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# Planckian scattering and high-dimensional gravity fixed points

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# Motivation

- TeV-scale gravity scenarios $\Rightarrow$ transplanckian scattering at high-energy colliders

- $S$-matrix computable by semiclassical methods (eikonal) for large $\sqrt{s}$ and large impact parameters

$$\Downarrow$$

insensitive to UV completion of gravity?

$\Rightarrow$ explore physical effects of UV fixed point on eikonal $S$ matrix in Weinberg's asymptotic safety hypothesis

# OUTLINE

**I**. Introduction: low scale gravity and high energy scattering

**II**. Eikonal $S$ matrix in fixed point gravity

**III**. Towards phenomenological applications

# I. Introduction: Transplanckian Scattering

♠ Scattering at center-of-mass energy $>$ quantum gravity scale

*['t Hooft 1987; Muzinich, Soldate 1988; Amati, Ciafaloni, Veneziano 1987; Gross, Mende 1988]*

- elastic small-angle scattering for $b \gg R_{Schwarzs.}$
  by (semiclassical) eikonal interactions
- strong inelastic corrections for $b \sim R_{Schwarzs.}$
  $\Longrightarrow$ black hole formation and evaporation?

♠ Large extra dimension scenarios of TeV-scale gravity

*[Antoniadis, Arkani-Hamed, Dimopoulos, Dvali 1998]*

$\Longrightarrow$ transplanckian scattering at colliders?

*[Giddings, Thomas 2002; Giudice, Rattazzi, Wells 2002]*

# LOW-SCALE QUANTUM GRAVITY

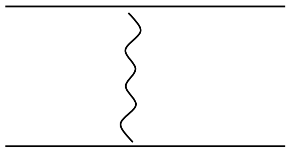
- SM in 4 dimensions; gravity in (compactified) $n$ extra dimensions of size $R_C$

$$M_P^2 \sim (R_C M_D)^n M_D^2$$

- effective theory couples massive KK gravitons to SM fields

*[Giudice, Rattazzi, Wells 1999; Han, Lykken, Zhang 1999]*

Born amplitude



$$= \frac{s^2}{M_D^{2+n}} \int d^n m \, \frac{1}{t - m^2}$$

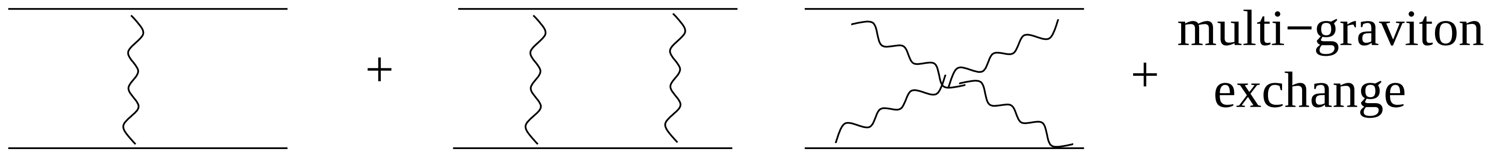
- Bounds on $n$ and $M_D$ from collider searches

e.g. $pp \to dijets$ @ LHC

*[Franceschini, Giardino, Giudice, Lodone & Strumia, arXiv:1101.4919 [hep-ph]]*

$\Rightarrow M_D \sim$ multi-TeV

# EIKONAL $S$-MATRIX FOR $|t|/s \ll 1, \ \sqrt{s} \gg M_D$




$$S(b,s) = \exp(i\chi) \quad , \quad \chi(b,s) = \frac{1}{s} \int d^2q \; e^{iq\cdot b} \; A_{\rm Born}$$

- $A_{\rm Born}$ UV-divergent for $n \geq 2 \ \Rightarrow$ UV regulator (DR or EFT)
  - amplitude rises with $t$ (after regularization)

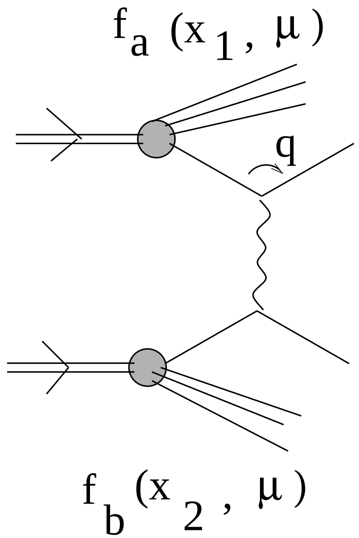
$$\Rightarrow \quad \text{eikonal phase} \quad \chi(b,s) = (b_c/b)^n \quad , \quad b_c \sim M_D^{-1} \left(s/M_D^2\right)^{1/n}$$

- arguably, still insensitive to UV gravity completion as long as

$$\text{eikonal integral} \quad = \quad \int d^2b \; e^{-iq\cdot b} \left(e^{i\chi} - 1\right)$$

is dominated by long-distance saddle point $b_s$

# JET PRODUCTION BY GRAVITATIONAL INTERACTIONS




$$b^{-1} \gg \Lambda_{\rm QCD} \sim 1 \text{ fm}^{-1}$$

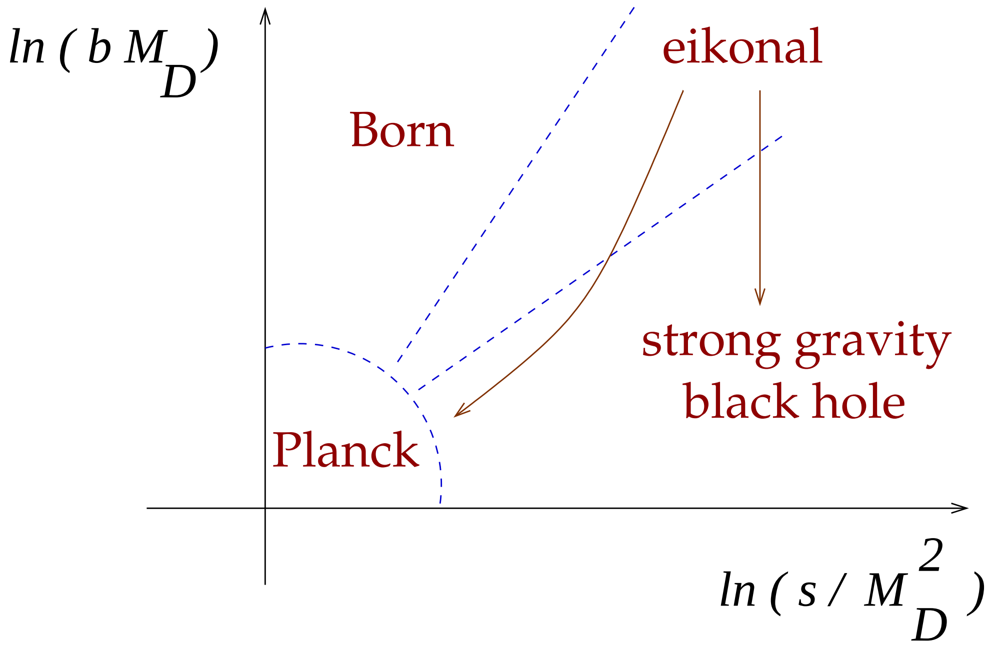
$$d\sigma = \sum_{a,b} \int dx_1 \; dx_2 \; f_a(x_1, \mu) \; d\sigma_{eik} \; f_b(x_2, \mu) + \mathcal{O}(1/q^2)$$

- large dijet invariant masses
- factorization scale $\mu \sim b_s^{-1}$

- rising parton densities at $x \to 0$ probe lower end of $\sqrt{s}$ spectrum in $d\sigma_{eik}$

# HIGH-ENERGY GRAVITATIONAL SCATTERING

*See "phase diagram" in S. Giddings, Erice lectures, arXiv:1105.2036.*




• strong gravity effects with decreasing $b$ at fixed $\sqrt{s}$

• growth of parton density at $x \to 0 \Rightarrow$ "slide-down" in $\sqrt{s}$

$\longrightarrow$ onset of $b \sim M_D$ effects in planckian region?

# II. Asymptotic safety scenario

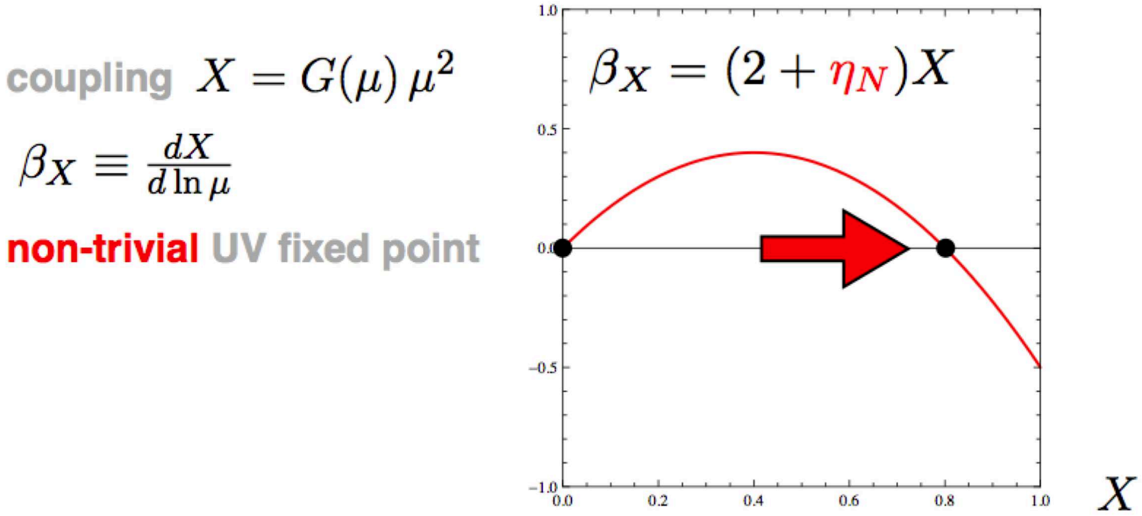
*[Weinberg 1979]*

$S = G_N^{-1} \int \sqrt{g}(-R + 2\Lambda)$ perturbatively nonrenormalizable $[G_N] = 2 - D$

$$G_N \;\rightarrow\; G(\mu) = G_N Z^{-1}(\mu) \quad , \quad g(\mu) = G(\mu)\,\mu^{D-2}$$

- Weinberg's scenario: $G(\mu) \sim \mu^{2-D}$ at high energies

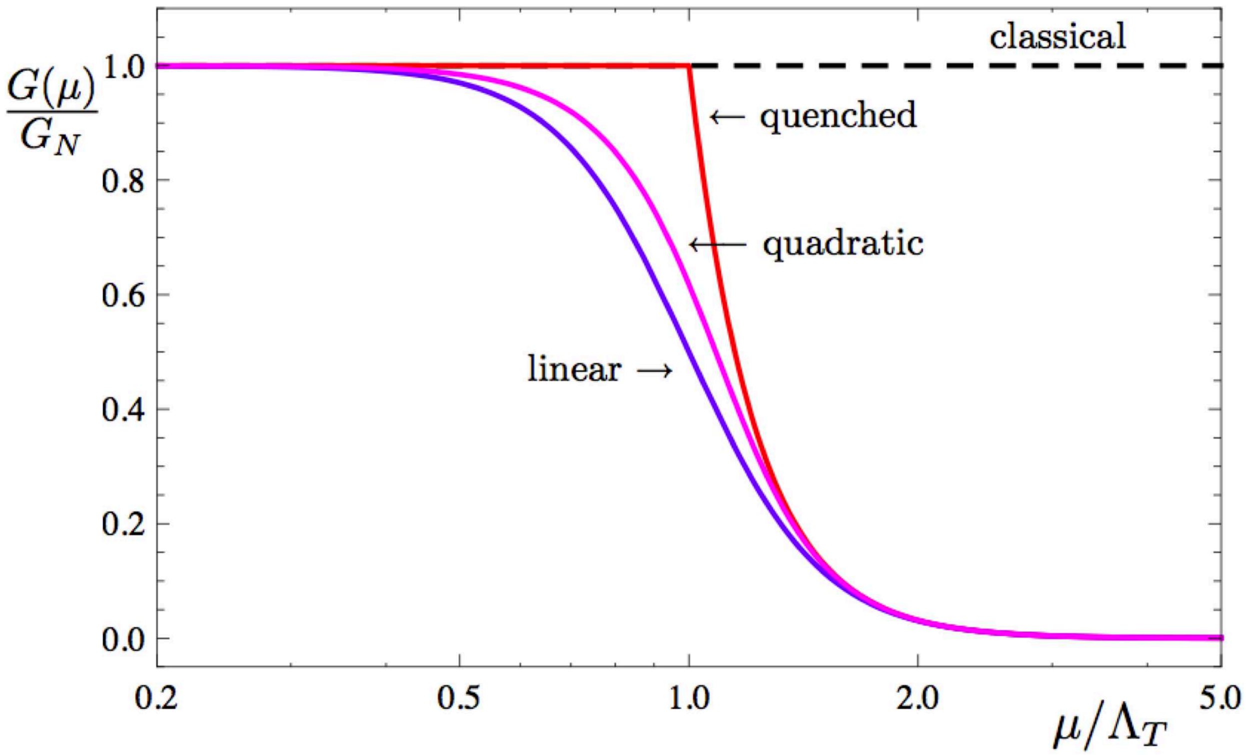
$g \rightarrow$ fixed point $g_*$




$\mu \rightarrow \infty :$ $G(\mu) \rightarrow g_* \mu^{2-D} \ll G_N$ [Litim 2004; 2011]

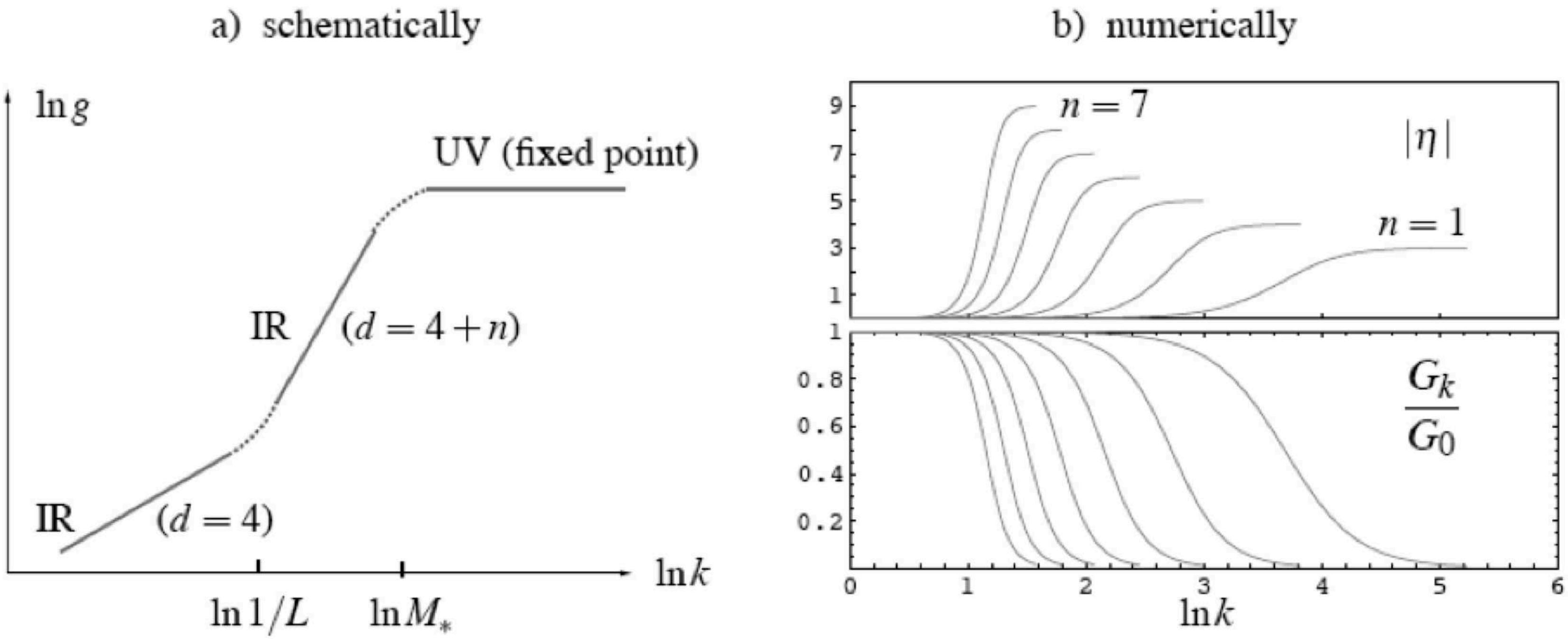
# RUNNING GRAVITATIONAL COUPLING

♣ cross-over scale $\Lambda_T$ between classical scaling and fixed-point scaling




*[Gerwick, Litim, Plehn 2011]*

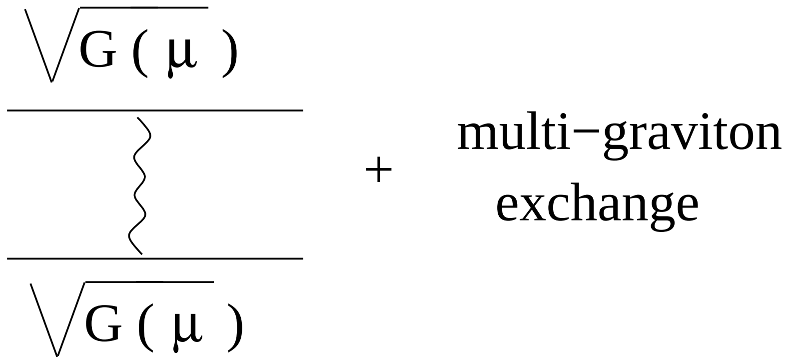
# RUNNING COUPLINGS




$L \sim$ compactification radius
$M_* \sim D$-dimensional gravity scale

*[Fischer, Litim 2005]*
*[Alkofer, Litim, Schaefer 2013]*

# III. Eikonal scattering in fixed-point gravity




- Born amplitude falls off with $t$
- eikonal phase finite at small $b$

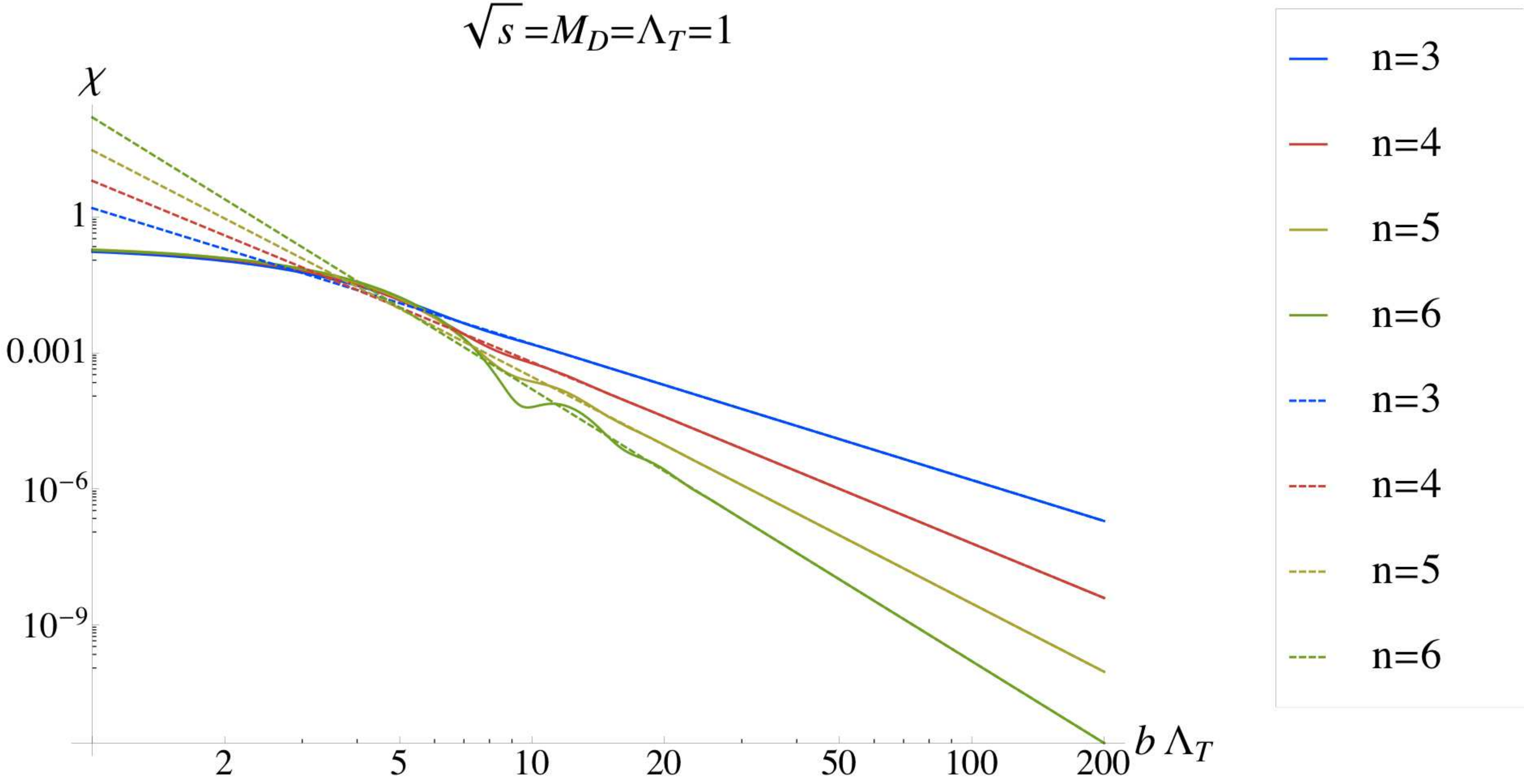
$\Rightarrow$ cross-over scale for $b \propto \Lambda_T^{-1}$

$$\chi(b,s) = (b_c/b)^n \qquad \text{at large } b \text{ (semiclassical)}$$

$$\chi(b,s) = \chi(0,s) + \mathcal{O}(b^2 \ln b) \qquad \text{for } b \leq b_T$$

- $b_T = b_c[\chi(0)]^{-1/n}$ new length scale

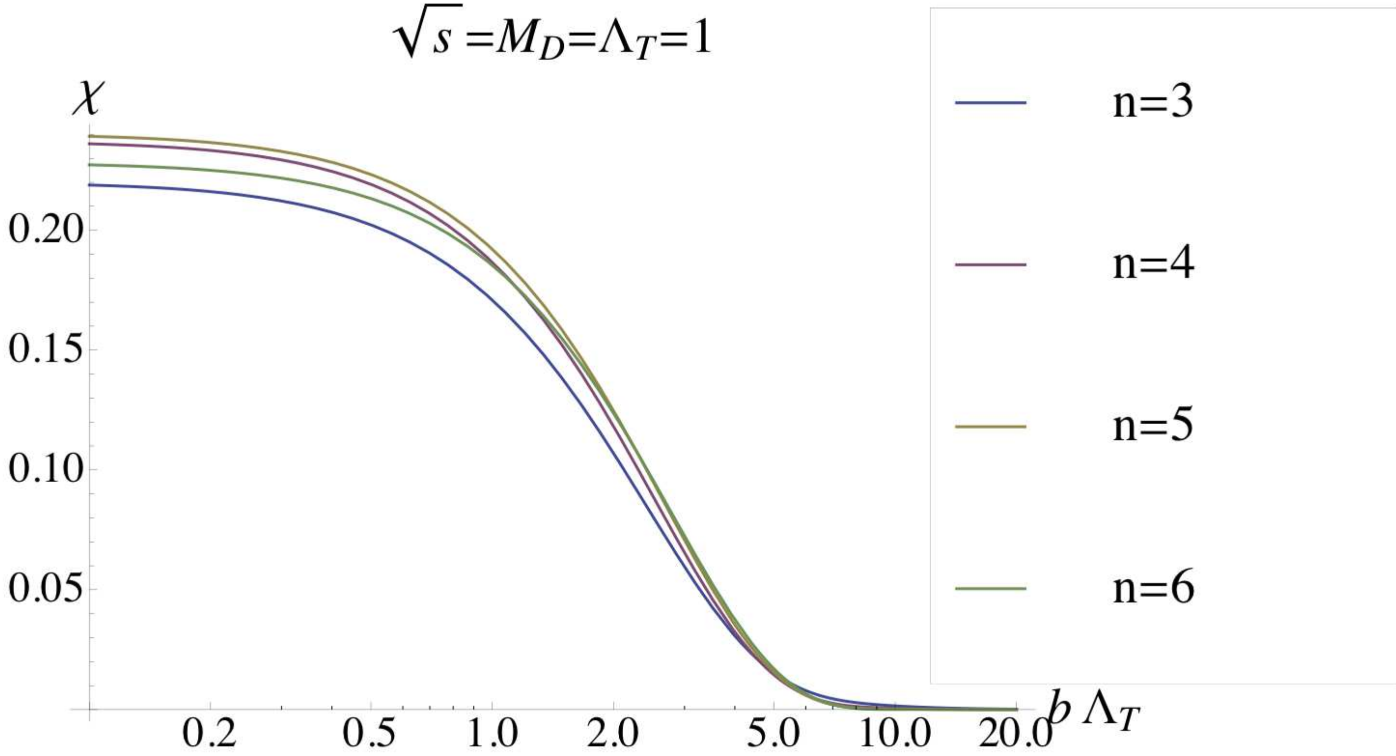
# THE PHASE $\chi(b, s)$




♢ diverges for $b \to 0$ in the effective theory (dashed lines)

♢ finite for $b \to 0$ in the fixed-point case (solid lines)

## THE PHASE $\chi(b, s)$



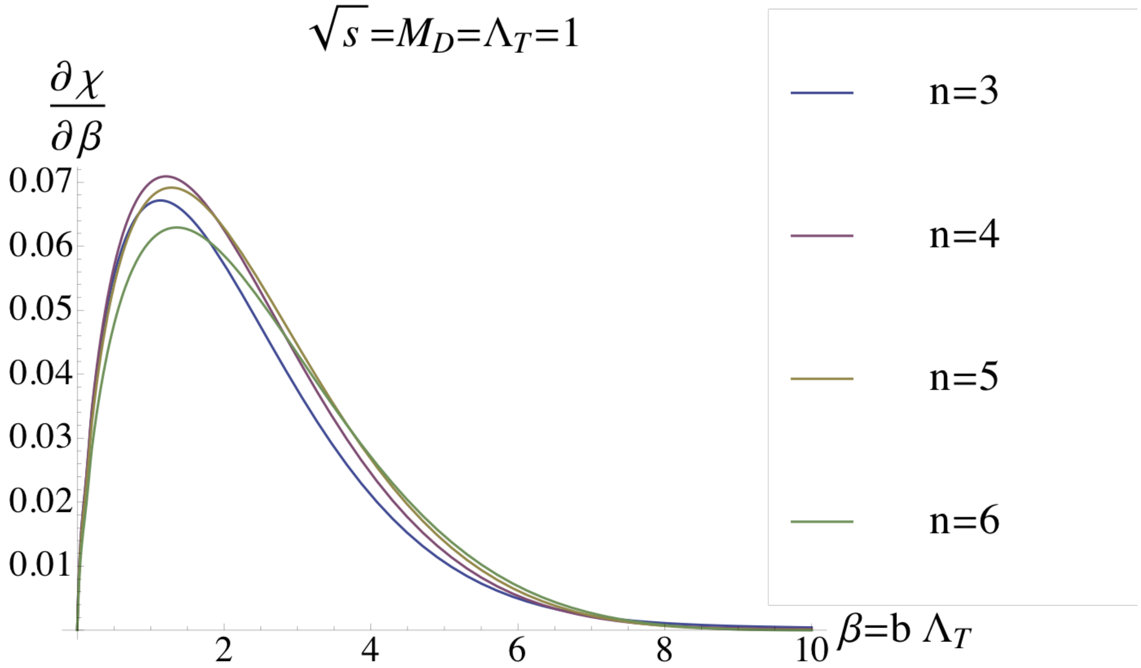

- Finite brane width would also soften the small-$b$ behavior but would still leave a logarithmic divergence

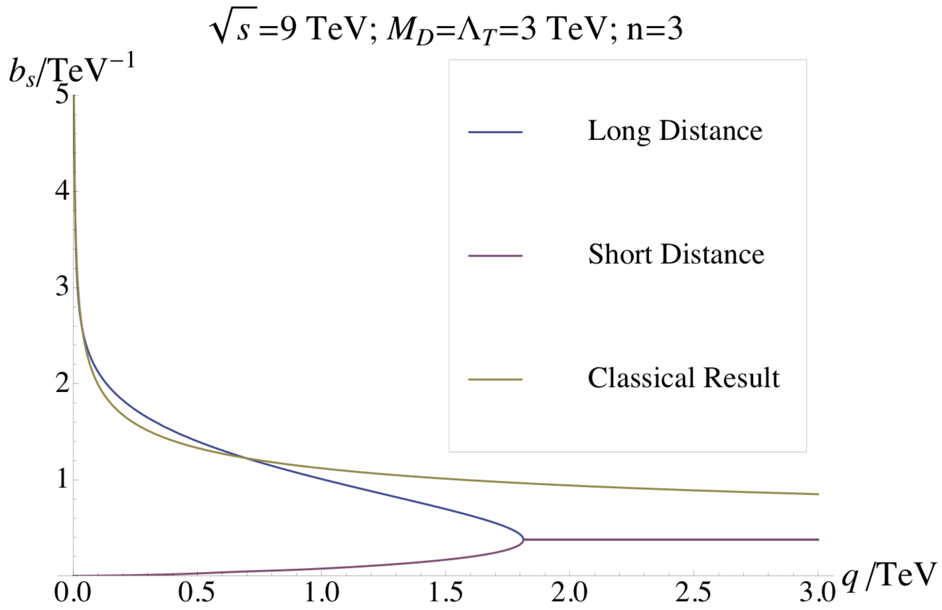
# EIKONAL INTEGRAL BY STATIONARY PHASE

$$\mathcal{M}_{eik}(s,q) = 2is \int d^2b\, e^{-iq\cdot b}\, [1 - S(b,s)] \quad , \qquad S(b,s) = \exp(i\chi)$$

- saturation of eikonal phase to finite $\chi(0)$ at small $b$

  $\Rightarrow$ saddle point arises at short distances




◇ onset of fixed-point scaling affects the eikonal through the low-$b$ saddle point

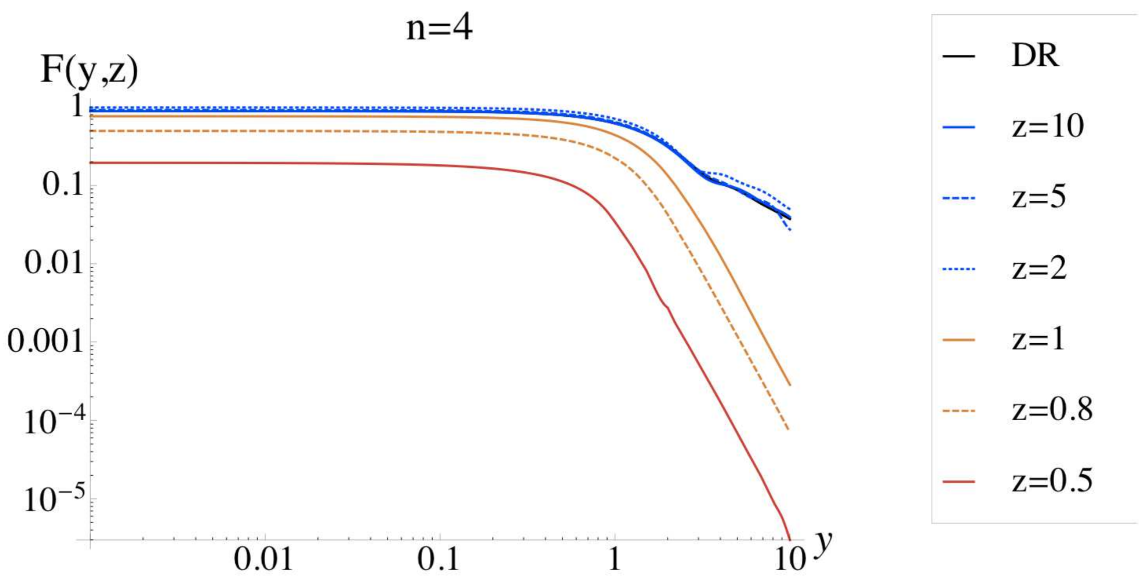
# MOMENTUM TRANSFER DEPENDENCE

$$\mathcal{M}_{eik}(s,q) = 4\pi b_c^2 s F(y,z)$$

$$\text{where} \quad y = qb_c \quad , \qquad b_c = \frac{\sqrt{4\pi}}{M_D} \left(\frac{\Gamma(n/2)s}{16\pi M_D^2}\right)^{1/n}$$

$$z = k_n \Lambda_T b_c \equiv [\chi(0)]^{1/n}$$

- $F_{semicl.} = F(y, z \to \infty)$

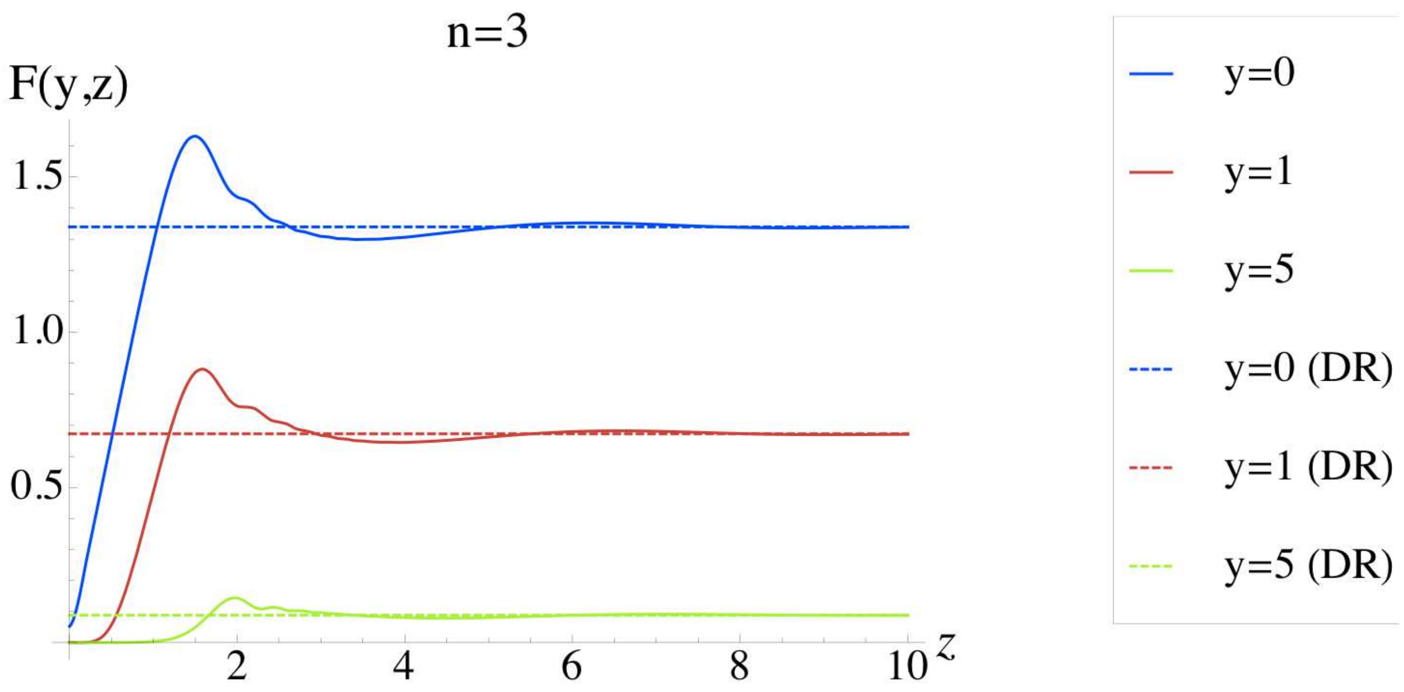



- faster fall-off with increasing momentum transfer $q$ from fixed-point coupling

# FIXED-POINT CORRECTIONS TO EIKONAL AMPLITUDE

$$\mathcal{M}_{eik}(s,q) = 4\pi b_c^2 s F(y,z)$$

$$y = qb_c \quad , \qquad z = k_n \Lambda_T b_c$$

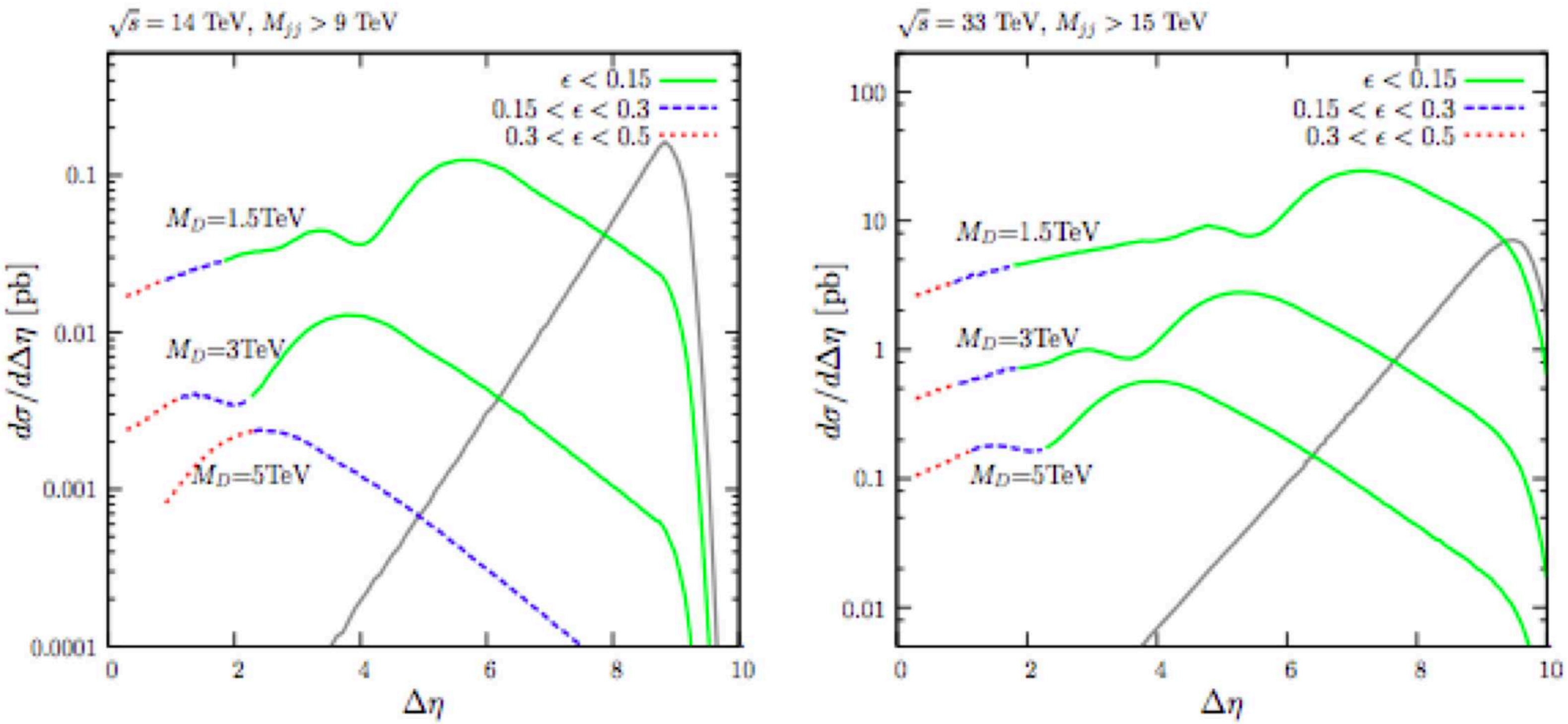



- for any given $y$, corrections to semiclassical $z \to \infty$ amplitude
  - $k_n = k_n(g_*, s/M_D^2) \ \Rightarrow$ effects on energy spectrum

# TOWARDS PHENOMENOLOGICAL APPLICATIONS

- Dijets at large invariant masses and large rapidity separations

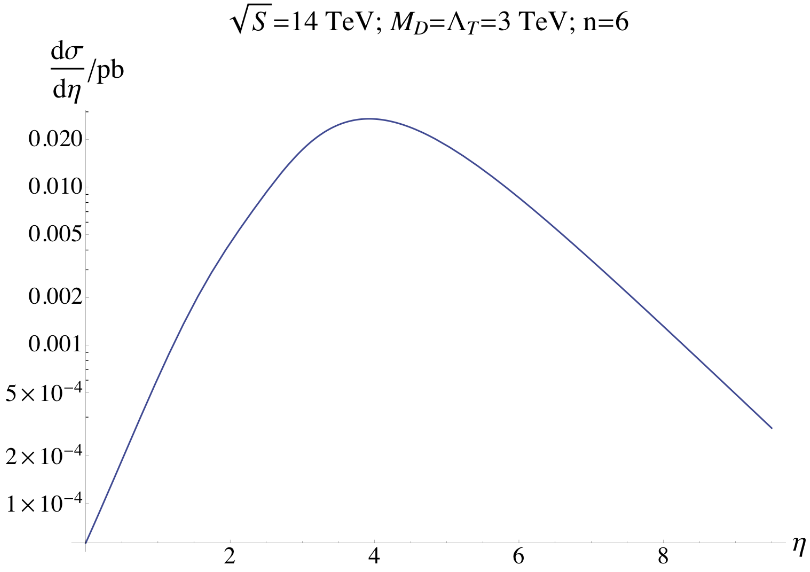
Effective theory:

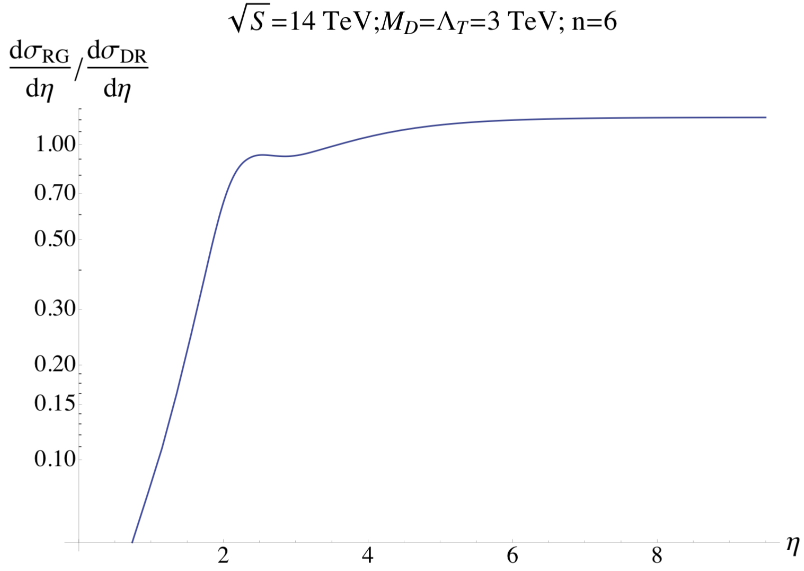



*[Stirling, Vryonidou, Wells 2011]*

# DIJET PRODUCTION

## Fixed-point gravity:




(left) dijet rapidity distribution; (right) ratio of the distribution to effective theory result

- fixed-point graviton coupling modifies jet signal in central region compared to effective theory
- semiclassical result is recovered as rapidity increases

# Conclusion

• Transplanckian scattering provides collider signatures
for low-scale quantum gravity

• In asymptotically safe scenarios
UV fixed-point scaling leads to finite eikonal phase
at small $b$ for any extra dimension $n$

$\Rightarrow$ eikonal integral no longer dominated by long-distance stationary point

• Is fixed-point "phenomenology" necessary
in collider searches for TeV-scale gravity?