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Event-by-event flow and initial geometry from LHC

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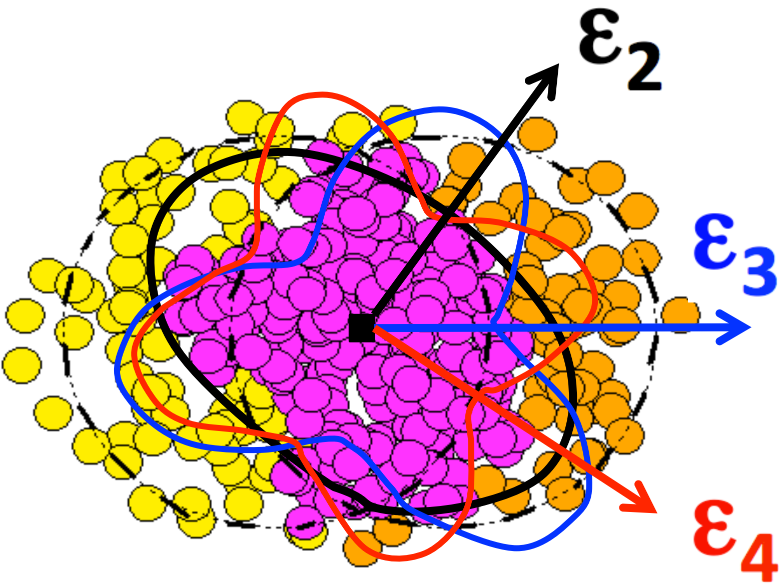
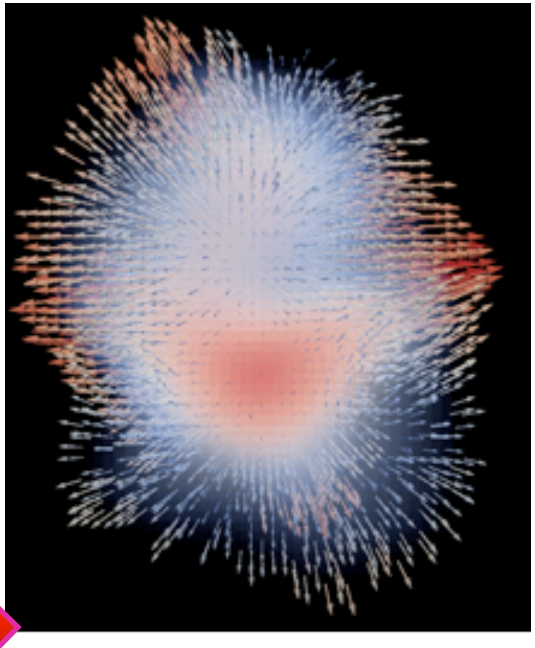
Initial geometry & momentum anisotropy



$$\varepsilon_n = \sqrt{\frac{\langle r^n \cos(n\phi)\rangle + \langle r^n \sin(n\phi)\rangle}{r^n}} \quad \tan(n\Phi_n^*) = \frac{\langle r^n \sin(n\phi)\rangle}{\langle r^n \cos(n\phi)\rangle}$$

pressure



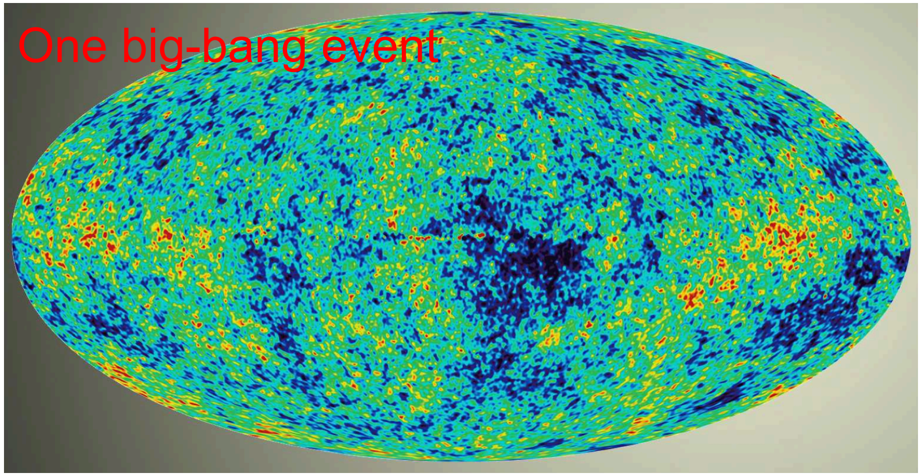
by MADAI.us

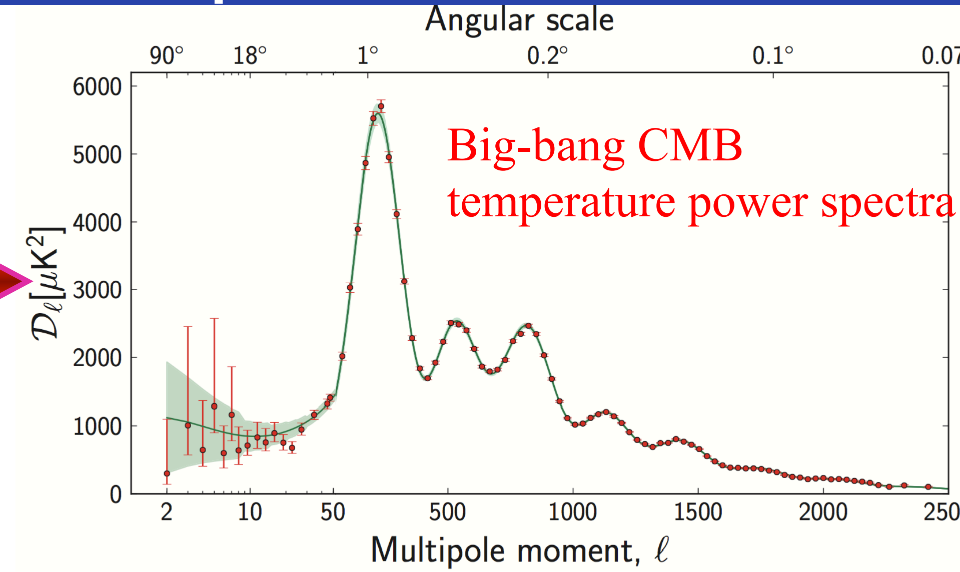
Single particle distribution
$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_n \cos n\left(\phi - \Phi_n\right)$$

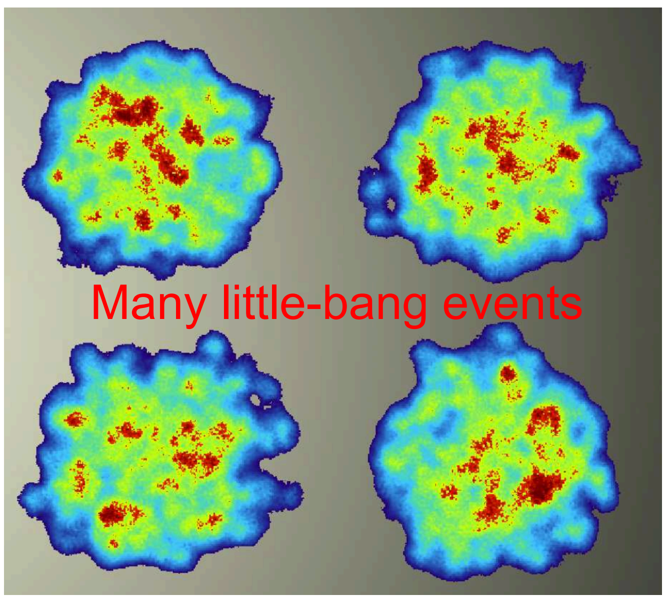
Pair distribution
$$\frac{dN}{d\Delta\phi} = \left[\frac{dN}{d\phi_a} * \frac{dN}{d\phi_b}\right] \propto 1 + \sum_n 2\mathrm{v}_n^a \mathrm{v}_n^b \cos\left(n\Delta\phi\right)$$

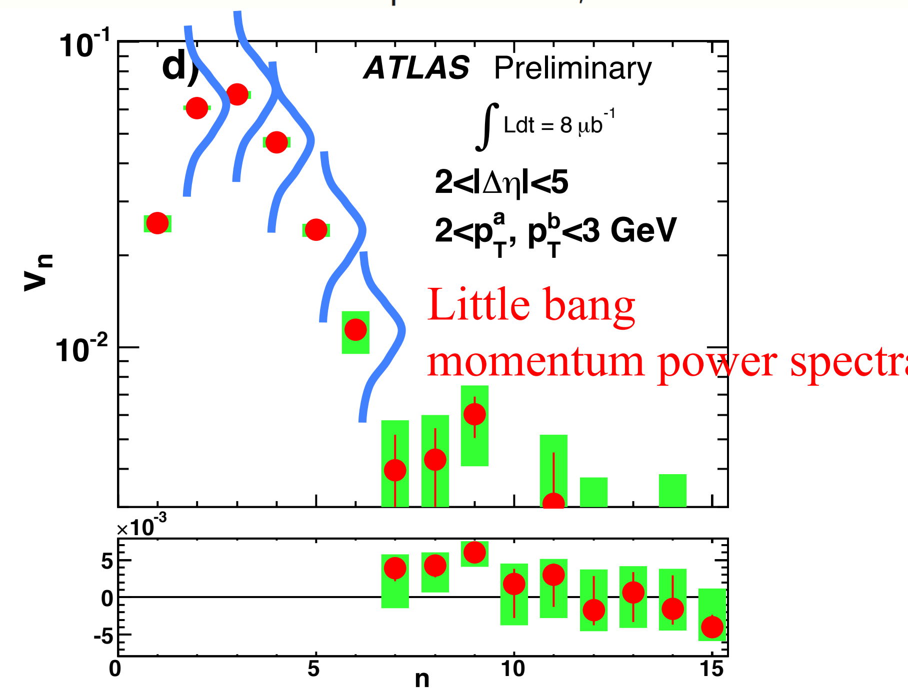
Momentum anisotropy probes:
initial geometry and transport properties of the QGP

Anisotropy power spectra





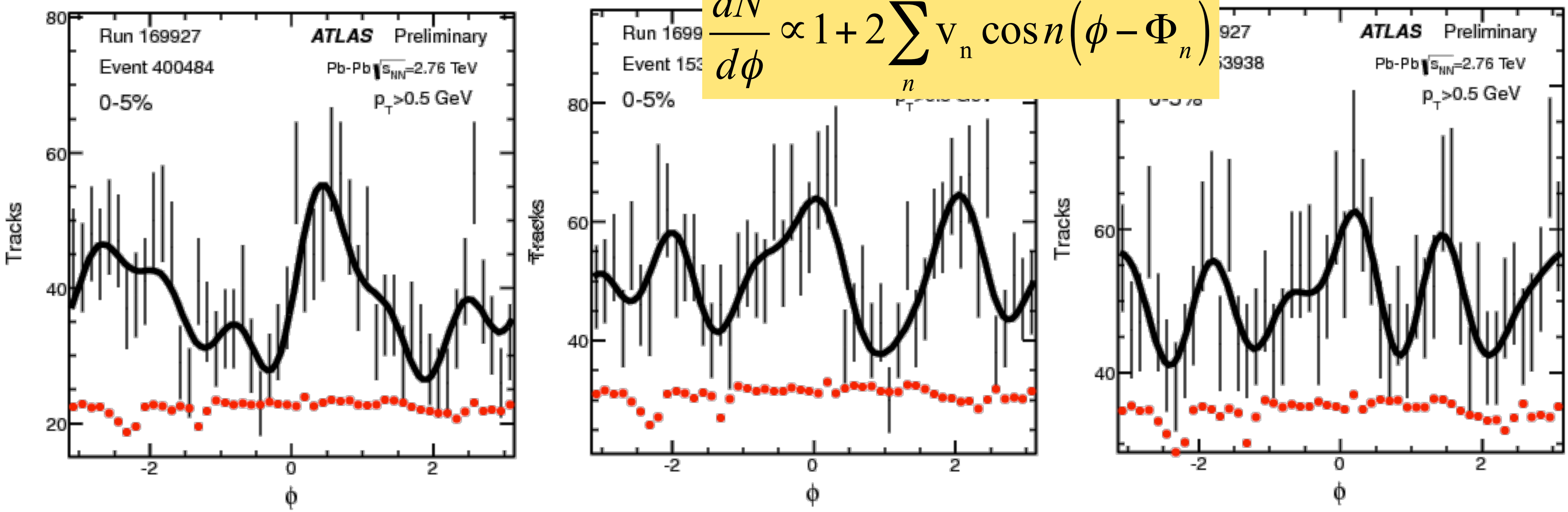




Many little-bang events→
probability distributions: $p(v_n, v_m, \ldots, \Phi_n, \Phi_m \ldots)$

Fluctuation event by event

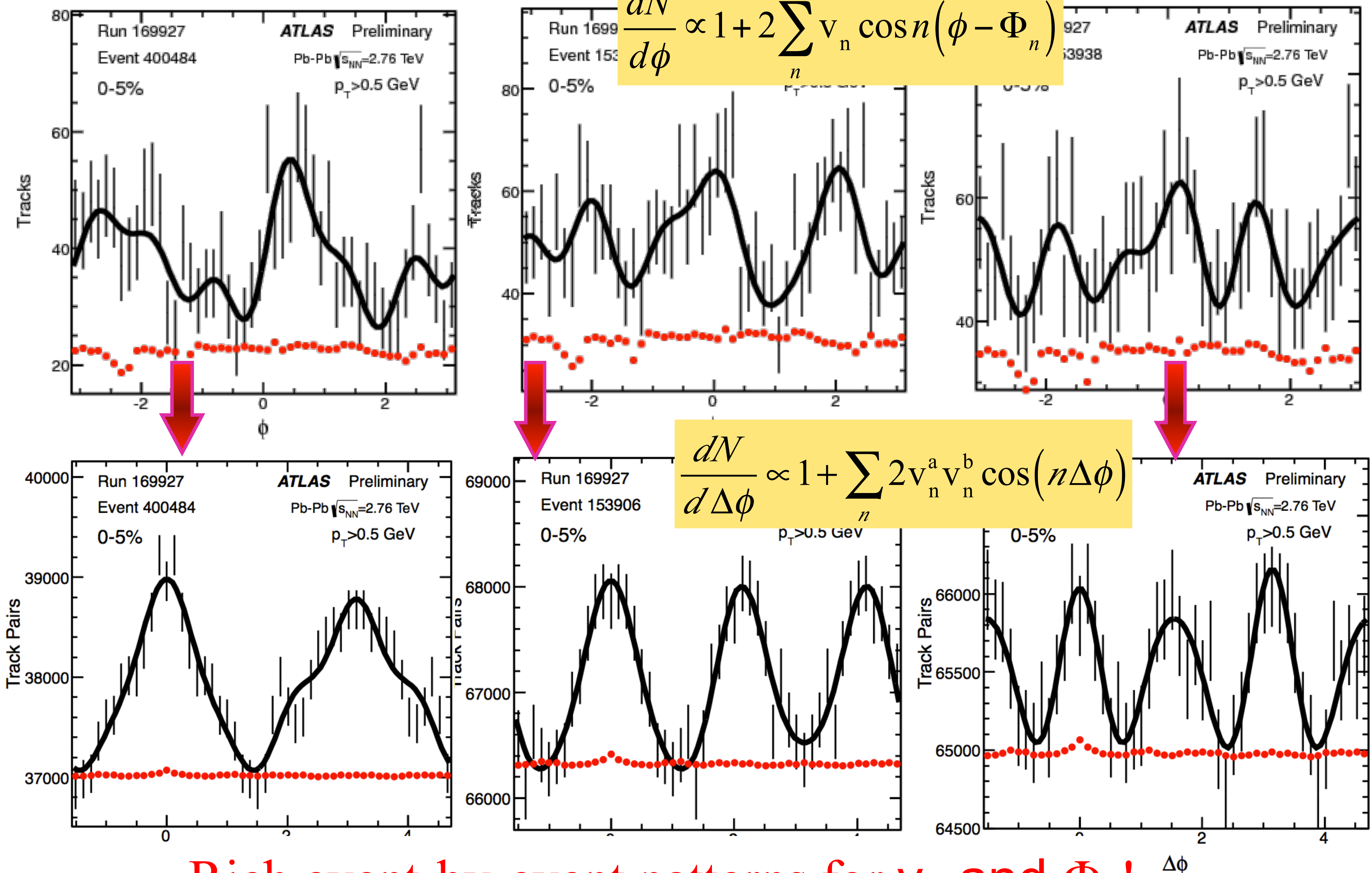
$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_\mathrm{n} \cos n\left(\phi - \Phi_n\right)$$



Fluctuation event by event

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_n \cos n\left(\phi - \Phi_n\right)$$

$$\frac{dN}{d\Delta\phi} \propto 1 + \sum_n 2\mathrm{v}_n^a \mathrm{v}_n^b \cos\left(n\Delta\phi\right)$$



Rich event-by-event patterns for v_n and Φ_n!

Outline

$$p(v_n, v_m, \ldots, \Phi_n, \Phi_m \ldots)$$

- Event-plane correlations → $p(\Phi_n, \Phi_m \ldots)$

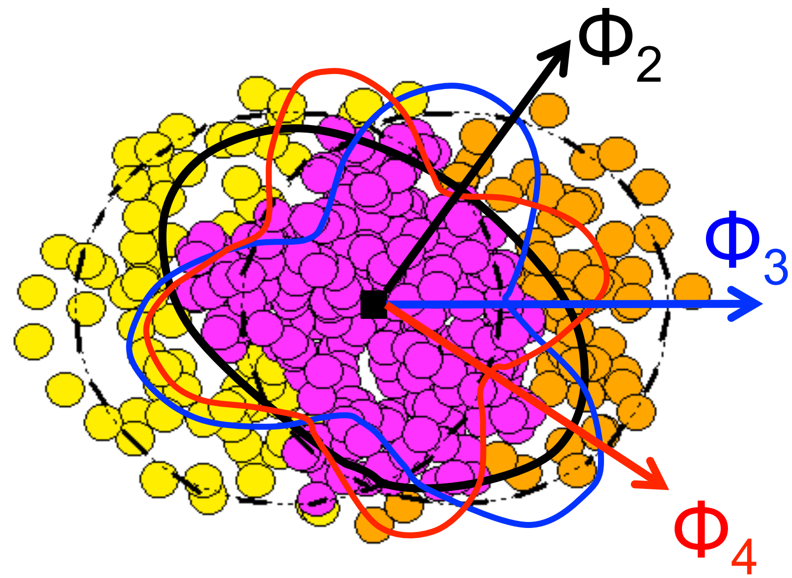
ATLAS-CONF-2012-49

- Event-by-event v_n distributions → $p(v_n)$

1305.2942

p(Φn,Φm.....)

- Correlation can exist in the initial geometry and also generated during hydro evolution



- The correlation quantified via correlators

$$\boxed{\frac{dN_{events}}{d(k(\Phi_n-\Phi_m))}=1+2\sum_{j=1}^{\infty}V_{n,m}^{j}\cos(jk(\Phi_n-\Phi_m))}$$

$$V_{n,m}^{j}=\left\langle\cos(jk(\Phi_n-\Phi_m))\right\rangle$$

arXiv:1205.3585
arXiv:1203.5095

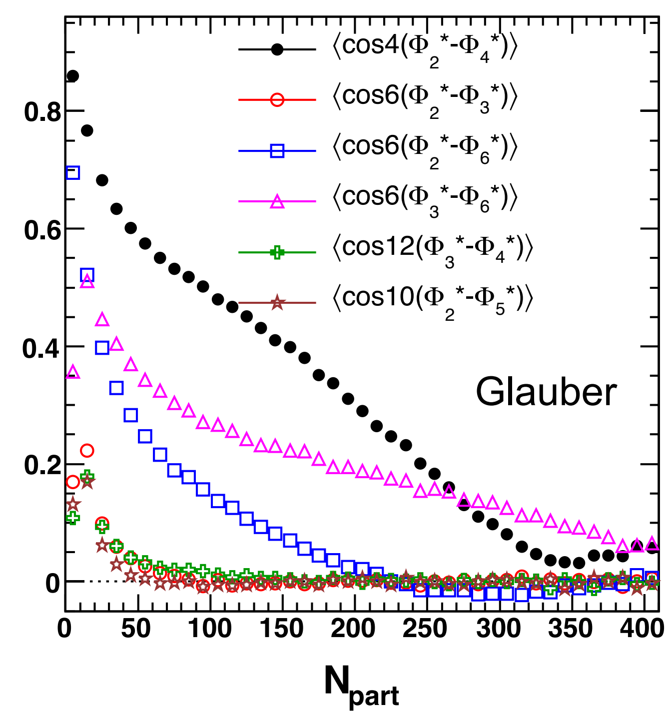
- Corrected by resolution

$$\left\langle\cos(jk(\Phi_n-\Phi_m))\right\rangle=\frac{\left\langle\cos(jk(\Psi_n-\Psi_m))\right\rangle}{\mathrm{Res}(jk\Psi_n)\mathrm{Res}(jk\Psi_m)}$$

$$\Phi_n=True,\ \Psi_n=Measured$$

- Generalize to multi-plane correlations

$$\left\langle\cos(c_1\Phi_1+2c_2\Phi_2...lc_l\Phi_l))\right\rangle\quad c_1+2c_2+...lc_l=0$$



A list of measured correlators

- List of two-plane correlators

$$\boxed{\begin{array}{c}\langle\cos 4(\Phi_2-\Phi_4)\rangle\\ \langle\cos 8(\Phi_2-\Phi_4)\rangle\\ \langle\cos 12(\Phi_2-\Phi_4)\rangle\end{array}}$$

$$\langle\cos 6(\Phi_2-\Phi_3)\rangle$$
$$\langle\cos 6(\Phi_2-\Phi_6)\rangle$$
$$\langle\cos 6(\Phi_3-\Phi_6)\rangle$$
$$\langle\cos 12(\Phi_3-\Phi_4)\rangle$$
$$\langle\cos 10(\Phi_2-\Phi_5)\rangle$$

- List of three-plane correlators

"2-3-5" $\boxed{\begin{array}{c}\langle\cos(2\Phi_2+3\Phi_3-5\Phi_5)\rangle\\ \langle\cos(-8\Phi_2+3\Phi_3+5\Phi_5)\rangle\end{array}}$

"2-4-6" $\boxed{\begin{array}{c}\langle\cos(2\Phi_2+4\Phi_4-6\Phi_6)\rangle\\ \langle\cos(-10\Phi_2+4\Phi_4+6\Phi_6)\rangle\end{array}}$

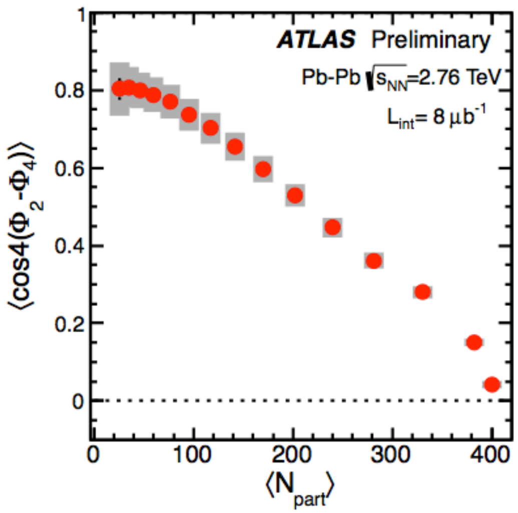
"2-3-4" $\boxed{\begin{array}{c}\langle\cos(2\Phi_2-6\Phi_3+4\Phi_4)\rangle\\ \langle\cos(-10\Phi_2+6\Phi_3+4\Phi_4)\rangle\end{array}}$

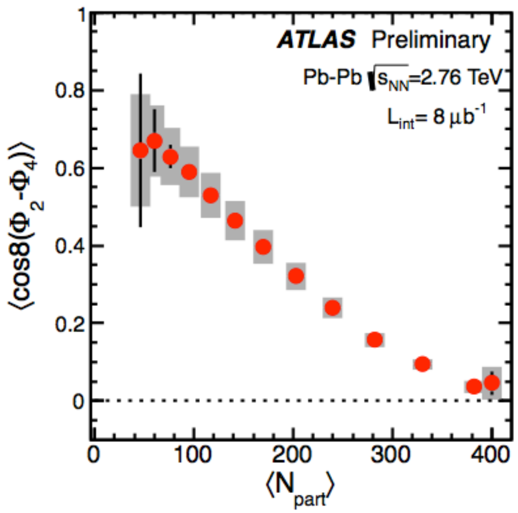
$$\begin{aligned} 2\Phi_2+4\Phi_4-6\Phi_6 &= 4(\Phi_4-\Phi_2)-6(\Phi_6-\Phi_2)\\ -10\Phi_2+4\Phi_4+6\Phi_6 &= 4(\Phi_4-\Phi_2)+6(\Phi_6-\Phi_2)\end{aligned}$$

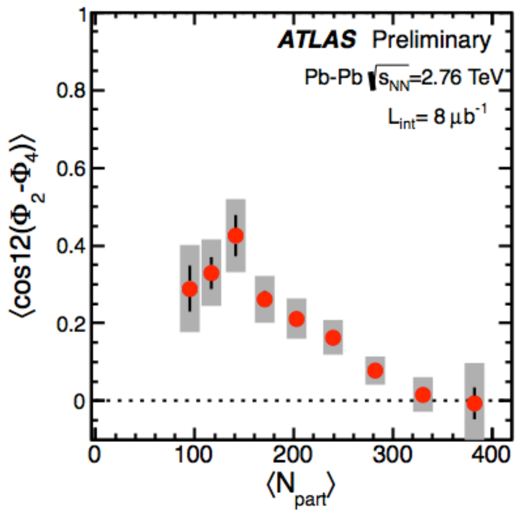
Reflects correlation of two Φ_n relative to the third

Two-plane correlations

$\left\langle \cos(1\times 4(\Phi_2-\Phi_4))\right\rangle$ $\left\langle \cos(2\times 4(\Phi_2-\Phi_4))\right\rangle$ $\left\langle \cos(3\times 4(\Phi_2-\Phi_4))\right\rangle$

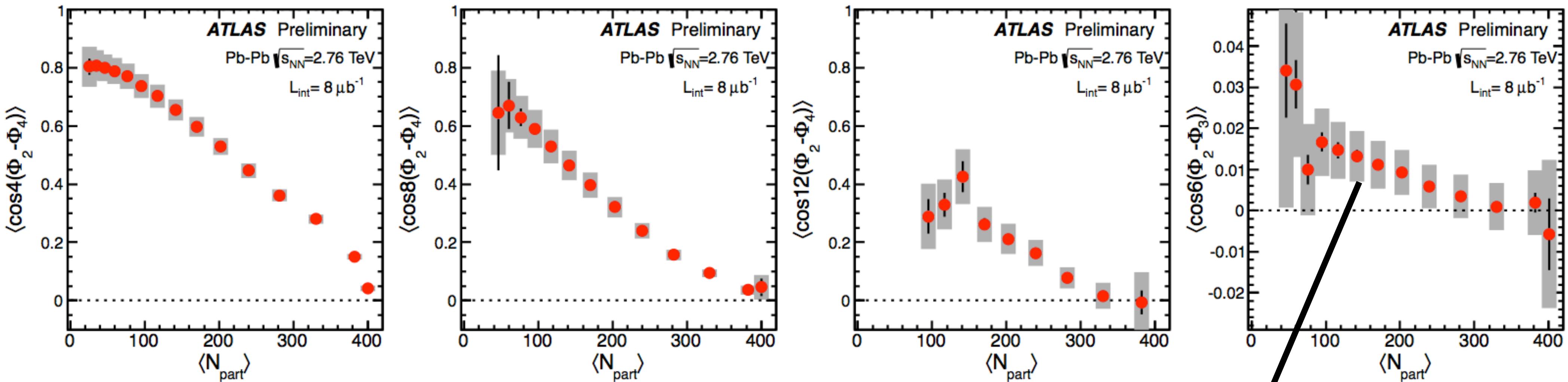






Two-plane correlations

$\langle \cos(1\times 4(\Phi_2-\Phi_4))\rangle$ $\langle \cos(2\times 4(\Phi_2-\Phi_4))\rangle$ $\langle \cos(3\times 4(\Phi_2-\Phi_4))\rangle$ $\langle \cos(1\times 6(\Phi_2-\Phi_3))\rangle$



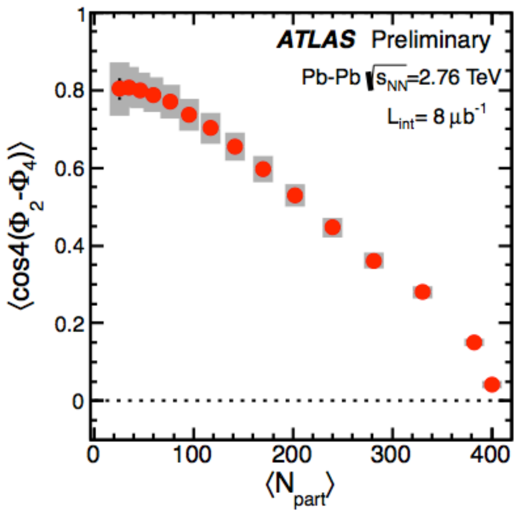
$$v_2 e^{-i2\Psi_2} = \text{geometry} + v_1 v_1 e^{-i2\Psi_1} + \ldots$$

$$v_3 e^{-i3\Psi_3} = \text{geometry} + v_1 v_2 e^{-i(\Psi_1+2\Psi_2)} + \ldots$$

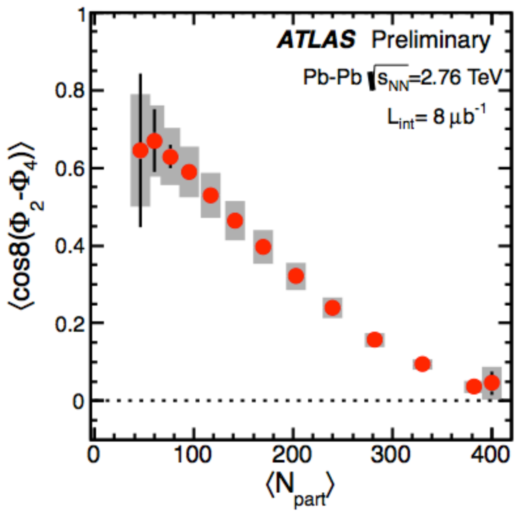
Teaney & Yan

Two-plane correlations

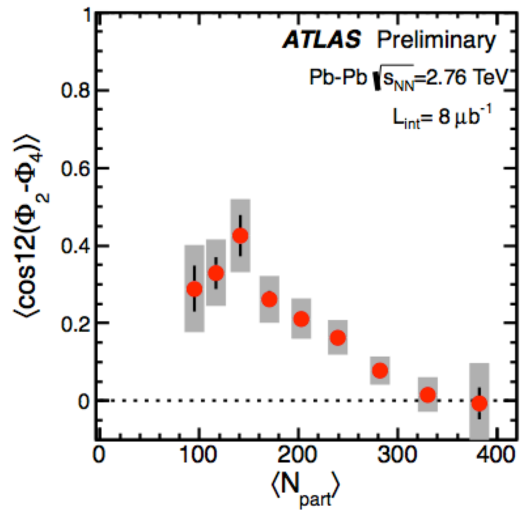
$\langle \cos(1\times 4(\Phi_2-\Phi_4)) \rangle$



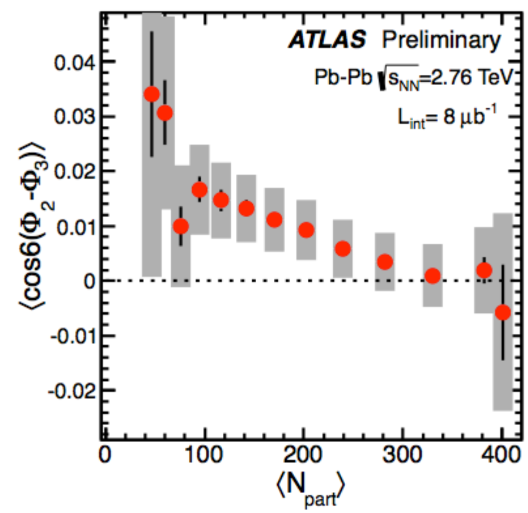
$\langle \cos(2\times 4(\Phi_2-\Phi_4)) \rangle$



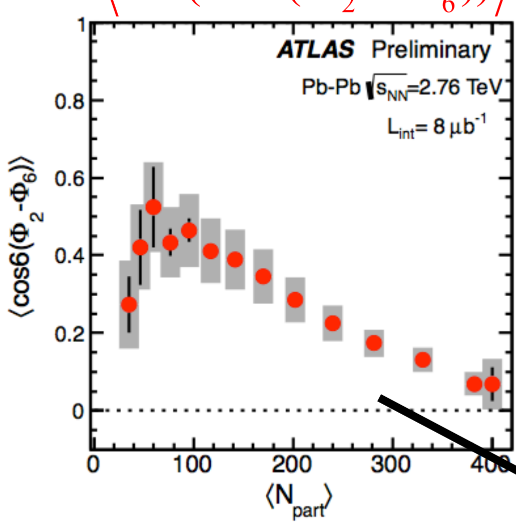
$\langle \cos(3\times 4(\Phi_2-\Phi_4)) \rangle$



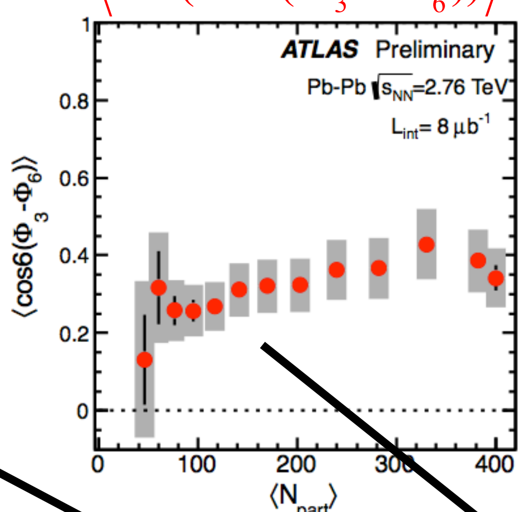
$\langle \cos(1\times 6(\Phi_2-\Phi_3)) \rangle$



$\langle \cos(1\times 6(\Phi_2-\Phi_6)) \rangle$



$\langle \cos(1\times 6(\Phi_3-\Phi_6)) \rangle$



$$v_6 e^{-i6\Psi_6} = \text{geometry} + v_2 v_2 v_2 e^{-i6\Psi_2} + v_3 v_3 e^{-i6\Psi_3} + \ldots$$

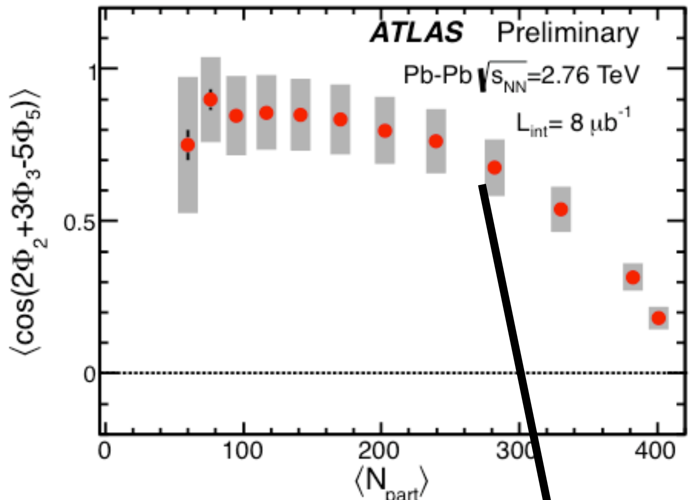
Teaney & Yan

Rich patterns for the centrality dependence

Three-plane correlations

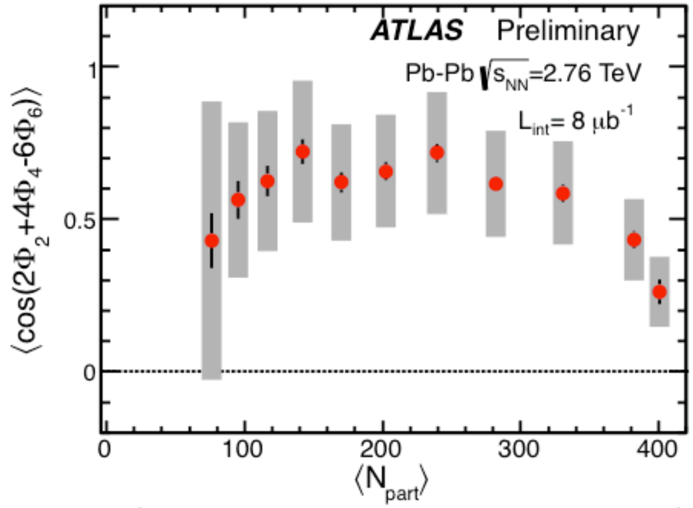
"2-3-5" correlation

$$\left\langle \cos\left(2\Phi_2 + 3\Phi_3 - 5\Phi_5\right)\right\rangle$$



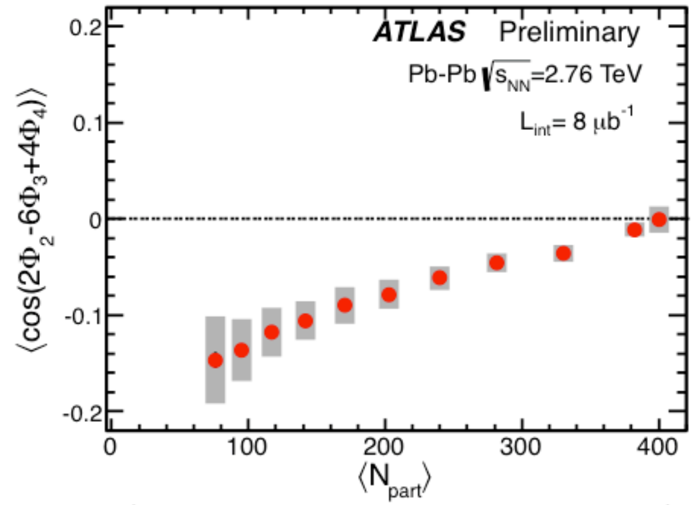
"2-4-6" correlation

$$\left\langle \cos\left(2\Phi_2 + 4\Phi_4 - 6\Phi_6\right)\right\rangle$$

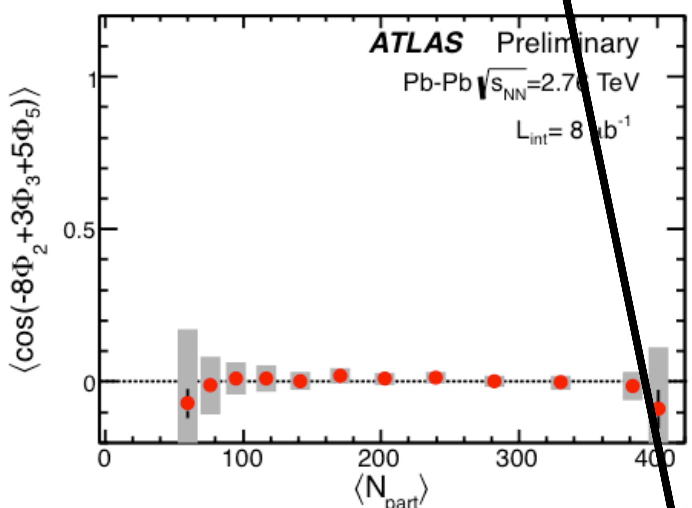


"2-3-4" correlation

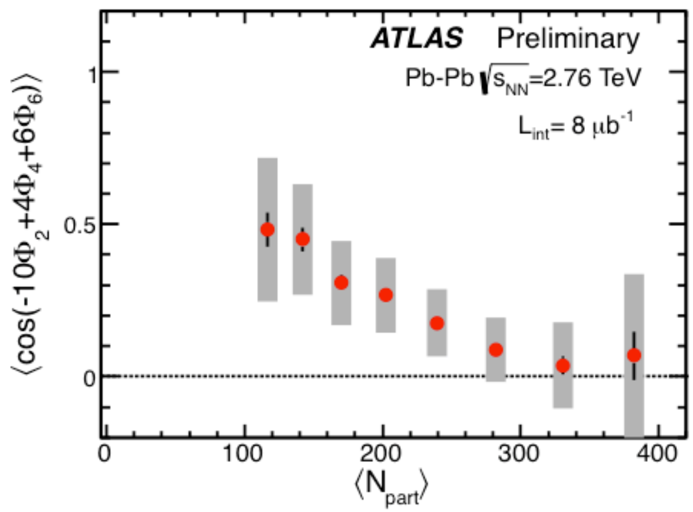
$$\left\langle \cos\left(2\Phi_2 - 6\Phi_3 + 4\Phi_4\right)\right\rangle$$



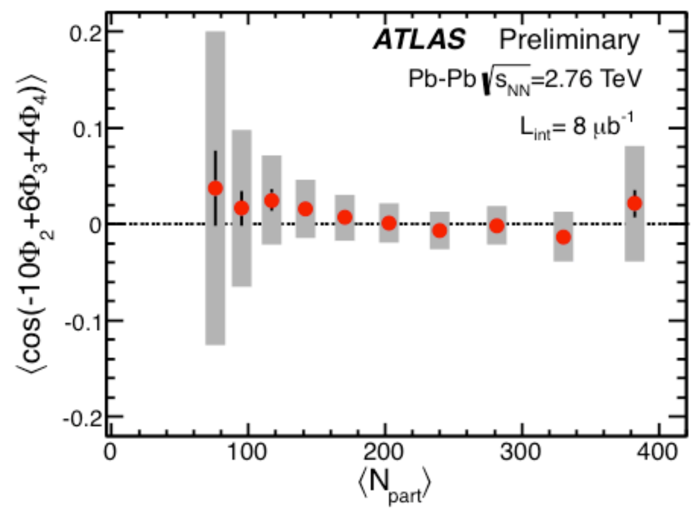
$$\left\langle \cos\left(-8\Phi_2 + 3\Phi_3 + 5\Phi_5\right)\right\rangle$$



$$\left\langle \cos\left(-10\Phi_2 + 4\Phi_4 + 6\Phi_6\right)\right\rangle$$



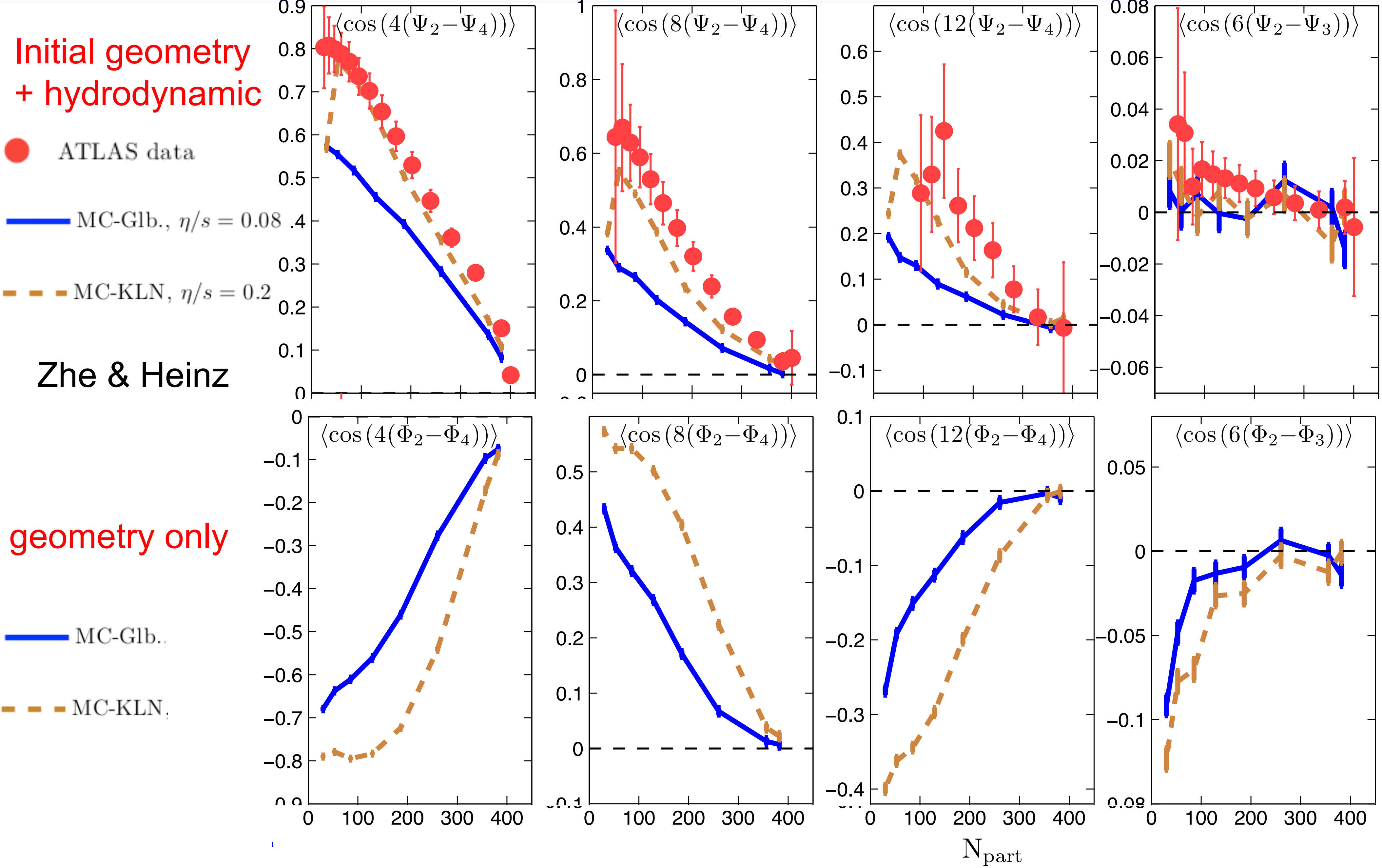
$$\left\langle \cos\left(-10\Phi_2 + 6\Phi_3 + 4\Phi_4\right)\right\rangle$$



$$v_5 e^{-i5\Psi_5} = \text{geometry} + v_2 v_3 e^{-i(2\Psi_2+3\Psi_3)} + \ldots$$

Rich patterns for the centrality dependence

Compare with EbE hydro calculation: 2-plane



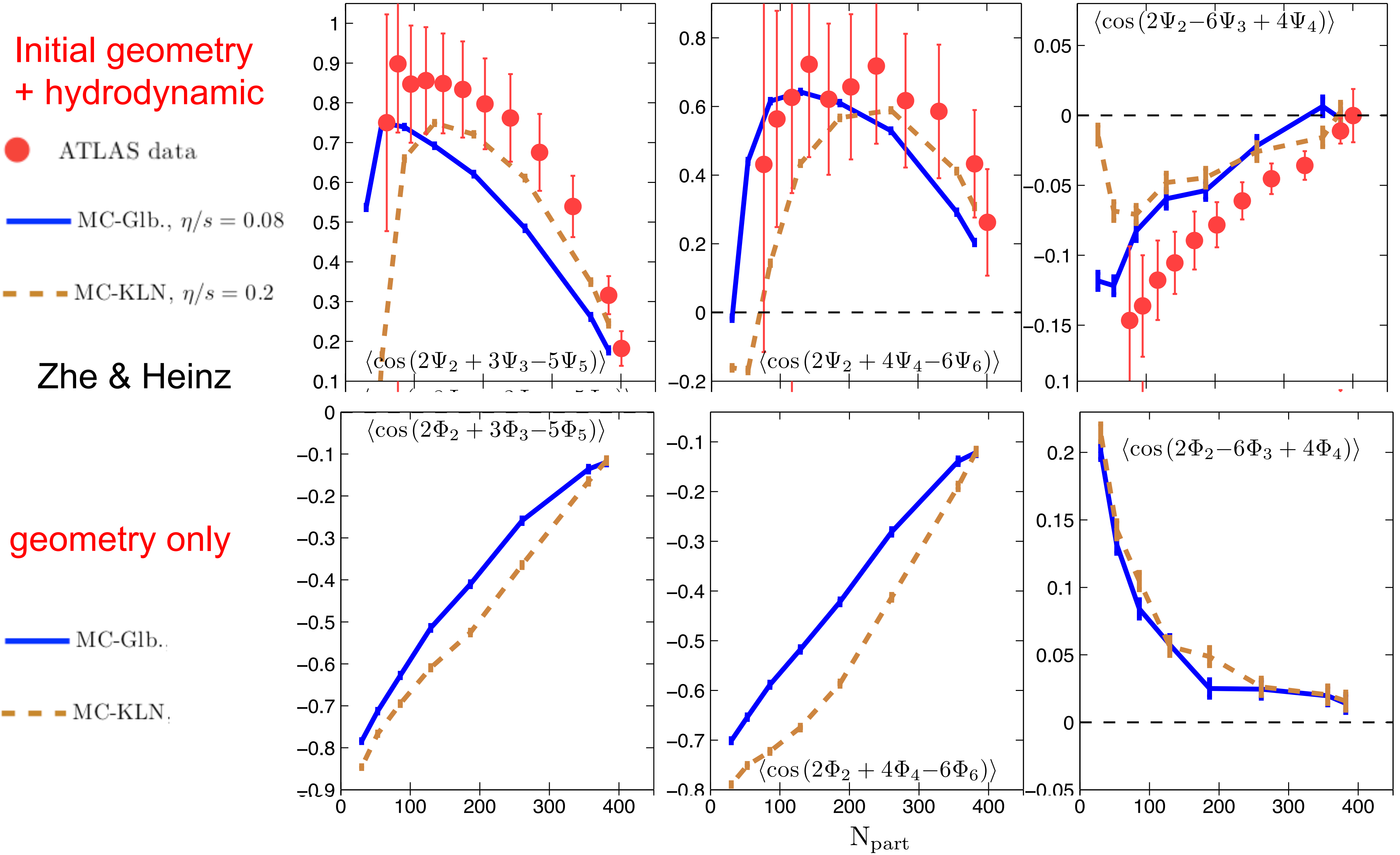
EbyE hydro qualitatively reproduce features in the data

Compare with EbE hydro calculation: 3-plane

Initial geometry + hydrodynamic

geometry only

Zhe & Heinz



Over-constraining the transport properties

Event-by-event v_n distributions

Gaussian model of v_n fluctuations

- Flow vector $$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_n \cos n(\phi - \Phi_n) \quad \vec{\mathrm{v}}_\mathrm{n} = (\mathrm{v}_\mathrm{n}\cos n\Phi_n, \mathrm{v}_\mathrm{n}\sin n\Phi_n) = \vec{\mathrm{v}}_\mathrm{n}^{\mathrm{RP}} + \vec{\mathrm{p}}_\mathrm{n}^{\mathrm{fluc}}$$

- Gaussian model $$p(\vec{v}_n) \propto \exp\left(\frac{-(\vec{v}_n - \vec{v}_n^{RP})^2}{2\delta_n^2}\right)$$

arXiv: 0708.0800

arXiv:0809.2949

$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^{RP})^2}{2\delta_n^2}\right) I_0\left(\frac{v_n v_n^{RP}}{\delta_n^2}\right)$$ Bessel-Gaussian function

$$v_n^{RP} = 0 \Rightarrow \quad p(v_n) \propto v_n \exp\left(\frac{-(v_n^2)}{2\delta_n^2}\right)$$ For pure fluctuations

- Multi-particle cumulants in Gaussian fluctuation limit

$$v_n\{2\} = \sqrt{\left(v_n^{RP}\right)^2 + 2\delta_n^2} \qquad v_n\{4\} = v_n\{6\} = v_n\{8\} = v_n^{RP}$$

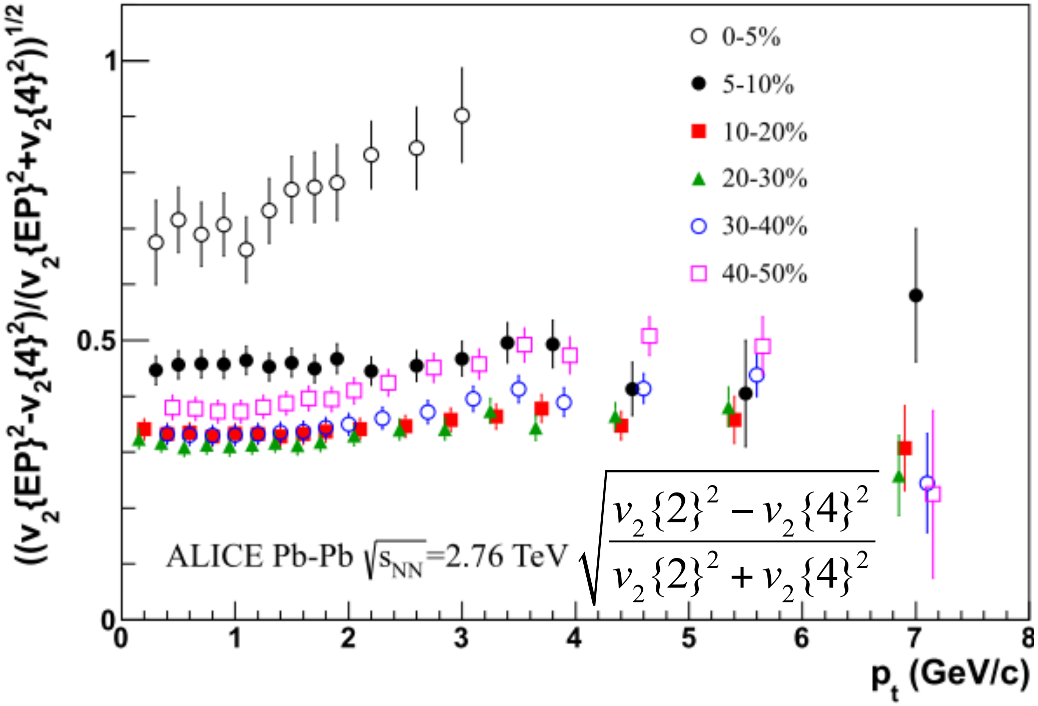
In general $v_2\{2\}$ and $v_2\{4\}$ can be different even in the absence of nonflow

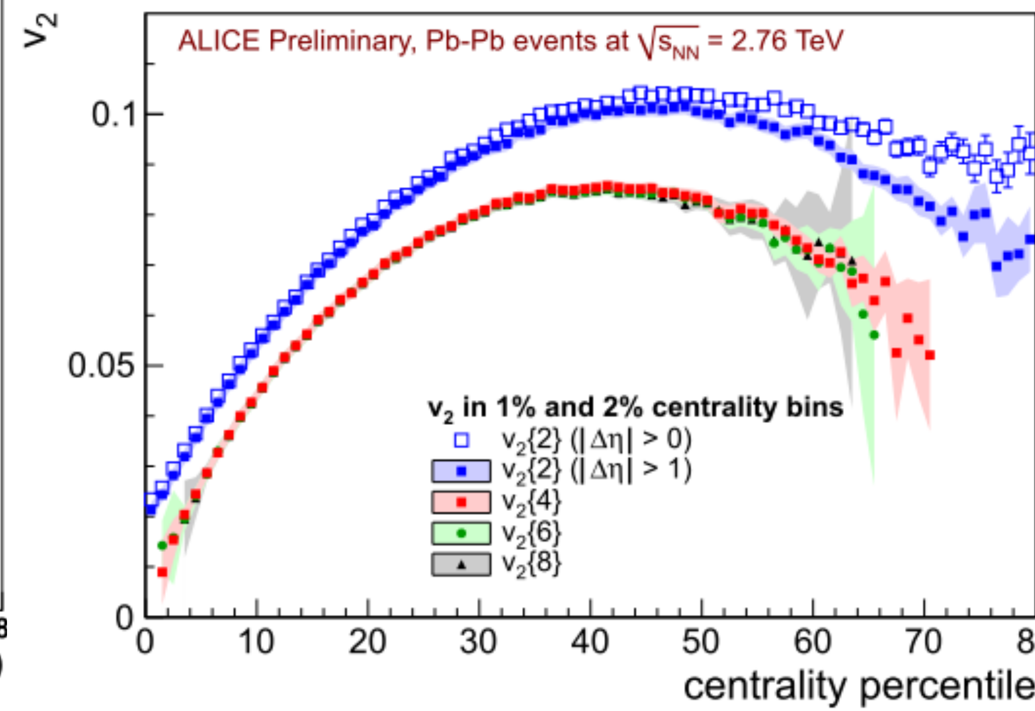
- Various estimators of the fluctuations:

$$\sqrt{\frac{v_n\{2\}^2 - v_n\{4\}^2}{v_n\{2\}^2 + v_n\{4\}^2}} \qquad \sqrt{\frac{v_n\{2\}^2 - v_n\{4\}^2}{2v_n\{4\}^2}} = \frac{\delta_n}{v_n^{RP}} \qquad \frac{\sigma_n}{\langle v_n \rangle}$$

$$v_n^{RP} = 0 \Rightarrow \qquad = 1 \qquad\qquad = \infty \qquad\qquad = \sqrt{4/\pi - 1} = 0.52$$

Measuring v_2 fluctuations with Cumulants





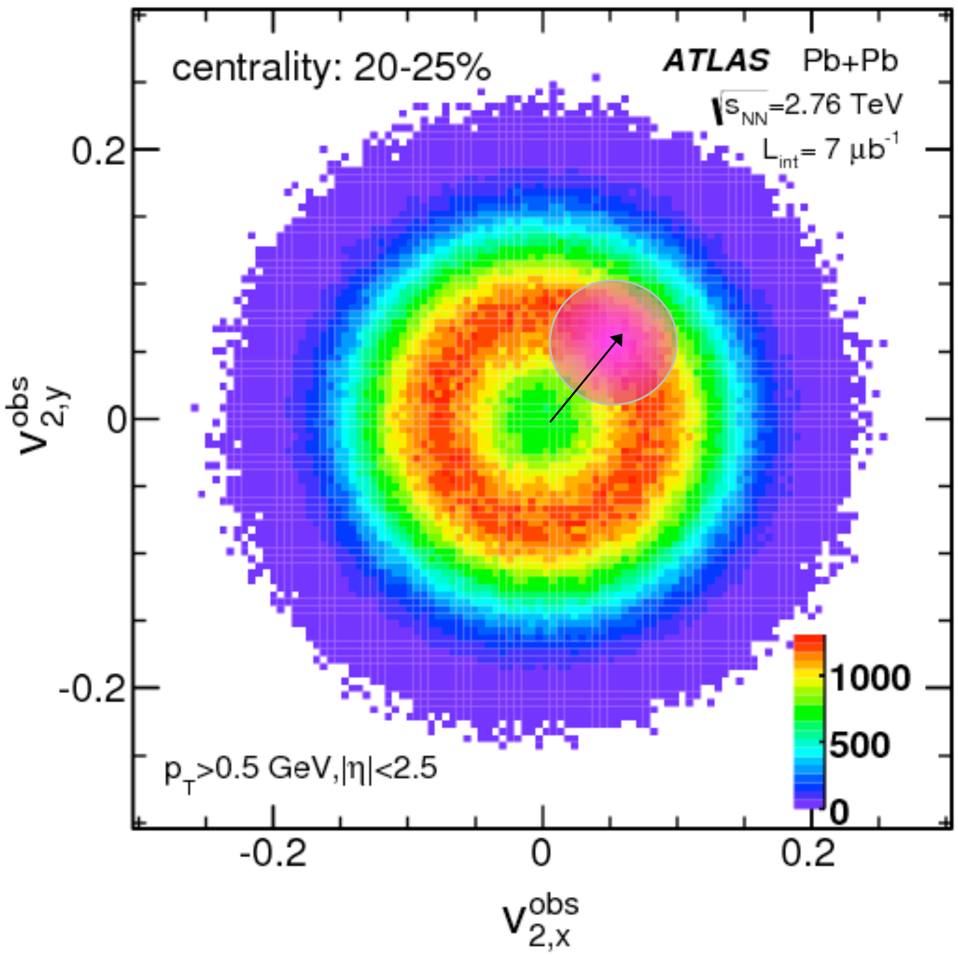
- Relative fluctuations increase with p_T (by 25%)
  - $v_2^{RP} \neq 0$ even in 0-5% central collisions
- Higher order cumulants such as $v_2\{6\}, v_2\{8\}$ all measure v_2^{RP}
  - Significant uncertainty in 0-5% centrality.
  - Fluctuation is described by Bessel-Gaussian function??

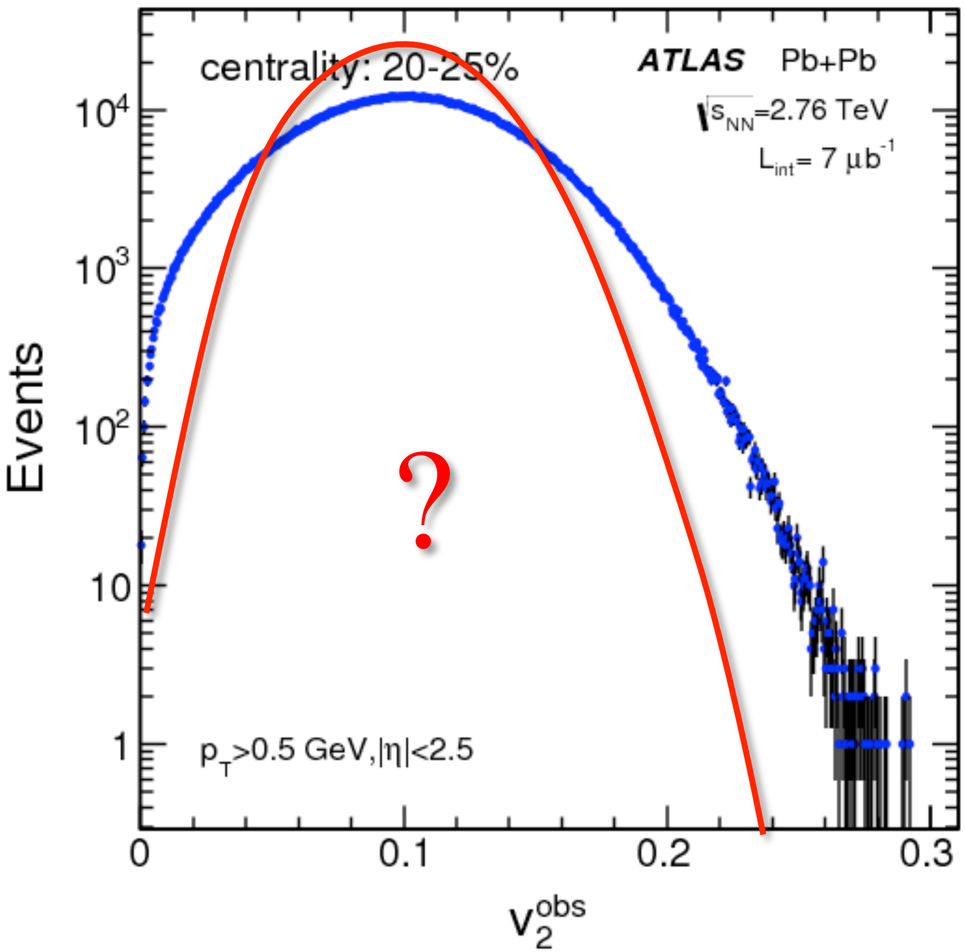
Flow vector and smearing

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n^{obs} \cos n\left(\phi - \Phi_n^{obs}\right)$$

$$\vec{v}_n^{\,obs} = (v_{n,x}^{obs}, v_{n,y}^{obs}) = \vec{v}_n + \vec{p}_n^{\,smear}$$

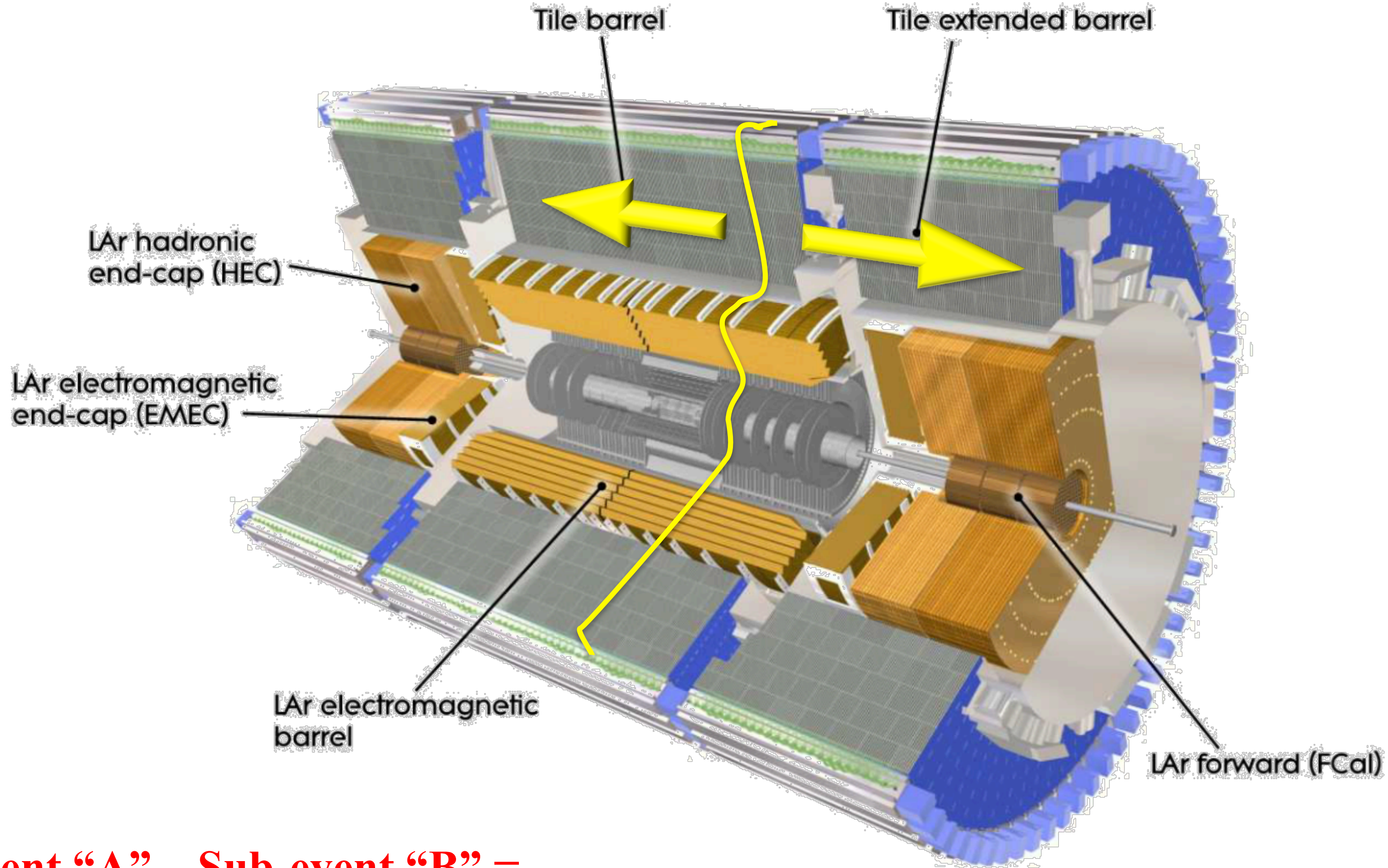
$$\vec{v}_n = \vec{v}_n^{\,RP} + \vec{p}_n^{\,fluc}$$





The key of unfolding is response function: $p(v_n^{obs}|v_n)$

Split the event into two: 2SE method



Sub-event "A" – Sub-event "B" =

Shown by simulation studies

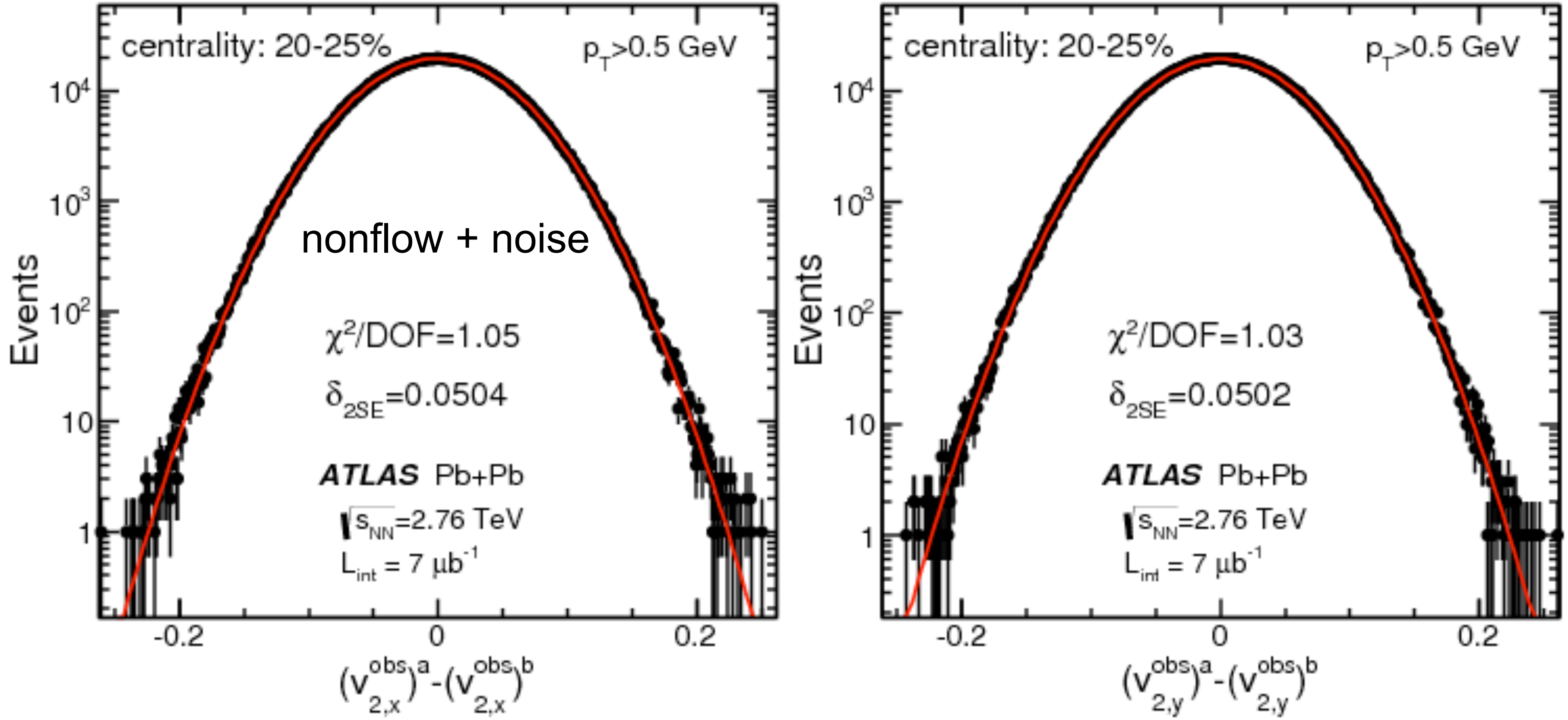
$$(\vec{v}_n^{\,\mathrm{obs}})^{\mathrm{a}} - (\vec{v}_n^{\,\mathrm{obs}})^{\mathrm{b}} = \text{nonflow} + \text{noise}$$

arxiv:1304.1471

$$(\vec{v}_n^{\,\mathrm{obs}})^{\mathrm{a}} + (\vec{v}_n^{\,\mathrm{obs}})^{\mathrm{b}} = \text{nonflow} + \text{noise} + 2\vec{v}_n$$

Width of $(\vec{v}_n^{\,\mathrm{obs}})^a - (\vec{v}_n^{\,\mathrm{obs}})^b \xrightarrow{1/\sqrt{2}}$ Width of $(\vec{v}_n^{\,\mathrm{obs}})^a \xrightarrow{1/\sqrt{2}}$ Width of $\vec{v}_n^{\,\mathrm{obs}}$

Obtaining the response function



Response function is a 2D Gaussian around truth

Nonflow is Gaussian!

$$p(\vec{v}_n^{\text{obs}}|\vec{v}_n) \propto e^{-\frac{|\vec{v}_n^{\text{obs}}-\vec{v}_n|^2}{2\delta^2}} \qquad \delta = \delta_{2\text{SE}}/2$$

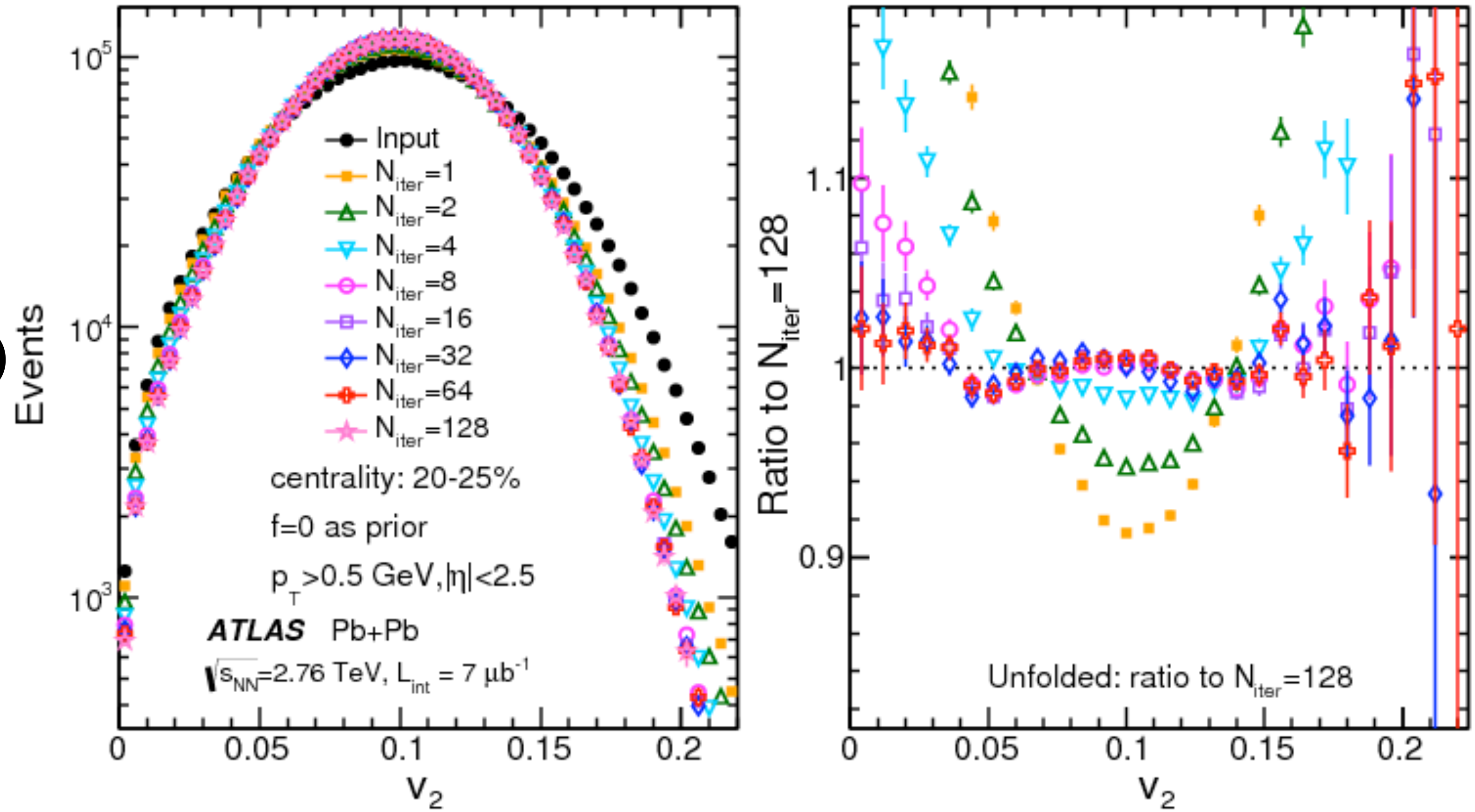
$$p(v_n^{\text{obs}}|v_n) \propto v_n^{\text{obs}} e^{-\frac{(v_n^{\text{obs}})^2+v_n^2}{2\delta^2}} I_0\left(\frac{v_n^{\text{obs}} v_n}{\delta^2}\right)$$

Data driven method

Unfolding performance: v_2, 20-25%

$$Av_n = v_n^{obs}$$

$$A_{ji} = p(v_{n,j}^{obs} | v_{n,i})$$



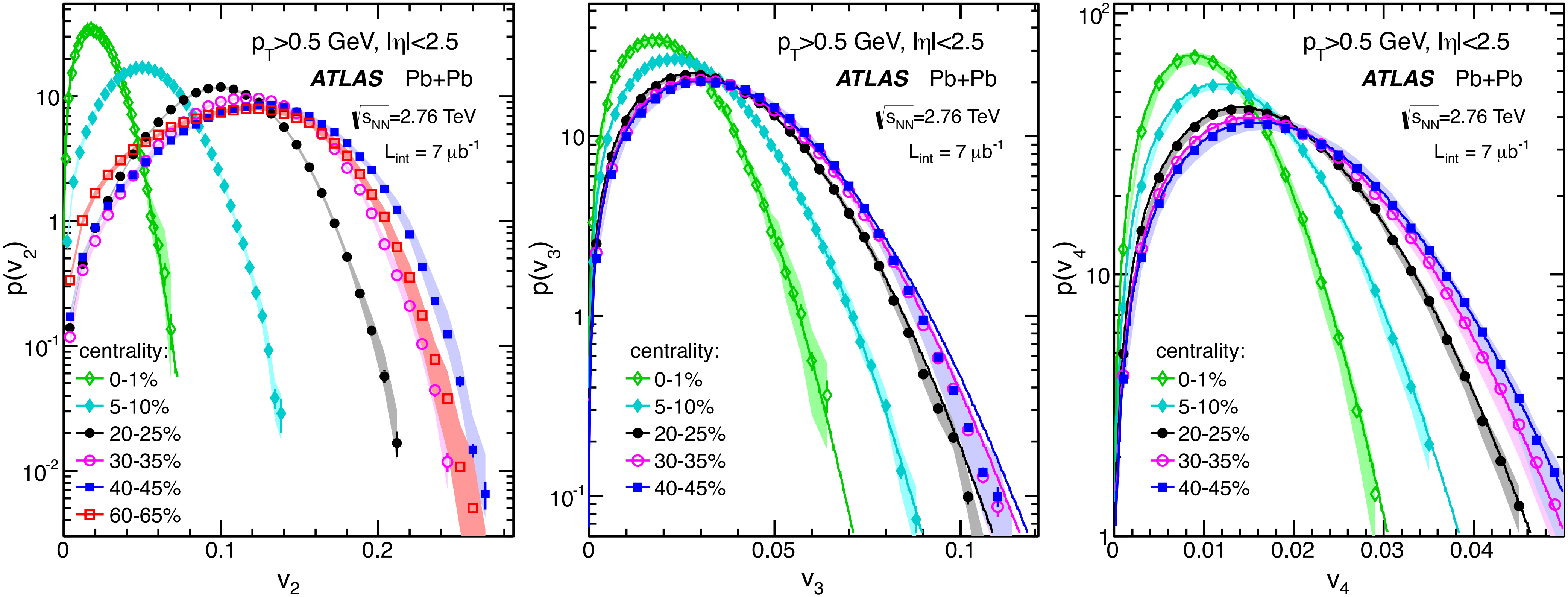
- Standard Bayesian unfolding technique
- Converges within a few % for N_{iter}=8, small improvements for larger N_{iter}.
- Many cross checks show good consistency
  - Unfolding with different initial distributions
  - Unfolding using tracks in a smaller detector
  - Unfolding based on the EbyE two-particle correlation.
  - Closure test using HIJING+flow simulation

Details in **arxiv:1305.2942**

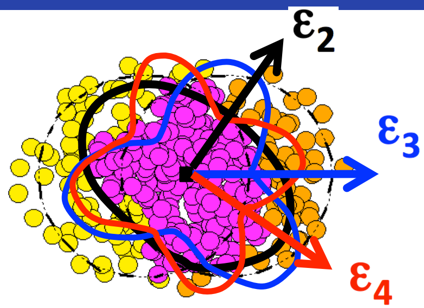
arxiv:1304.1471

$p(v_2)$, $p(v_3)$ and $p(v_4)$ distributions

- Measured in broad centrality over large v_n range
  - The fraction of events in the tails is less than 0.2% for v_2 and v_3, and ~1-2% for v_4.



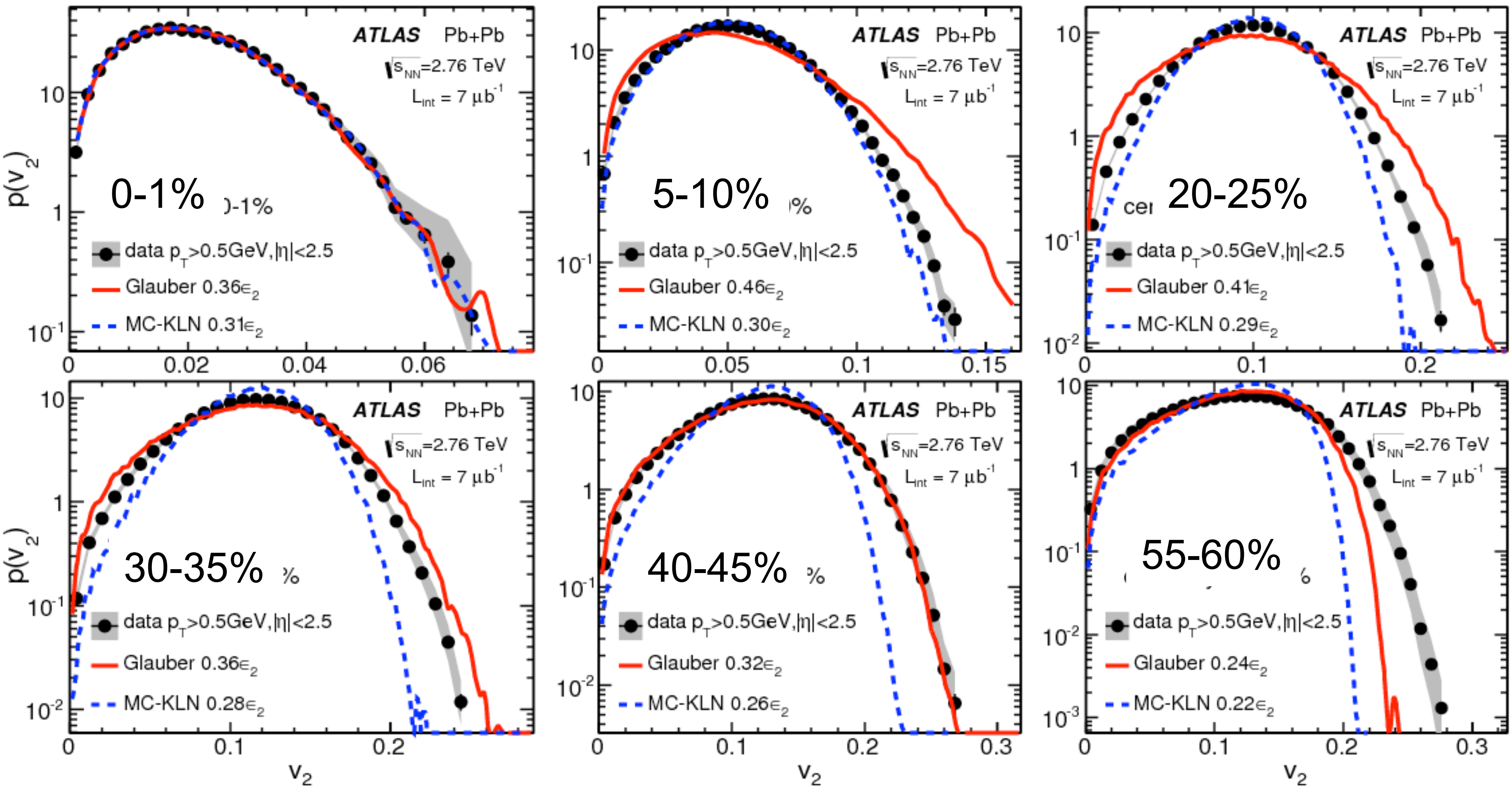
Compare with initial geometry models



$$v_n \propto \epsilon_n = \frac{\sqrt{\langle r^n \cos n\phi \rangle^2 + \langle r^n \sin n\phi \rangle^2}}{\langle r^n \rangle}$$

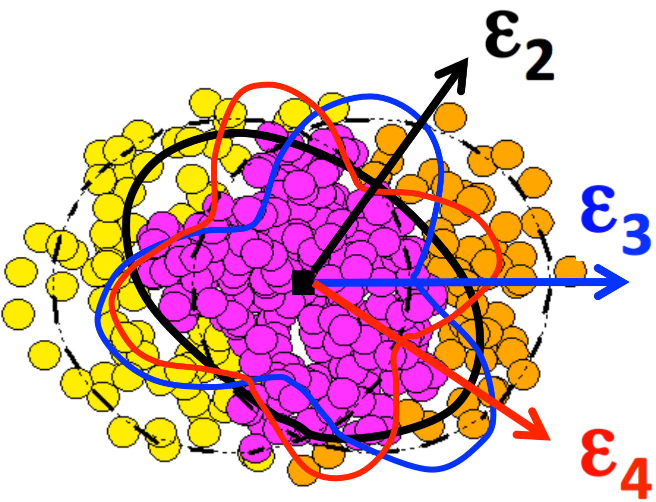
Glauber and CGC mckln

Rescale ε_n distribution to the mean of data

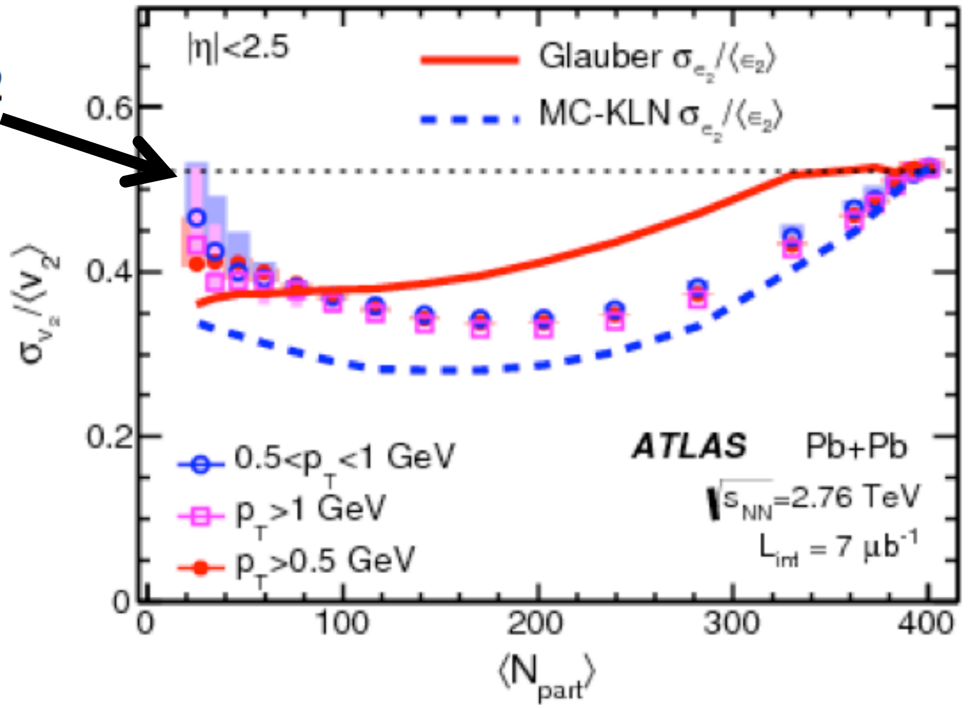


Both models fail describing p(v_2) across the full centrality range

Compare with initial geometry models



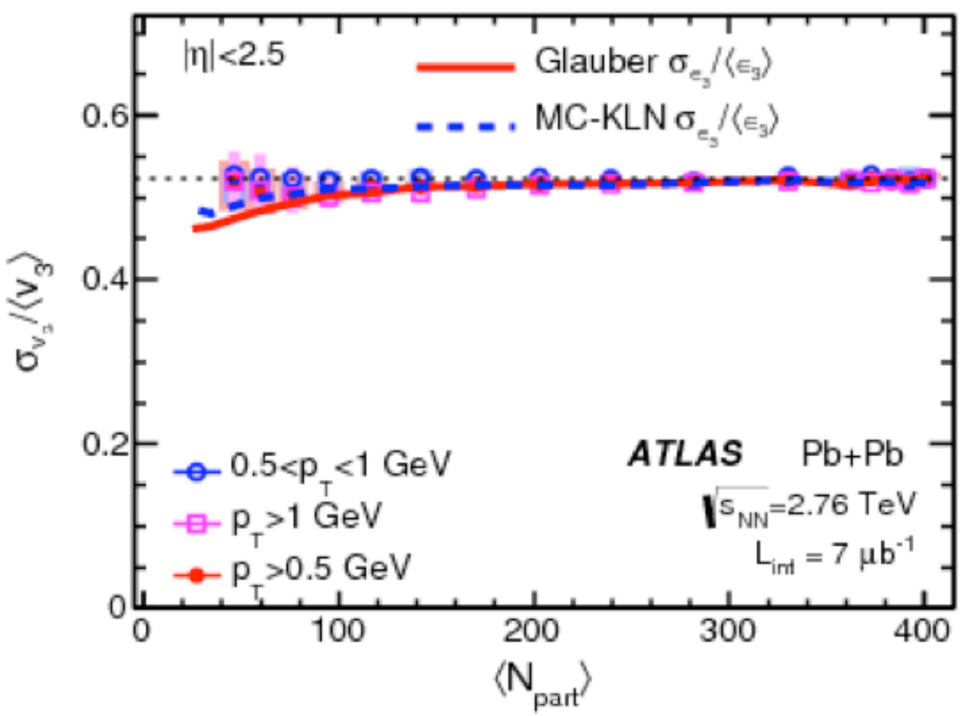
$$\frac{\sigma_{v_n}}{\langle v_n \rangle} = \sqrt{\frac{4}{\pi} - 1} = 0.52$$



$$\varepsilon_n = \sqrt{\frac{\langle r^n \cos(n\phi)\rangle + \langle r^n \sin(n\phi)\rangle}{r^n}} \qquad v_n \propto \varepsilon_n$$

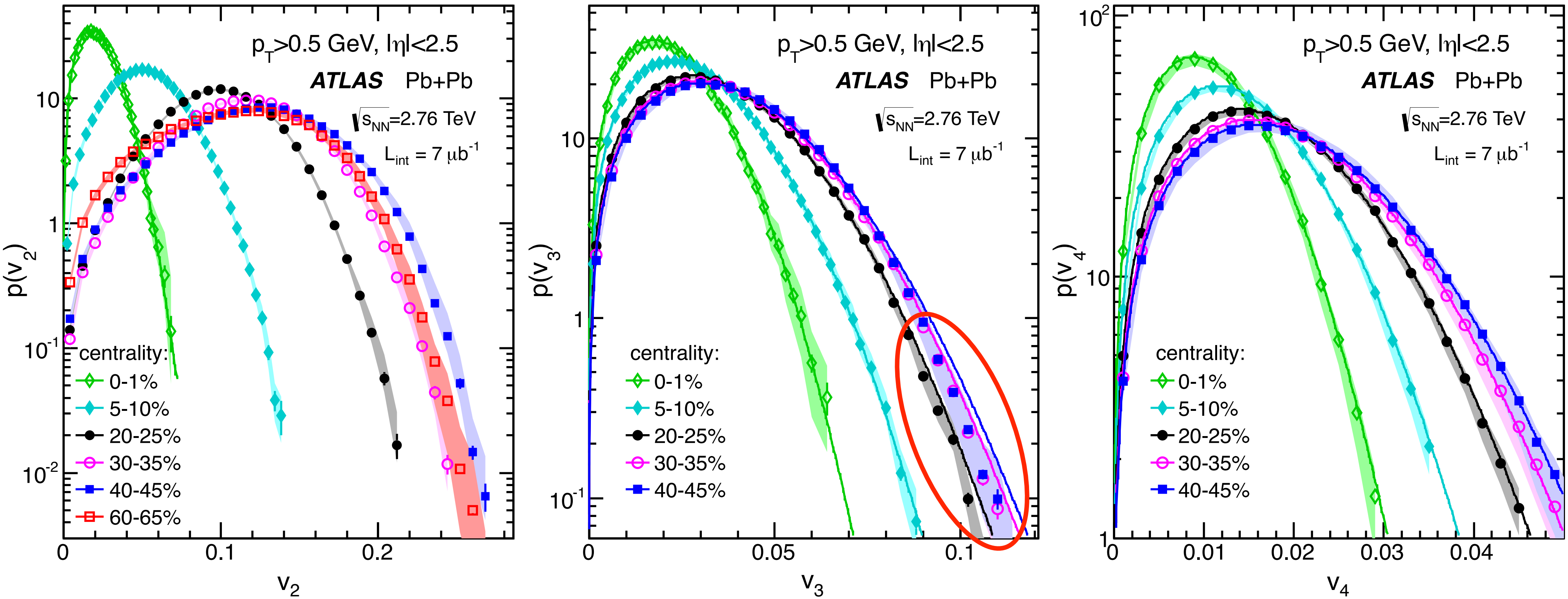
- Test relation $\boxed{\dfrac{\sigma_{\varepsilon_n}}{\langle \varepsilon_n \rangle} = \dfrac{\sigma_{v_n}}{\langle v_n \rangle}}$

Both models failed.



$p(v_2)$, $p(v_3)$ and $p(v_4)$ distributions

Parameterize with pure fluctuation scenario: $p(v_n) \propto v_n \exp\left(\frac{-v_n^2}{2\delta_n^2}\right)$

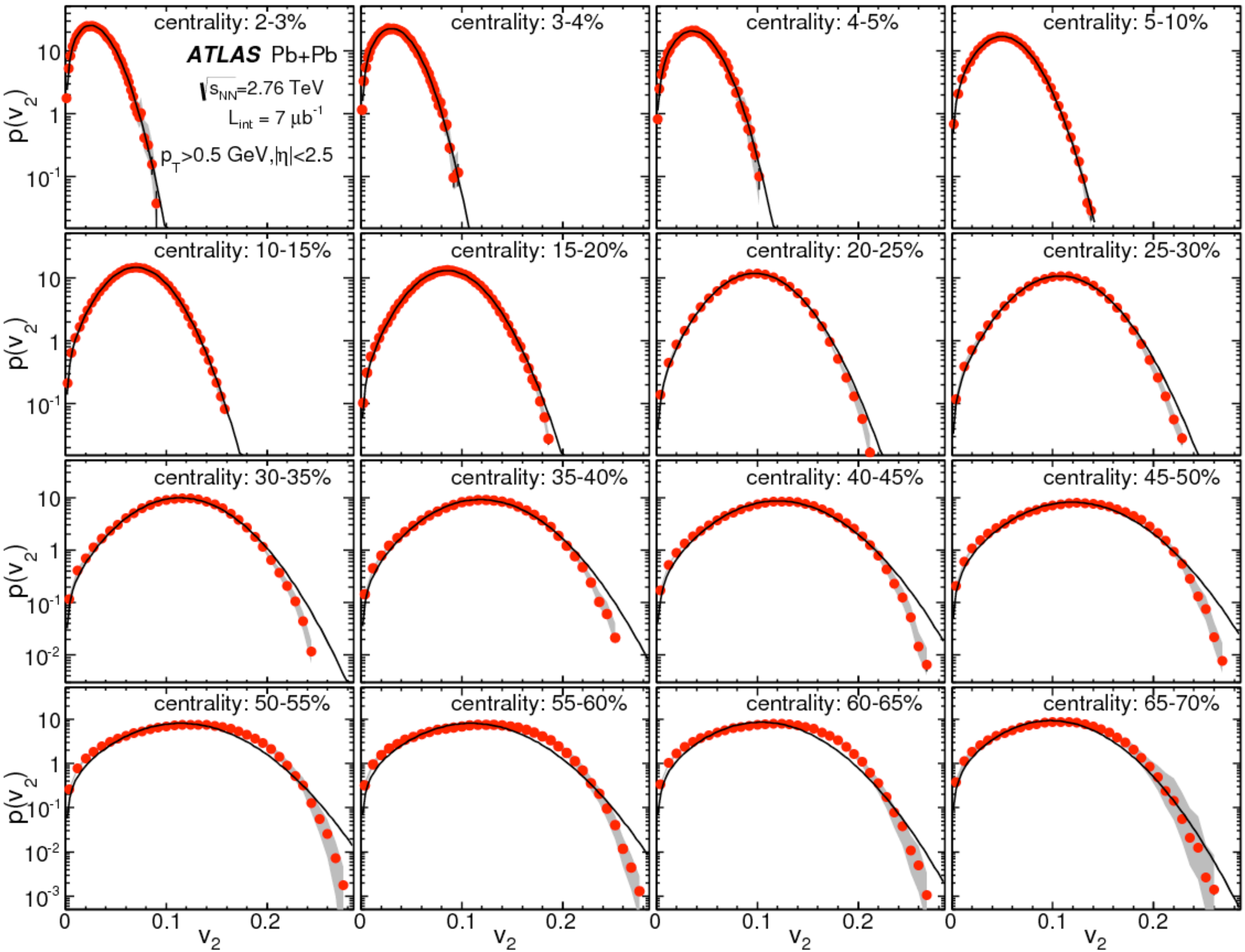


Only works in 0-2% centrality
Require non zero v_2^{RP} in others

Deviations in the tails:
non zero v_3^{RP}

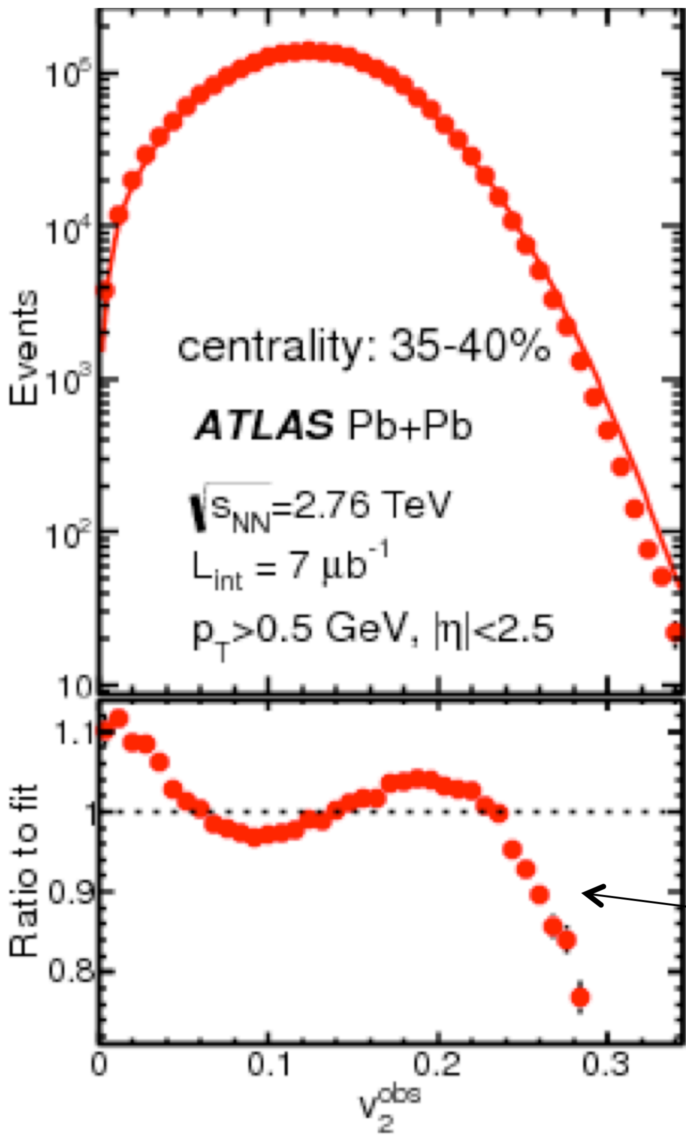
No deviation is observed,
however v_4 range is limited.

Bessel-Gaussian fit to $p(v_2)$



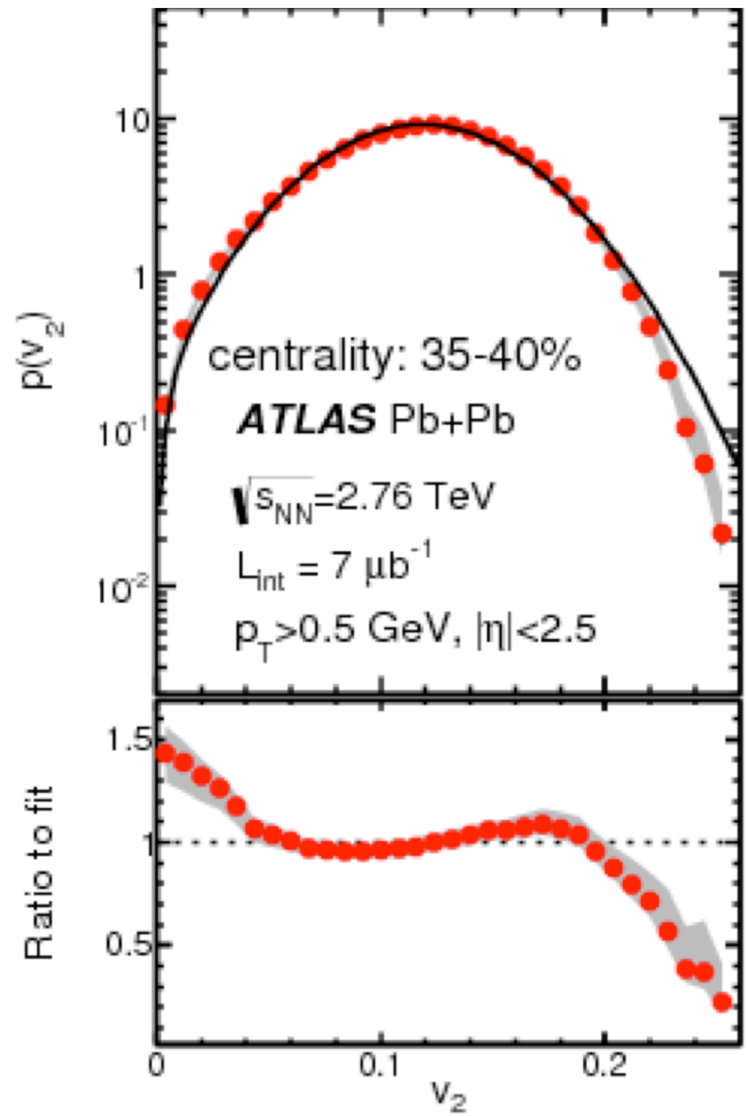
The deviation from B-G before/after unfolding

Before unfolding

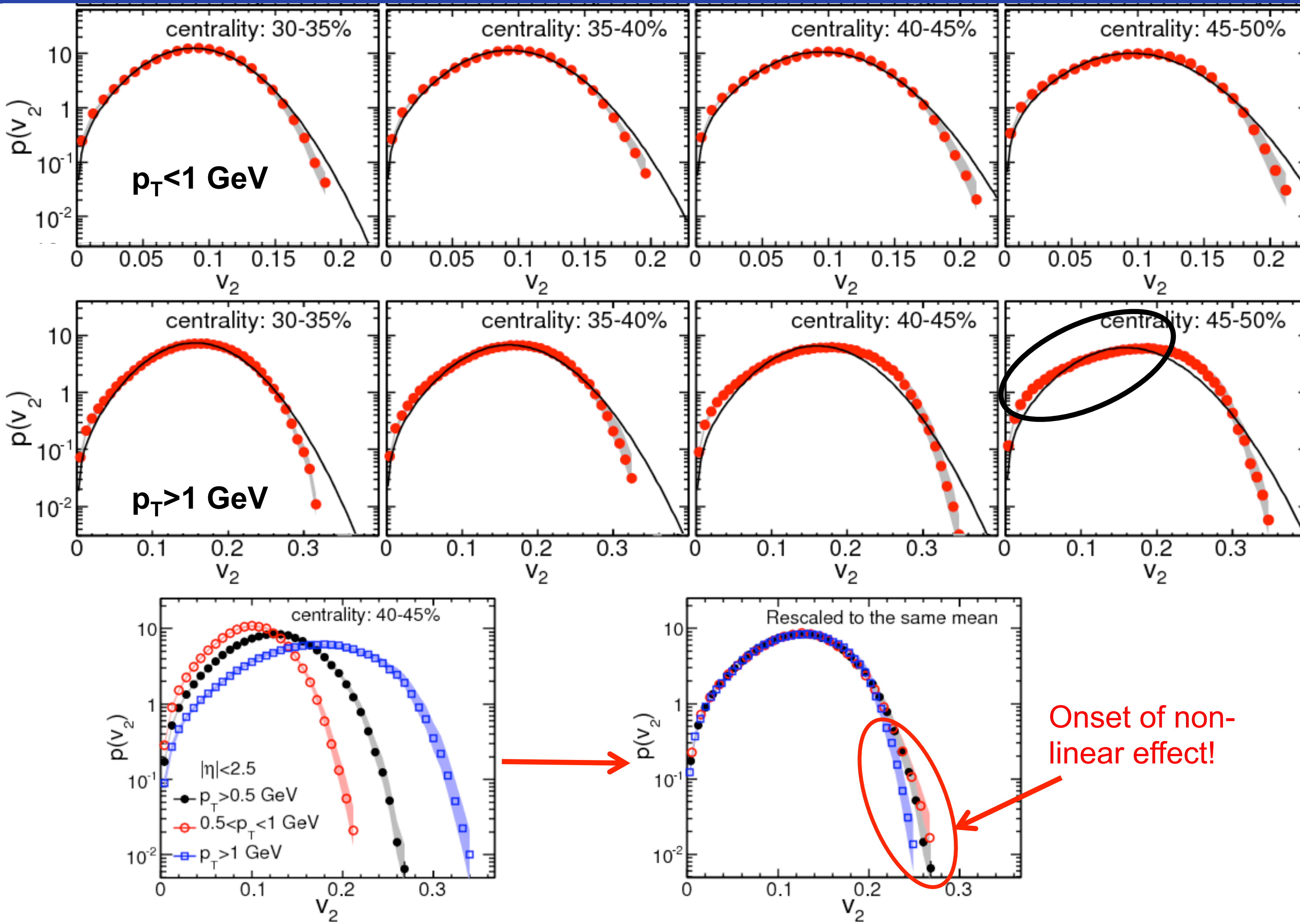


Deviation from BG is already clear before unfolding

After unfolding



Deviation grows with p_T



Are cumulants sensitive to these deviations?

$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^{RP})^2}{2\delta_n^2}\right) I_0\left(\frac{v_n v_n^{RP}}{\delta_n^2}\right)$$

$$v_2^{\rm calc}\{4\}^4 \equiv -\langle v_2^4\rangle + 2\langle v_2^2\rangle^2 \approx \left(v_2^{\rm RP}\right)^4 \ ,$$

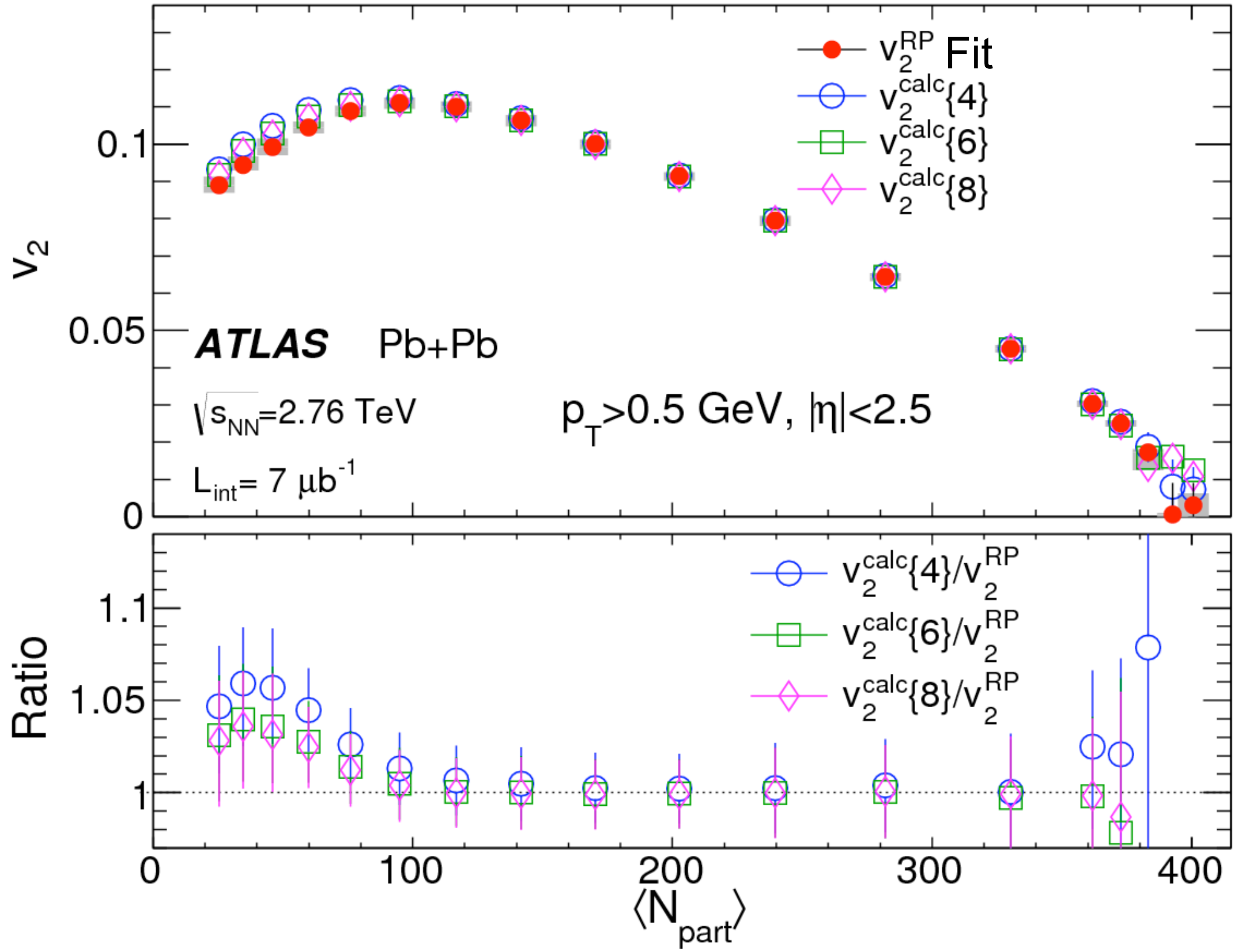
$$v_2^{\rm calc}\{6\}^6 \equiv \left(\langle v_2^6\rangle^2 - 9\langle v_2^4\rangle\langle v_2^2\rangle + 12\langle v_2^2\rangle^3\right)/4 \approx \left(v_2^{\rm RP}\right)^6 \ ,$$

$$v_2^{\rm calc}\{8\}^8 \equiv -\left(\langle v_2^8\rangle^2 - 16\langle v_2^6\rangle\langle v_2^2\rangle - 18\langle v_2^4\rangle^2 + 144\langle v_2^4\rangle\langle v_2^2\rangle^2 - 144\langle v_2^2\rangle^4\right)/33 \approx \left(v_2^{\rm RP}\right)^8$$

Cumulants are not sensitive to the tails, presumably because their values are dominated by the v_2^{RP}:

$$v_2\{6\}^6 \sim \left(v_2^{RP}\right)^6 + \left(\Delta\right)^6$$

If Δ=0.5 v_2^{RP}, $v_2\{6\}$ only change by 2%



$v_2\{4,6,8\}$ without unfolding

$$\vec{v}_2^{\,obs} = \vec{v}_2^{\,RP} + \vec{p}_2^{\,fluc} + \vec{p}_2^{\,smear}$$

← **Removed by unfolding**

Initial geometry fluctuations

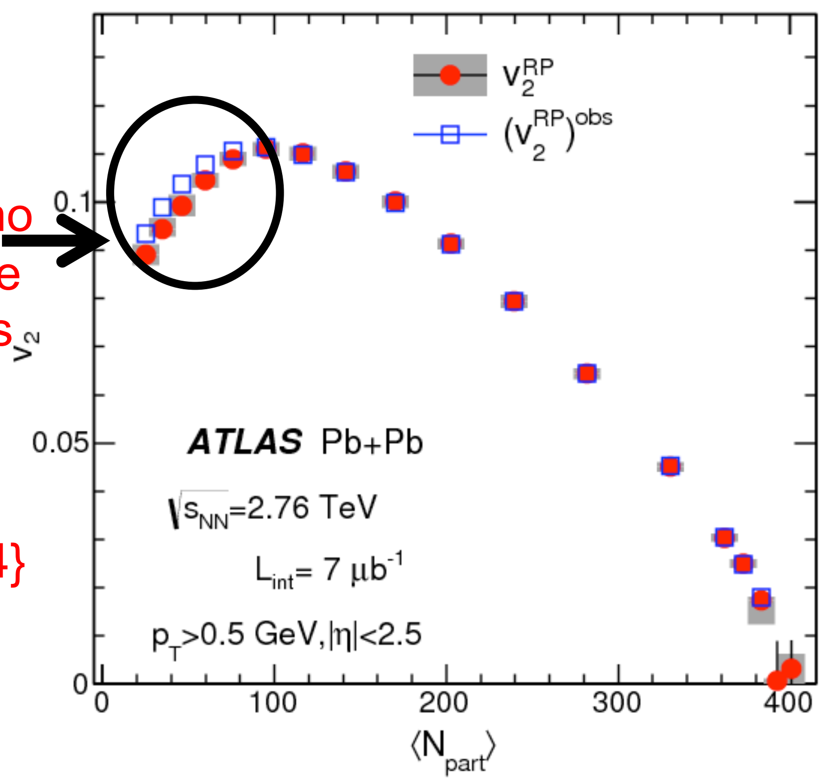
$$p(\vec{v}_2) \propto \exp\left(\frac{-(\vec{v}_2 - \vec{v}_2^{\,RP})^2}{2\delta_2^2}\right)$$

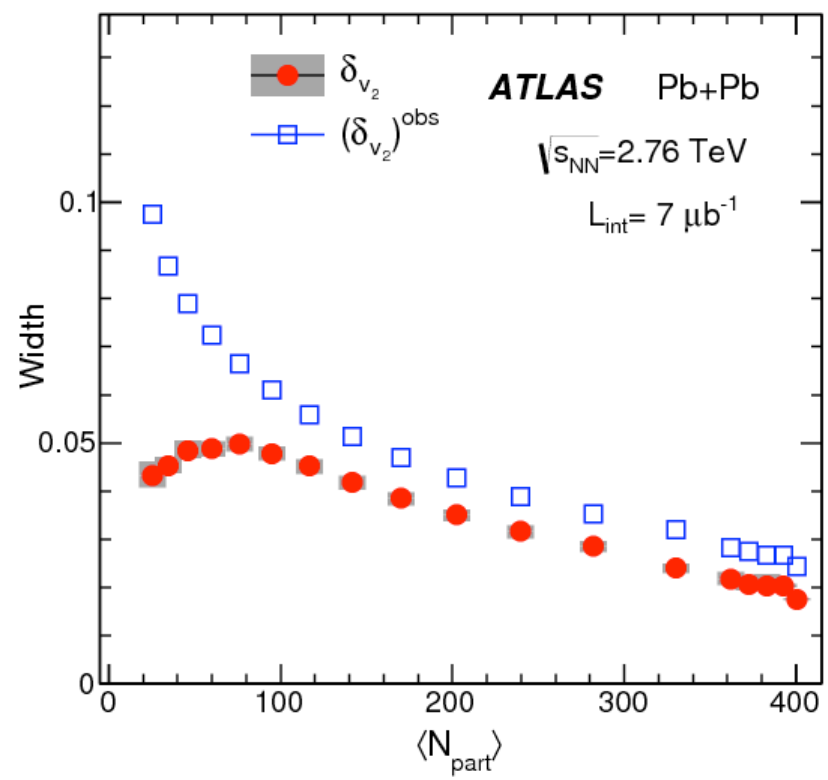
Additional Gaussian smearing won't change the v_2^{RP}.

Indeed the response function is Gaussian

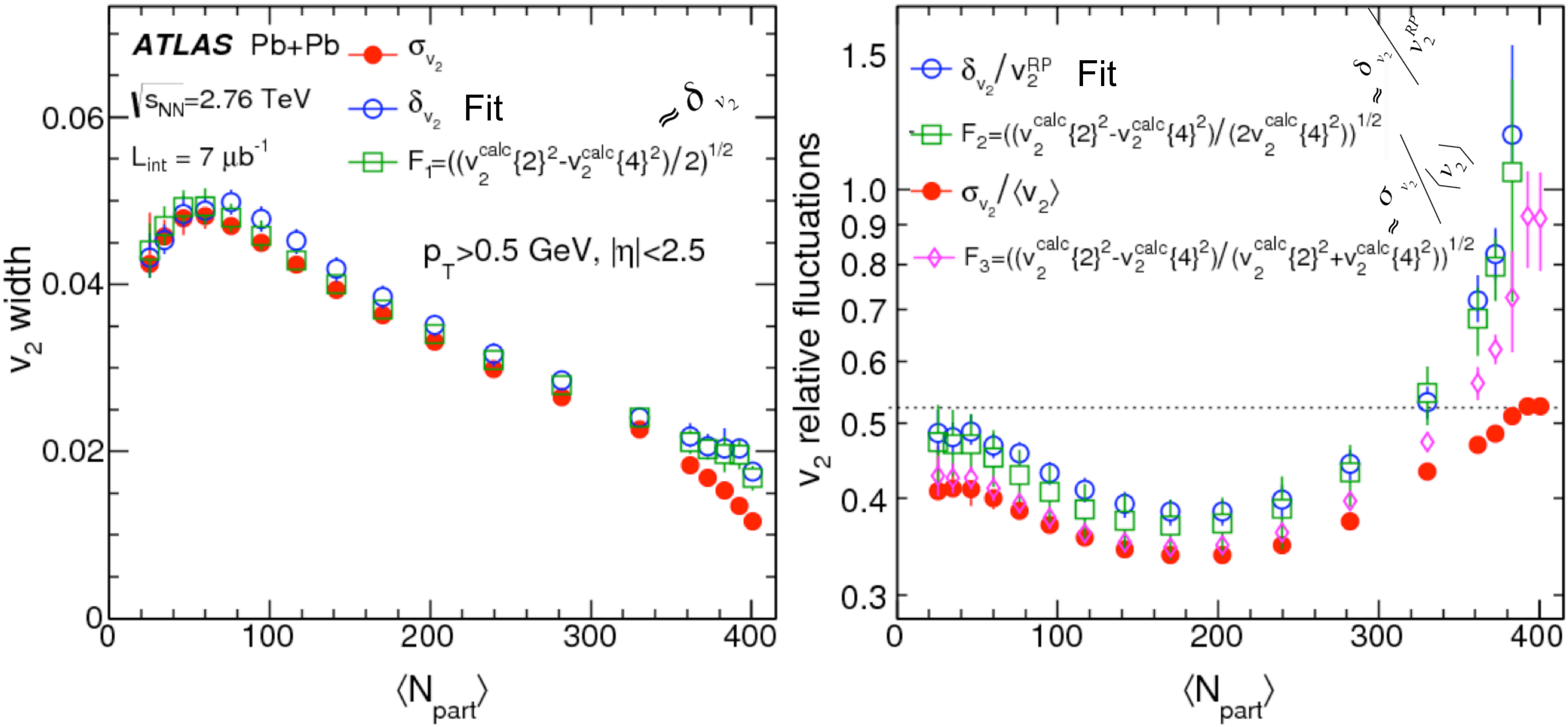
Response function (nonflow+noise) is no longer Gaussian, the real flow fluctuations may also be non-Gaussian.

The meaning of $v_2\{4\}$ is non-trivial in this limit (also in pPb)



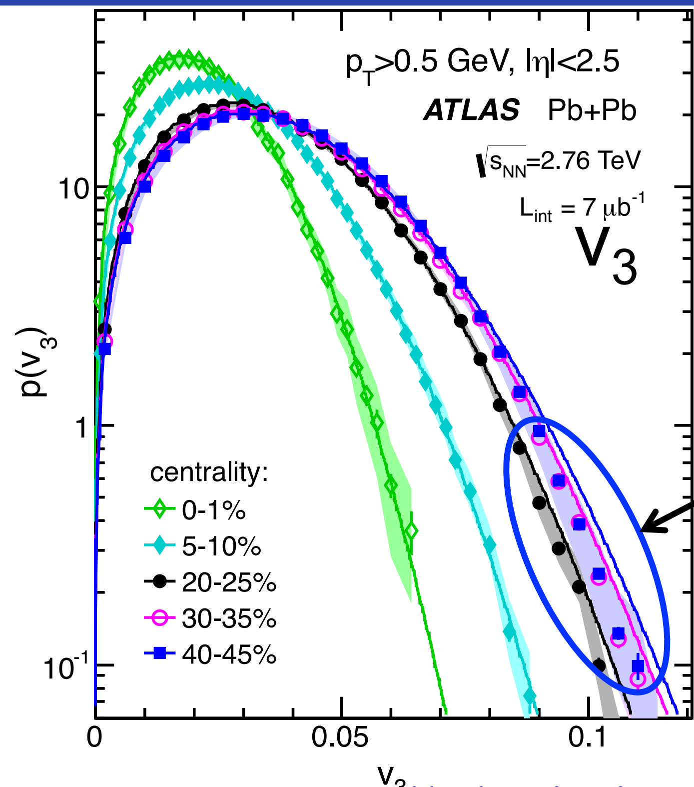


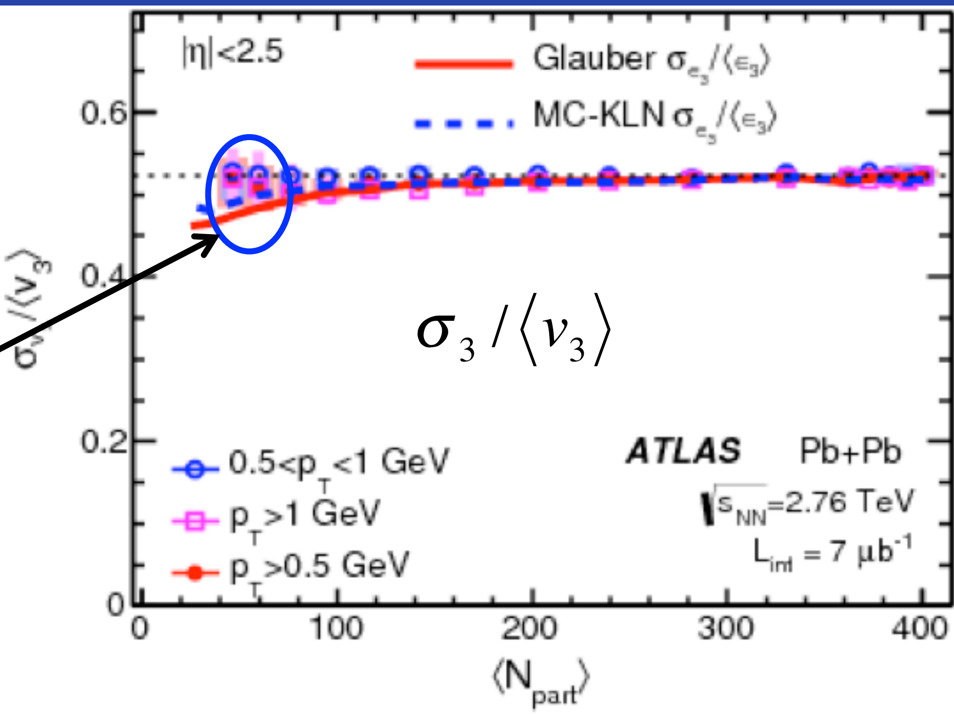
Extracting relative fluctuations



- Different estimator gives different answer, especially in central collisions
  - Expected since they have different limit.
  - Stick to one convention?

Flow fluctuation & $v_3\{4\}$



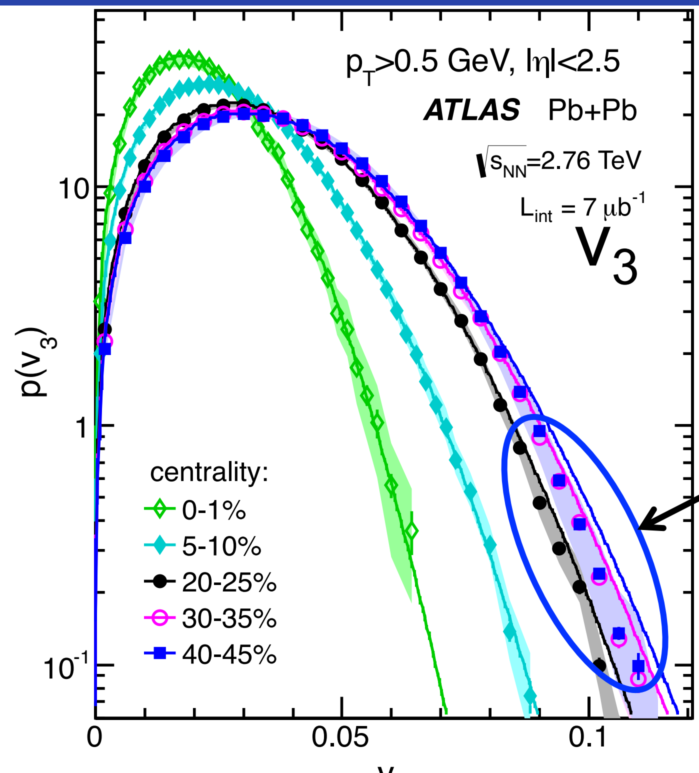


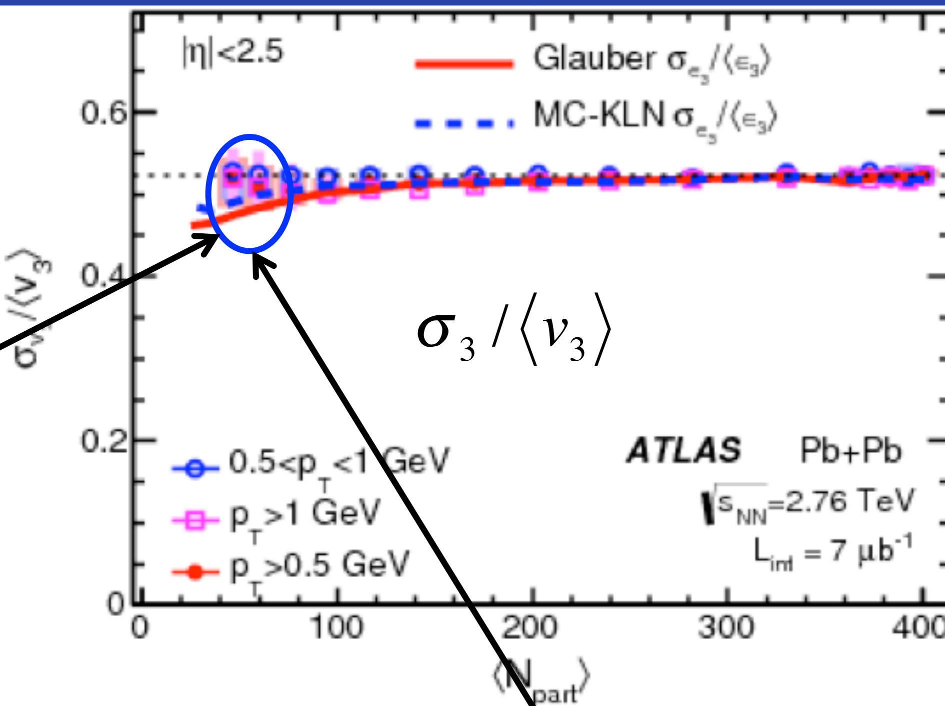
- Even a small deviation will imply a v_n^{RP} or $v_n\{4\}$ value comparable to δ_{vn}

$$v_n\{4\} = \left[2\left\langle v_n^2\right\rangle^2 - \left\langle v_n^4\right\rangle\right]^{1/4}$$

a 4% difference gives a $v_n\{4\}$ value of about 45% of $v_n\{2\}$

Flow fluctuation & $v_3\{4\}$





$\sigma_3 / \langle v_3 \rangle$

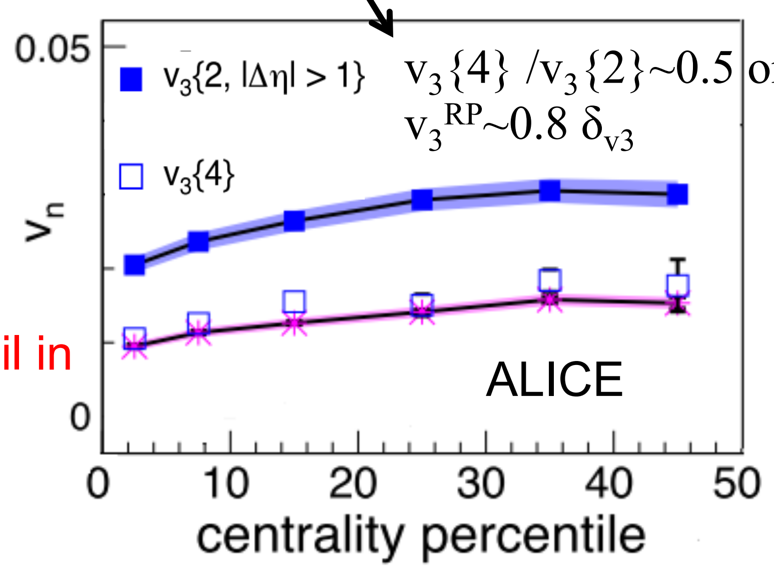
- Even a small deviation will imply a v_n^{RP} or $v_n\{4\}$ value comparable to δ_{vn}

$$v_n\{4\} = \left[2\left\langle v_n^2 \right\rangle^2 - \left\langle v_n^4 \right\rangle \right]^{1/4}$$

a 4% difference gives a $v_n\{4\}$ value of about 45% of $v_n\{2\}$

Due to a non-Gaussian tail in the $p(v_3)$ distribution?.

$v_3\{4\} / v_3\{2\} \sim 0.5$ or $v_3^{RP} \sim 0.8\ \delta_{v3}$



Summary

- Event-by-event fluctuation of the QGP and its evolution can be accessed via $p(v_n,v_m,\ldots,\Phi_n,\Phi_m\ldots)$

- Detailed correlation measurement 2- and 3- event planes → the Fourier coefficients of $p(\Phi_n,\Phi_m)$ and $p(\Phi_n,\Phi_m,\Phi_L)$
  - Strong proof of mode-mixing/non-linear effects of the hydro response to initial geometry fluctuations.
  - New set of constraints on geometry models and η/s.

- First measurements of the $p(v_2)$, $p(v_3)$ and $p(v_4)$
  - Glauber and MC-KLN models ruled out
  - $p(v_2)$ show significant deviation of the fluctuation from Gaussian, also suggestive of strong non-linear effects.
  - $v_2\{4,6,8\}$ are not sensitive to these deviations, except in peripheral collisions.
  - $p(v_3)$ distribution suggests a non-zero v_3^{RP}.

- Look into other correlations.