3$).

$\mathbb{Z}_3$ is explicitly broken in the presence of light (dynamic) quarks, but remains an approximate symmetry useful to consider the deconfinement phase transition ; when quark mass $\to \infty$ the QCD Lagrangian is invariant under $\mathbb{Z}_3$ because the mass term $\gg$ kinetic term.

The Polyakov loop: an order parameter for deconfinement

Partition function of QCD in the presence of a static quark Q at position $\vec{R}$:

$$Z_Q = \int \mathcal{D}A_\mu e^{-S_E(pure\ gauge)} \times \mathrm{Tr}_c\, e^{ig\int_0^\beta dx_4\, A_4(\vec{R},x_4)}.$$

One define the Polyakov loop: $$L(\vec{R}) = \mathrm{Tr}_c\, e^{ig\int_0^\beta dx_4\, A_4(\vec{R},x_4)}\ ;$$

it is a color singlet with a $\mathbb{Z}_3$ charge ($L \to zL$).
Its thermal expectation value is the so-called Polyakov loop:

$$\Phi(\vec{R}) = \frac{1}{N_c} < L(\vec{R}) >_\beta = \frac{Z_Q}{Z_{pureglue}} = e^{-\beta F_Q(\vec{R})}.$$

F_Q is the free energy associated to a static quark (test charge) added to the pure glue.

Two limit cases:

- $F \to +\infty$ i.e. there is a confinement of color charges: $\Phi \to 0$
- $F \to 0$ i.e. asymptotic freedom: $\Phi \to 1$

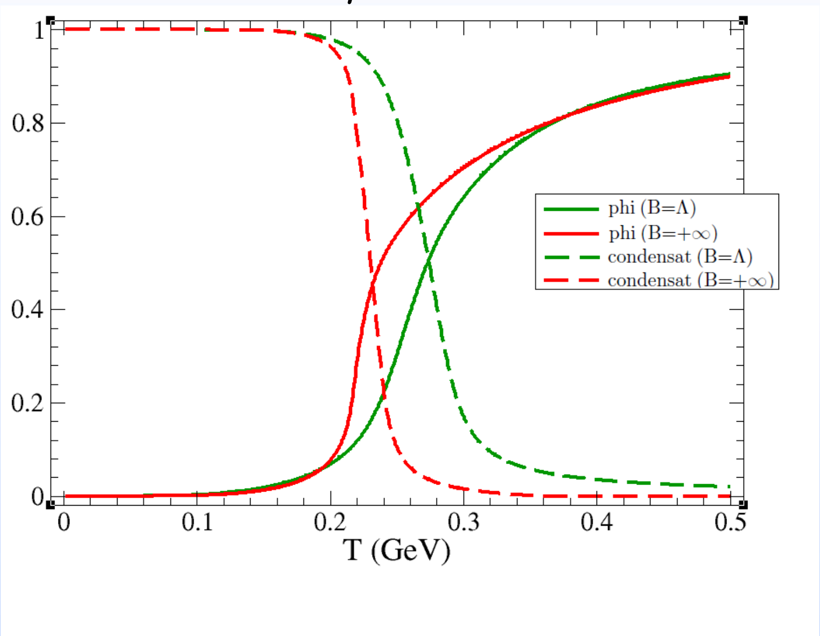
As a consequence, Φ can be seen as an order parameter for the confined ($\Phi \simeq 0$) - deconfined ($\Phi \simeq 1$) phase transition.

It is easy to anticipate that at high temperature, $\Phi \to 1$ (from the definition of $L(\vec{R})$, $T \to +\infty$, $\beta \to 0$ hence the integral and $L(\vec{R}) \simeq 1$).

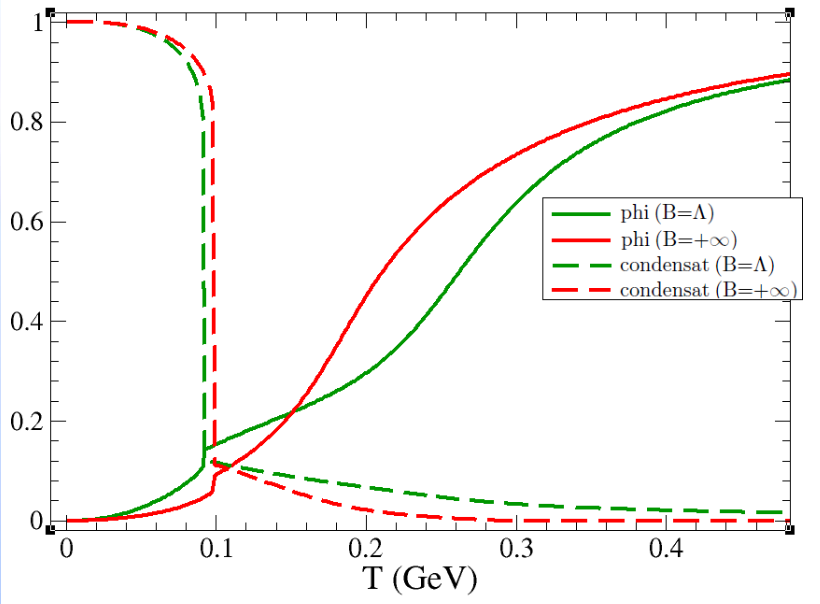
Effect of the cutoff

Consequence on the Polyakov loop: With $\Lambda_T = +\infty$: "faster" (stronger) crossover ; Also at high density, the deconfinement crossover is larger ("slower" transition) because of the Fermi motion that acts as a large crossover field.

$\mu = 0$



$\mu = 0.37$ GeV (NB: small jump in Φ)



Link between chiral restoration and deconfinement

Hence one can solve the mean field equation for Φ for any mass (between m_0 and $m_{Hartree}$) without significant changes ; at the contrary, it is wrong for m or the condensate. It depends strongly on Φ: when $\Phi \to 1$ it generates more quarks and more screening that breaks the condensate hence a faster chiral restoration.

But it does **not** mean that chiral symmetry restoration cannot occurs **before** deconfinement at finite density.

A simple argument (Casher) states that chiral restoration cannot happen within a confined phase. As we have seen it is not the case at high density. At zero density the Casher argument implies that with increasing T first there is deconfinement then chiral symmetry restoration (or at most, a coincidence).

The Fermi motion plays an important role here: if $\mu > 0$ ($\mu > \mu_{CEP}$) it breaks $\mathbb{Z}_3$ (it is no more a very good symmetry of the system) $\Rightarrow$ slow increases of Φ that does not influence $< \bar{q}q >$ a lot anymore. On the other hand $\mu < q^\dagger q >$ is enough to restore the chiral symmetry.

Hence the two transition seems to decouple at high density and the argument is not holding.