The Trailing String in Confining Holographic Theories

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Work with E. Kiritsis, L. Mazzanti, to appear soon

Introduction

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In this talk, I will discuss the trailing string solution in $T = 0$ vacuum geometries, in particular those dual to a confining medium.
(Notice that it does make sense to consider an infinitely massive, non-dynamical quark in the confining vacuum).

Motivation and results

- From a practical point of view, in order to correctly obtain the dynamics of the probe in the deconfined medium, one needs a subtraction procedure to make basic quantities (Boundary retarded correlators) well defined. The natural way to operate this subtraction is through the vacuum correlator. Whence the need of the vacuum trailing string solution.

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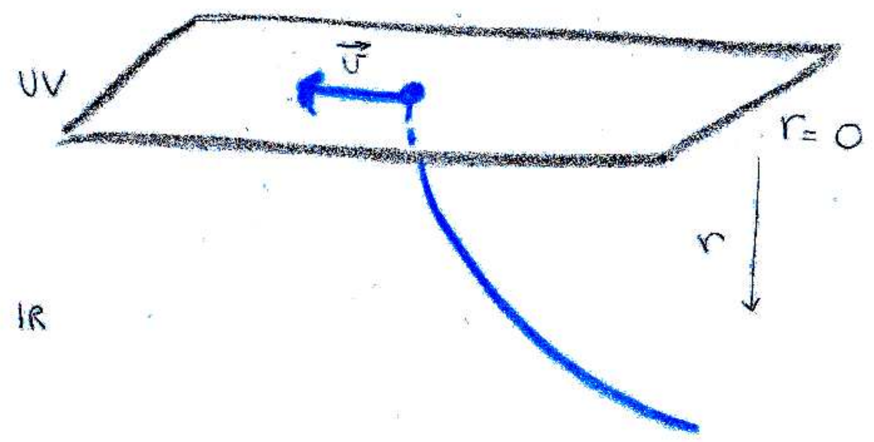
- Interesting and unexpected properties of the vacuum dynamics
  - emergent temperature
  - dissipation effects in vacuum propagation
  - These effects are the result of entanglement of a system at large-N

The Trailing String



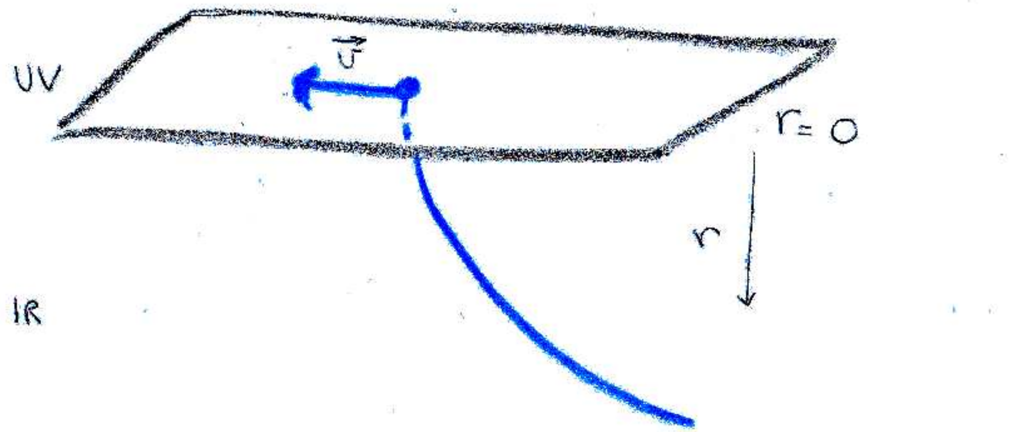
Probe quark on the boundary a 5D asymptotically AdS spacetime

$$\Updownarrow$$

Classical string attached at the boundary and extending in the interior.

(Gubser ’06)

The Trailing String

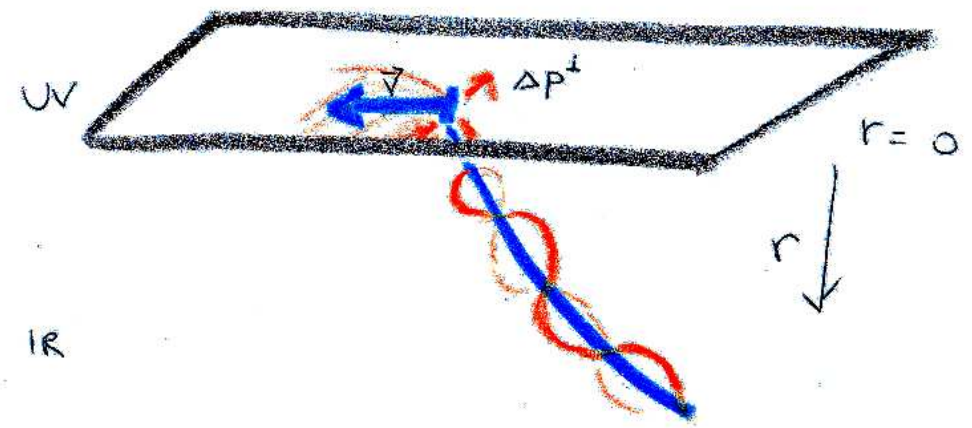


The string profile is found by extremizing the surface spanned by the string

$$S = \frac{1}{2\pi\ell_s^2}\int dtdr\sqrt{-\det g_{ind}}\,,$$

with respect to the embedding coordinates: $\vec{X}(t,r) = \vec{v}t + \vec{\xi}(r)$.
The string exerts a drag force which causes the quark to lose energy:
dual description of in-medium energy loss

The Trailing String Fluctuations

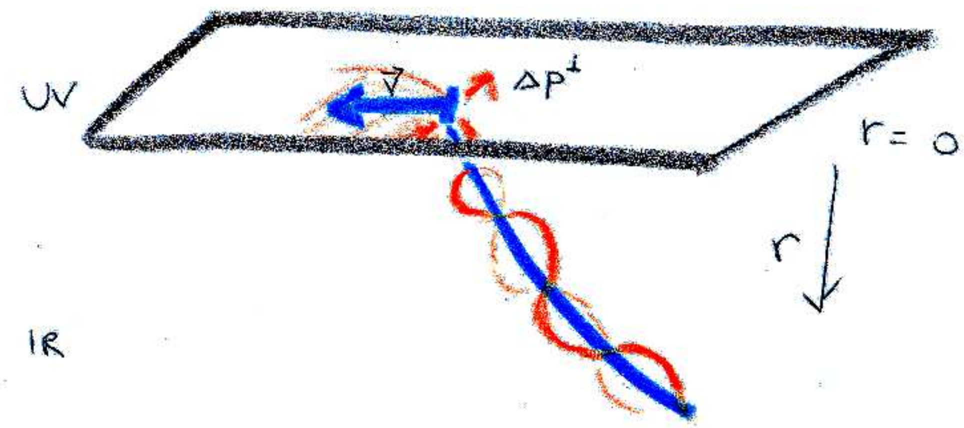


Add small fluctuations along the string:

$$X(t,r) = \xi(r) + \delta X(t,r)$$

they induce a Brownian-like dynamics for the boundary quark, governed by a Langevin equation, and leading to a spread in momentum. (Gubser '05, De Boer *et al* 06, Herzog *et al* 06, Son and Teaney 09) .
Dual description of transverse momentum broadening.

The Trailing String Fluctuations



$$\dot{P}(t) + \int dt'\, \Gamma_R(t-t')P(t') = \zeta(t), \qquad \langle \zeta(t)\zeta(t') \rangle = G_s(t-t')$$

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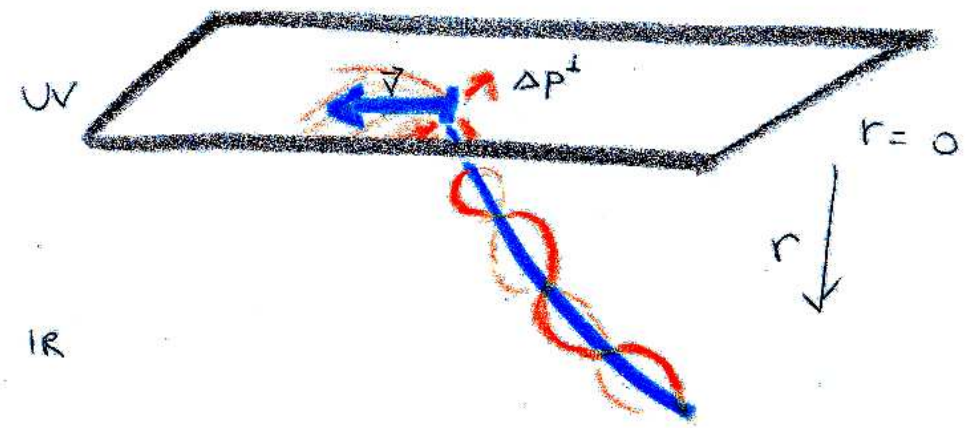

$$\dot{P}(t) + \int dt'\, \Gamma_R(t-t')P(t') = \zeta(t), \qquad \langle \zeta(t)\zeta(t') \rangle = G_s(t-t')$$

- $G_R(t) = \dot{\Gamma}_R(t)$ is the retarded boundary correlator associated to the fluctuations $\delta X(t,r)$ around the classical trailing string.
- $G_s(t)$ is the associated by the symmetric correlator, obtained from G_R via a Fluctuation-Dissipation relation, characteristic of the ensemble.

Trailing string in 5D black hole

Consider a generic asymptotically AdS 5D black hole:

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right]$$

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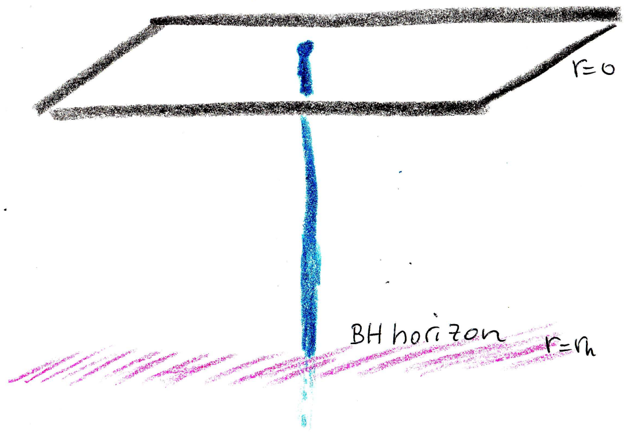
- Dual to a non-conformal gauge theory in thermal equilbrium at a temperature T_h, in a deconfined phase.

Trailing String in a 5D black hole

Let us focus on the static case.



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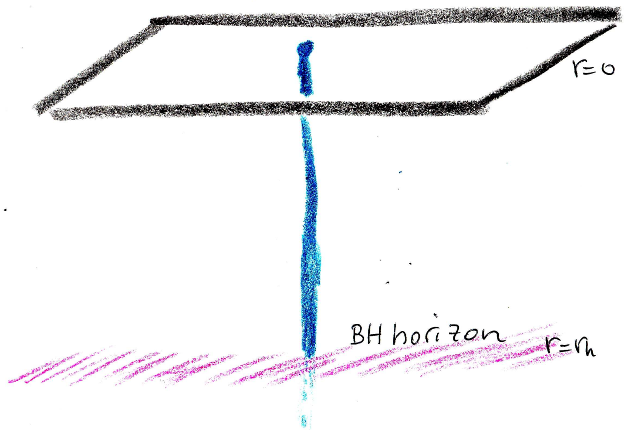
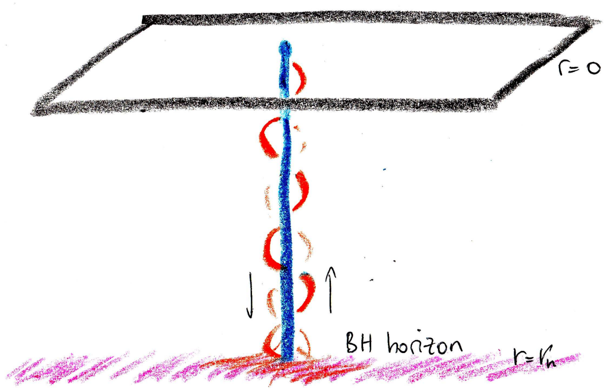


The string falls straight down into the horizon.
The induced 2D worldsheet metric is a 2D black hole with horizon r_h and temperature T_h.

$$ds^2_{ind} = b^2(r)\left[-f(r)dt^2 + f^{-1}(r)dr^2\right]$$

Fluctuations

Add fluctuations: modes going in/out of the horizon.



The fluctuation equation close to the horizon is

$$\delta X'' - \frac{1}{(r_h - r)}\delta X' + \frac{\hat{\omega}^2}{(r_h - r)^2}\delta X = 0, \qquad \hat{\omega} \equiv \frac{\omega}{4\pi T_h}$$

The solutions have infalling/outgoing behavior near r_h,

$$\delta X(\omega, r) \simeq (r_h - r)^{\pm i\hat{\omega}}$$

Correlators

The retarded correlator is found by the Policastro-Son-Starinetz prescription

$$G_R(\omega) = \left[\mathcal{G}(r)\,\delta X'_R(\omega, r)\right]_{r\to 0}, \qquad \delta X_R(\omega, r) \to \begin{cases} 1 & r \to 0 \\ (r - r_h)^{-i\hat{\omega}} & r \to r_h \end{cases}$$

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Two qualitatively very different cases:

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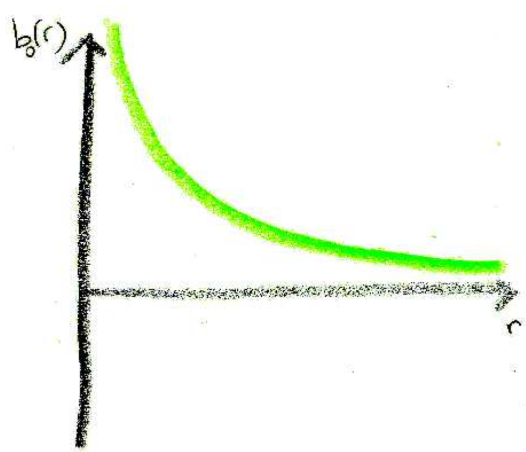
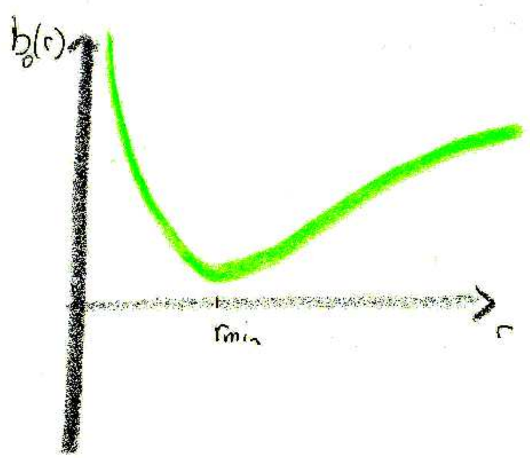
Confinement is essentially equivalent to the presence of a minimum of the bulk scale factor $b(r)$ (cfr. holographic Wilson Loop)





non-confining

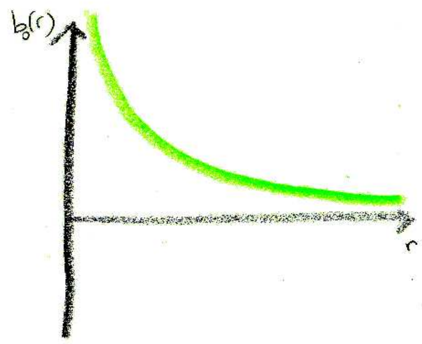
confining $\sigma_c = b^2(r_m)$

Non-confining case

$$ds^2 = b^2(r)\left[dr^2 + dx^\mu dx_\mu\right],$$

the string profile satisfies:

$$\xi'(r) = \frac{C}{\sqrt{b^4(r) - C^2}}$$



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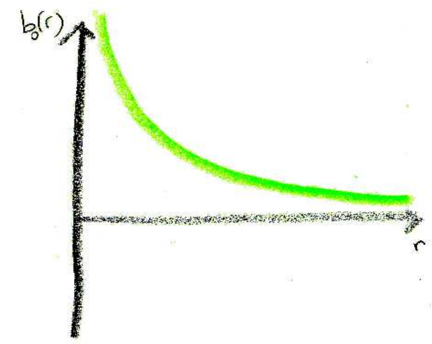
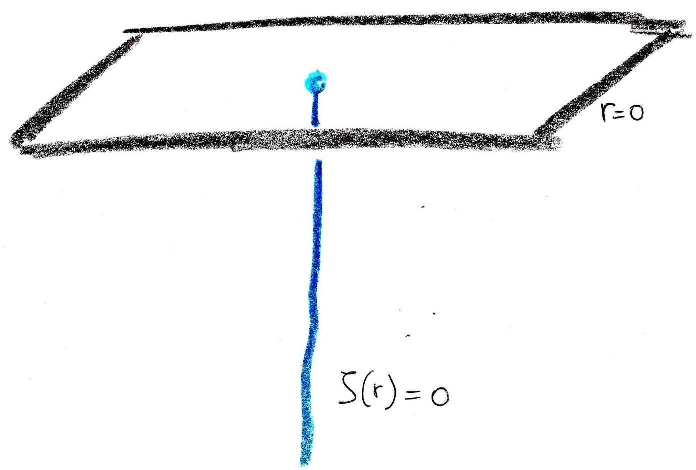


As $b \to 0$, regularity requires $C = 0$: the embedding is trivial, $\xi = 0$



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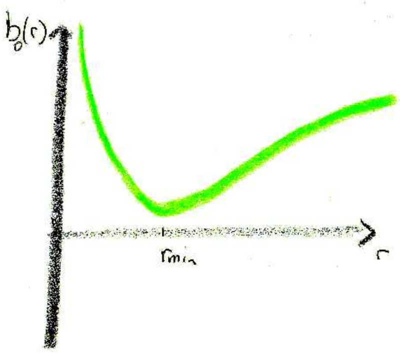

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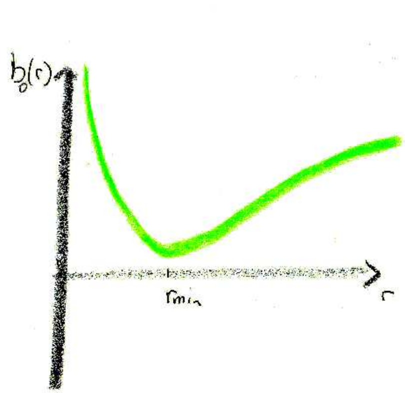

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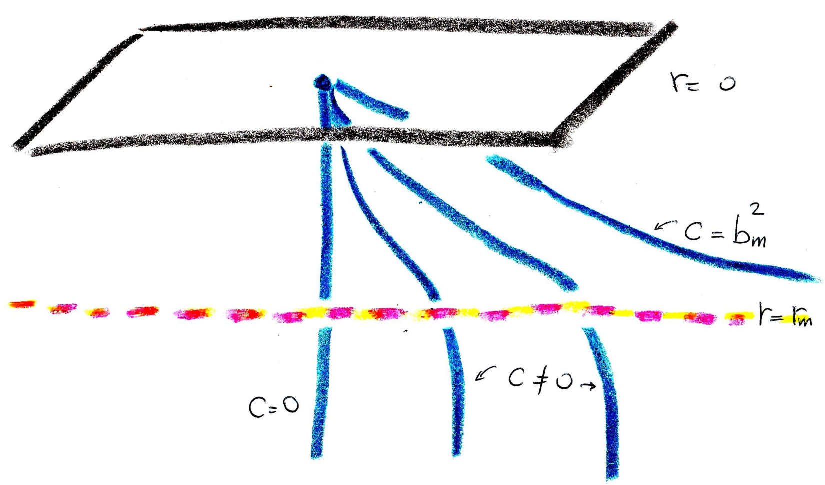
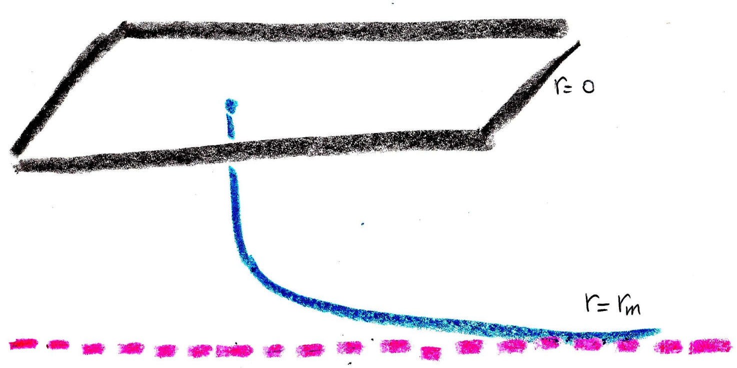
One-parameter family of solutions with $0 \leq C \leq b^2(r_m)$



Confining Trailing String

The extremal $C = b_m^2$ string is the one with lowest action. It does not extend beyond the *confining horizon* $r = r_m$, and it extend to infinity along one of the spatial directions.



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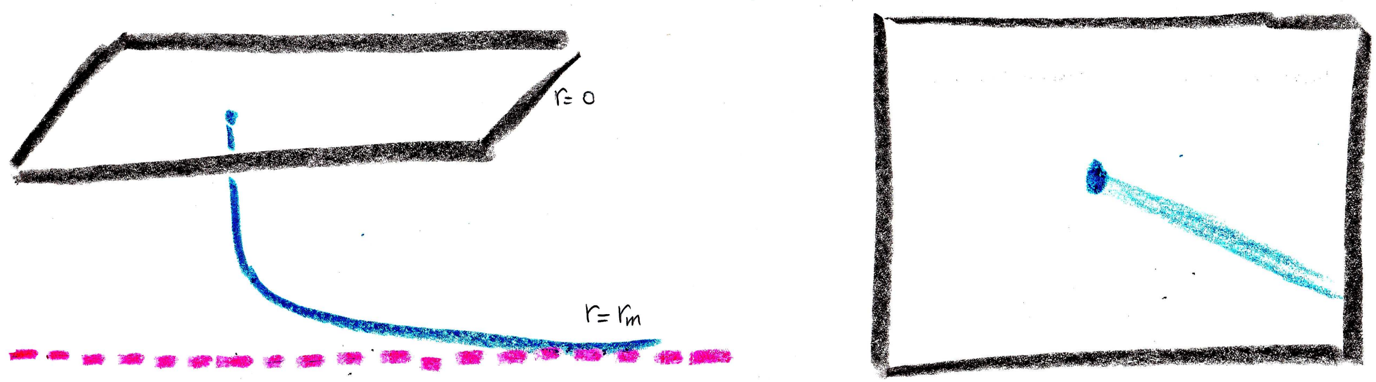

Asymptotically it looks like a straight string with fixed tension b_m^2 i.e. the confining string tension of the dual theory: it is the QCD flux-tube.

Confining string geometry

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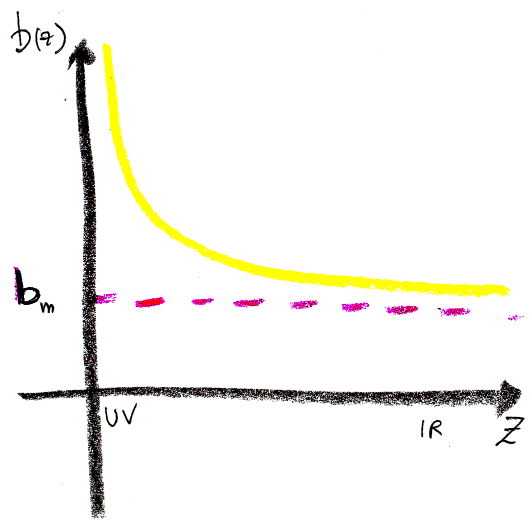

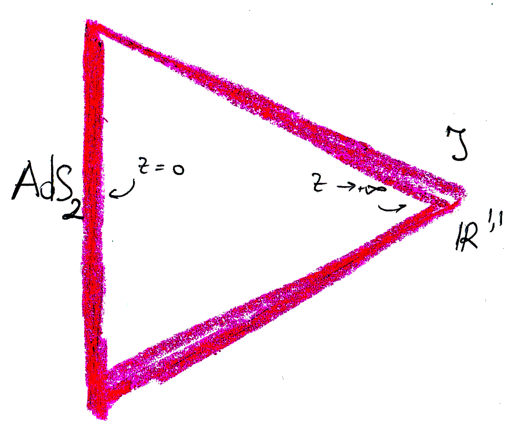

Emergent Temperature

The equation for transverse fluctuations close to r_m is:

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This is *not* just an analogy: one can show, using the method of the horizon action at r_m (Caron-Huot, Chesler, Teaney 2011) that the fluctuation-dissipation relation is thermal at the temperature T_m, provided we define the retarded correlator by choosing ingoing waves at infinity:

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Emergence of an *effective temperature set by the confinement scale*.

Dissipation at zero temperature

The boundary correlator at small frequency starts as:

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The confining vacuum is dissipative for a single quark and the dissipation time scale is again set by the confinement scale.

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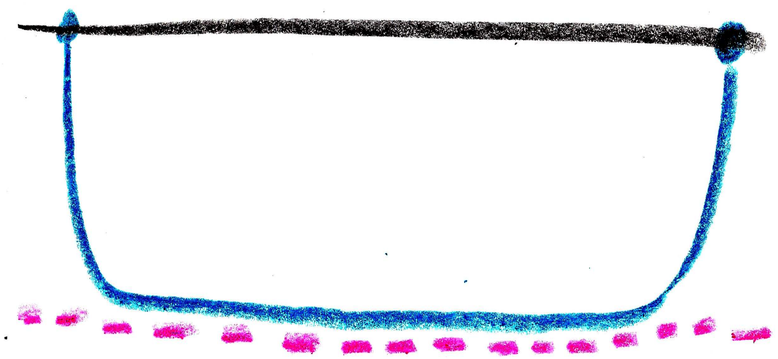

look at the trailing string as half of the confining string connecting two quarks, one of which is observed, the other (shadow quark) infinitely far.

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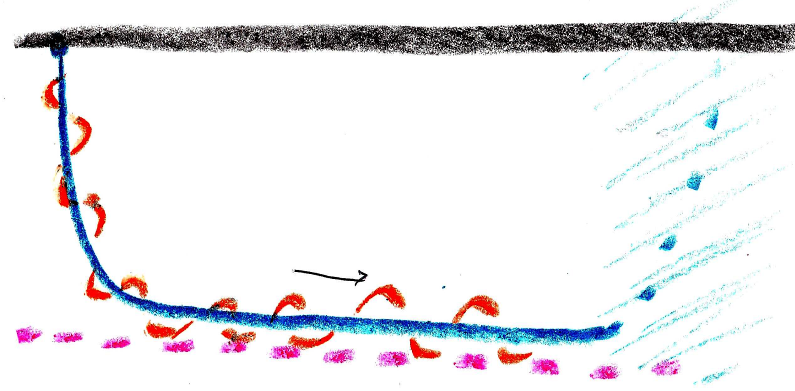

All calculation done on a single (observed) quark should be done by assuming that no information is available or comes from the shadow quark. E.g. the infalling wave condition at $z \longrightarrow \infty$

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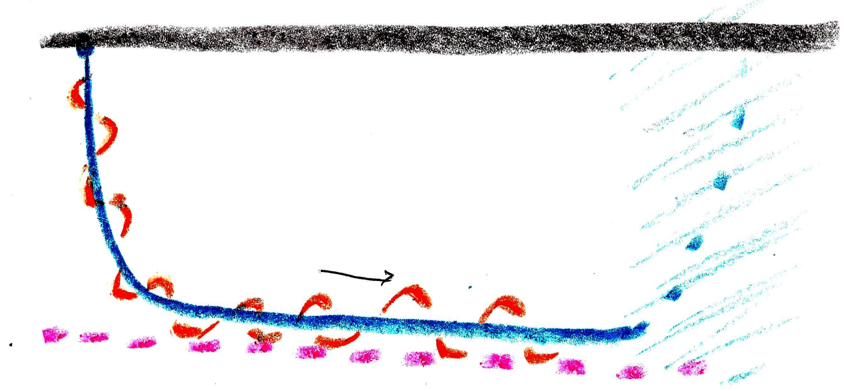

The ensemble nature of the correlators arises by tracing over the hidden quark d.o.f. The surprising fact that this ensemble is thermal is probably due to large-N.

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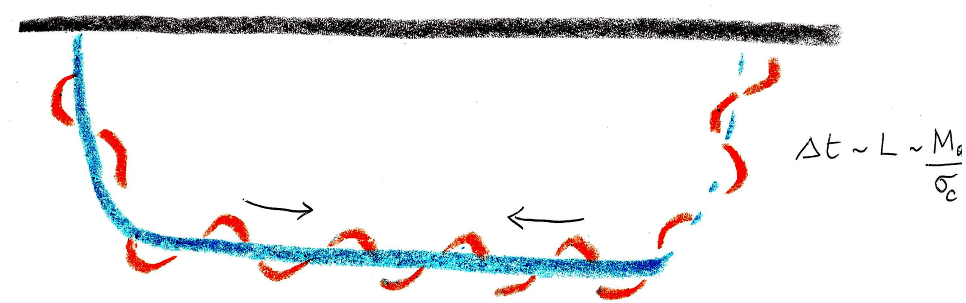

In practice, for any finite string, sooner or later information will come back and the system will become non-dissipative: a finite length of the string destroys the small-ω linear term in ImG_R. A finite quark mass M will introduce an IR cutoff to the string length.

Conclusions and open questions

- The trailing string picture of a single quark in a confining holographic theory displays some non-trivial and surpsing dynamics:
  - From a statistical mechanics standpoint: an emergent thermal ensemble and dissipation over long-times in the vacuum.
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  - From a GR standpoint: a thermal ensemble NOT associated to a horizon.
- Are there other examples of these phenomena ?
- Can these features be understood in terms of the *shadow quark* picture in a 4D perspective?

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  - From a statistical mechanics standpoint: an emergent thermal ensemble and dissipation over long-times in the vacuum.
  - From a GR standpoint: a thermal ensemble NOT associated to a horizon.
- Are there other examples of these phenomena ?
- Can these features be understood in terms of the *shadow quark* picture in a 4D perspective?

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One can use the resulting correlators as originally planned, for a subtraction of the finite-temperature correlators.