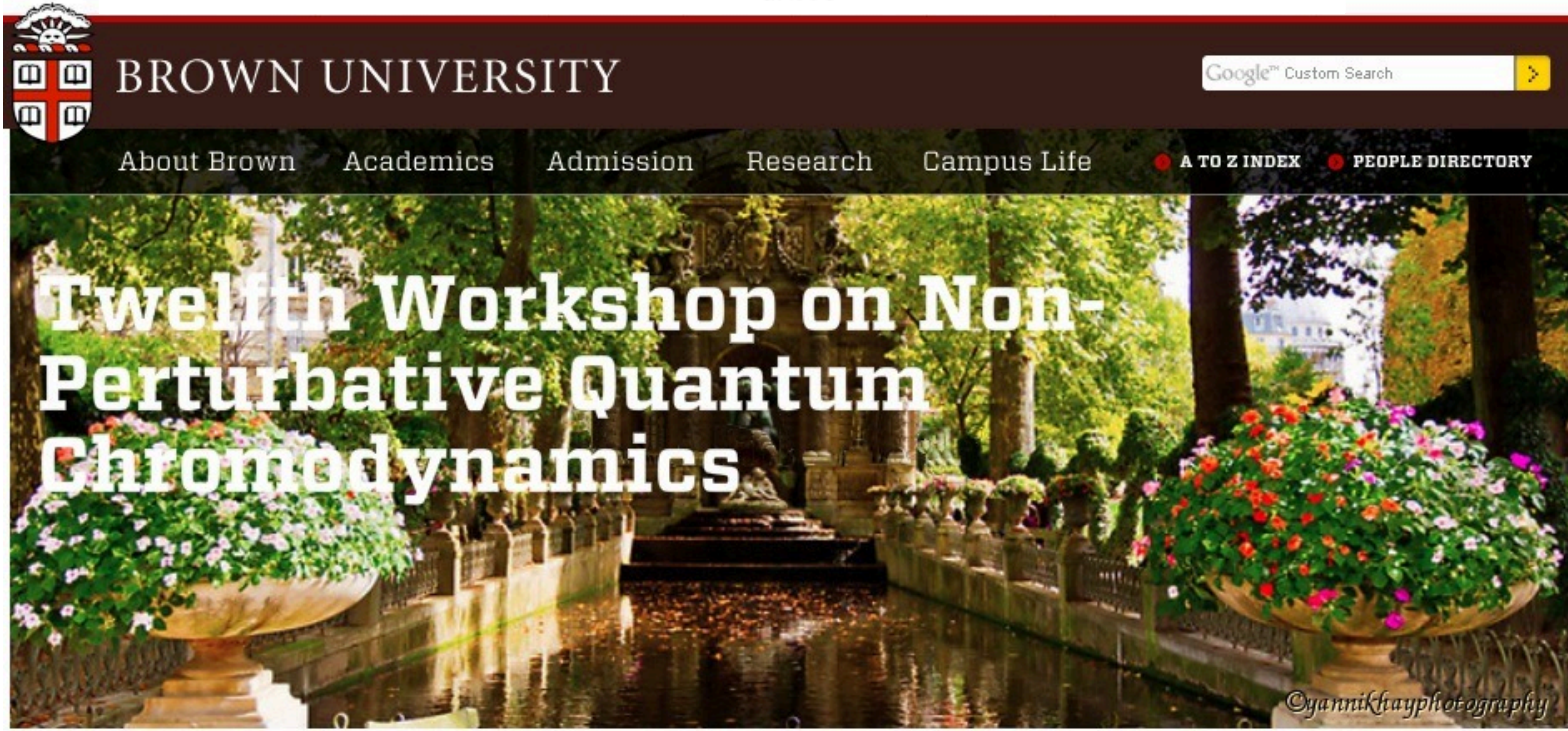
The Quantum Measurement Problem in Cosmology

Paris
June 13th, 2013

Patrick Peter
Institut d'Astrophysique de Paris
GReCO



Twelfth Workshop on Non-Perturbative QCD, Paris - June 13th, 2013

The Quantum Measurement Problem in Cosmology

Paris
June 13th, 2013

Patrick Peter
Institut d'Astrophysique de Paris
GReCO

J. Martin, V. Vennin and P. P., *Phys. Rev.* **D86**, 103524 (2012) [arXiv:1207.2086]

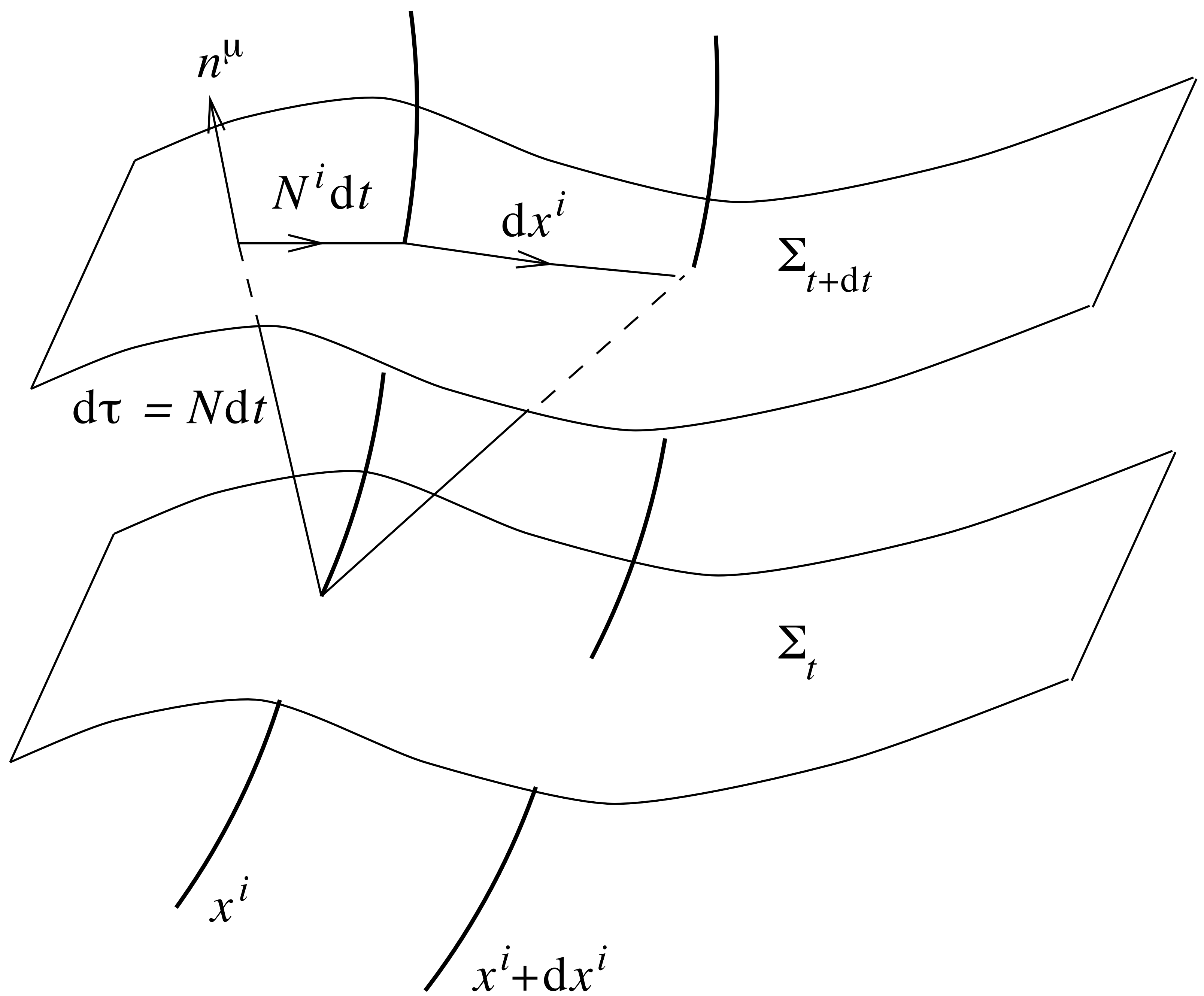
+ collaborations with N. Pinto-Neto & A. Valentini (2001...)

The Universe as a closed quantum system: Quantum cosmology

- Hamiltonian GR

The Universe as a closed quantum system: Quantum cosmology

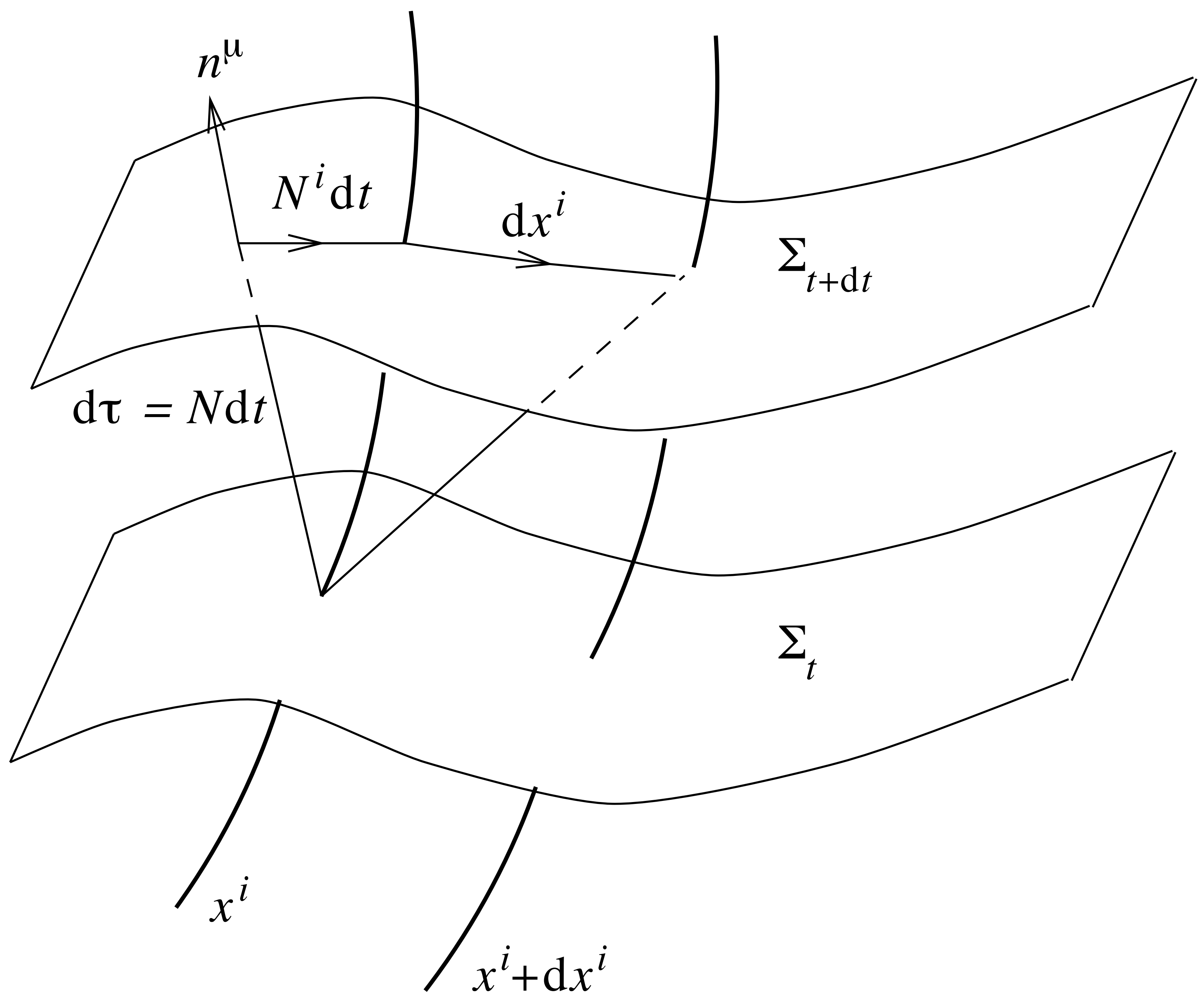
- Hamiltonian GR



The Universe as a closed quantum system: Quantum cosmology

- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

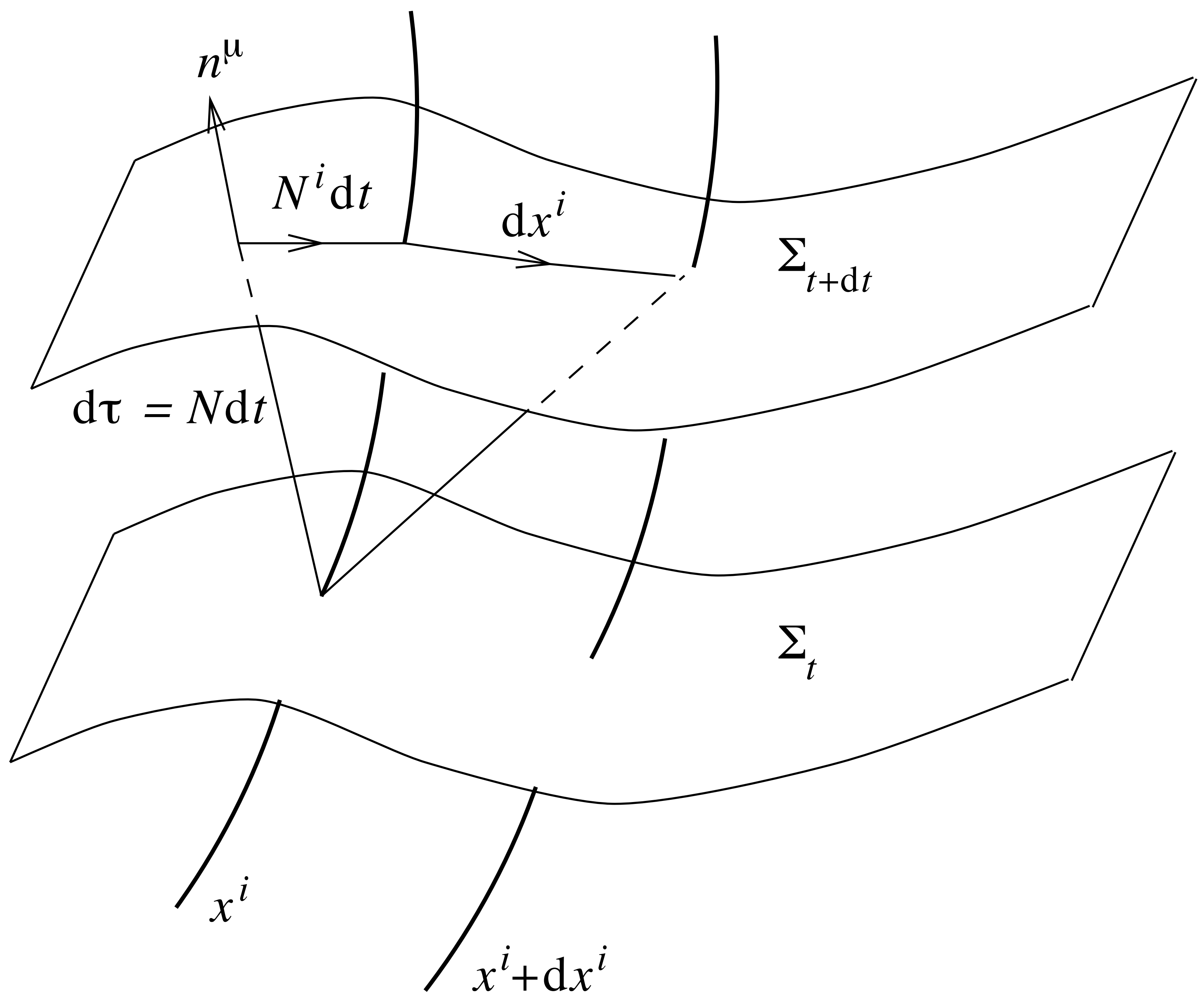


The Universe as a closed quantum system: Quantum cosmology

- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Lapse function



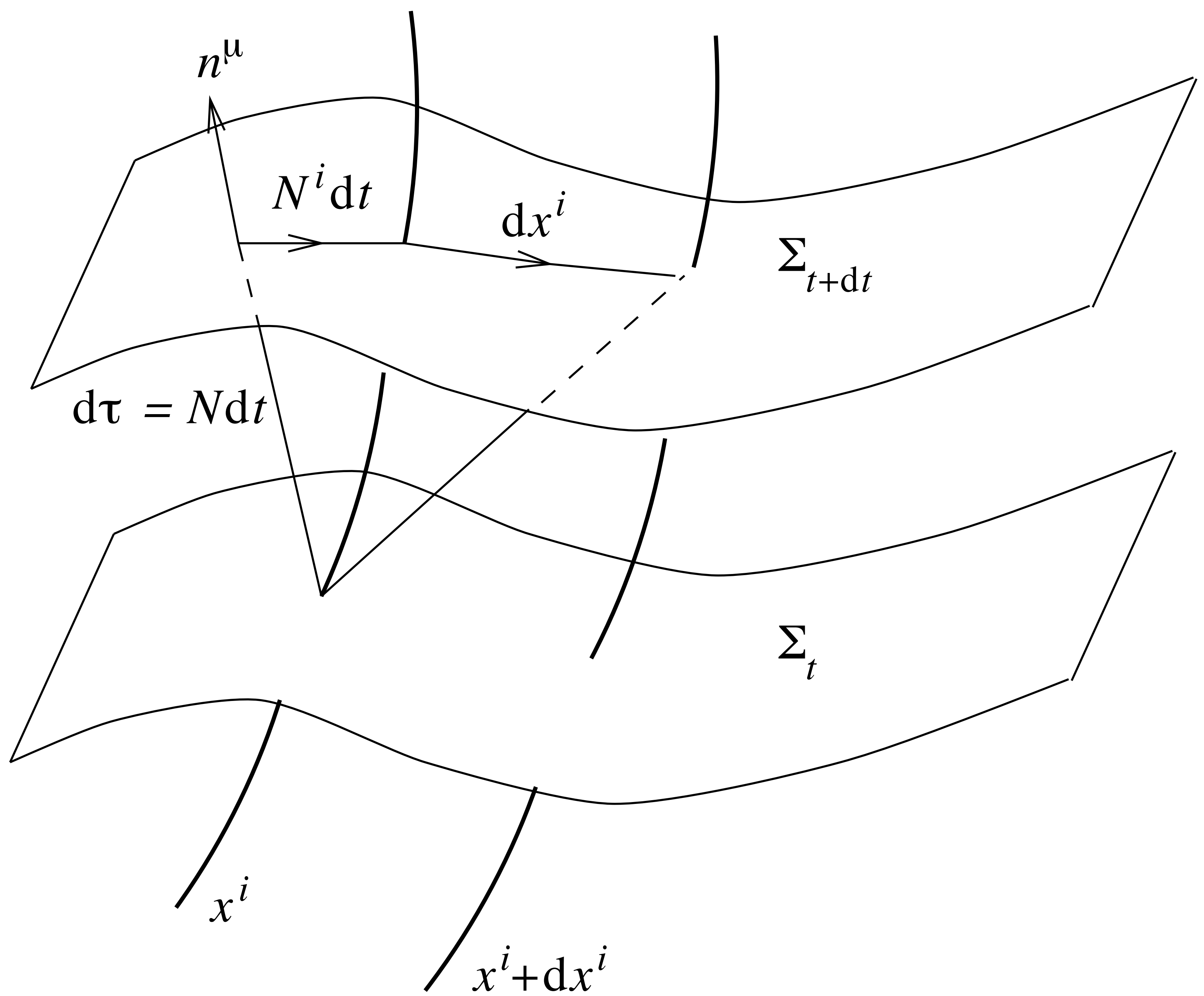
The Universe as a closed quantum system: Quantum cosmology

- Hamiltonian GR

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\mathcal{N}^2 dt^2 + h_{ij}\left(dx^i + \mathcal{N}^i dt\right)\left(dx^j + \mathcal{N}^j dt\right)$$

Lapse function

Intrinsic metric
= first fundamental form



The Universe as a closed quantum system: Quantum cosmology

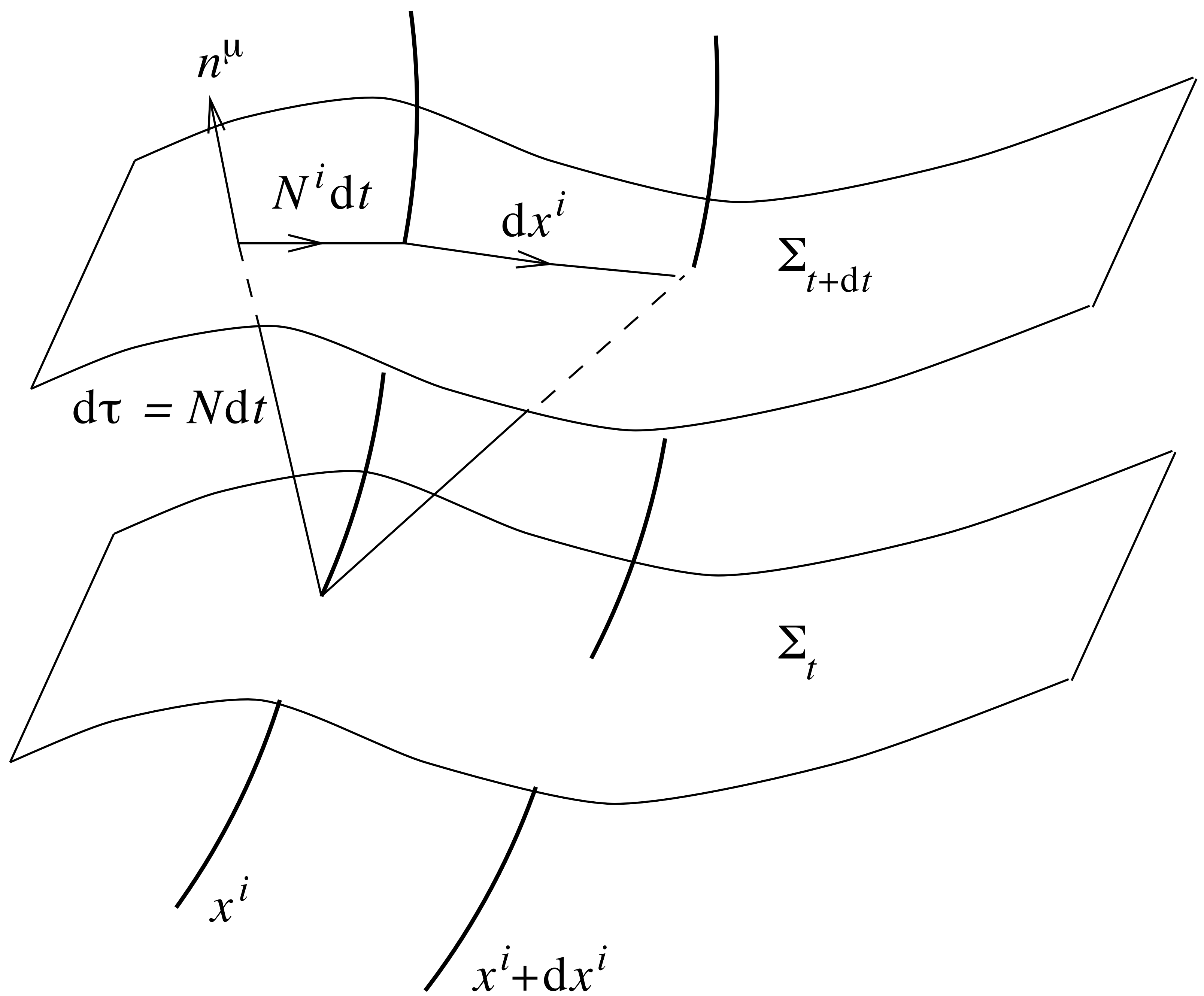
- Hamiltonian GR

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\mathcal{N}^2 dt^2 + h_{ij}\left(dx^i + \mathcal{N}^i dt\right)\left(dx^j + \mathcal{N}^j dt\right)$$

Shift vector

Lapse function

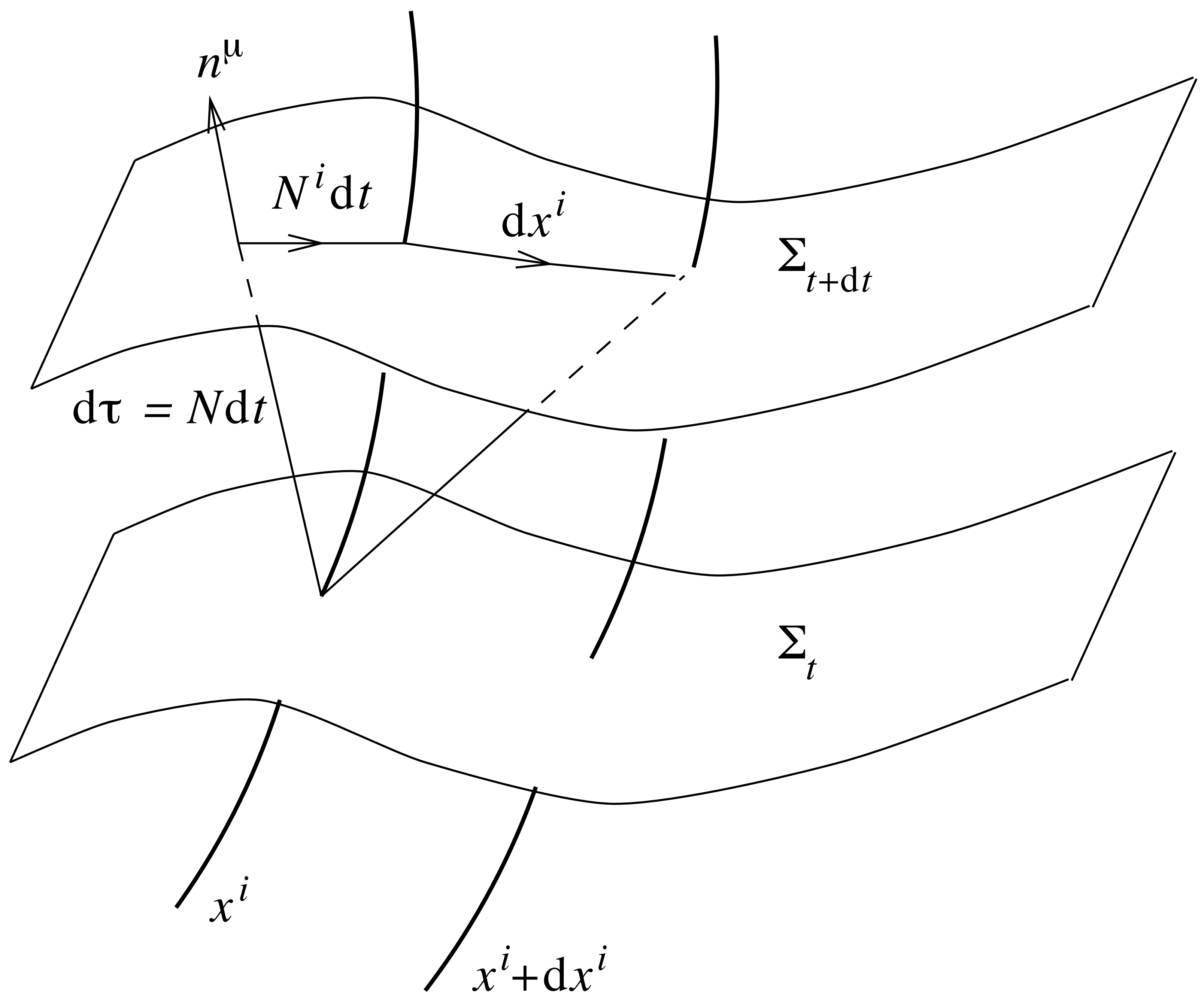
Intrinsic metric
= first fundamental form



The Universe as a closed quantum system: Quantum cosmology

- Hamiltonian GR

$$ds^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$



Lapse function

Intrinsic metric
= first fundamental form

Shift vector

Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

The Universe as a closed quantum system: Quantum cosmology

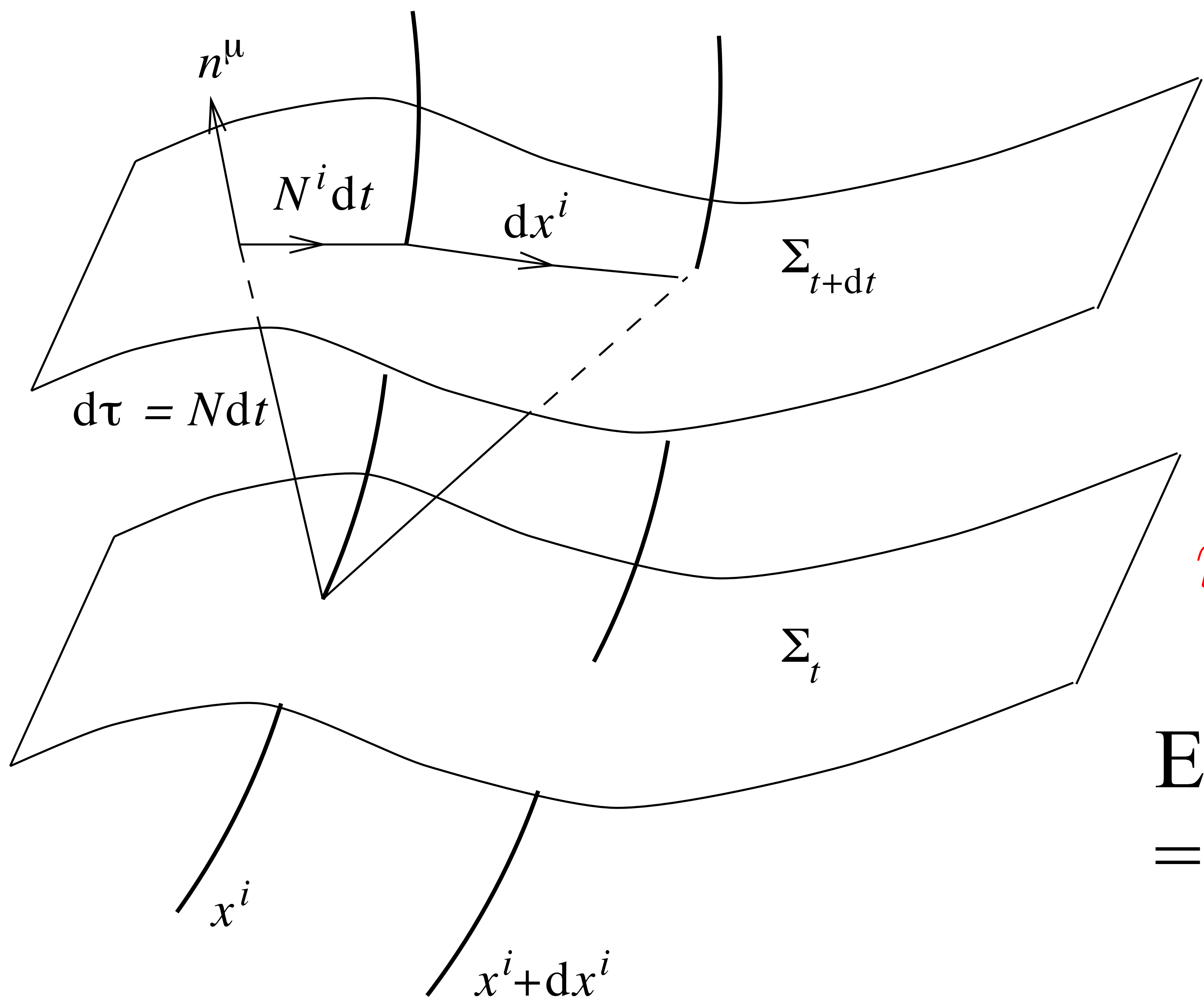
- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Shift vector

Lapse function

Intrinsic metric
= first fundamental form



n^μ Normal to Σ_t

Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

Extrinsic curvature
= second fundamental form

$$K_{ij} \equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0 = \frac{1}{2\mathcal{N}}\left(\nabla_j \mathcal{N}_i + \nabla_i \mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right)$$

The Universe as a closed quantum system: Quantum cosmology

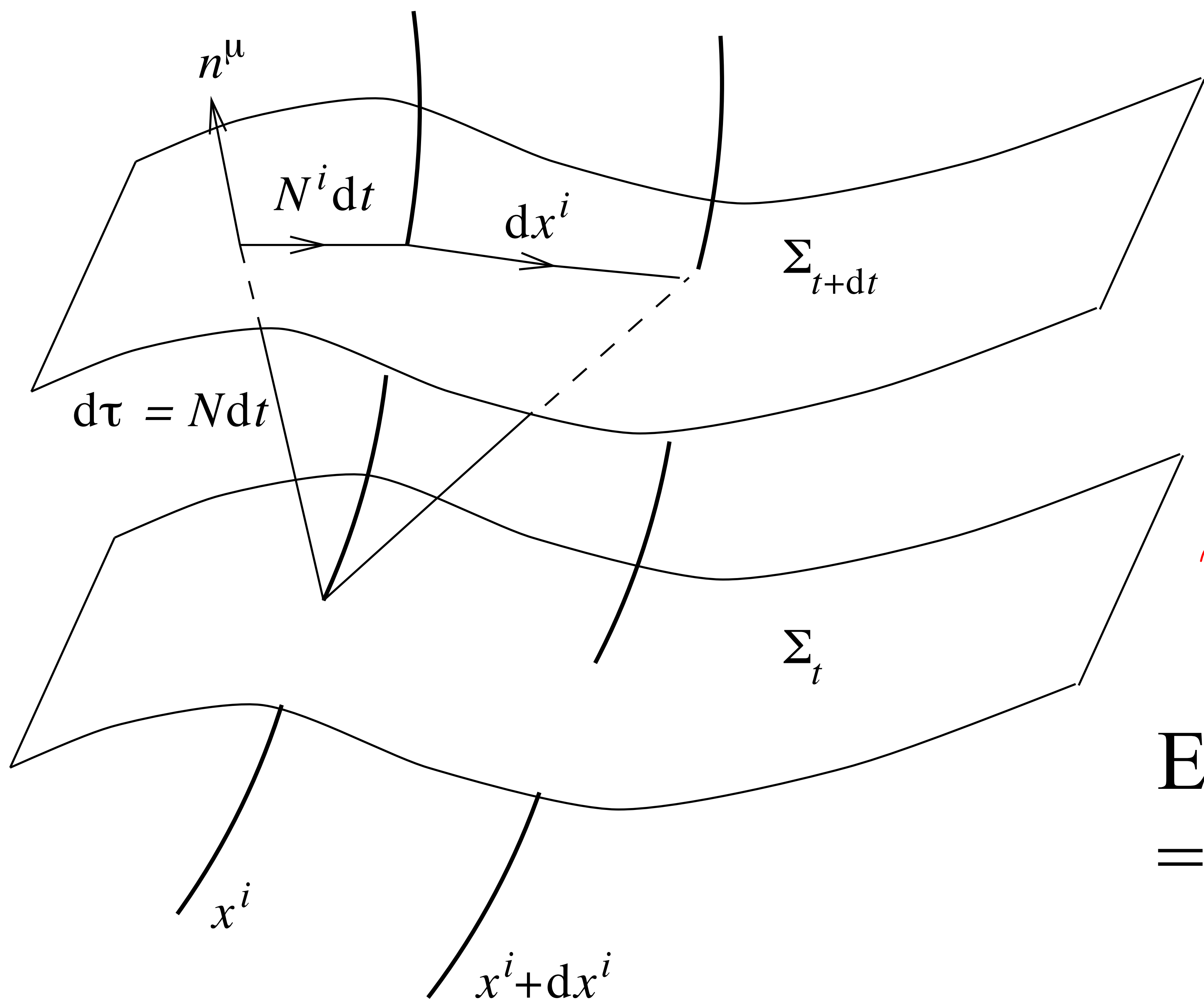
- Hamiltonian GR

$$ds^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Shift vector

Lapse function

Intrinsic metric
= first fundamental form



n^μ Normal to Σ_t

Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

Extrinsic curvature
= second fundamental form

$$K_{ij} \equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0 = \frac{1}{2\mathcal{N}}\left(\nabla_j\mathcal{N}_i + \nabla_i\mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action: $$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}}\left[\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K^i{}_i\right] + \mathcal{S}_{\mathrm{matter}}$$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(K^{ij} - h^{ij}K\right)$$

$$\pi_{\Phi} \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}}\left(\dot{\Phi} - \mathcal{N}^i \frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0 \end{aligned}\right\}$$ Primary constraints

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\rm N}} \left(K^{ij} - h^{ij} K\right)$$

$$\pi_\Phi \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}} \left(\dot{\Phi} - \mathcal{N}^i \frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0 \end{aligned}\right\} \text{Primary constraints}$$

Hamiltonian $H \equiv \int \mathrm{d}^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \pi^{ij} \dot{h}_{ij} + \pi_\Phi \dot{\Phi}\right) - L = \int \mathrm{d}^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \mathcal{N}\mathcal{H} + \mathcal{N}_i \mathcal{H}^i\right)$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\rm N}}\left(K^{ij} - h^{ij}K\right)$$

$$\pi_\Phi \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}}\left(\dot{\Phi} - \mathcal{N}^i \frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0 \end{aligned}\right\} \text{Primary constraints}$$

↓

Hamiltonian $H \equiv \int \mathrm{d}^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \pi^{ij}\dot{h}_{ij} + \pi_\Phi \dot{\Phi}\right) - L = \int \mathrm{d}^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \mathcal{N}\mathcal{H} + \mathcal{N}_i \mathcal{H}^i\right)$

Variation wrt lapse $\mathcal{H} = 0$ Hamiltonian constraint

Variation wrt shift $\mathcal{H}^i = 0$ momentum constraint

} Secondary constraints

$\Longrightarrow$ Classical description

• Superspace & canonical quantisation

Relevant configuration space?

matter fields

$$\mathrm{Riem}(\Sigma) \equiv \left\{ h_{ij}\,(x^\mu), \Phi\,(x^\mu) \;\; | \; x \in \Sigma \right\}$$

parameters

GR $\Longrightarrow$ invariance / diffeomorphisms $\Longrightarrow$ $\mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$ superspace

Wave functional $\Psi\left[h_{ij}(x), \Phi(x)\right]$

Dirac canonical quantisation

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}} \qquad \pi_\Phi \to -i\frac{\delta}{\delta \Phi} \qquad \pi^0 \to -i\frac{\delta}{\delta \mathcal{N}} \qquad \pi^i \to -i\frac{\delta}{\delta \mathcal{N}_i}$$

Primary constraints

$$\hat{\pi}\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}} = 0$$

$$\hat{\pi}^i\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}_i} = 0$$

Momentum constraint $\hat{\mathcal{N}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\frac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\rm N}\hat{T}^{0i}\Psi$

Primary constraints

$$\hat{\pi}\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}} = 0$$

$$\hat{\pi}^i\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}_i} = 0$$

Momentum constraint $\hat{\mathcal{N}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\frac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\rm N}\hat{T}^{0i}\Psi$

Hamiltonian constraint

$$\hat{\mathcal{H}}\Psi = \left[-16\pi G_{\rm N}\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\rm N}}\left(-{}^3R + 2\Lambda + 16\pi G_{\rm N}\hat{T}^{00}\right)\right]\Psi = 0$$

Wheeler - De Witt equation

$$\mathcal{G}_{ijkl} = \frac{1}{2}h^{-1/2}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

DeWitt metric...

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini - superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t), \phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
QM = minisuperspace of QFT

- Minisuperspace

  Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini - superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t), \phi(t)\right]$

However, one can actually make calculations!

Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70)

$$p = p_0 \left[\frac{\dot{\varphi} + \theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi, \theta, s) =$ Velocity potentials

Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70) $\quad p = p_0\left[\frac{\dot{\varphi}+\theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$

$(\varphi, \theta, s) =$ Velocity potentials

canonical transformation: $\quad T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega} \quad$ ...

+ rescaling (volume…) + units… : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$N \rightarrow a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

Wheeler-De Witt $H\Psi = 0$

$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}}$$

space defined by $\chi > 0$ ⟶ constraint $\bar{\Psi}\frac{\partial\Psi}{\partial\chi} = \Psi\frac{\partial\bar{\Psi}}{\partial\chi}$

Wheeler-De Witt $H\Psi = 0$

$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial \Psi}{\partial T} = \frac{1}{4}\frac{\partial^2 \Psi}{\partial \chi^2}}$$

space defined by $\chi > 0$ ⟶ constraint $\bar{\Psi}\dfrac{\partial \Psi}{\partial \chi} = \Psi\dfrac{\partial \bar{\Psi}}{\partial \chi}$

Gaussian wave packet

$$\Rightarrow \quad \Psi = \left[\frac{8T_0}{\pi\left(T_0^2 + T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2 + T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $S = \dfrac{T\chi^2}{T_0^2 + T^2} + \dfrac{1}{2}\arctan\dfrac{T_0}{T} - \dfrac{\pi}{4}$

What do we do with the wave function of the Universe???

What do we do with the wave function of the Universe???

Measurement problem...
worst in a cosmological setup!

Quantum mechanics of closed systems

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution

Wavepacket reduction = non linear / stochastic

Quantum mechanics of closed systems

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution

Wavepacket reduction = non linear / stochastic

+ External observer

Quantum mechanics of closed systems

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

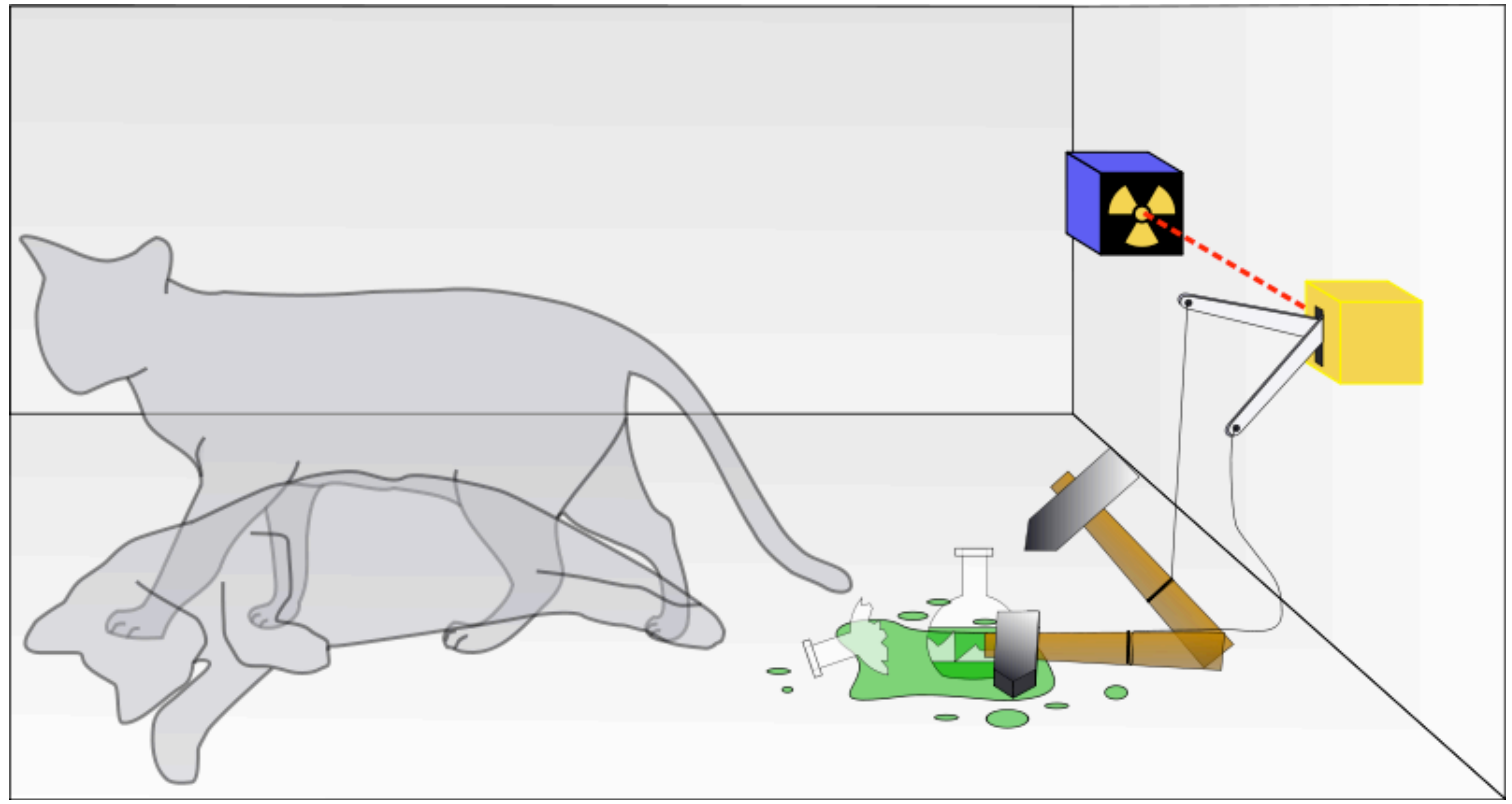
Schrödinger equation = linear (superposition principle) / unitary evolution

Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ External observer

The measurement problem in quantum mechanics

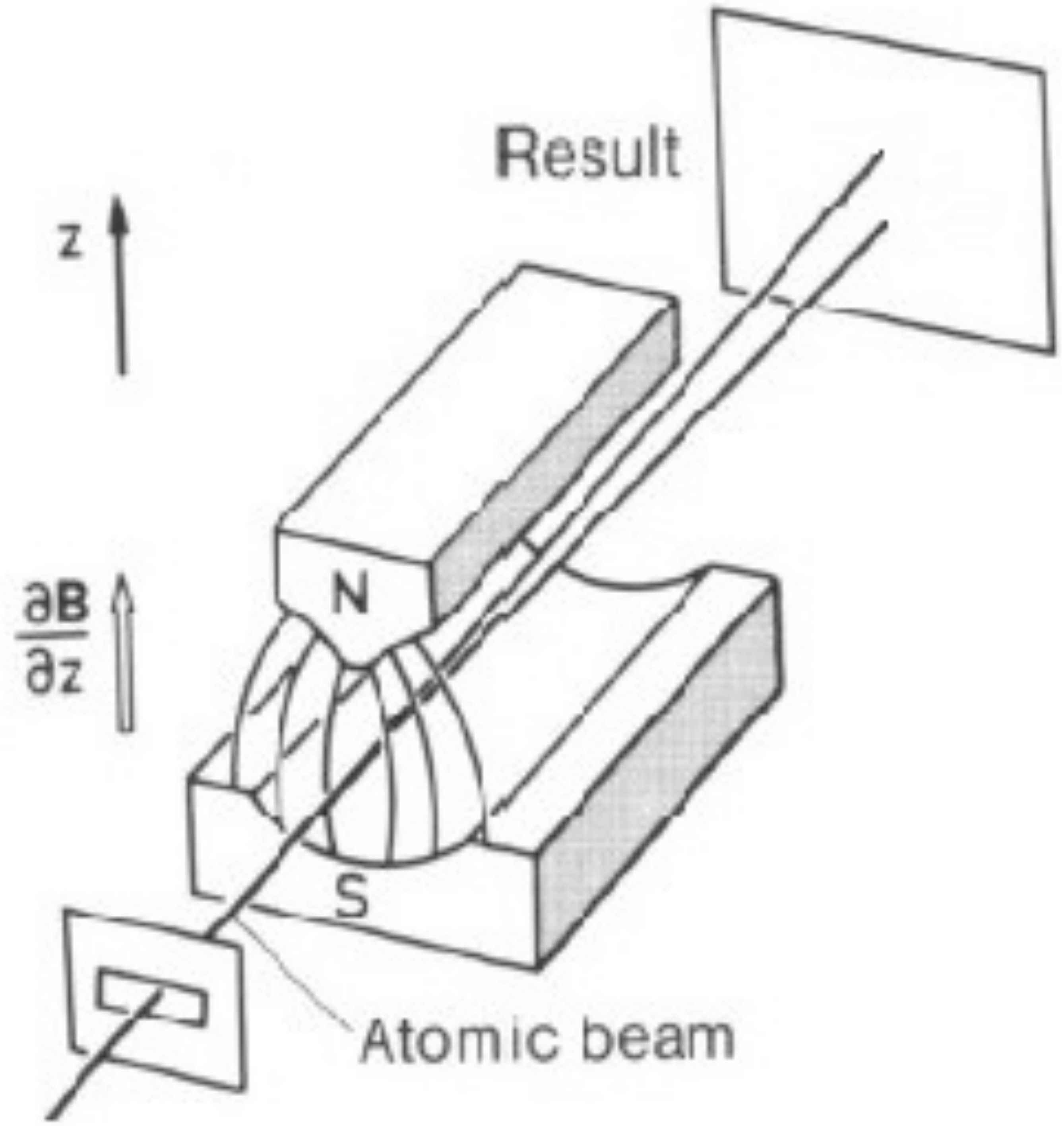


Preferred basis: no unique definition of measured observables

Definite outcome: we don't measure superpositions

→ collapse of the wave function

The measurement problem in quantum mechanics



Stern-Gerlach

pure state

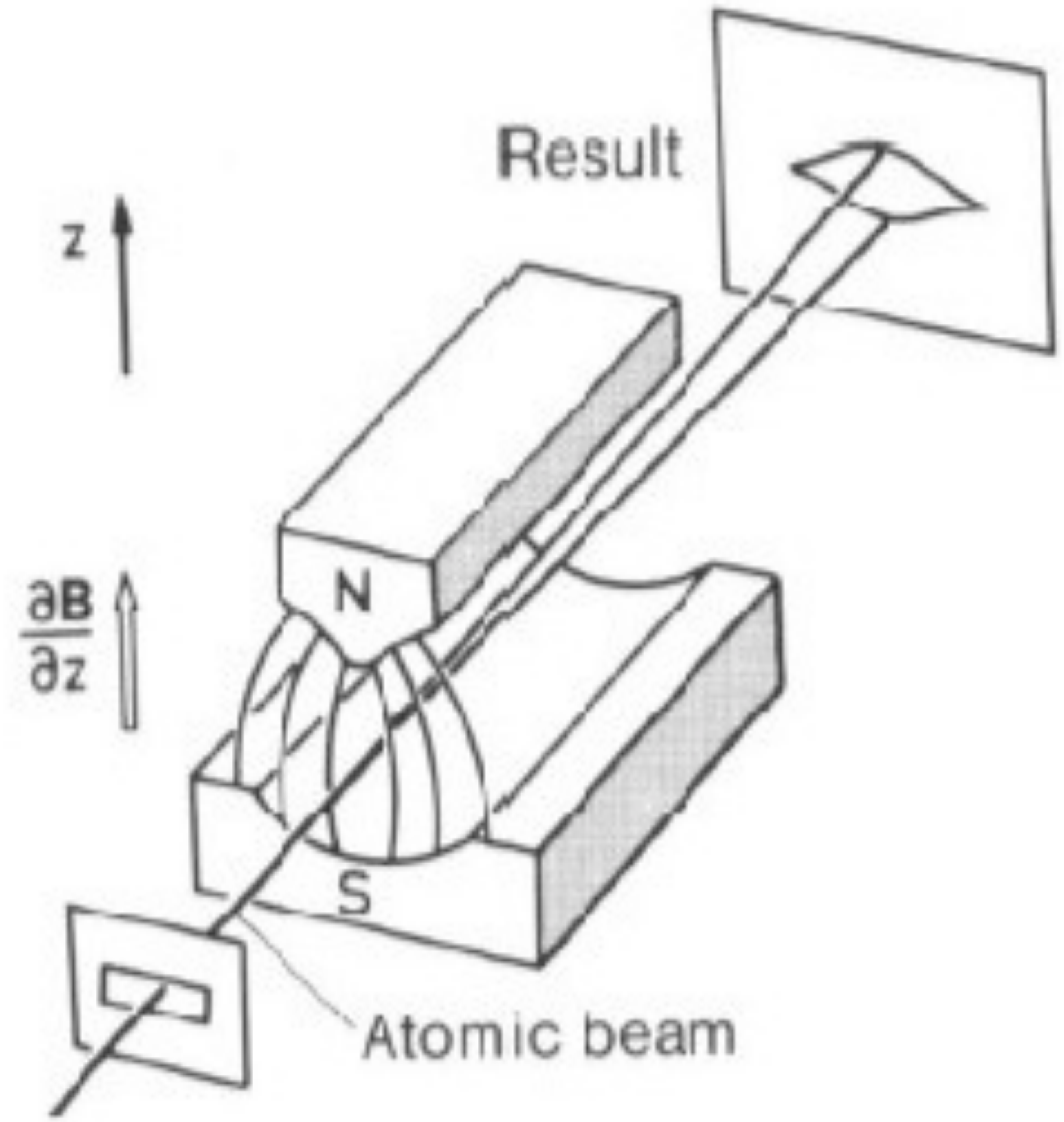
$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}\underbrace{(|\uparrow\rangle + |\downarrow\rangle)}_{}\otimes|\text{SG}_{\text{in}}\rangle$$

Unitary, deterministic Schödinger evolution

$$|\Psi_{\text{f}}\rangle = \exp\left[\int_{t_{\text{in}}}^{t_{\text{f}}}\hat{H}(\tau)\,\text{d}\tau\right]|\Psi_{\text{in}}\rangle$$
$$= \frac{1}{\sqrt{2}}(|\uparrow\rangle\otimes|\text{SG}_{\uparrow}\rangle + |\downarrow\rangle\otimes|\text{SG}_{\downarrow}\rangle)$$

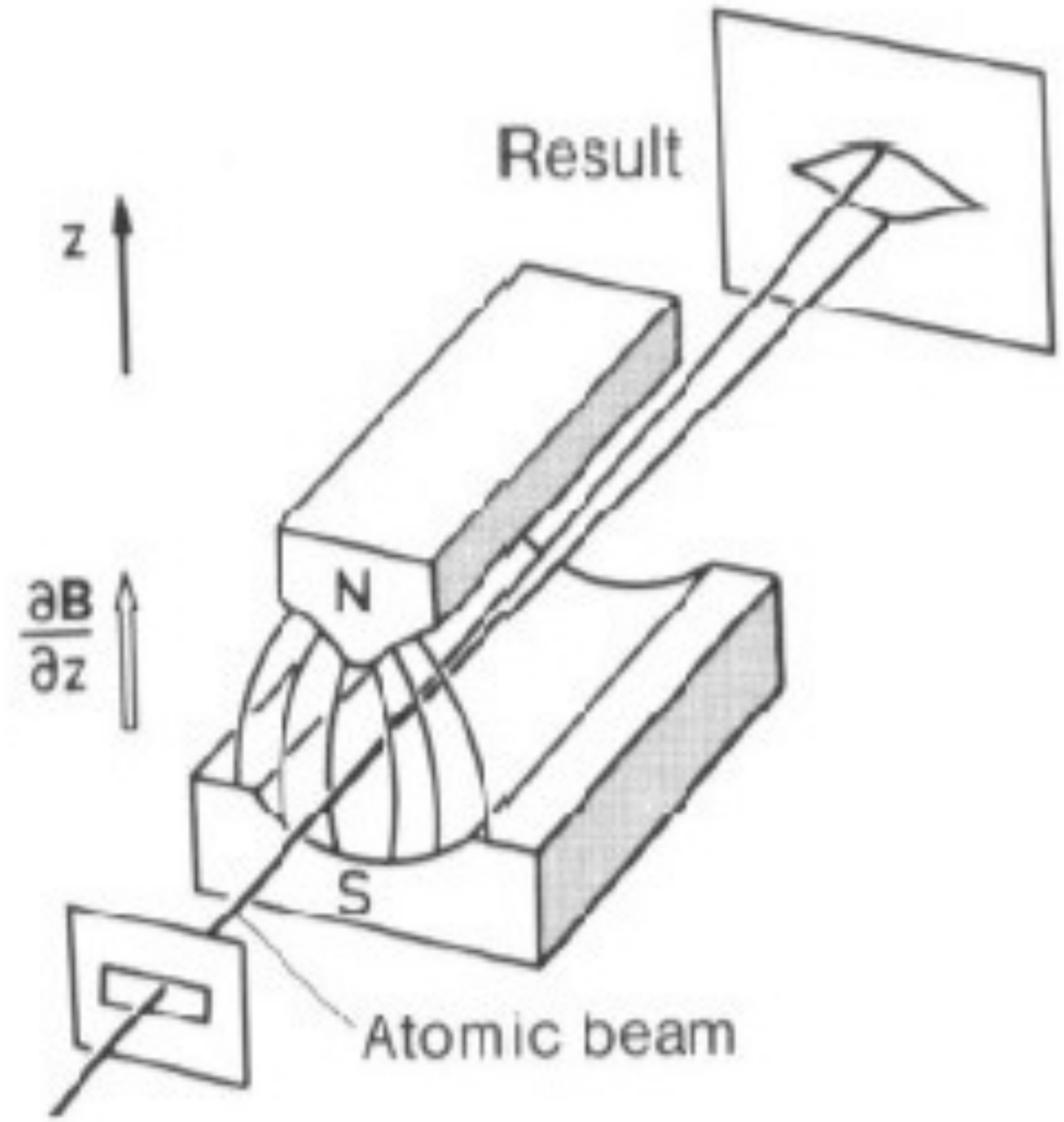
Problem: how to reach the actual measurement $|\uparrow\rangle\otimes|\text{SG}_{\uparrow}\rangle$ or $|\downarrow\rangle\otimes|\text{SG}_{\downarrow}\rangle$?

The measurement problem in quantum mechanics



Stern-Gerlach

The measurement problem in quantum mechanics

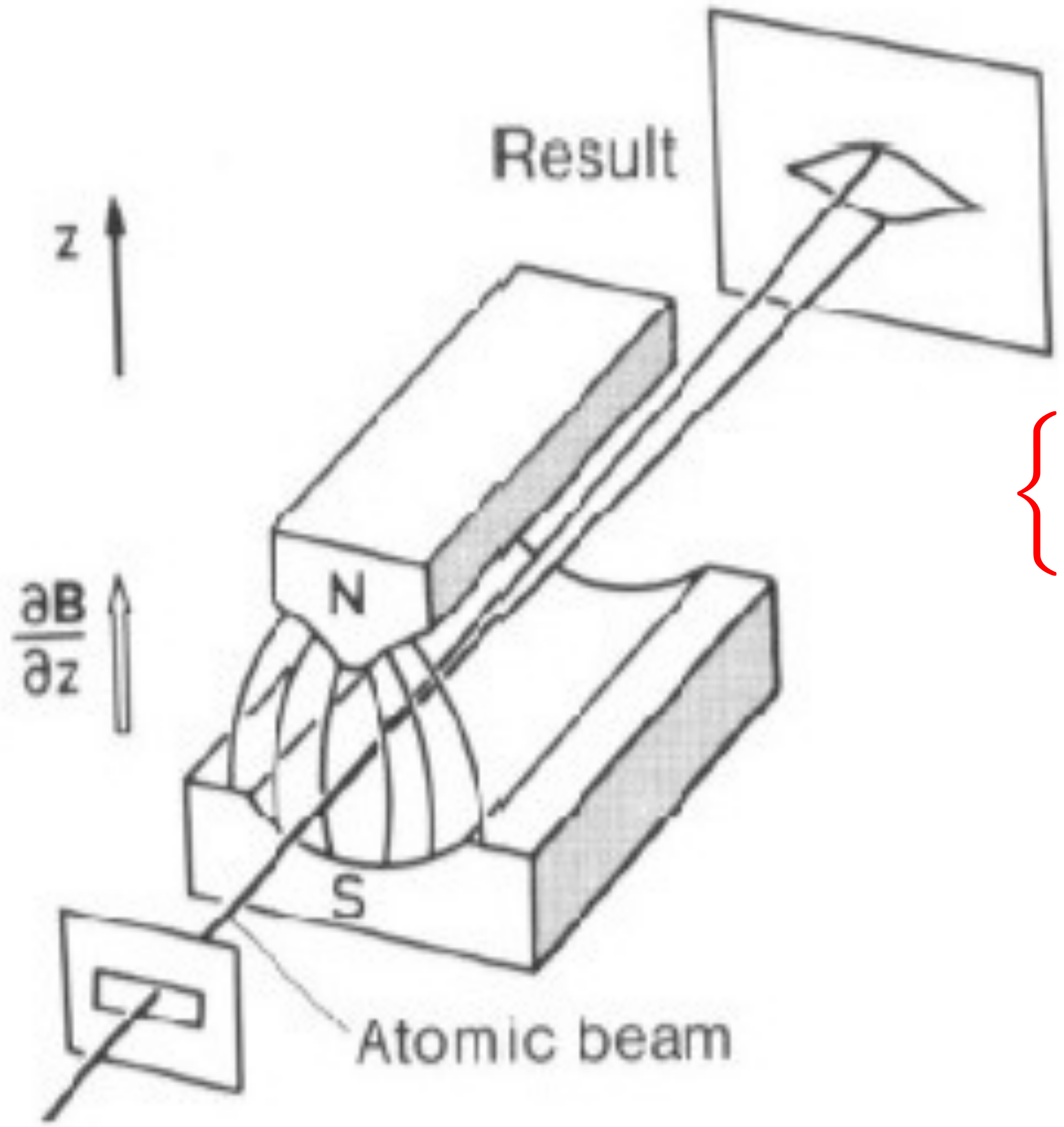


Stern-Gerlach

Statistical mixture

$$\left\{ |\uparrow\rangle \otimes |SG_\uparrow\rangle \right\} \cup \left\{ |\downarrow\rangle \otimes |SG_\downarrow\rangle \right\}$$

The measurement problem in quantum mechanics



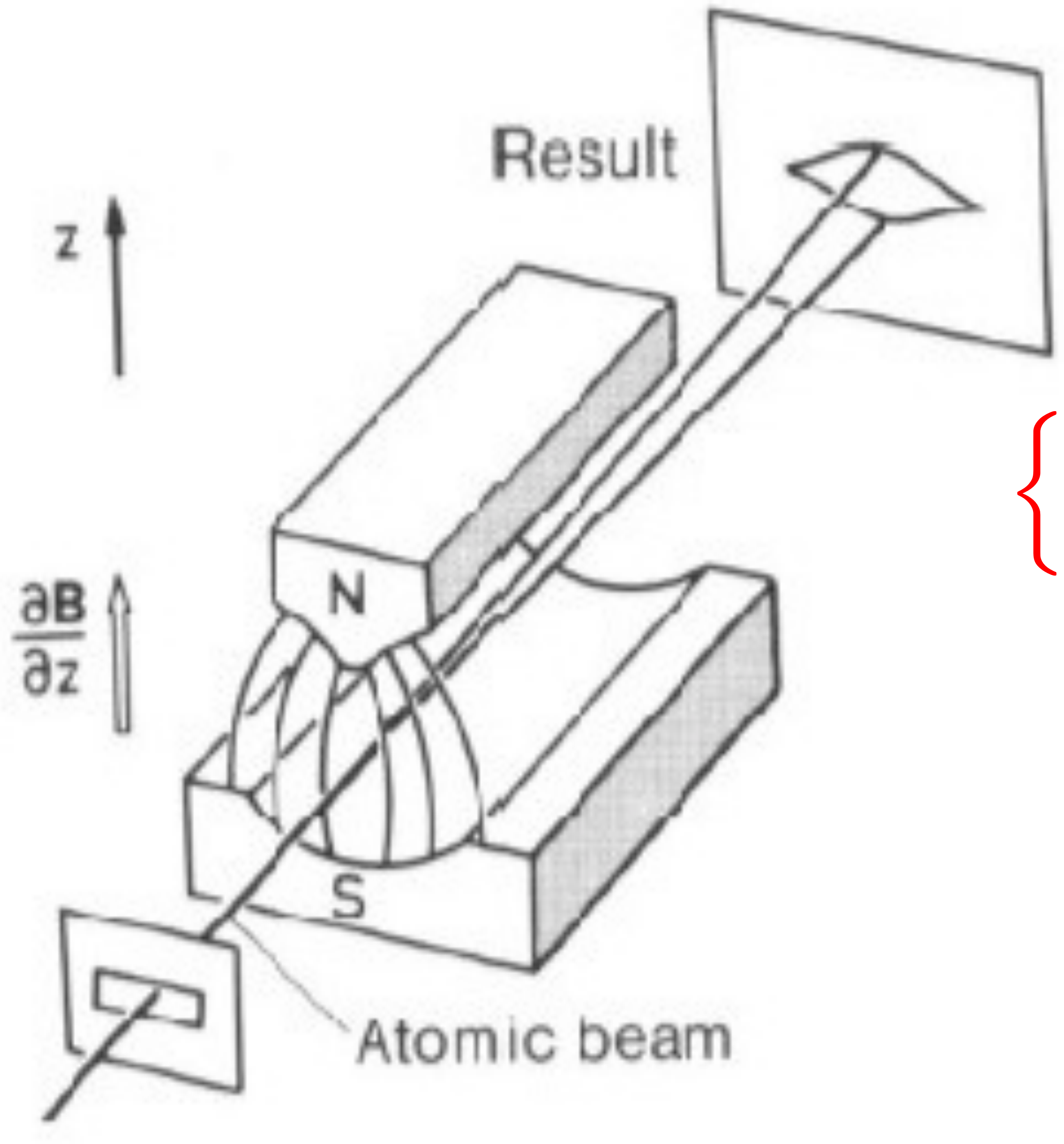
Statistical mixture

$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle\right\}$$

$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \qquad \left\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$

Stern-Gerlach

The measurement problem in quantum mechanics

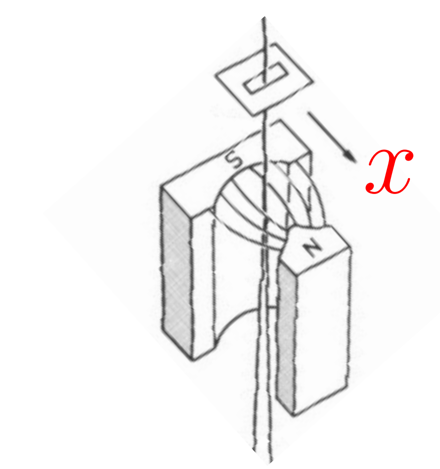


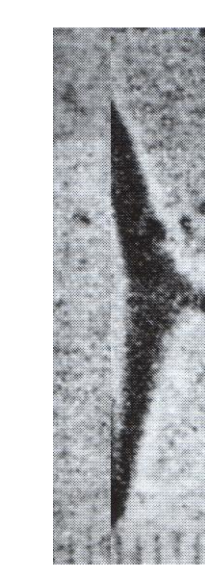
Statistical mixture

$$\left\{ |\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle \right\} \cup \left\{ |\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle \right\}$$

$$\left\{ |\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\} \cup \left\{ |\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\}$$

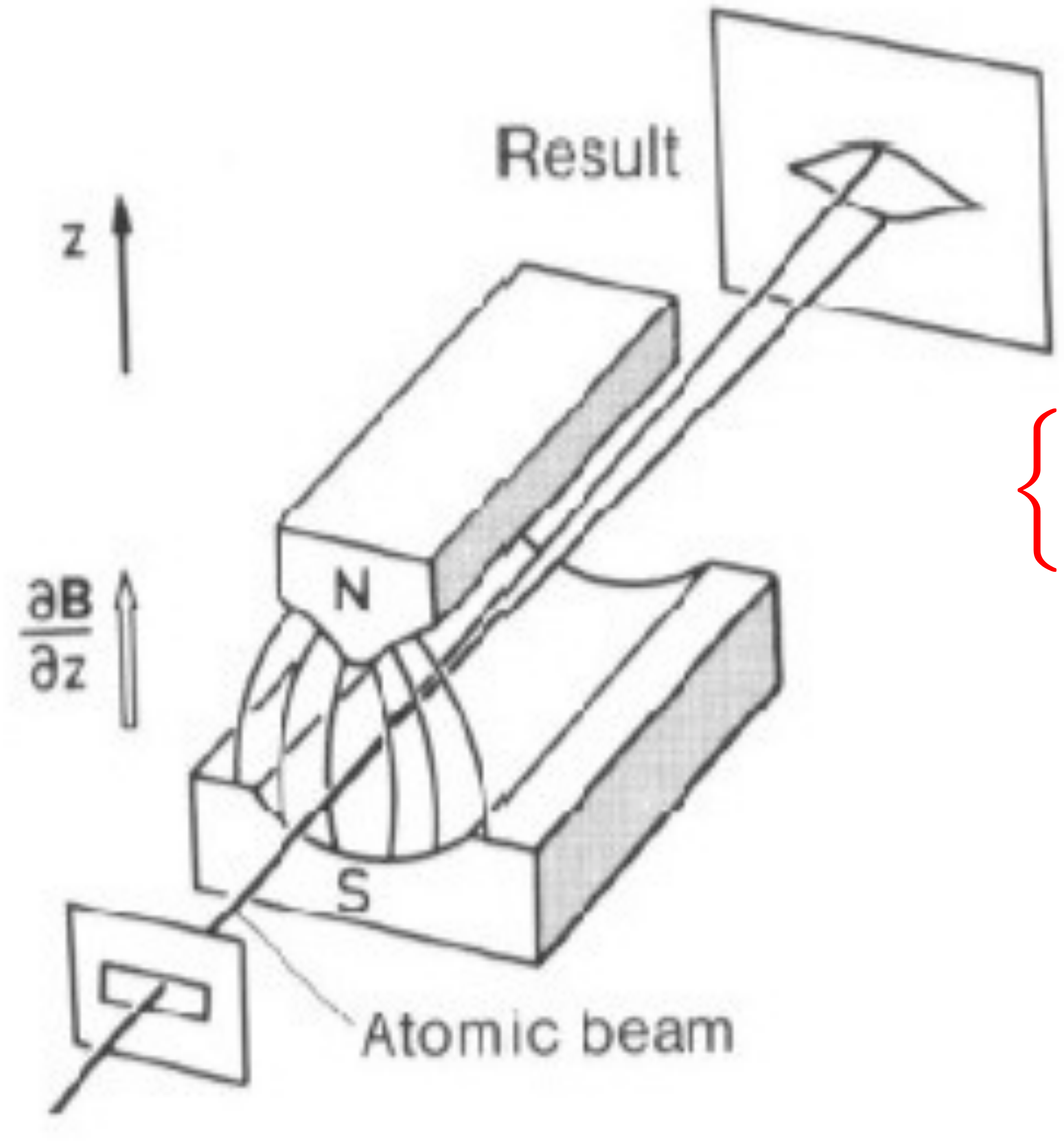
$$\left\{ (|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\}$$





Stern-Gerlach

The measurement problem in quantum mechanics



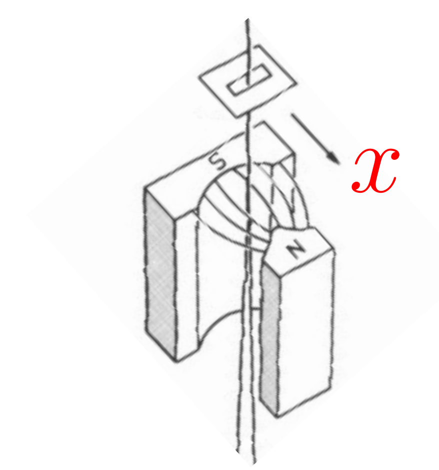
Stern-Gerlach

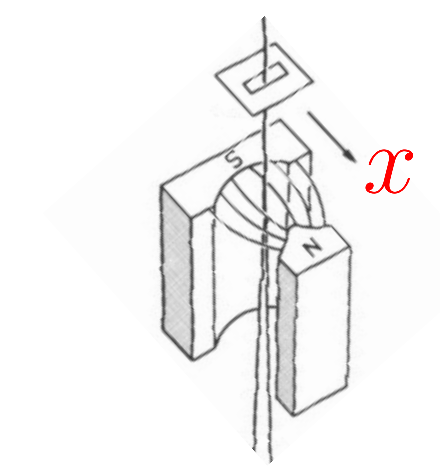
Statistical mixture

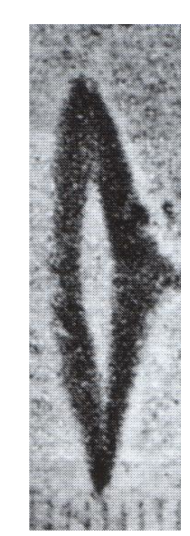
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle\right\}$$

$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$

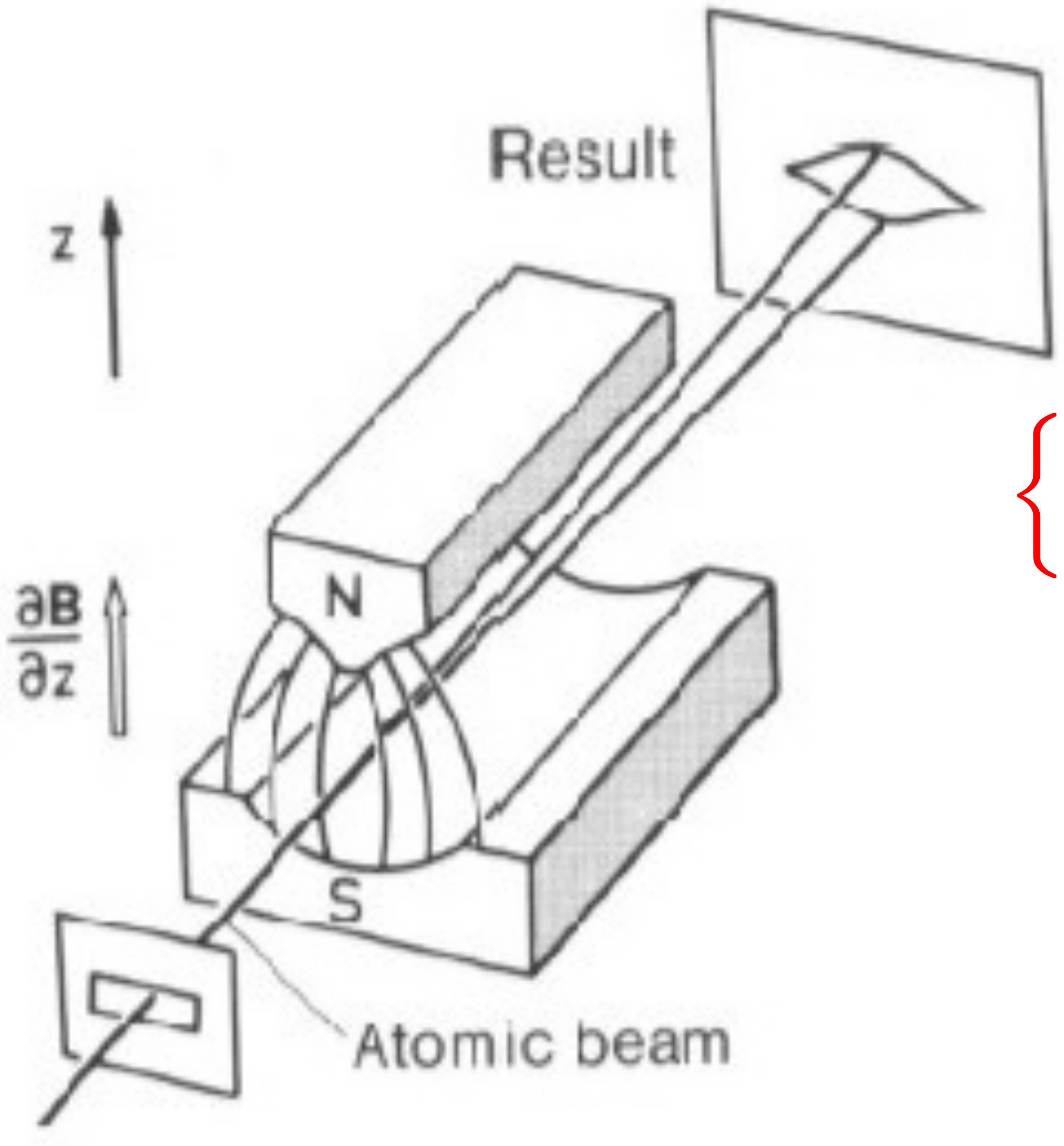
$$\left\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$







The measurement problem in quantum mechanics

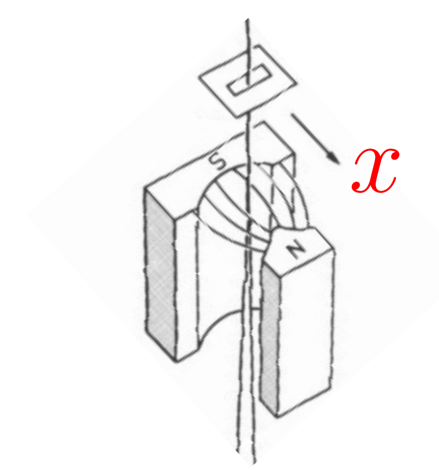


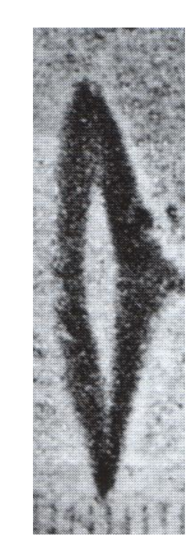
Stern-Gerlach

Statistical mixture

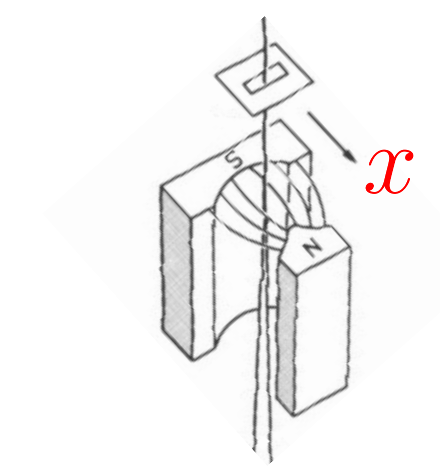
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_\uparrow\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_\downarrow\rangle\right\}$$

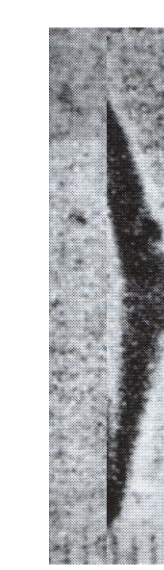
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$





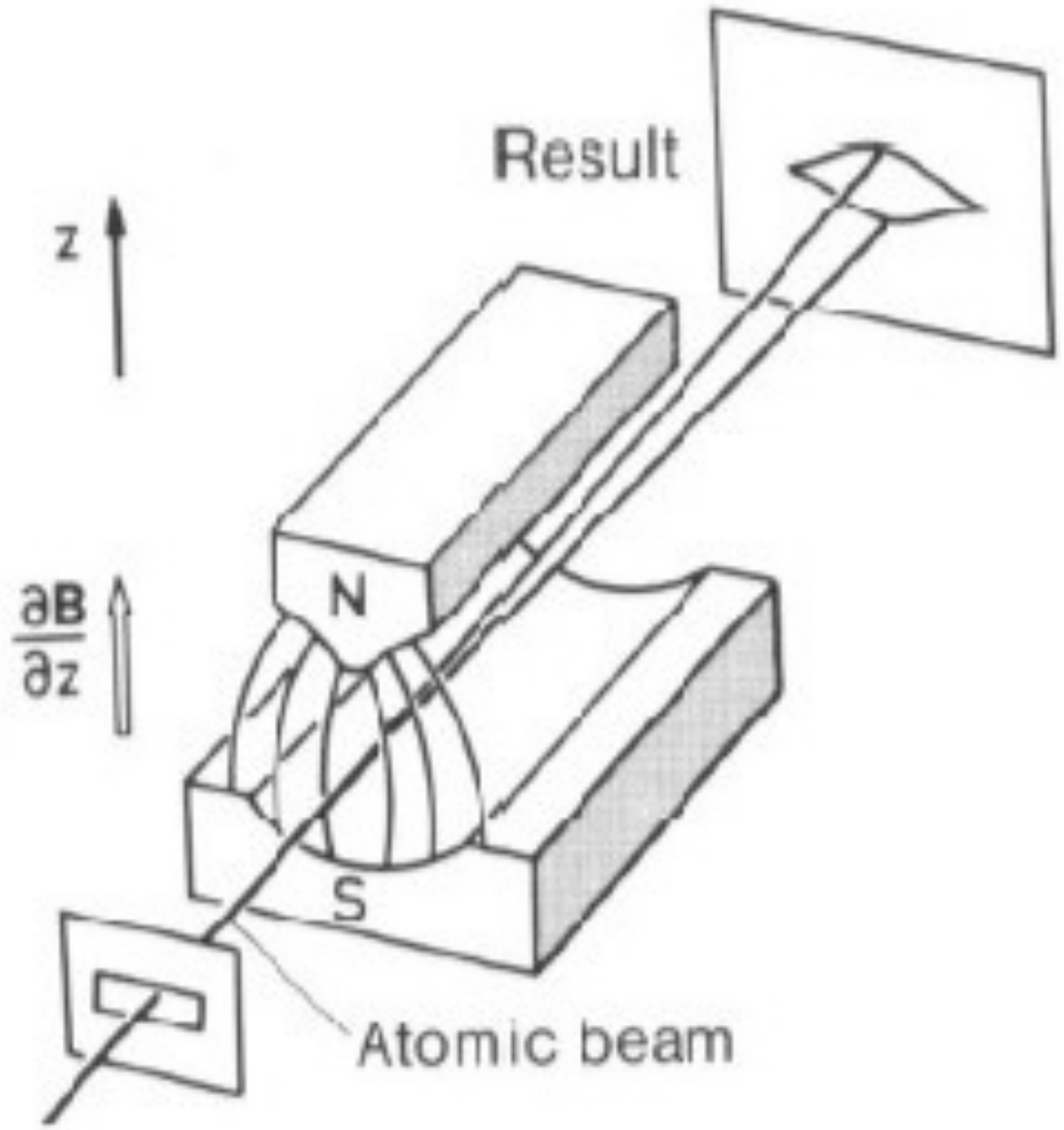
$$\left\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$





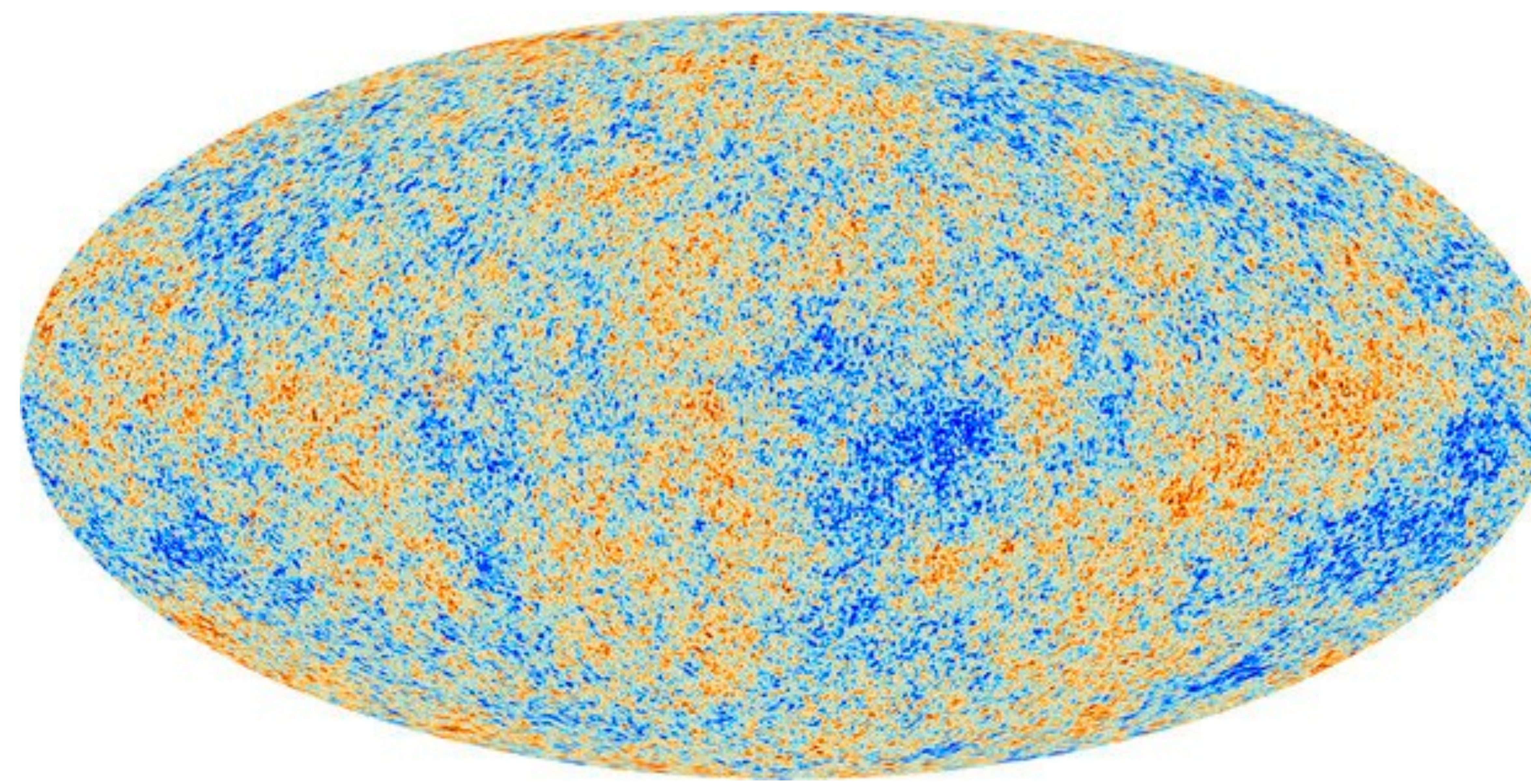
What about situations in which one has only one realization?

The measurement problem in quantum mechanics



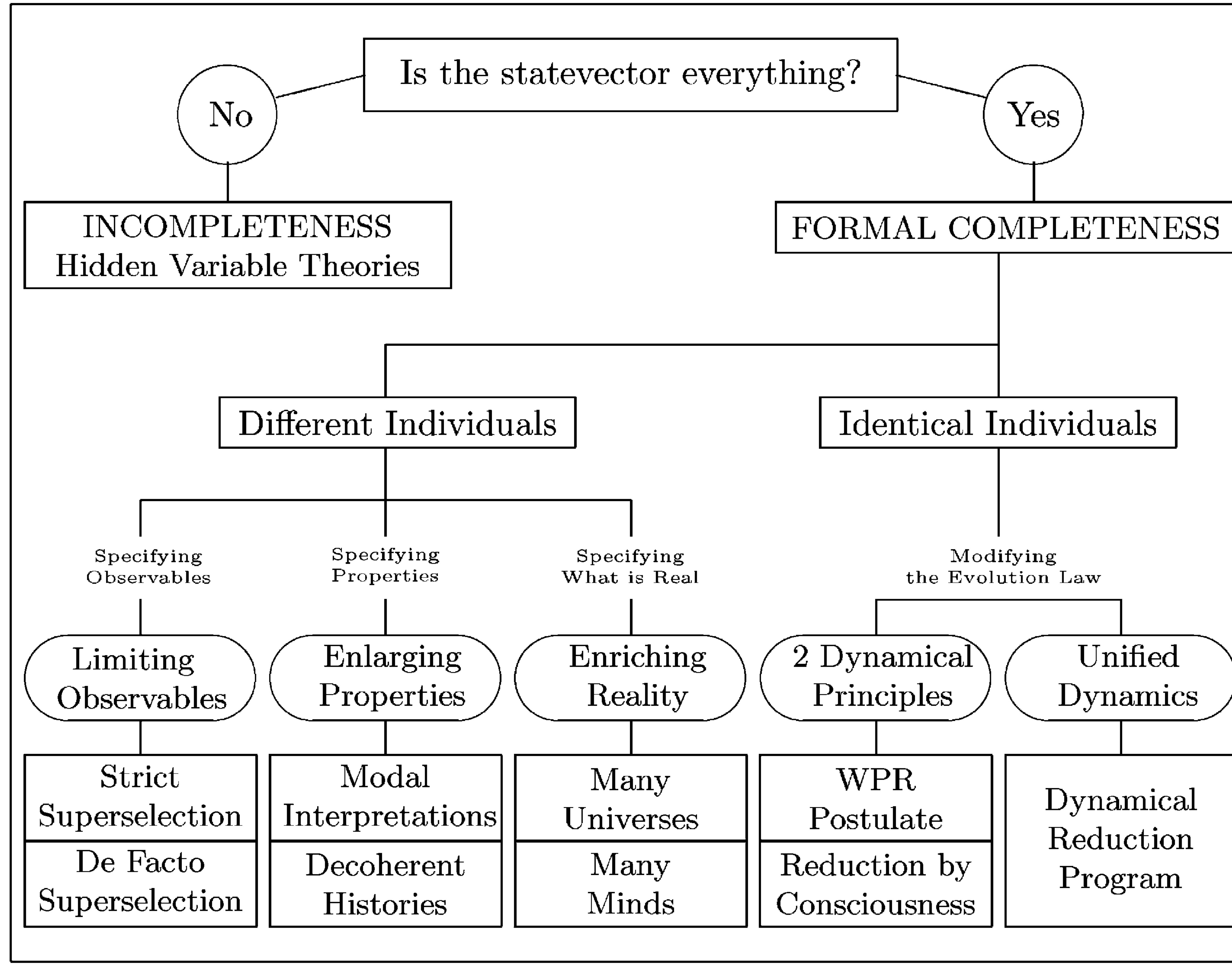
What about the Universe itself?

Stern-Gerlach



What about situations in which one has only one realization?

- Possible solutions and a criterion: the Born rule



A. Bassi and G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003)

▲ ***Superselection rules***

▲ ***Modal interpretation***

▲ ***Decoherent histories***

▲ ***Many worlds / many minds***

▲ ***Hidden variables***

▲ ***Modified Schrödinger dynamics***

} Born rule not put by hand!

Hidden Variable Theories

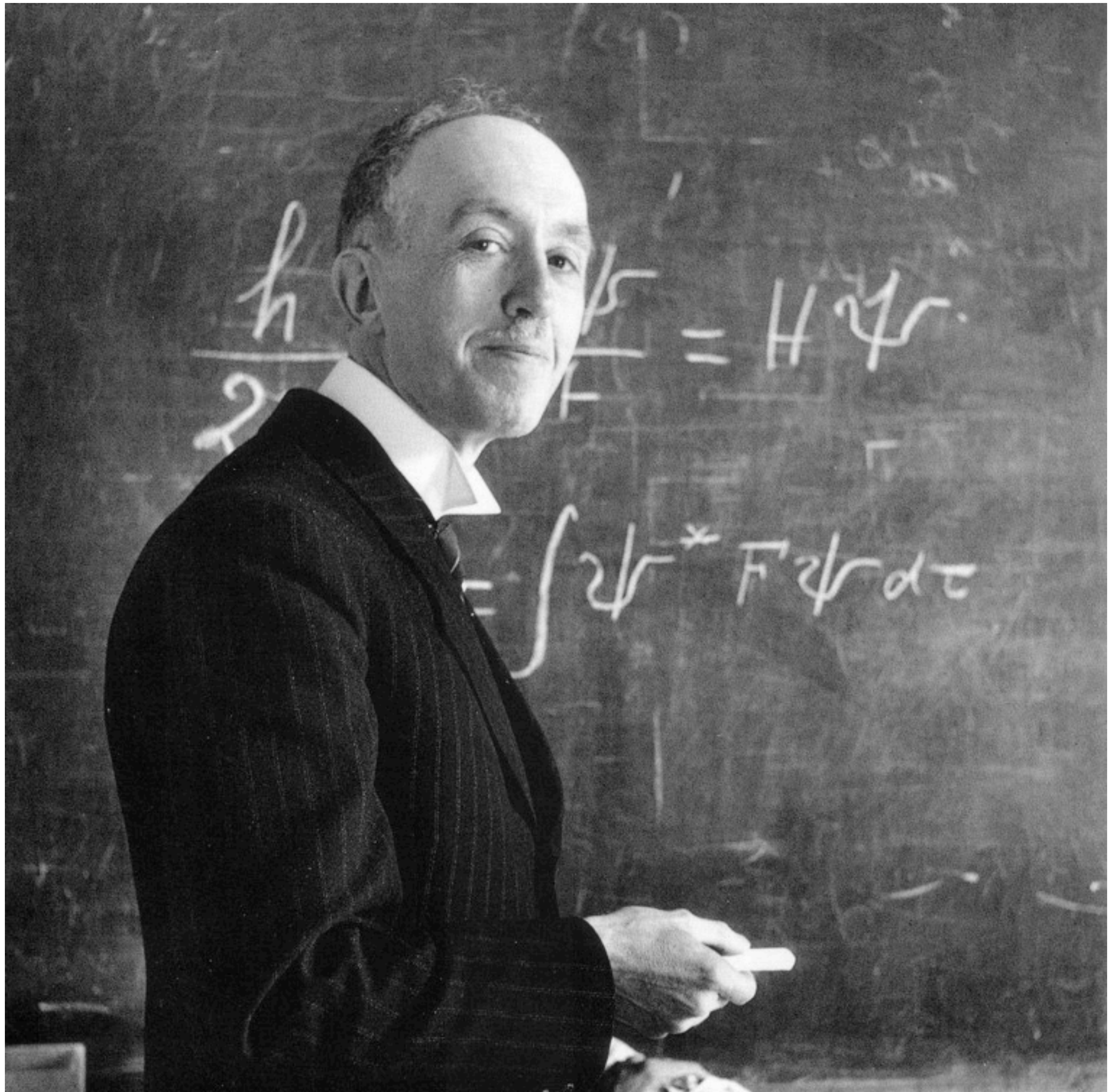
Schrödinger
$$i\frac{\partial \Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V(\boldsymbol{r})\right]\Psi$$

Polar form of the wave function
$$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi
$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential
$$\equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Ontological interpretation (dBB)

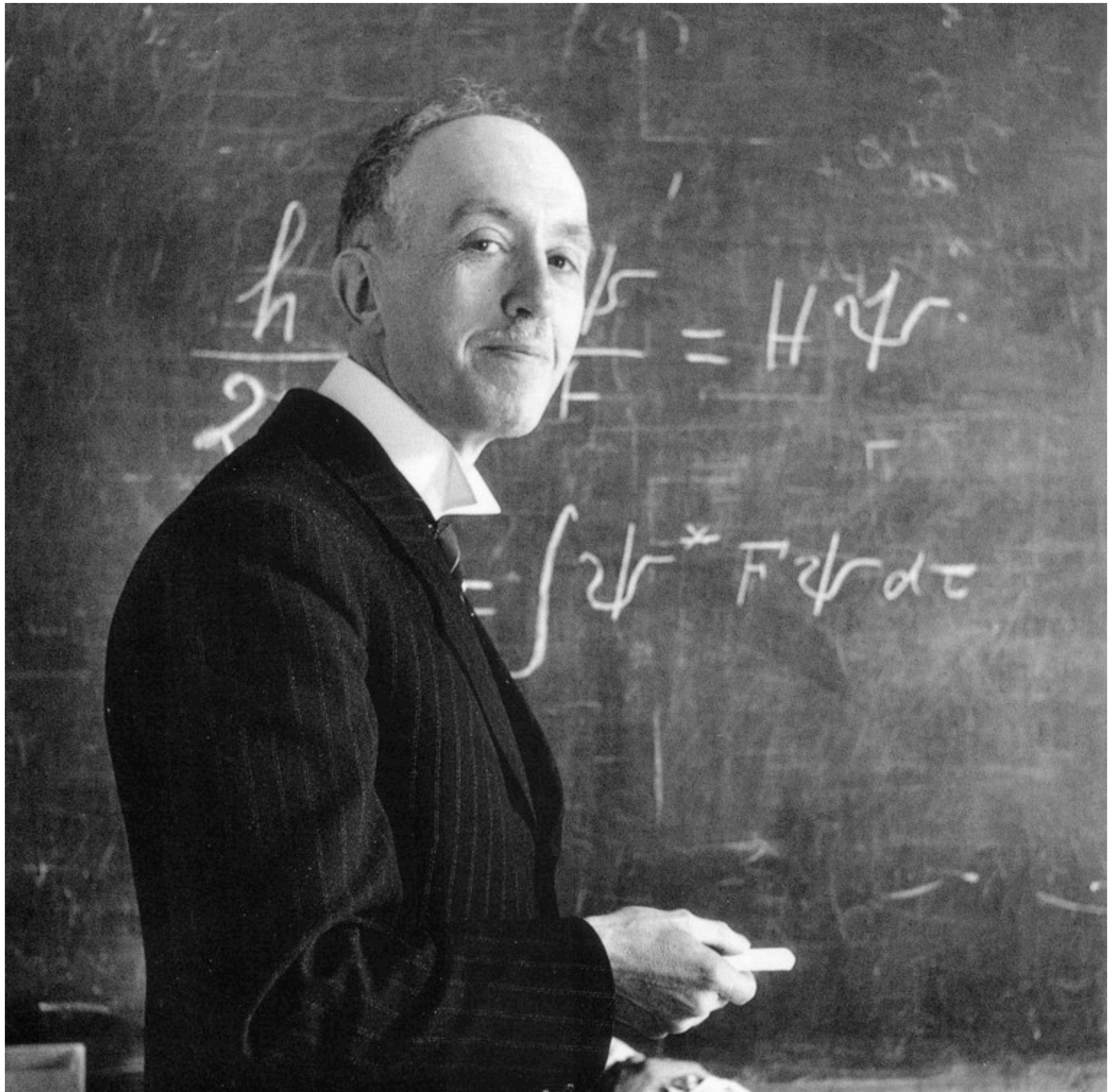


Louis de Broglie

David Bohm

1927 Solvay meeting and von Neuman mistake ... ‘*In 1952, I saw the impossible done*’ (J. Bell)

Ontological interpretation (dBB)

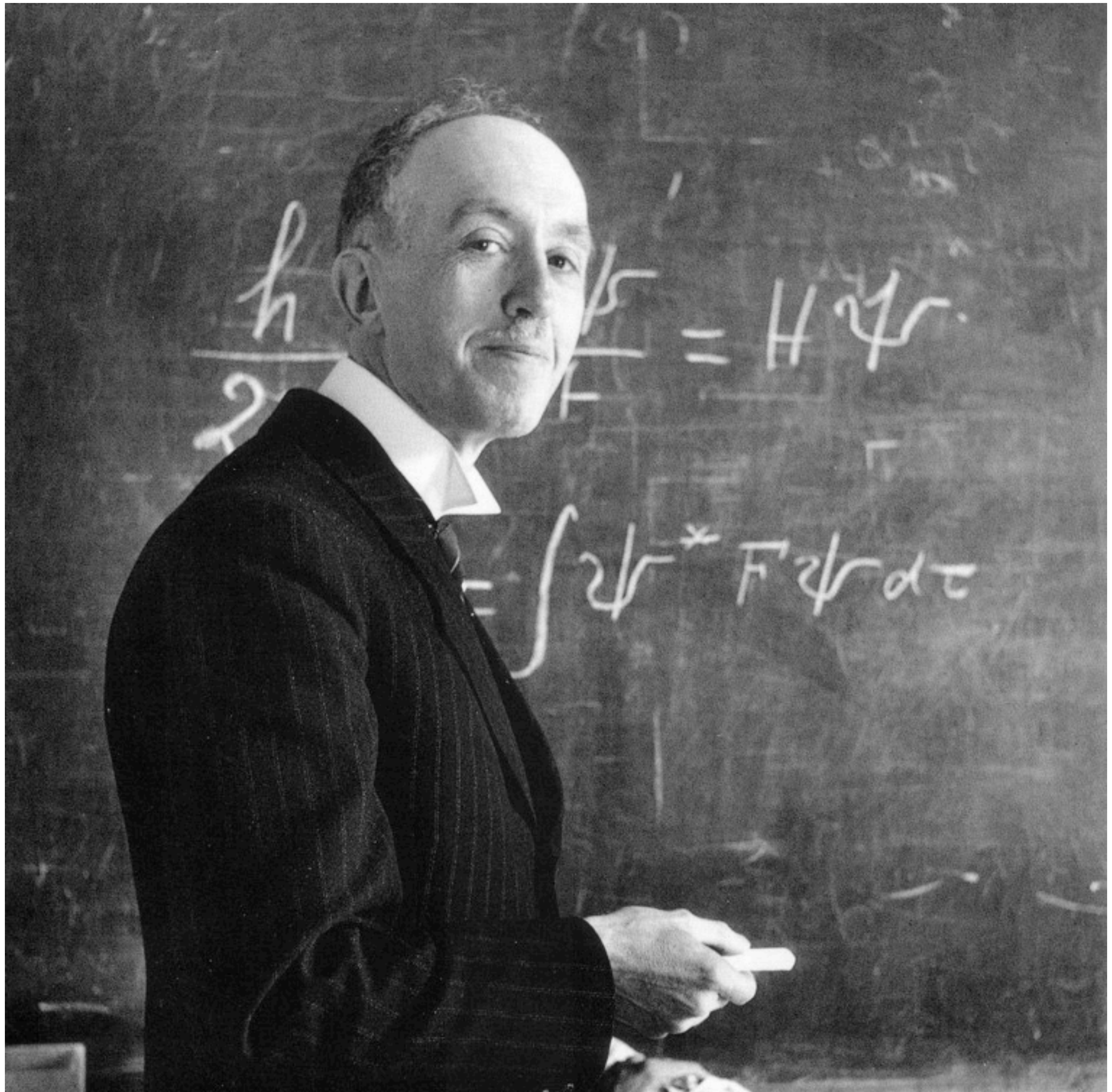


Louis de Broglie (Prince, duke ...)

David Bohm

1927 Solvay meeting and von Neuman mistake ... ‘*In 1952, I saw the impossible done*’ (J. Bell)

Ontological interpretation (dBB)

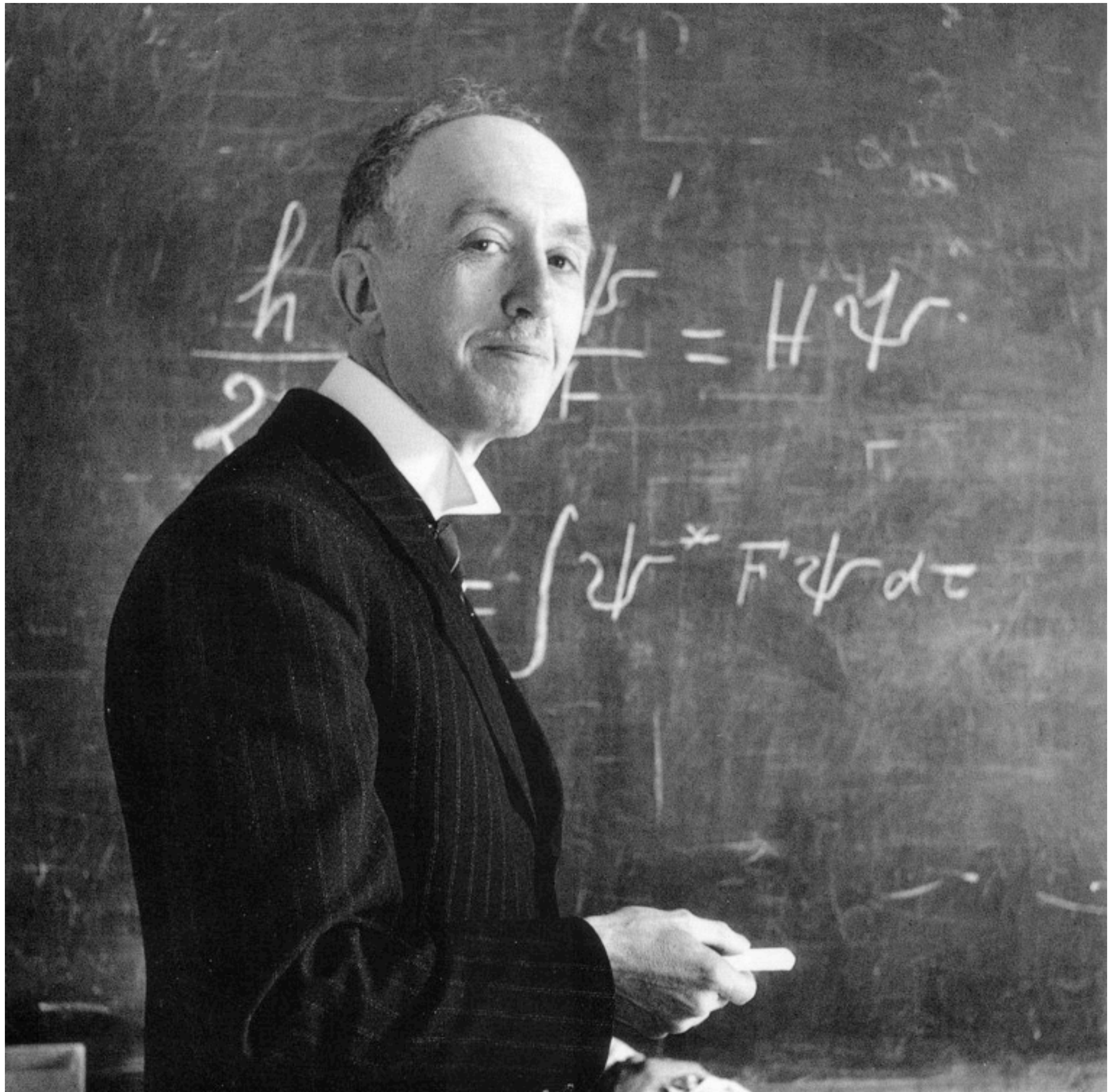


Louis de Broglie (Prince, duke ...)

David Bohm (Communist)

1927 Solvay meeting and von Neuman mistake ... '*In 1952, I saw the impossible done*' (J. Bell)

Ontological *formulation* (dBB)



Louis de Broglie (Prince, duke ...)

David Bohm (Communist)

1927 Solvay meeting and von Neuman mistake ... ‘*In 1952, I saw the impossible done*’ (J. Bell)

Hidden Variable Theories

Schrödinger $$i\frac{\partial\Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V(\boldsymbol{r})\right]\Psi$$

Polar form of the wave function $$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential $$Q \equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

$$\Psi = A\,(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla}S$$

$$\Psi = A(\boldsymbol{r}, t)\, \mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (Bohm)

$$m \frac{\mathrm{d}^2 \boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V + Q) \qquad Q \equiv -\frac{1}{2m} \frac{\boldsymbol{\nabla}^2 |\Psi|}{|\Psi|}$$

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla}S$$

$\Psi = A(\boldsymbol{r}, t)\, \mathrm{e}^{iS(\boldsymbol{r},t)}$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\, \frac{\Psi^* \boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla} S$$

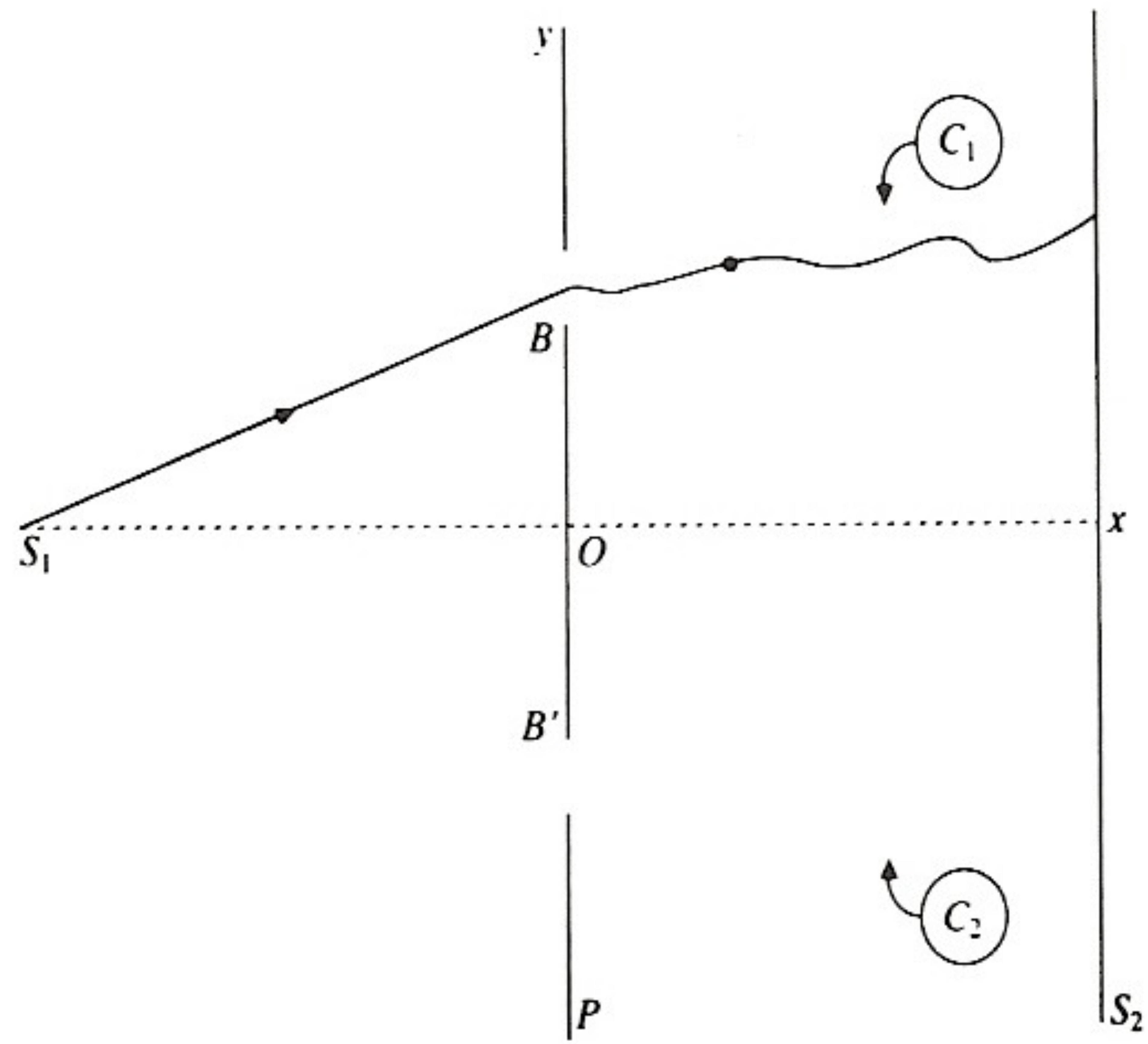
Properties:

- ☺ strictly equivalent to Copenhagen QM
  - ➡ probability distribution (attractor)

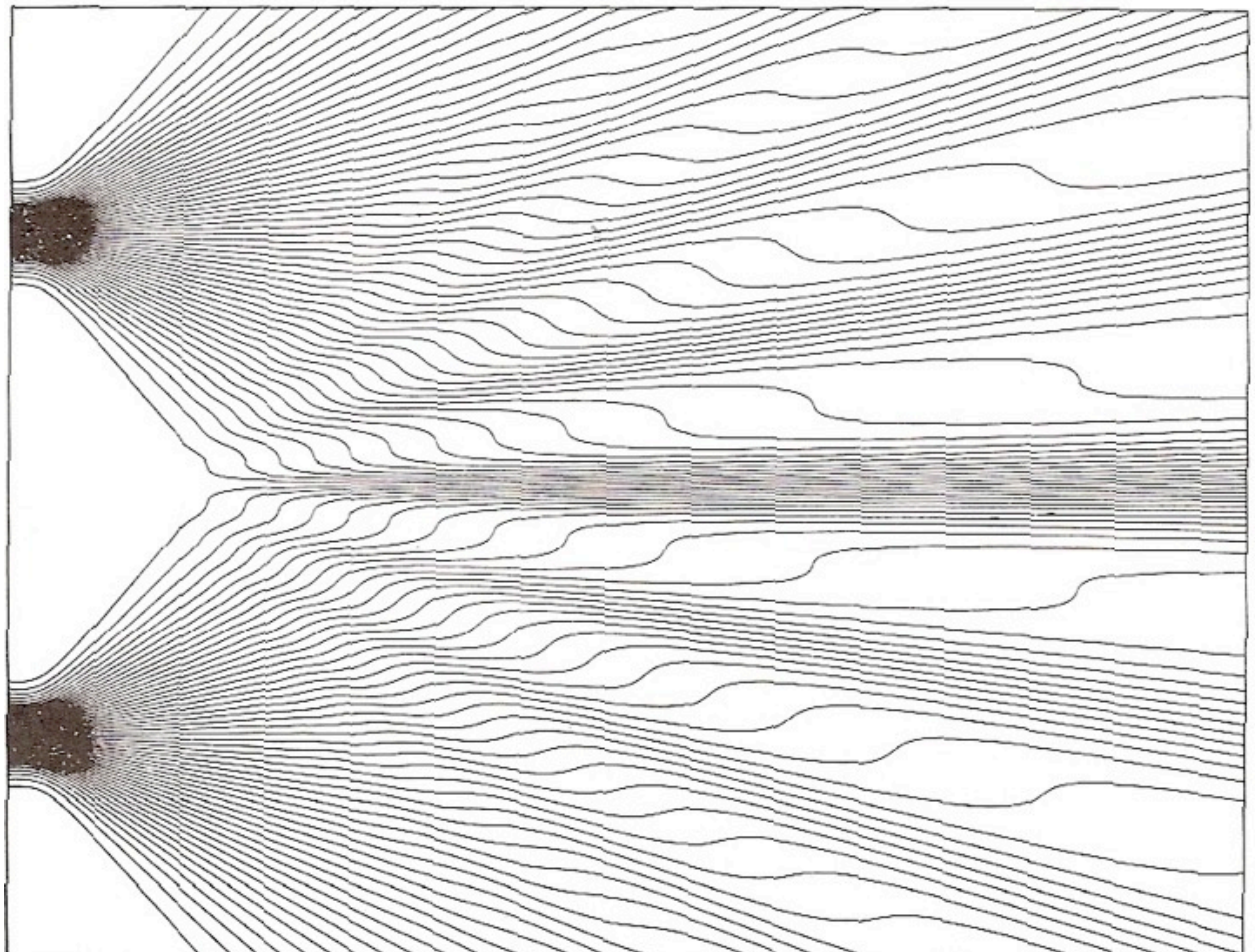
$$\exists t_0 ; \rho(\boldsymbol{x}, t_0) = |\Psi(\boldsymbol{x}, t_0)|^2$$

- ☺ classical limit well defined $Q \longrightarrow 0$
- ☺ state dependent
- ☺ $\exists$ intrinsic reality
  - ➡ non local …
- ☺ no need for external classical domain/observer!

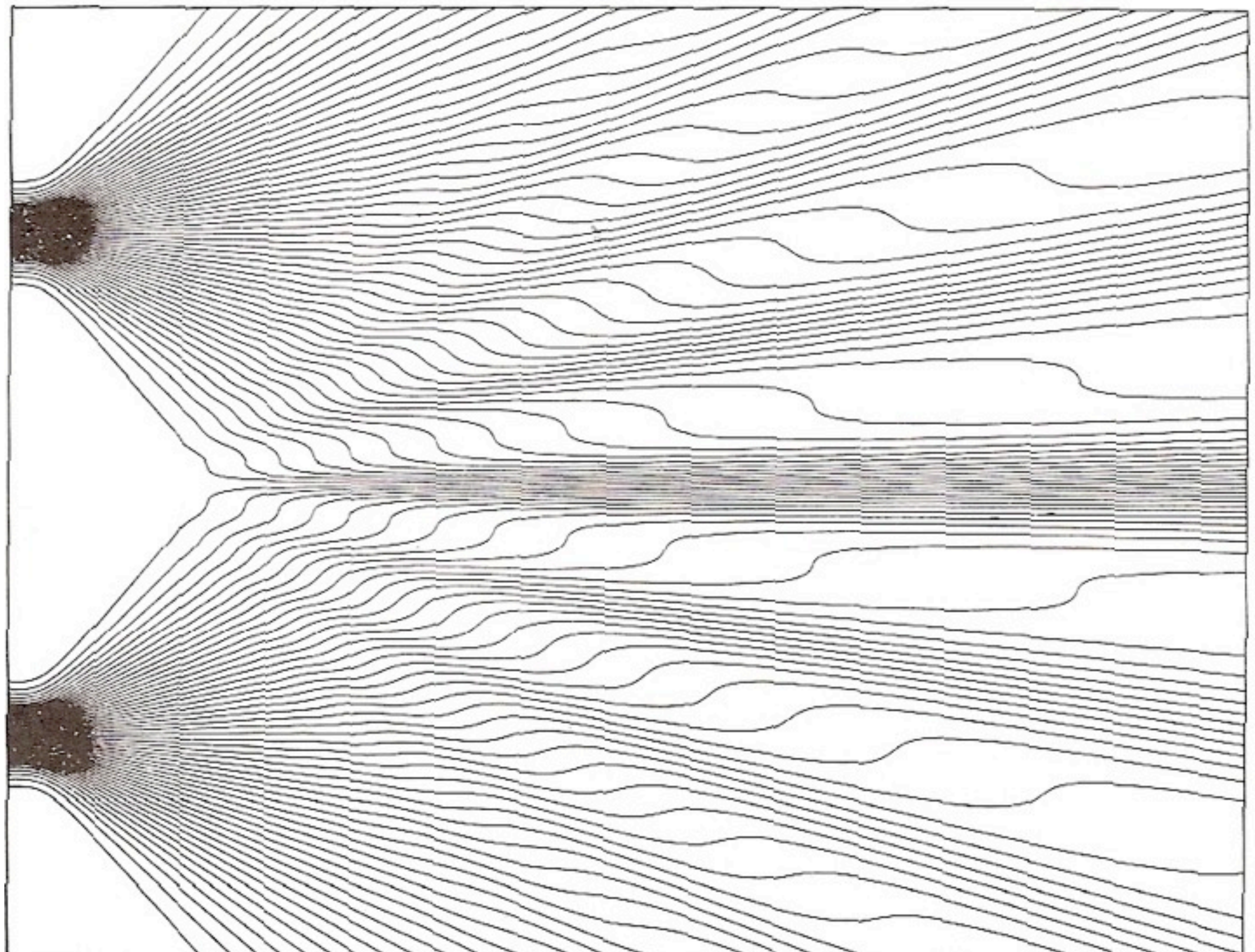
The two-slit experiment:



The two-slit experiment:



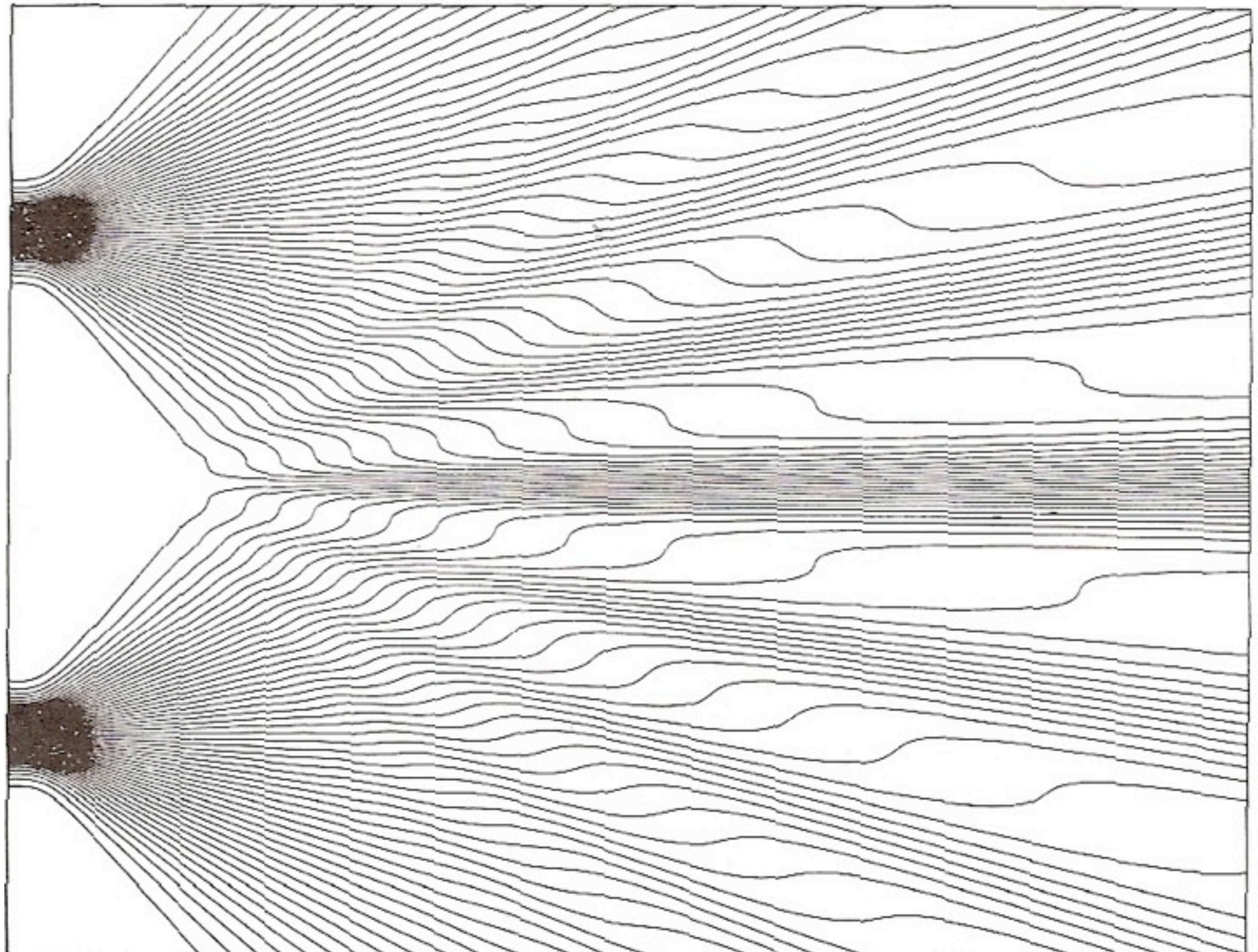
The two-slit experiment:



Surrealistic trajectories?

Non straight in vacuum...

The two-slit experiment:

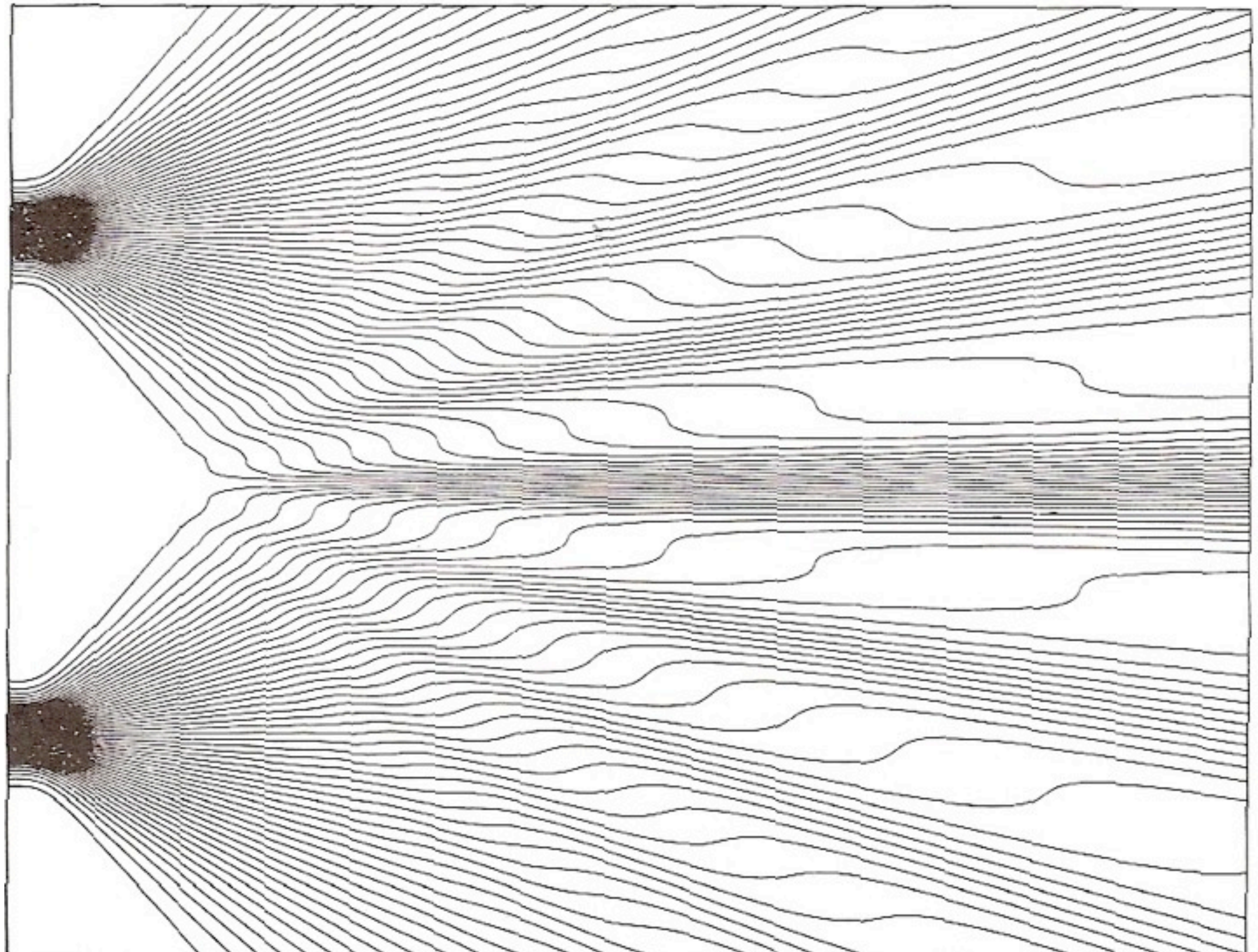


Surrealistic trajectories?

Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(V+Q\right)$$

The two-slit experiment:



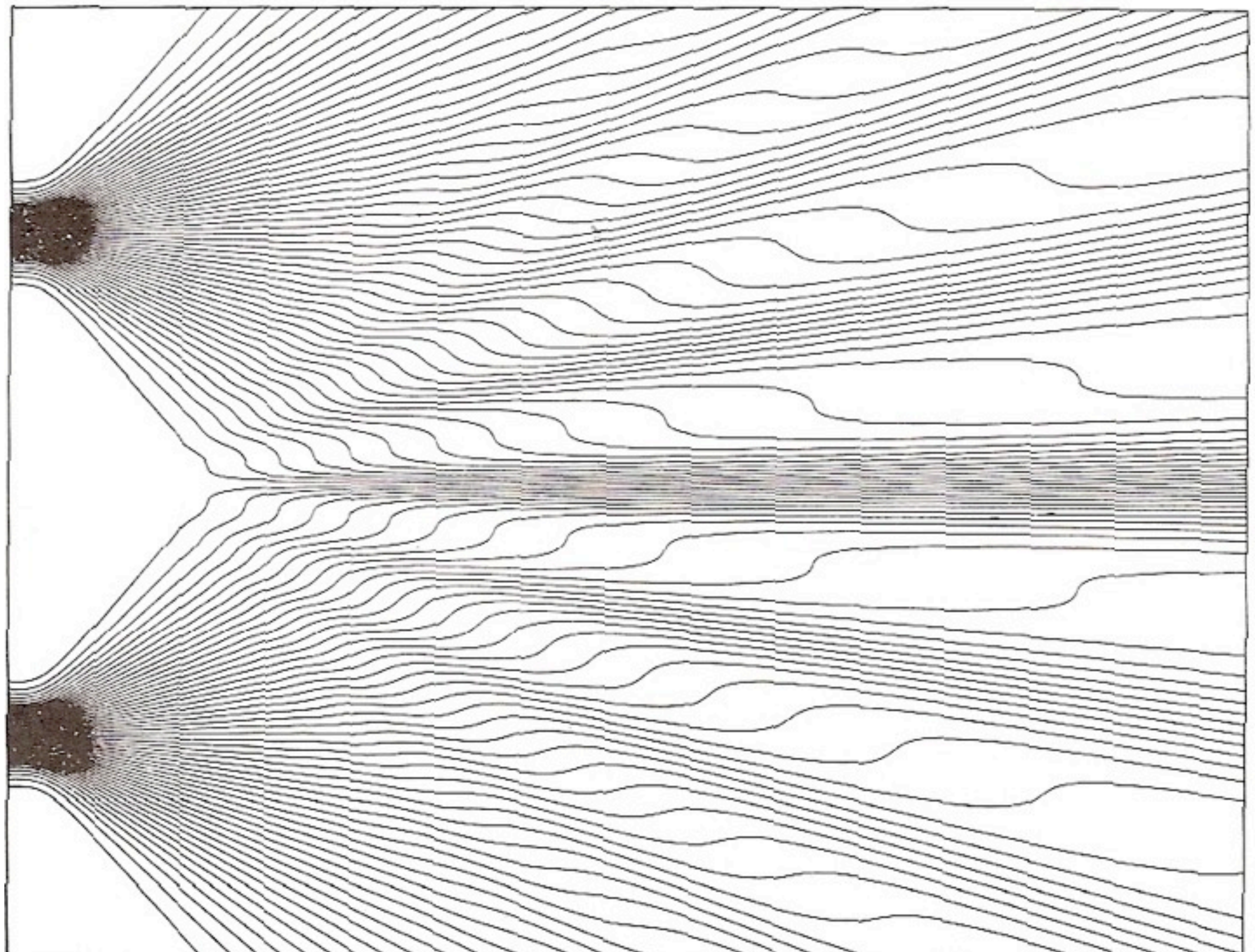
Surrealistic trajectories?

Non straight in vacuum...

$$m \frac{d^2 x(t)}{dt^2} = -\nabla \left(V + Q \right)$$

(The V is crossed out with an X, with an arrow pointing to it from "Non straight in vacuum...")

The two-slit experiment:



Surrealistic trajectories?

Non straight in vacuum...

$$m \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = -\nabla \left(V + Q \right)$$

Back to the QC wave function

Back to the QC wave function

Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

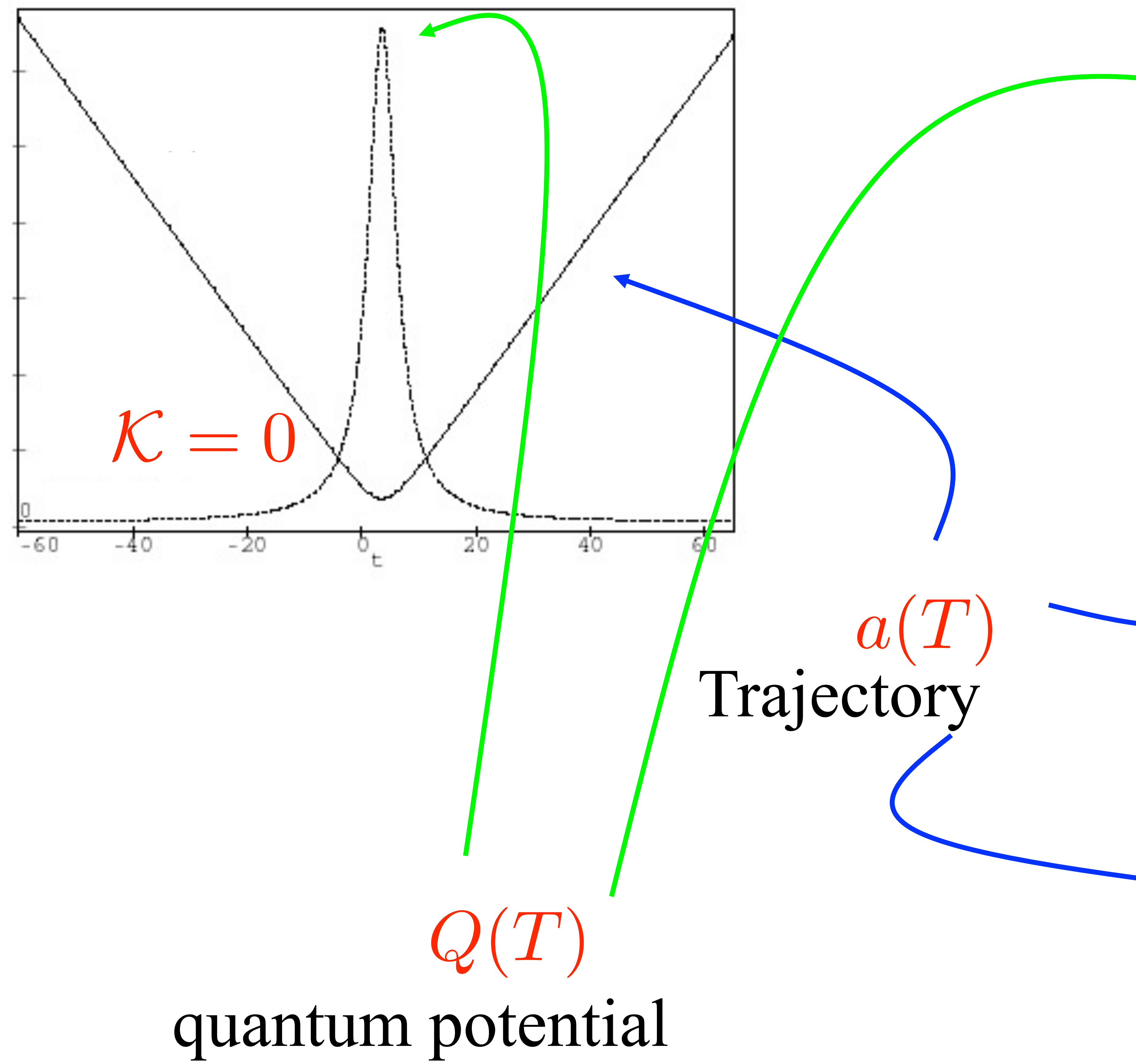
Back to the QC wave function

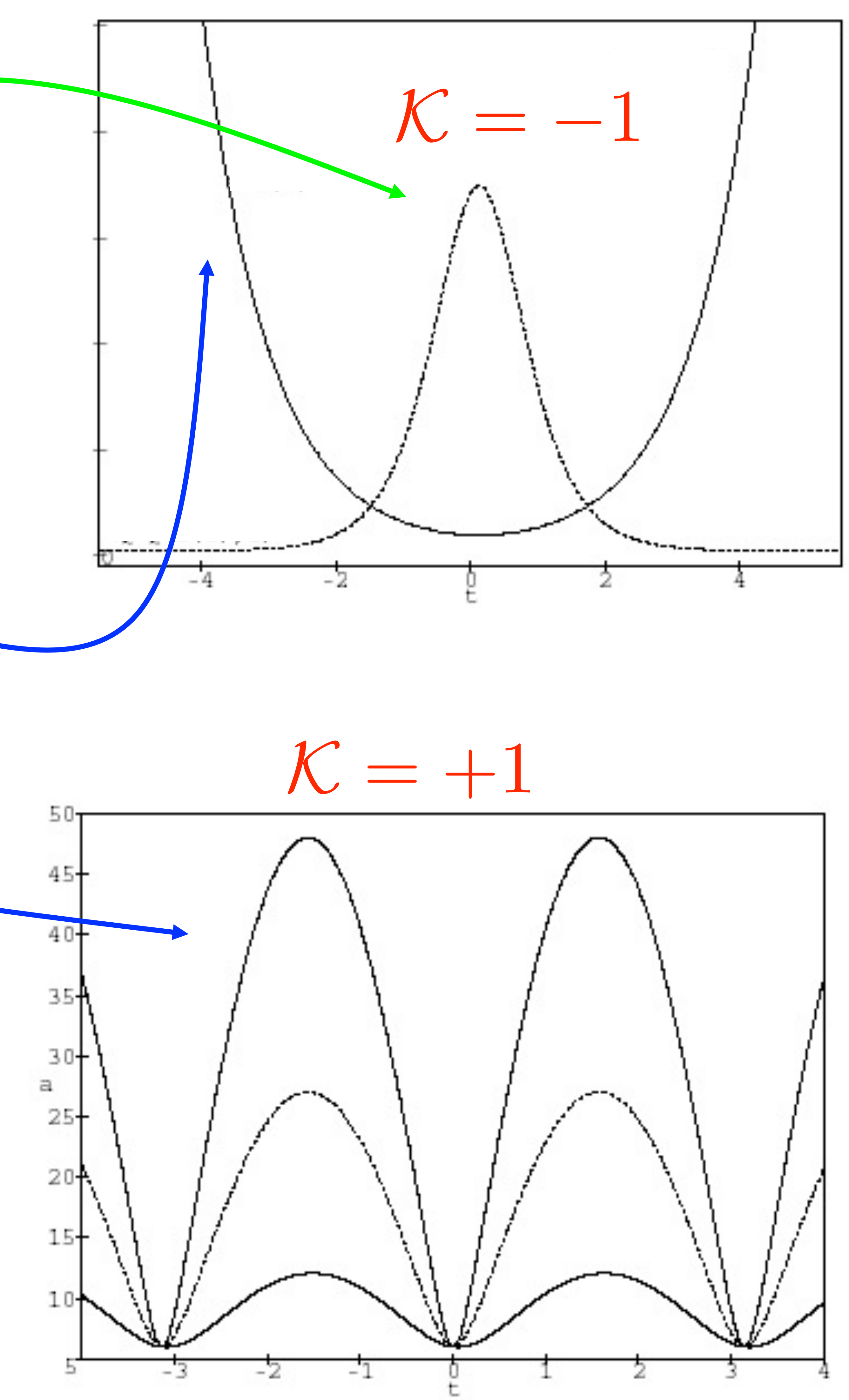
Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

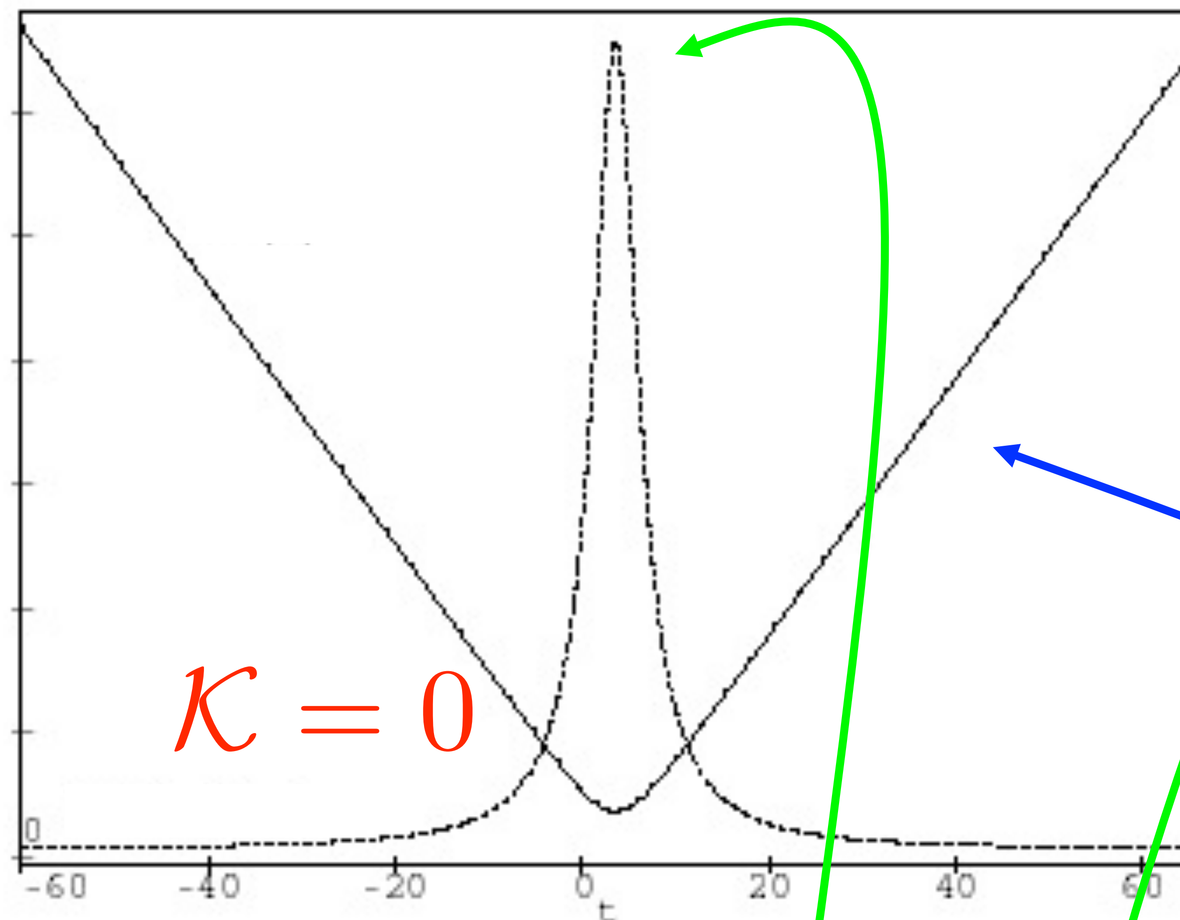
phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

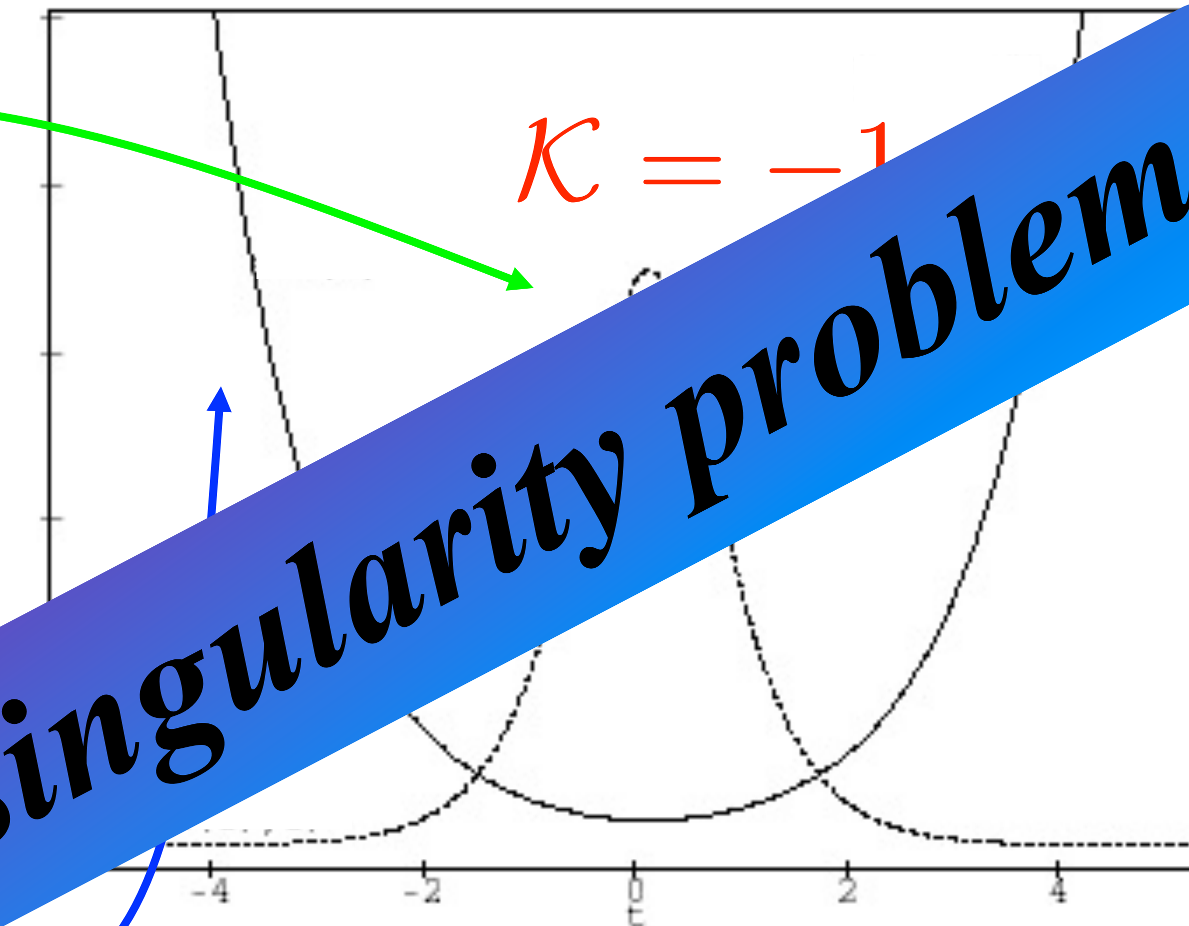
Bohmian trajectory $$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$





J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,, *Phys*. *Lett*. **A241**, 229 (1998)





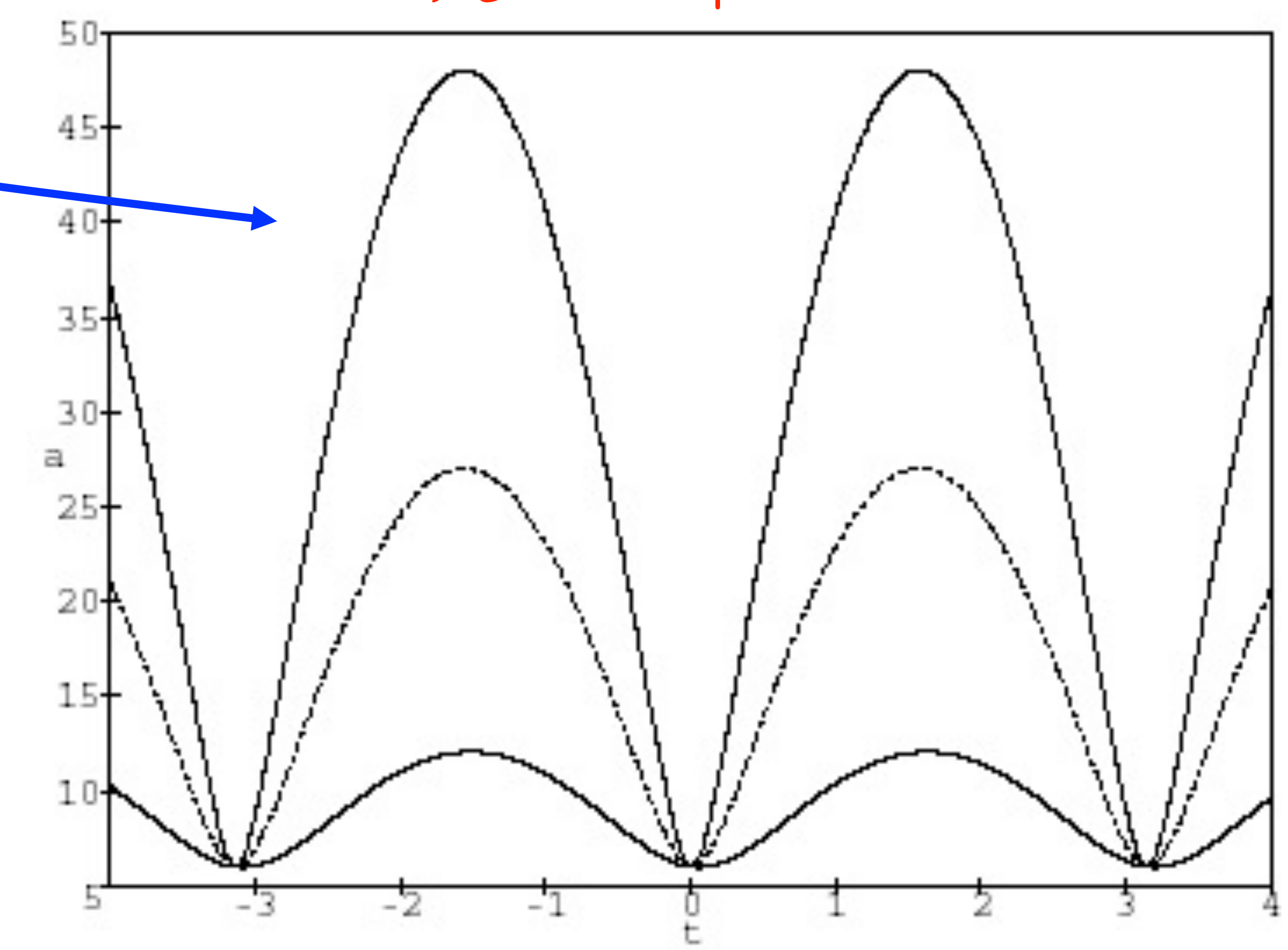
Natural quantum solution to the singularity problem!

y

(T)

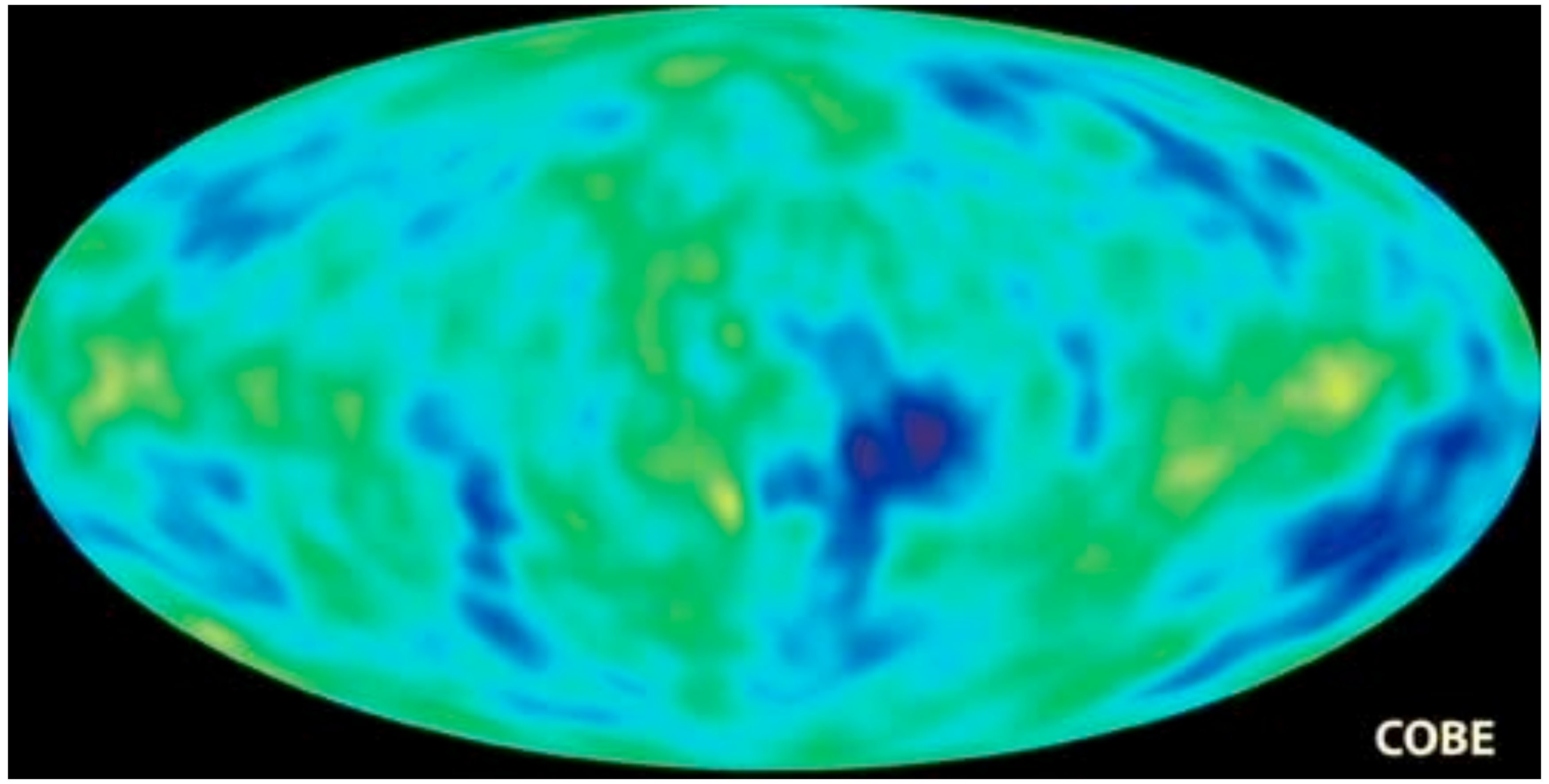
um potential

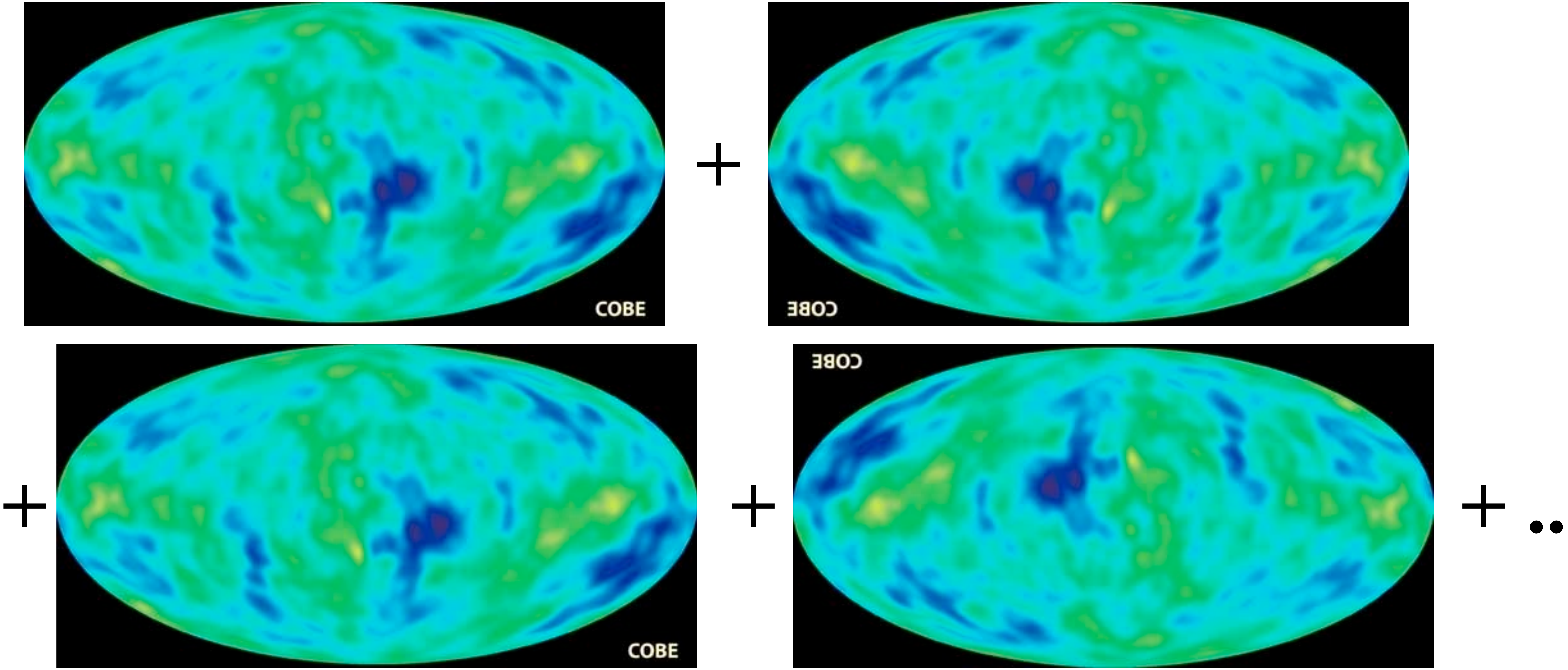
$\mathcal{K} = +1$



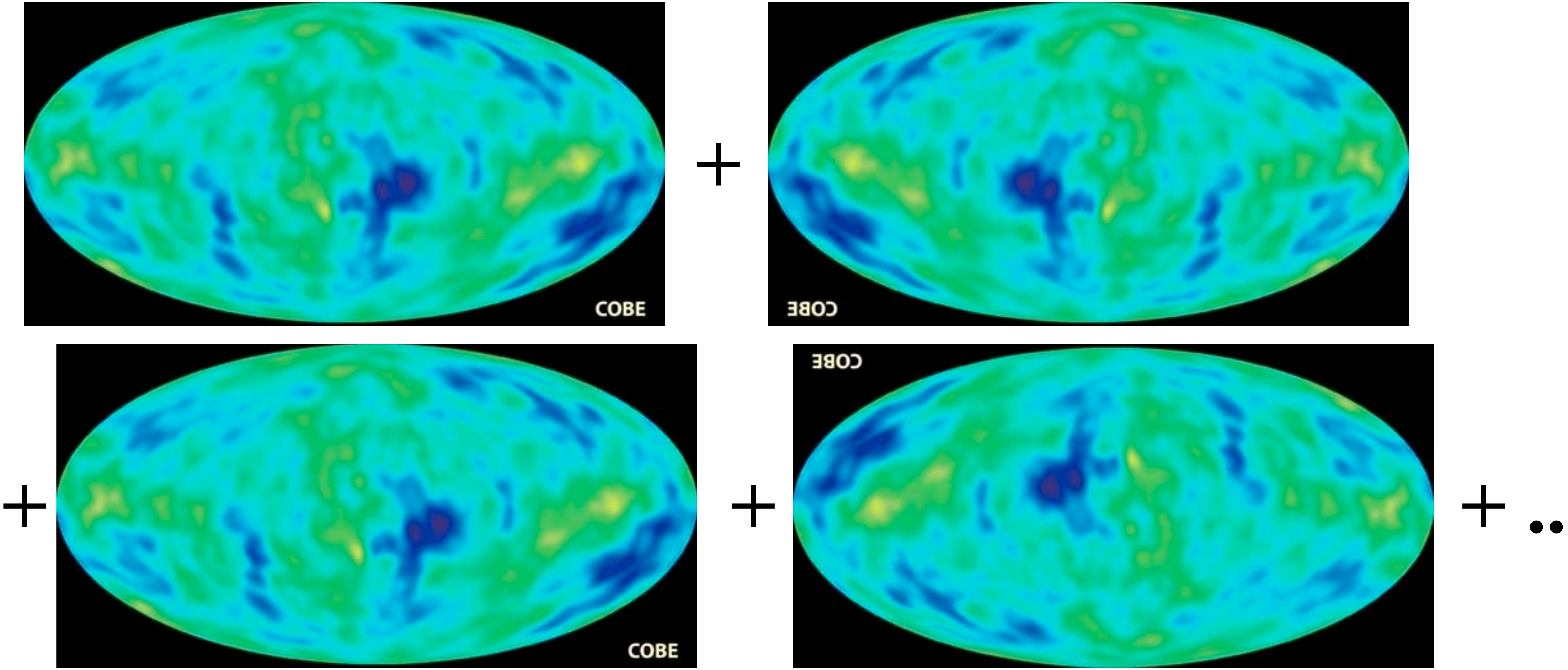
J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,, *Phys. Lett.* **A241**, 229 (1998)

What about perturbations?


COBE

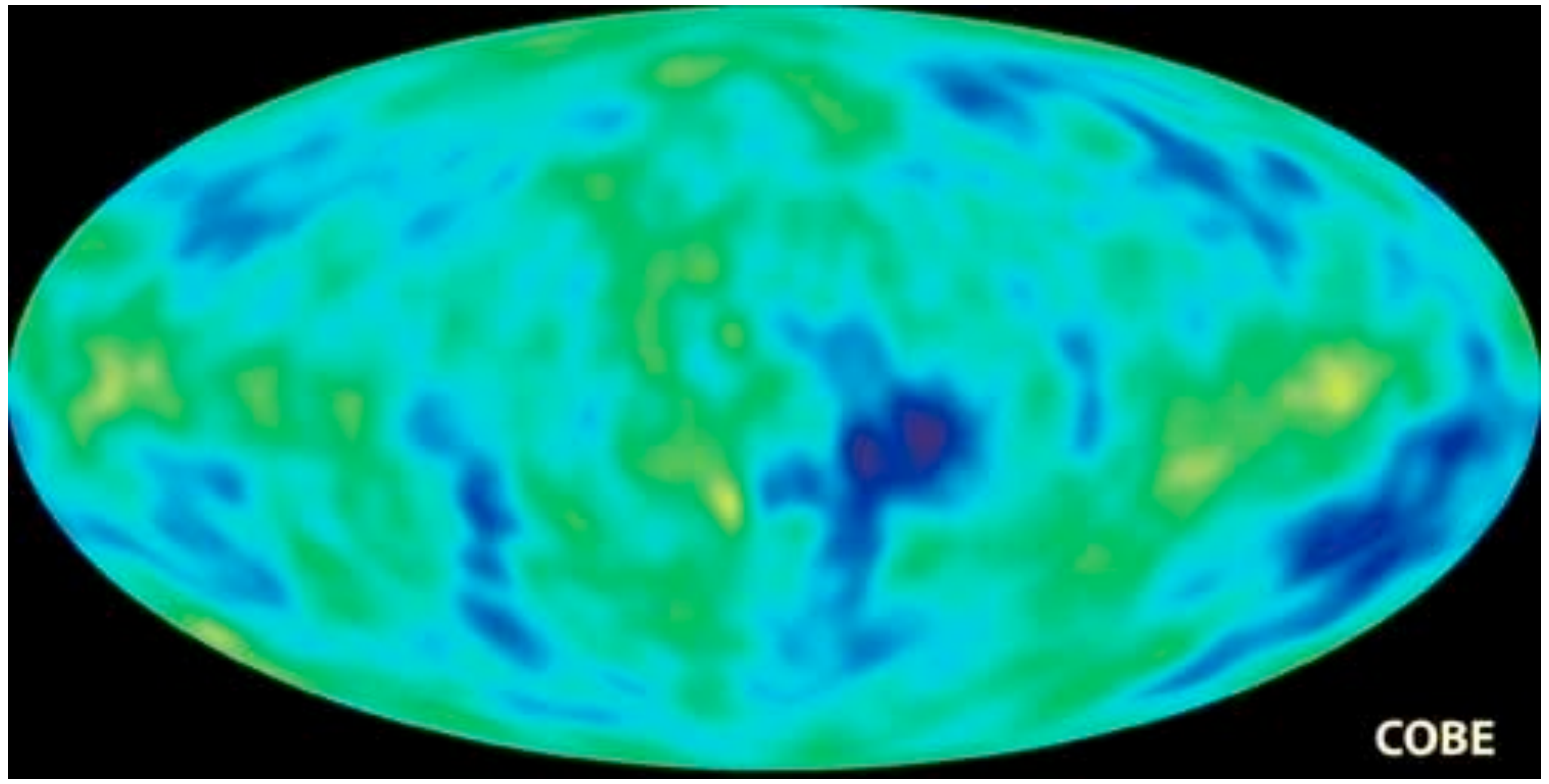


Superposition

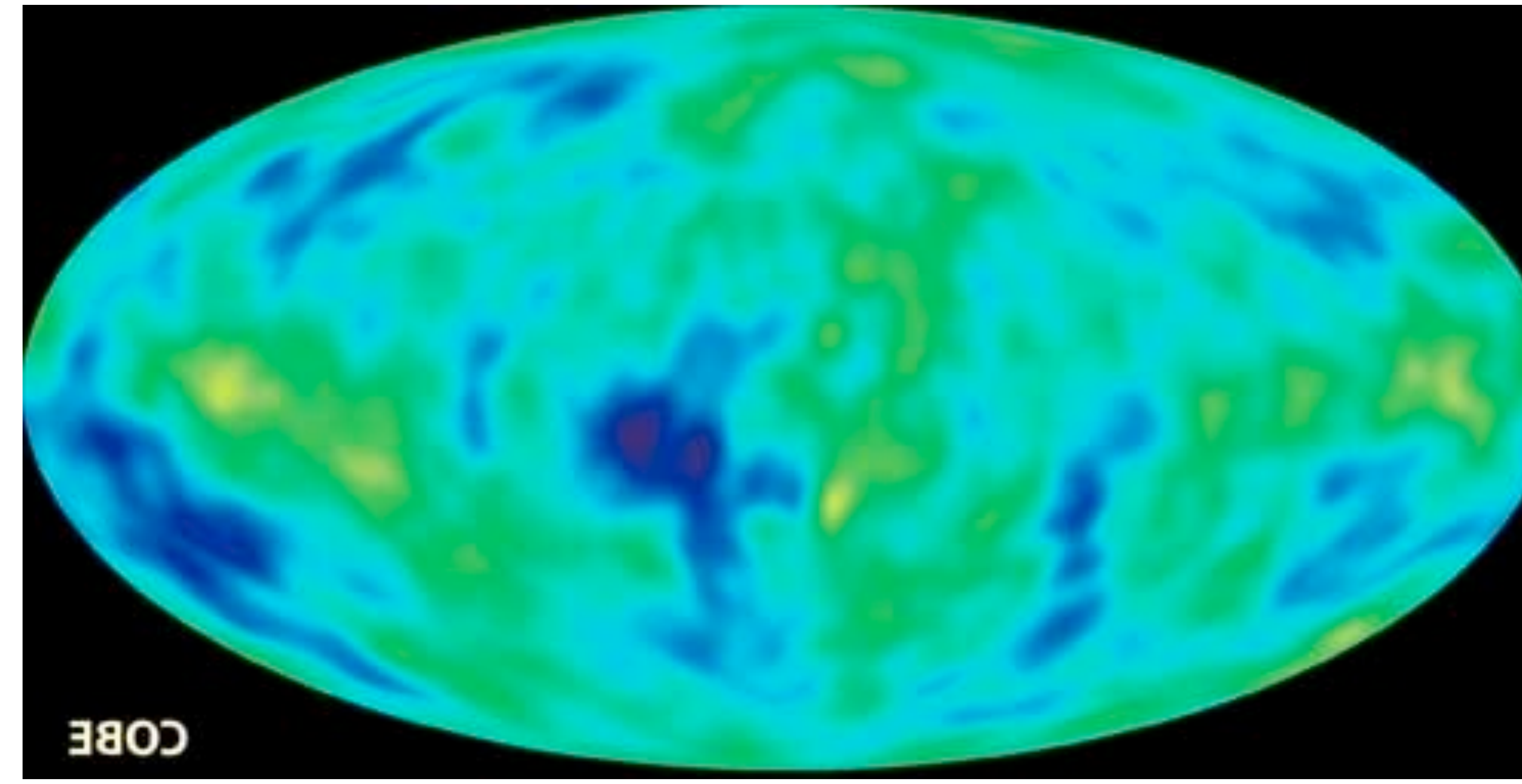


Superposition

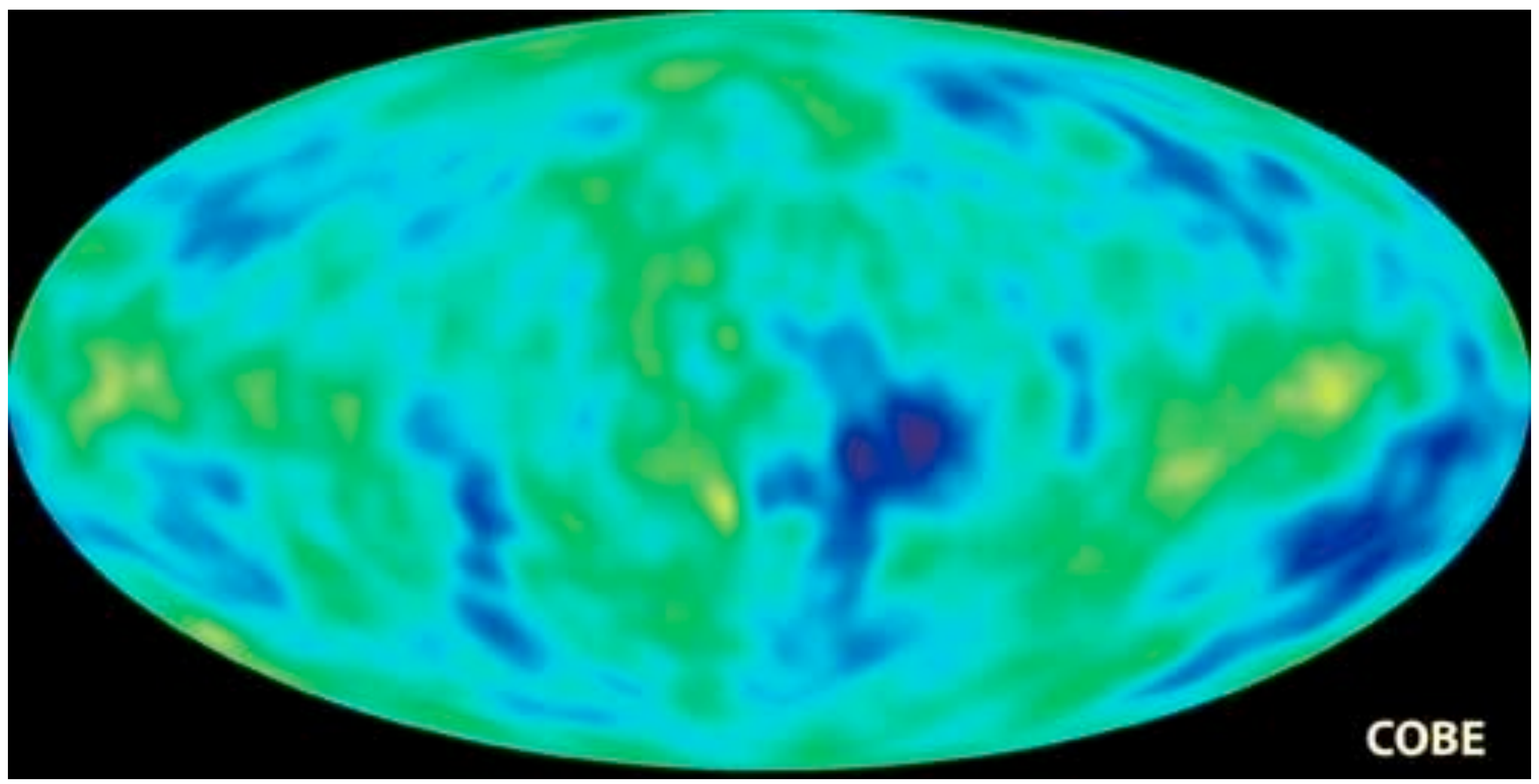
Collapse in 1992 ???



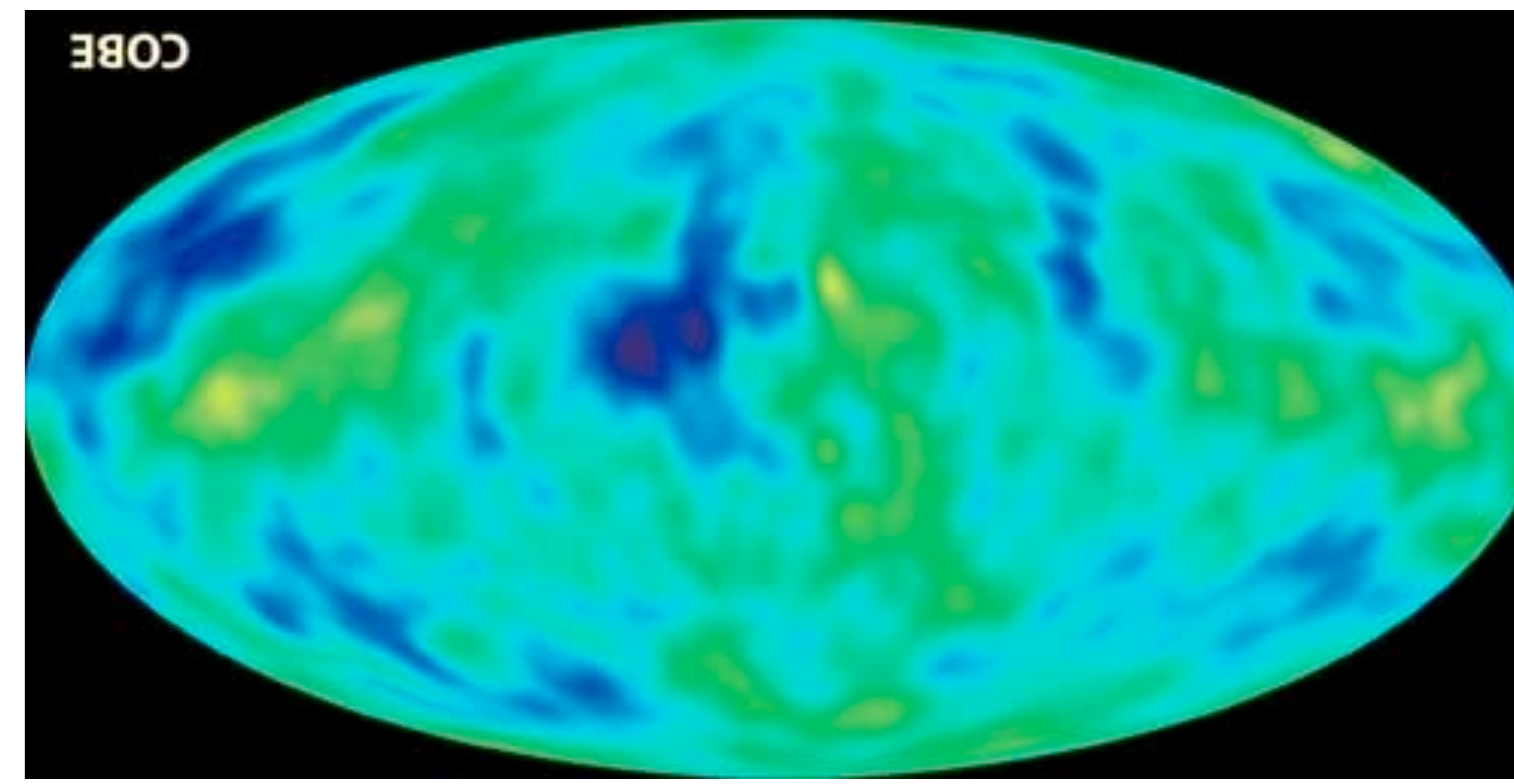
+



+

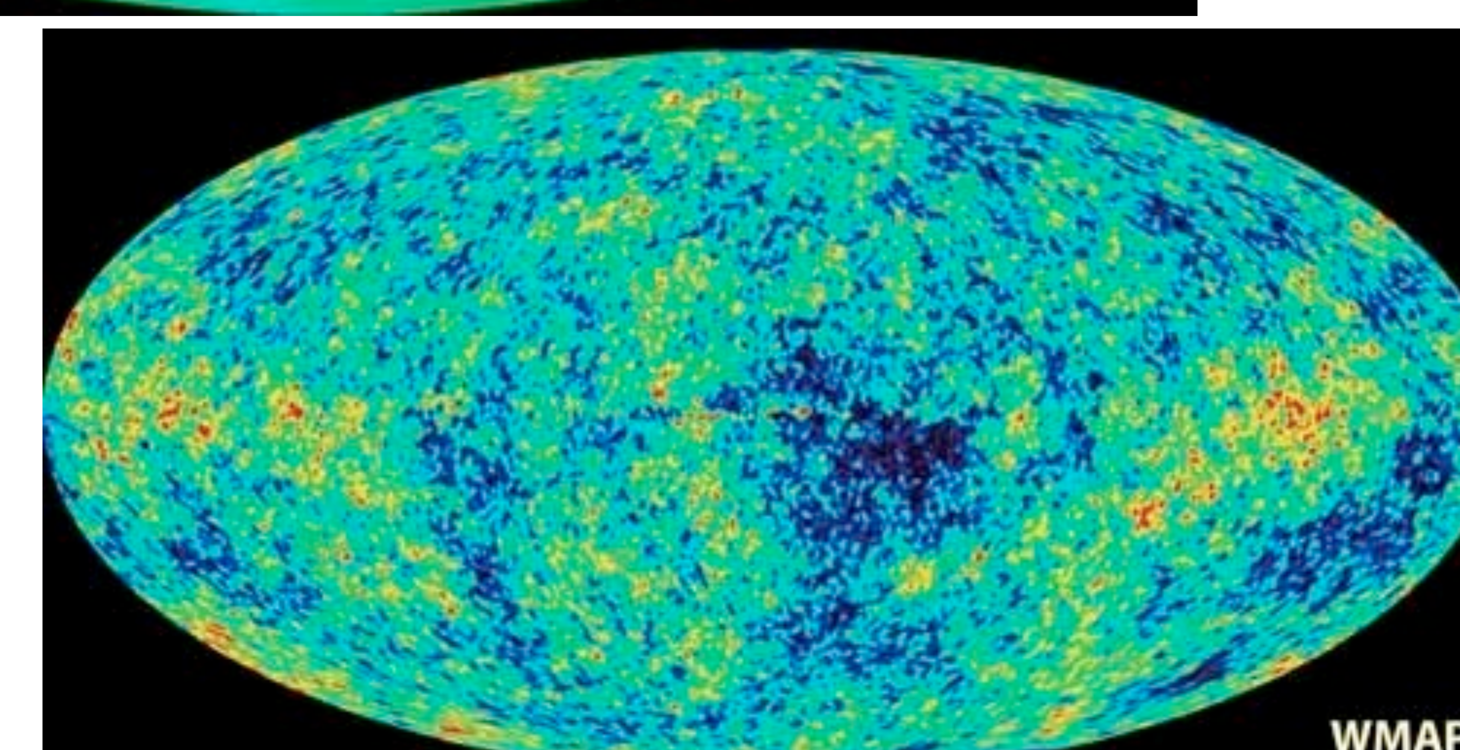


+



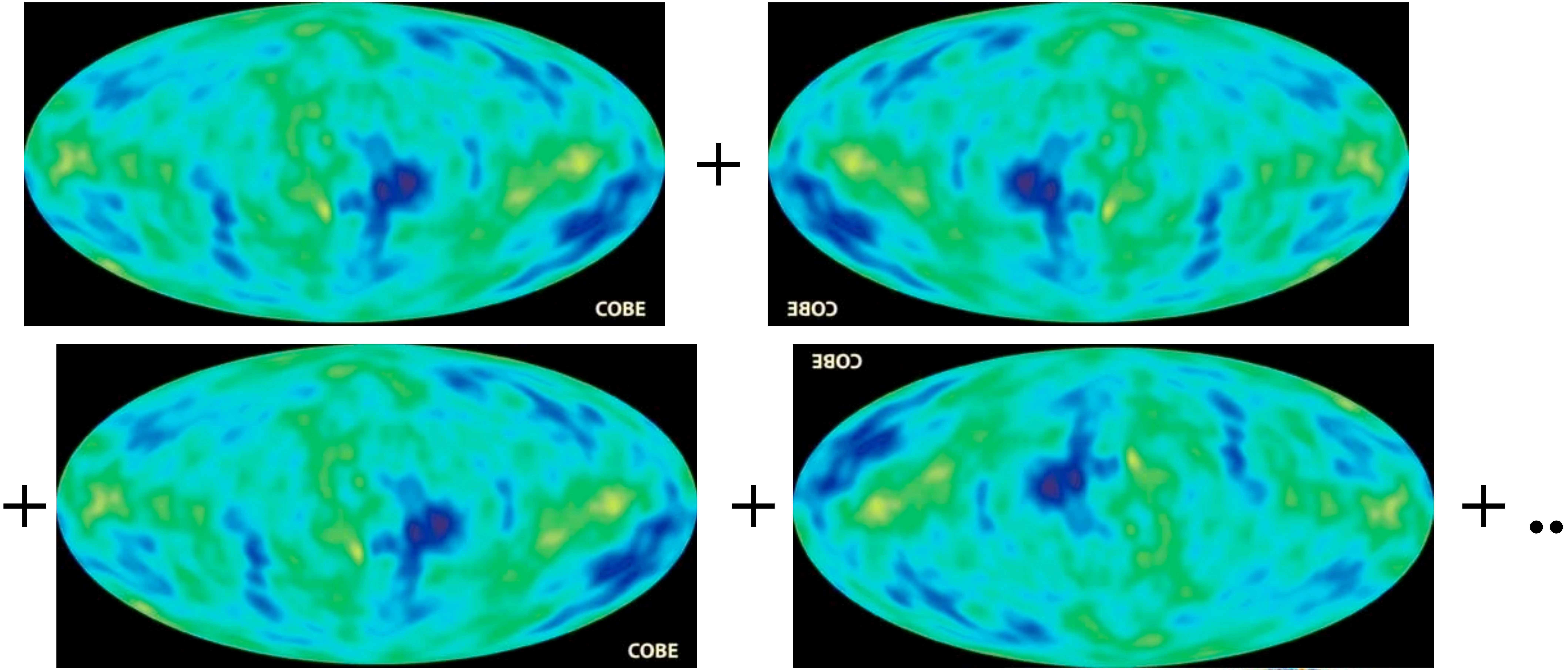
+ ...

Superposition



Collapse in 1992 ???

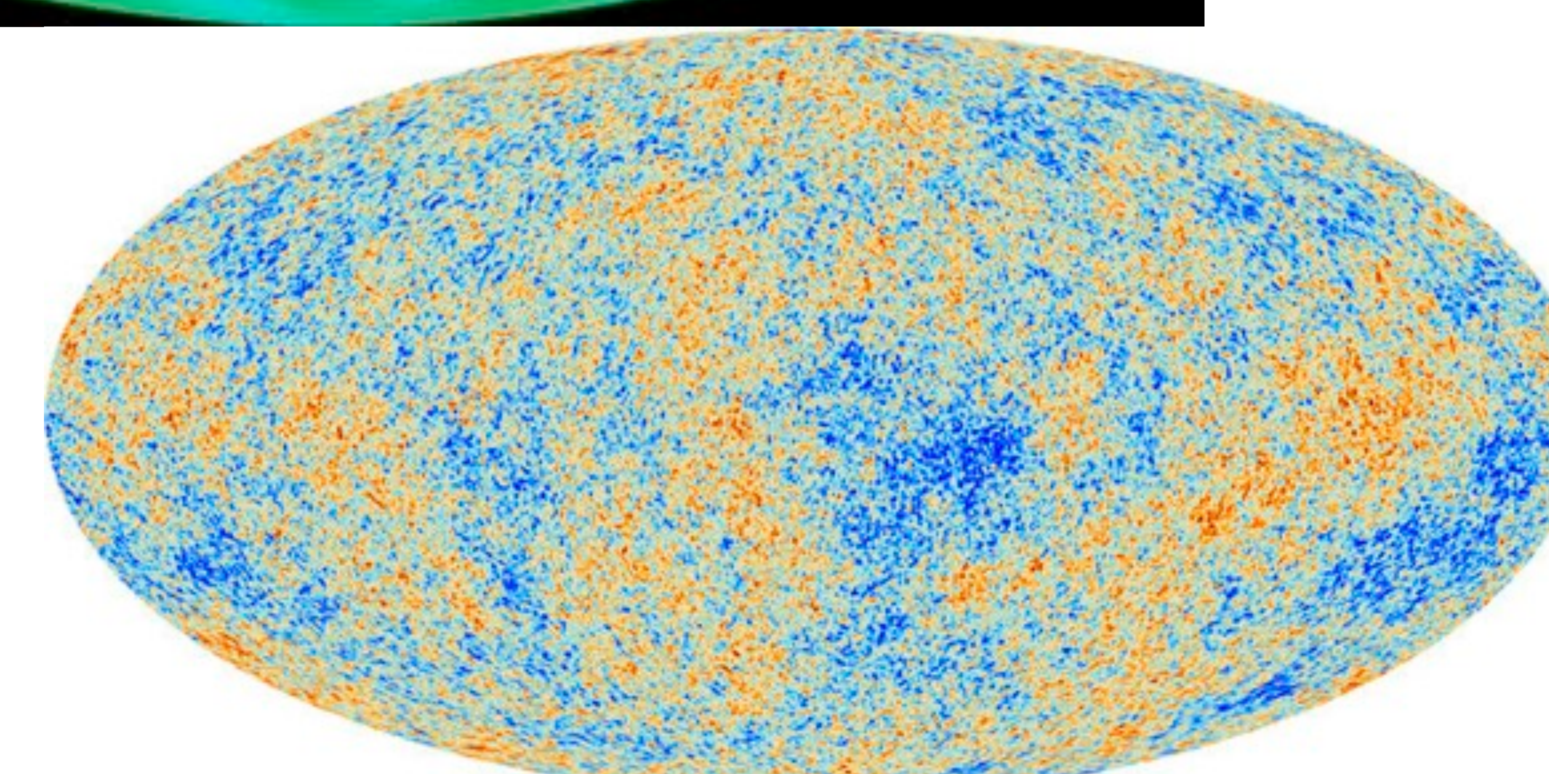
Further collapse in 2003
on smaller scales???



Superposition

Final (ultimate!) collapse in 2012?

Collapse in 1992 ???



- Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order $\mathcal{S}_{\mathrm{E-H}} = \int \mathrm{d}^4x \left[R^{(0)} + \delta^{(2)} R\right]$

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a(t)^{-1}\mathrm{d}t$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\,\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

Mukhanov-Sasaki variable

$$\int \mathrm{d}^4x\,\delta^{(2)}\mathcal{L} = \frac{1}{2}\int \sqrt{\gamma}\,\mathrm{d}^3\boldsymbol{x}\,\mathrm{d}\eta\left[(\partial_\eta v)^2 - \gamma^{ij}\partial_i v\partial_j v + \frac{z''}{z}v^2\right]$$

V. F. Mukhanov, H. A. Feldman & R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992)

Simple scalar field with varying mass in Minkowski space!!! $z = z[a(\eta)]$

- Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order $\mathcal{S}_{\mathrm{E-H}} = \int \mathrm{d}^4 x \left[R^{(0)} + \delta^{(2)} R\right]$

Classical

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i \mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a(t)^{-1}\mathrm{d}t$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\, a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

Mukhanov-Sasaki variable

$$\int \mathrm{d}^4 x\, \delta^{(2)}\mathcal{L} = \frac{1}{2}\int \sqrt{\gamma}\mathrm{d}^3\boldsymbol{x}\,\mathrm{d}\eta \left[(\partial_\eta v)^2 - \gamma^{ij}\partial_i v \partial_j v + \frac{z''}{z}v^2\right]$$

V. F. Mukhanov, H. A. Feldman & R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992)

Simple scalar field with varying mass in Minkowski space!!! $z = z[a(\eta)]$

- Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order

$$\mathcal{S}_{\rm E-H} = \int \mathrm{d}^4 x \, \left[R^{(0)} + \delta^{(2)} R\right]$$

Classical ($R^{(0)}$) — Quantum ($\delta^{(2)} R$)

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta) \left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right] \mathrm{d}x^i \mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a(t)^{-1}\mathrm{d}t$

$$\Delta\Phi = -\frac{3\ell_{\rm Pl}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\, a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

v: Mukhanov-Sasaki variable

$$\int \mathrm{d}^4 x\, \delta^{(2)}\mathcal{L} = \frac{1}{2}\int \sqrt{\gamma}\mathrm{d}^3\boldsymbol{x}\,\mathrm{d}\eta\, \left[(\partial_\eta v)^2 - \gamma^{ij}\partial_i v \partial_j v + \frac{z''}{z} v^2\right]$$

V. F. Mukhanov, H. A. Feldman & R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992)

Simple scalar field with varying mass in Minkowski space!!! $z = z[a(\eta)]$

Self-consistent treatment of the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

$$\Delta\Phi = -\frac{3\ell_{\rm Pl}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a,T\right)\Psi_{(2)}\left[v,T;a\left(T\right)\right]$$

comes from 0th order

Self-consistent treatment of the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

$$\Delta\Phi = -\frac{3\ell_{\rm Pl}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a,T\right)\Psi_{(2)}\left[v,T;a\left(T\right)\right]$$

comes from 0th order

Use dBB or...

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Schrödinger equation

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Schrödinger equation $\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle\mathrm{d}t$

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Schrödinger equation $\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t$

Hamiltonian

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle\mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle$$

Hamiltonian

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right) \mathrm{d}W_t |\Psi\rangle$$

Hamiltonian

non linear

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle$$

Hamiltonian

non linear

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle$$

Hamiltonian

non linear

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

break superposition principle

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle$$

Hamiltonian

non linear stochastic

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t \mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

break superposition principle

Wiener process

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle\mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle$$

Hamiltonian

non linear stochastic → random outcomes → Born rule

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

break superposition principle

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t\mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

Wiener process

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle - \frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2 \mathrm{d}t|\Psi\rangle$$

Hamiltonian

non linear stochastic

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

random outcomes → Born rule

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t \mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

break superposition principle

Wiener process

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle\mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle - \frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2\mathrm{d}t|\Psi\rangle$$

Hamiltonian

normalization

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

non linear stochastic

random outcomes

Born rule

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t\mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

break superposition principle

Wiener process

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle - \frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2 \mathrm{d}t|\Psi\rangle$$

Hamiltonian

normalization

non linear stochastic → random outcomes → Born rule

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t \mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

break superposition principle

Wiener process

BONUS: Amplification mechanism

→ Big objects are classical
small objects are quantum!

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle - \frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2 \mathrm{d}t|\Psi\rangle$$

Hamiltonian

normalization

non linear stochastic

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

break superposition principle

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t \mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

Wiener process

random outcomes

Born rule

BONUS: Amplification mechanism

Big objects are classical
small objects are quantum!

Primordial perturbations

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle\mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle - \frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2\mathrm{d}t|\Psi\rangle$$

Hamiltonian

normalization

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

non linear stochastic → random outcomes → Born rule

break superposition principle

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t\mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

Wiener process

BONUS: Amplification mechanism

Big objects are classical
small objects are quantum!

Grown perturbations

Primordial perturbations

Year	first author [ref.]	interfering object	m/m_p	τ	d	in GRW $\lambda <$	in GRW $\lambda/\sigma^2 <$	in CSL $\lambda <$	in CSL $\lambda/\sigma^2 <$
1927	Davisson [13]	electron	5×10^{-4}	N/A	2×10^{-10}m	10^{14} s^{-1}	3×10^{33} m^{-2}s^{-1}	10^{17} s^{-1}	5×10^{36} m^{-2}s^{-1}
1930	Estermann [15]	He	4	N/A	4×10^{-10}m	10^{11} s^{-1}	6×10^{29} m^{-2}s^{-1}	3×10^{10} s^{-1}	10^{29} m^{-2}s^{-1}
1959	Möllenstedt [28]	electron	5×10^{-4}	3×10^{-9} s	2×10^{-6} m	7×10^{11} s^{-1}	10^{23} m^{-2}s^{-1}	10^{15} s^{-1}	3×10^{26} m^{-2}s^{-1}
1987	Tonomura [37]	electron	5×10^{-4}	10^{-8} s	10^{-4} m	2×10^{11} s^{-1}	2×10^{19} m^{-2}s^{-1}	4×10^{14} s^{-1}	4×10^{22} m^{-2}s^{-1}
1988	Zeilinger [40]	neutron	1	10^{-2} s	10^{-4} m	2×10^{2} s^{-1}	2×10^{10} m^{-2}s^{-1}	2×10^{2} s^{-1}	2×10^{10} m^{-2}s^{-1}
1991	Carnal [9]	He	4	6×10^{-4} s	10^{-5} m	4×10^{2} s^{-1}	4×10^{12} m^{-2}s^{-1}	10^{2} s^{-1}	10^{12} m^{-2}s^{-1}
1999	Arndt [4]	C_{60}	720	6×10^{-3} s	10^{-7} m	2×10^{-1}s^{-1}	2×10^{13} m^{-2}s^{-1}	3×10^{-4} s^{-1}	3×10^{10} m^{-2}s^{-1}
2001	Nairz [29]	C_{70}	840	10^{-2} s	3×10^{-7} m	10^{-1}s^{-1}	10^{12} m^{-2}s^{-1}	10^{-4} s^{-1}	10^{9} m^{-2}s^{-1}
2004	Hackermüller [24]	C_{70}	840	2×10^{-3} s	10^{-6} m	10^{0} s^{-1}	10^{12} m^{-2}s^{-1}	10^{-3} s^{-1}	10^{9} m^{-2}s^{-1}
2007	Gerlich [17]	$C_{30}H_{12}F_{30}N_2O_4$	10^3	10^{-3} s	3×10^{-7} m	10^{0} s^{-1}	10^{13} m^{-2}s^{-1}	10^{-3} s^{-1}	10^{10} m^{-2}s^{-1}
2011	Gerlich [18]	$C_{60}[C_{12}F_{25}]_{10}$	7×10^3	10^{-3} s	3×10^{-7} m	10^{-1}s^{-1}	10^{12} m^{-2}s^{-1}	10^{-5} s^{-1}	10^{8} m^{-2}s^{-1}
	Proposed future experiments								
	Romero-Isart [35]	$[SiO_2]_{150,000}$	10^7	10^{-1} s	4×10^{-7} m	10^{-6}s^{-1}	6×10^{6} m^{-2}s^{-1}	10^{-13}s^{-1}	6×10^{-1} m^{-2}s^{-1}
	Nimmrichter [30]	$Au_{500,000}$	10^8	6×10^{0} s	10^{-7} m	2×10^{-9}s^{-1}	2×10^{5} m^{-2}s^{-1}	2×10^{-17}s^{-1}	2×10^{-3} m^{-2}s^{-1}

Table 1: Bounds on σ, λ obtained from different diffraction experiments. For each experiment, m = mass of the interfering object, m_p = proton mass, τ = time of flight between grating and image plane, d = period of grating (or transverse coherence length in [37]), N/A = not applicable. For each theory (GRW or CSL), two bounds are obtained. This table is the basis for Fig. 3.

Feldmann & Tumulka (2011)

→ γ constrained...

Year	first author [ref.]	interfering object	m/m_p	τ	d	in GRW $\lambda <$	in GRW $\lambda/\sigma^2 <$	in CSL $\lambda <$	in CSL $\lambda/\sigma^2 <$
1927	Davisson [13]	electron	5×10^{-4}	N/A	2×10^{-10}m	10^{14} s^{-1}	3×10^{33} $m^{-2}s^{-1}$	10^{17} s^{-1}	5×10^{36} $m^{-2}s^{-1}$
1930	Estermann [15]	He	4	N/A	4×10^{-10}m	10^{11} s^{-1}	6×10^{29} $m^{-2}s^{-1}$	3×10^{10} s^{-1}	10^{29} $m^{-2}s^{-1}$
1959	Möllenstedt [28]	electron	5×10^{-4}	3×10^{-9} s	2×10^{-6} m	7×10^{11} s^{-1}	10^{23} $m^{-2}s^{-1}$	10^{15} s^{-1}	3×10^{26} $m^{-2}s^{-1}$
1987	Tonomura [37]	electron	5×10^{-4}	10^{-8} s	10^{-4} m	2×10^{11} s^{-1}	2×10^{19} $m^{-2}s^{-1}$	4×10^{14} s^{-1}	4×10^{22} $m^{-2}s^{-1}$
1988	Zeilinger [40]	neutron	1	10^{-2} s	10^{-4} m	2×10^{2} s^{-1}	2×10^{10} $m^{-2}s^{-1}$	2×10^{2} s^{-1}	2×10^{10} $m^{-2}s^{-1}$
1991	Carnal [9]	He	4	6×10^{-4} s	10^{-5} m	4×10^{2} s^{-1}	4×10^{12} $m^{-2}s^{-1}$	10^{2} s^{-1}	10^{12} $m^{-2}s^{-1}$
1999	Arndt [4]	C_{60}	720	6×10^{-3} s	10^{-7} m	2×10^{-1} s^{-1}	2×10^{13} $m^{-2}s^{-1}$	3×10^{-4} s^{-1}	3×10^{10} $m^{-2}s^{-1}$
2001	Nairz [29]	C_{70}	840	10^{-2} s	3×10^{-7} m	10^{-1} s^{-1}	10^{12} $m^{-2}s^{-1}$	10^{-4} s^{-1}	10^{9} $m^{-2}s^{-1}$
2004	Hackermüller [24]	C_{70}	840	2×10^{-3} s	10^{-6} m	10^{0} s^{-1}	10^{12} $m^{-2}s^{-1}$	10^{-3} s^{-1}	10^{9} $m^{-2}s^{-1}$
2007	Gerlich [17]	$C_{30}H_{12}F_{30}N_2O_4$	10^3	10^{-3} s	3×10^{-7} m	10^{0} s^{-1}	10^{13} $m^{-2}s^{-1}$	10^{-3} s^{-1}	10^{10} $m^{-2}s^{-1}$
2011	Gerlich [18]	$C_{60}[C_{12}F_{25}]_{10}$	7×10^3	10^{-3} s	3×10^{-7} m	10^{-1} s^{-1}	10^{12} $m^{-2}s^{-1}$	10^{-5} s^{-1}	10^{8} $m^{-2}s^{-1}$
	Proposed future experiments								
	Romero-Isart [35]	$[SiO_2]_{150,000}$	10^7	10^{-1} s	4×10^{-7} m	10^{-6} s^{-1}	6×10^{6} $m^{-2}s^{-1}$	10^{-13} s^{-1}	6×10^{-1} $m^{-2}s^{-1}$
	Nimmrichter [30]	$Au_{500,000}$	10^8	6×10^{0} s	10^{-7} m	2×10^{-9} s^{-1}	2×10^{5} $m^{-2}s^{-1}$	2×10^{-17} s^{-1}	2×10^{-3} $m^{-2}s^{-1}$

Table 1: Bounds on σ, λ obtained from different diffraction experiments. For each experiment, m = mass of the interfering object, m_p = proton mass, τ = time of flight between grating and image plane, d = period of grating (or transverse coherence length in [37]), N/A = not applicable. For each theory (GRW or CSL), two bounds are obtained. This table is the basis for Fig. 3.

Feldmann & Tumulka (2011)

γ constrained...

Example: free particle evolution $\hat{H} = \frac{\hat{p}^2}{2m}$

and projection on position operator $\hat{C} = \hat{x}$

initial double gaussian wave function

$\Psi^*\Psi$

x

$\Psi^*\Psi$

x

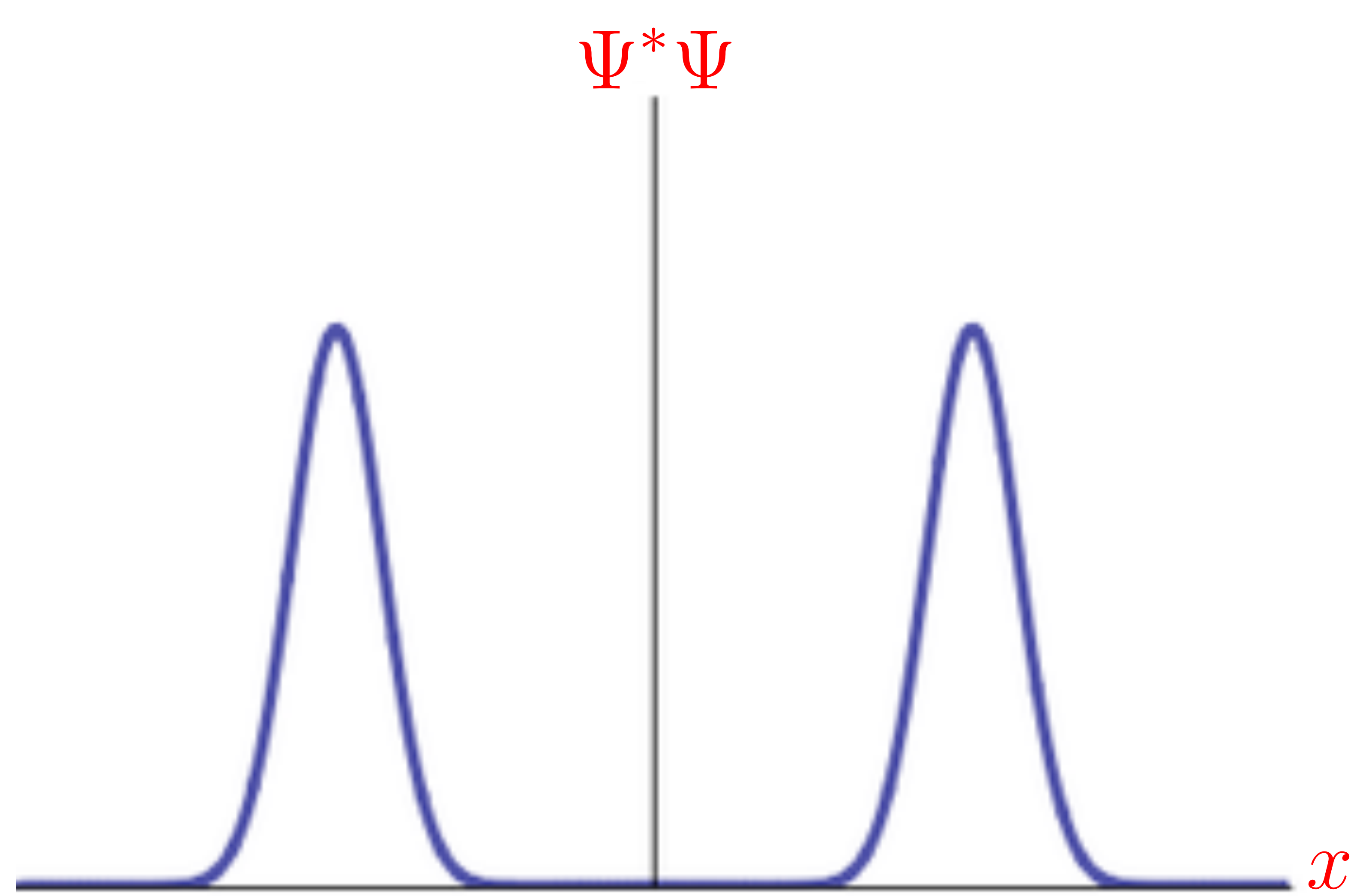
standard wave function
time evolution

modified wave function
time evolution with collapse

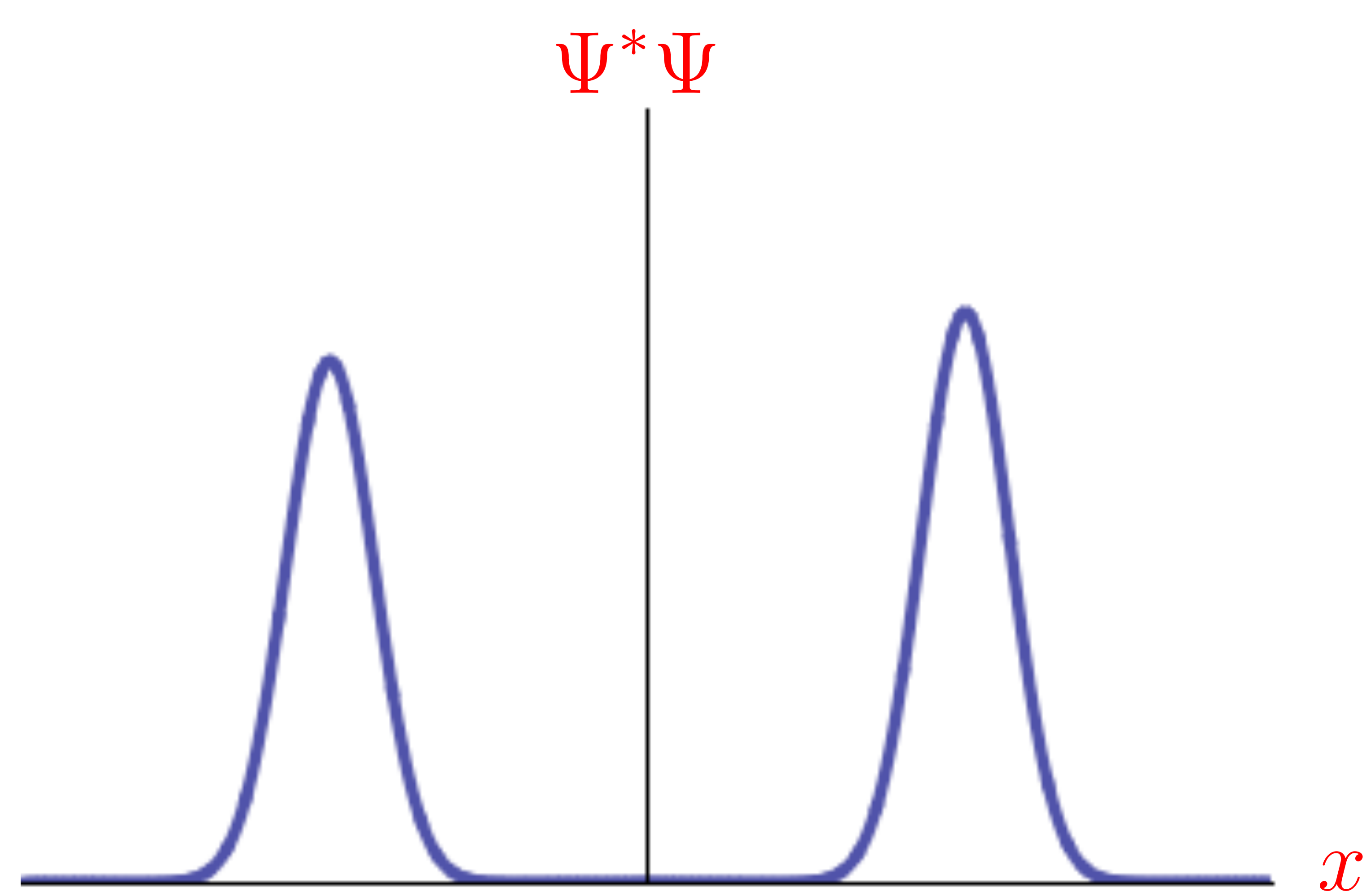
Example: free particle evolution $\hat{H} = \frac{\hat{p}^2}{2m}$

and projection on position operator $\hat{C} = \hat{x}$

initial double gaussian wave function



standard wave function
time evolution

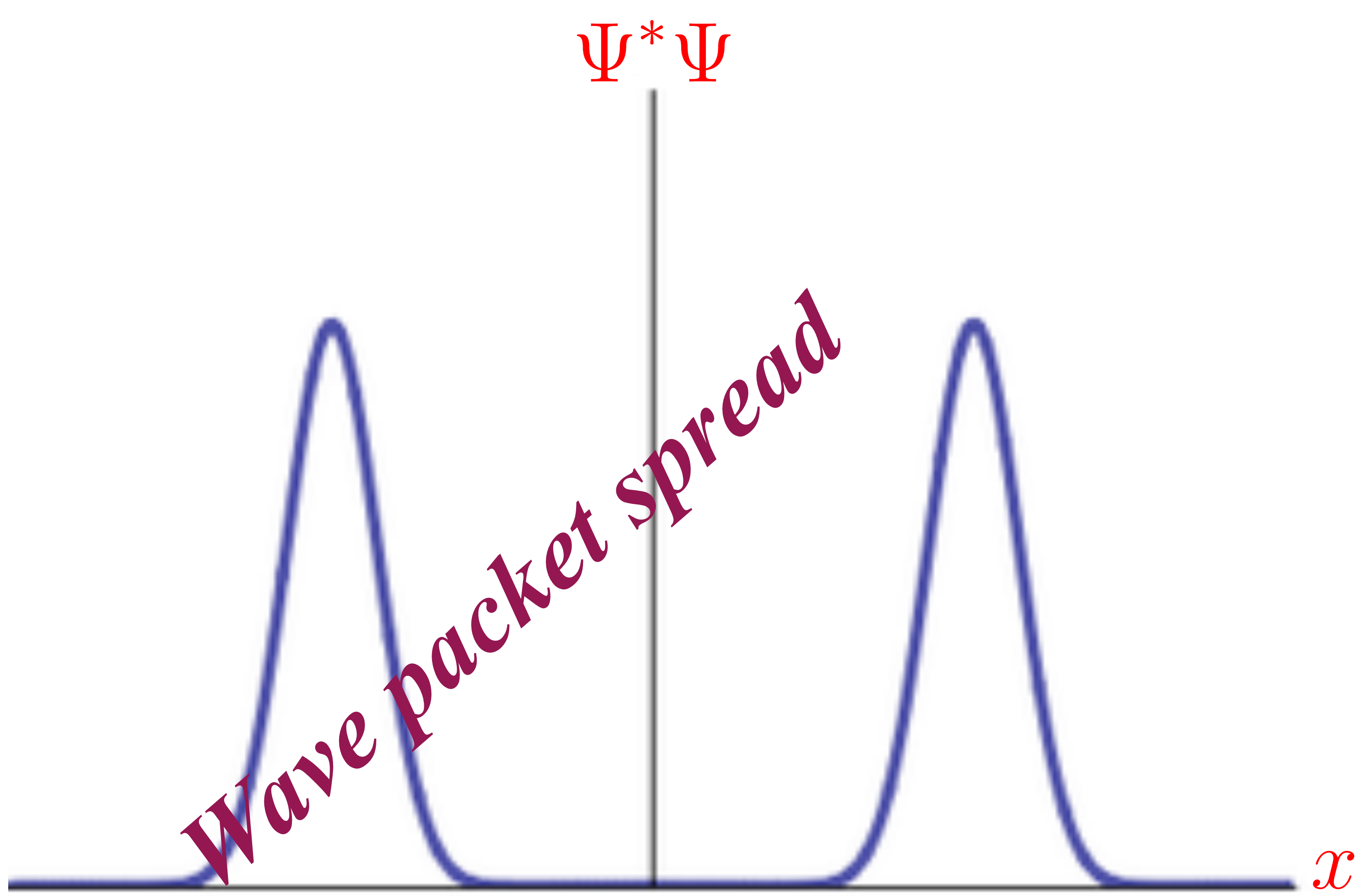


modified wave function
time evolution with collapse

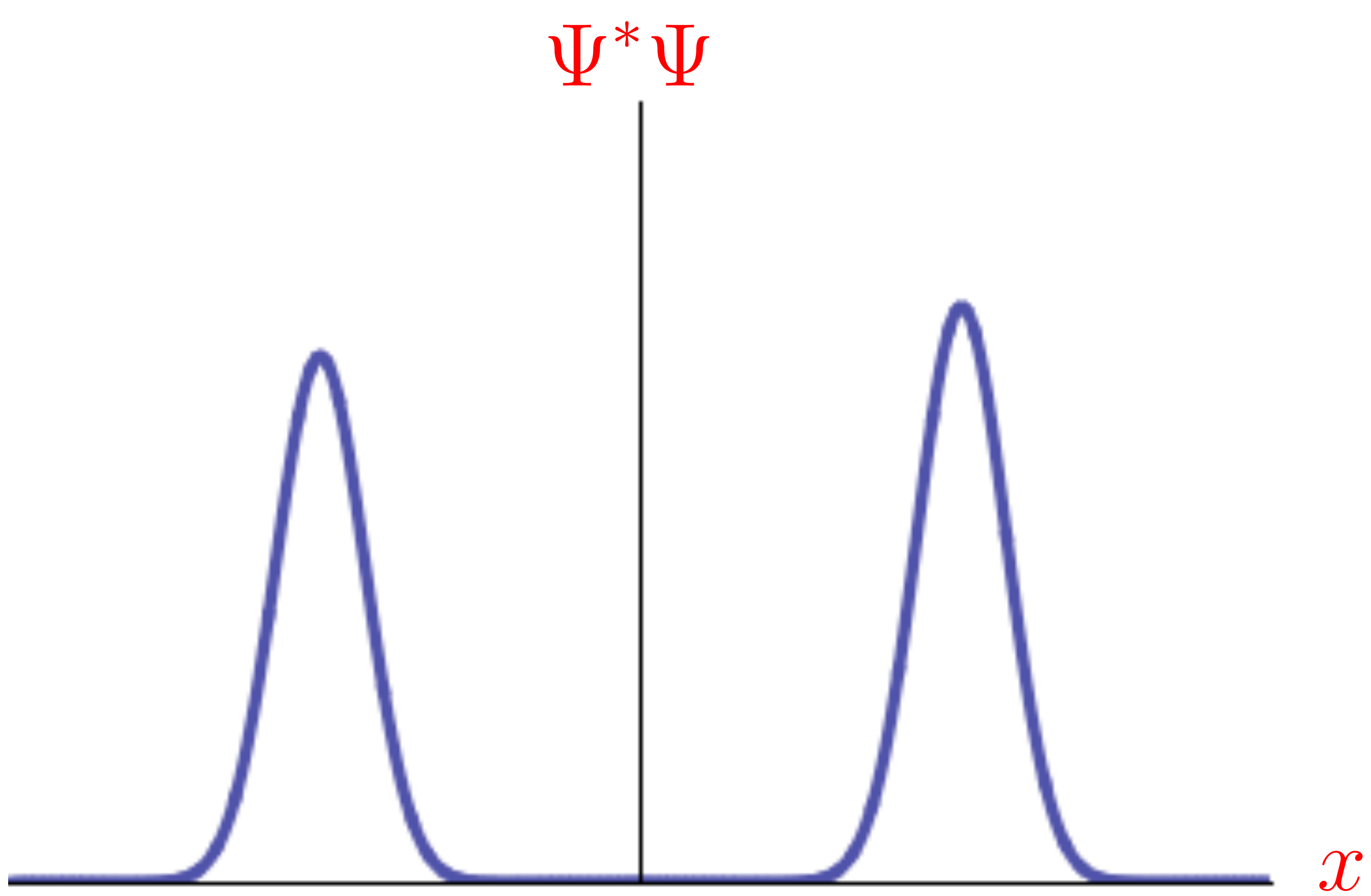
Example: free particle evolution $\hat{H} = \frac{\hat{p}^2}{2m}$

and projection on position operator $\hat{C} = \hat{x}$

initial double gaussian wave function



standard wave function
time evolution

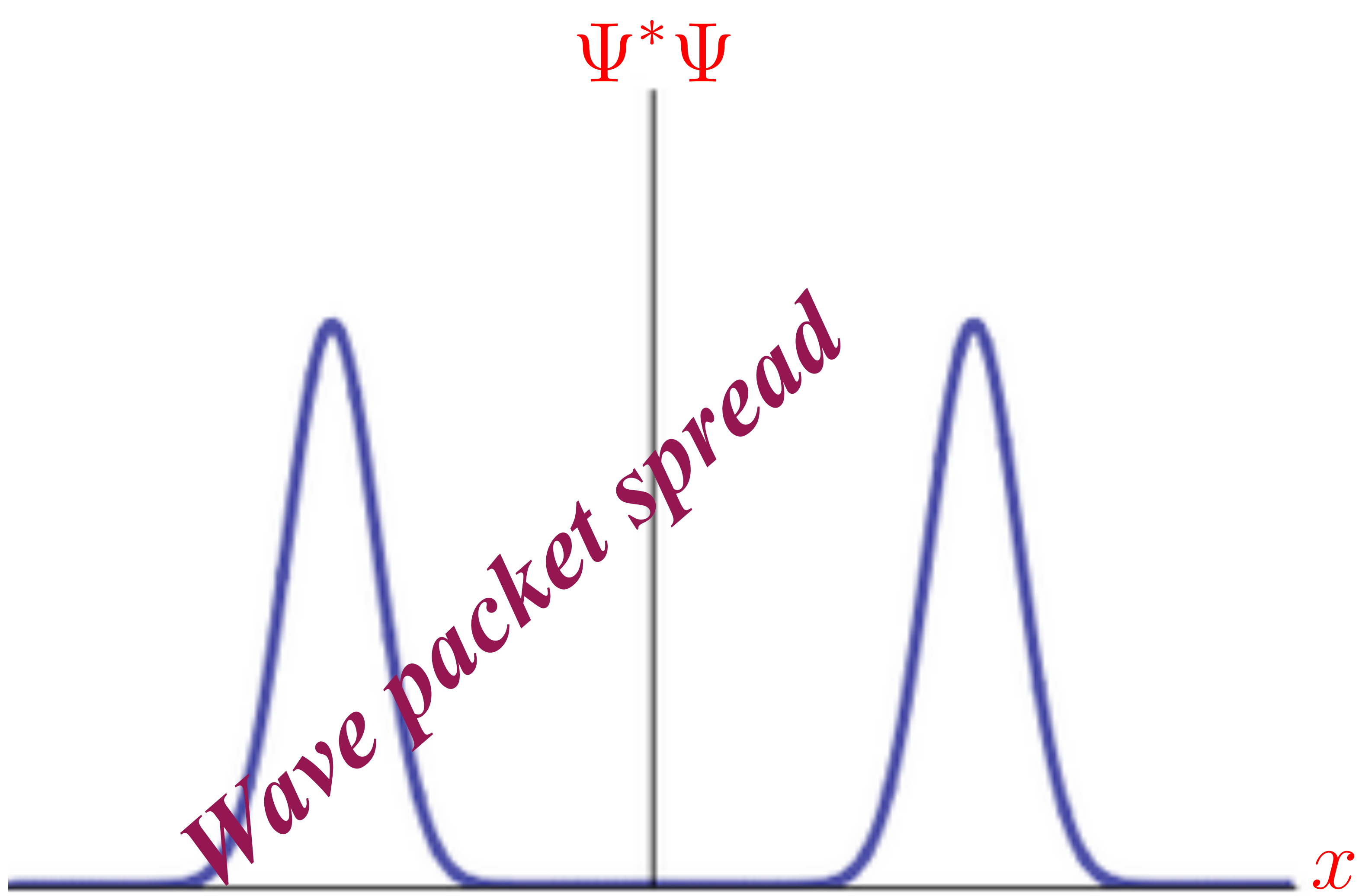


modified wave function
time evolution with collapse

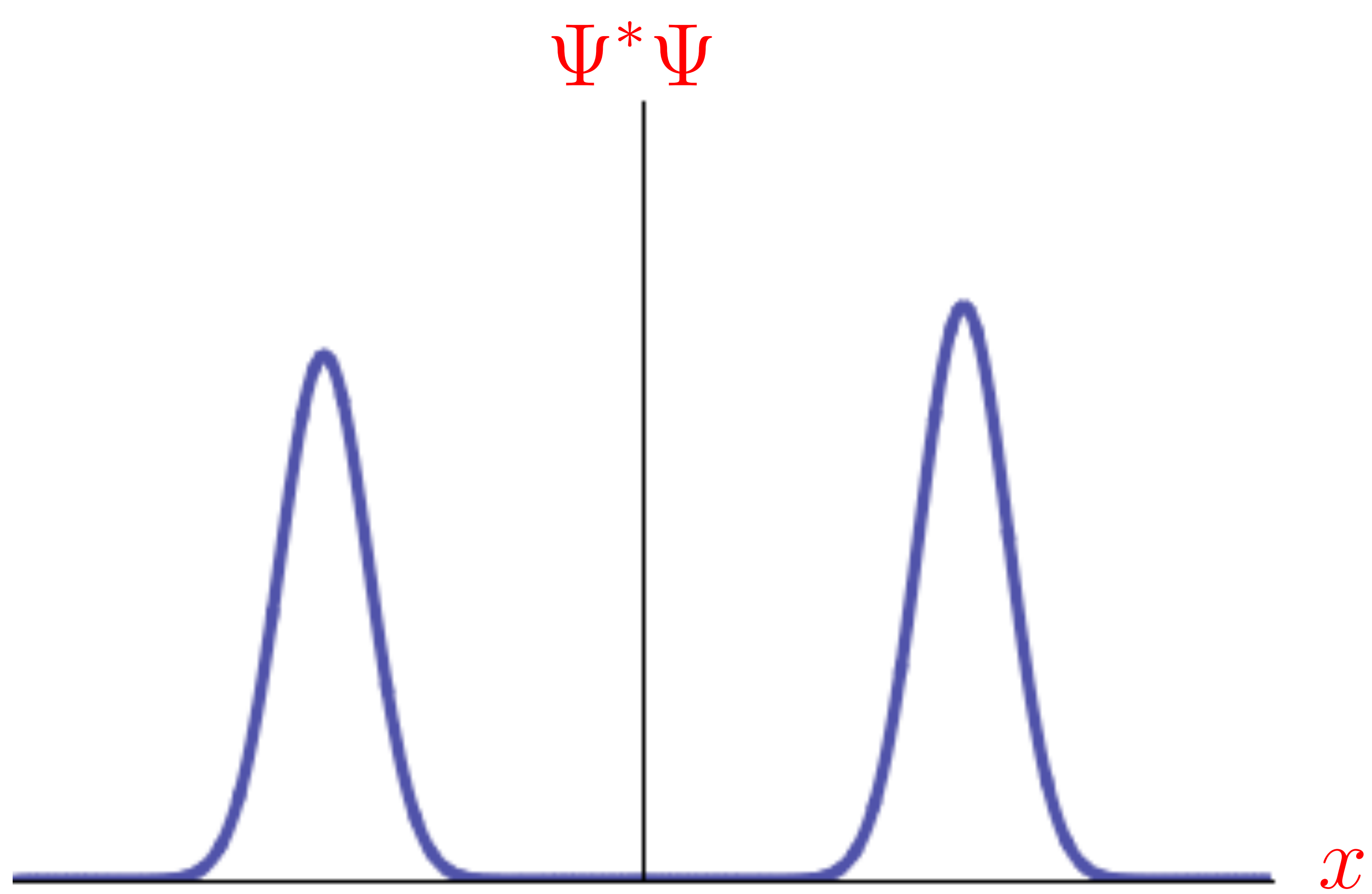
Example: free particle evolution $\hat{H} = \frac{\hat{p}^2}{2m}$

and projection on position operator $\hat{C} = \hat{x}$

initial double gaussian wave function



standard wave function
time evolution



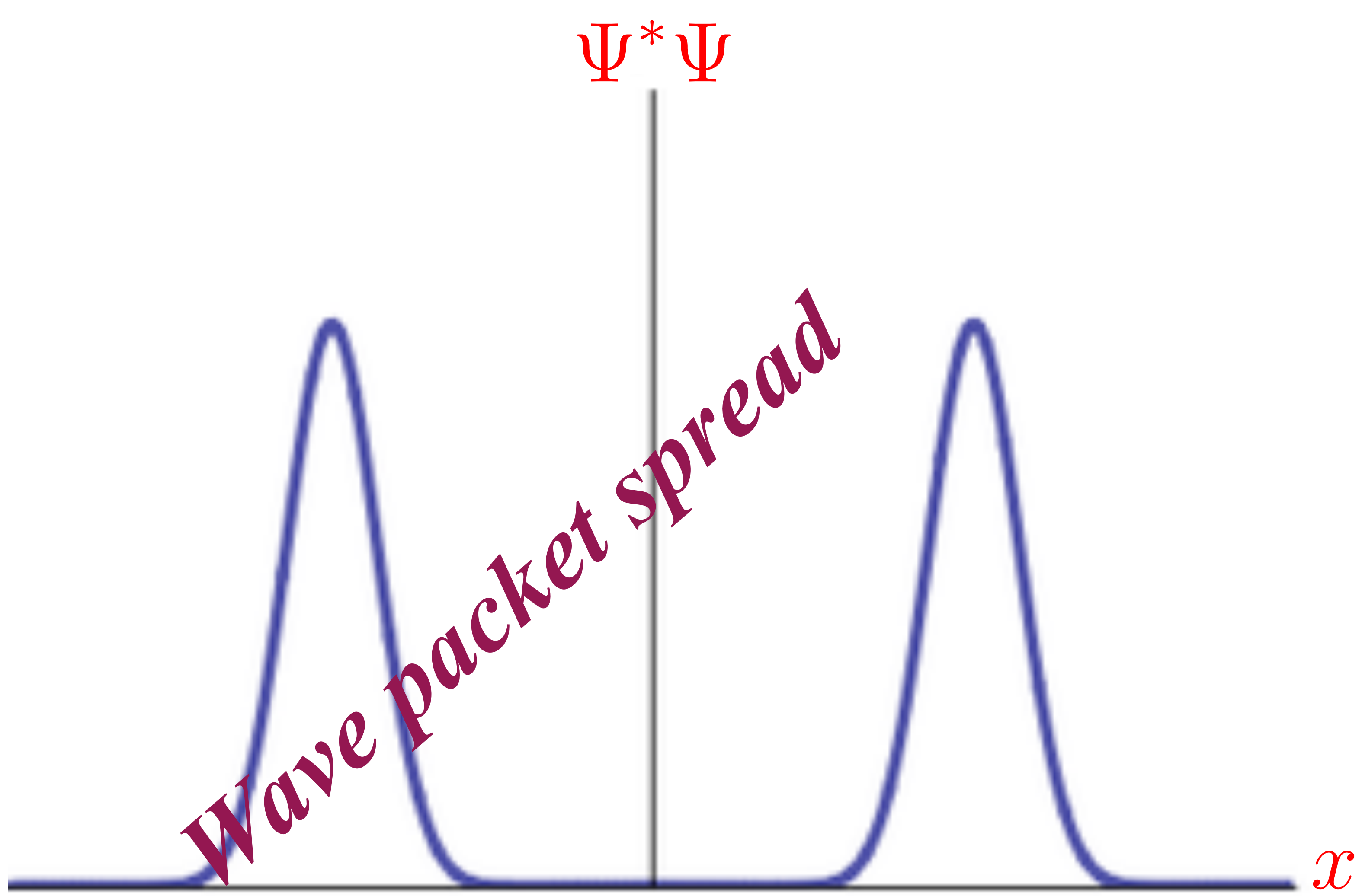
modified wave function
time evolution with collapse

Example: free particle evolution $\hat{H} = \frac{\hat{p}^2}{2m}$

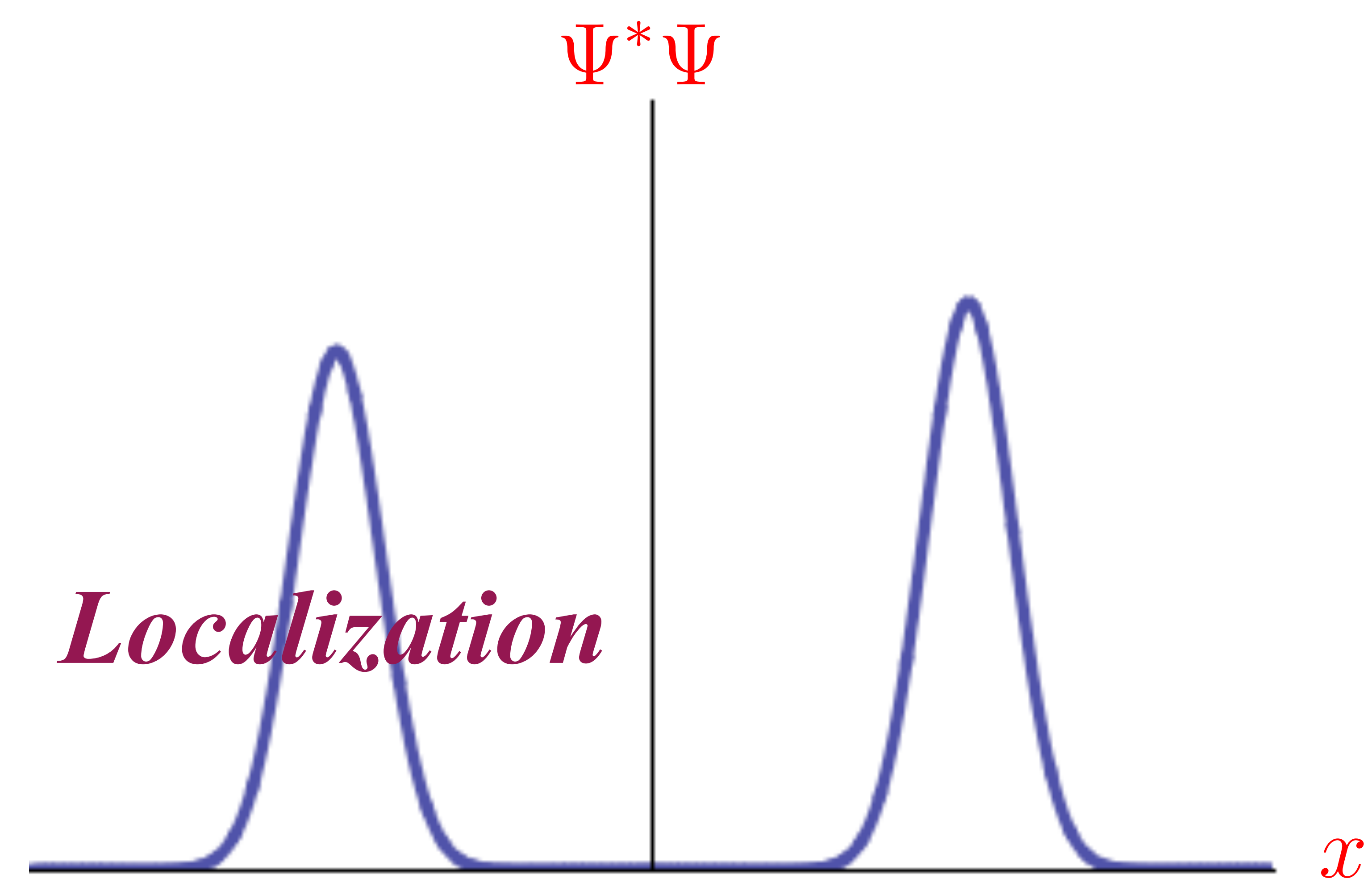
Animations provided by V. Vennin... thx!

and projection on position operator $\hat{C} = \hat{x}$

initial double gaussian wave function



standard wave function
time evolution



modified wave function
time evolution with collapse

Spontaneous collapse amplification mechanism

N identical particles

collapse operator: $\hat{C} = \sum_{i=1}^{N} \hat{x}_i$ acting on $|\Psi(\{x_i\})\rangle = |\Psi_{\text{CM}}(R)\rangle \otimes |\Psi_{\text{rel}}(\{r_i\})\rangle$

Spontaneous collapse amplification mechanism

N identical particles

collapse operator: $\hat{C} = \sum_{i=1}^{N} \hat{x}_i$ acting on $|\Psi(\{x_i\})\rangle = |\Psi_{\text{CM}}(R)\rangle \otimes |\Psi_{\text{rel}}(\{r_i\})\rangle$

$$\mathrm{d}|\Psi_{\text{rel}}(\{r_i\})\rangle = \left\{ \left[-i\hat{H}_{\text{rel}} - \frac{\gamma}{2} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)^2 \right] \mathrm{d}t + \sqrt{\gamma} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)\, \mathrm{d}W_t^{(i)} \right\} |\Psi_{\text{rel}}(\{r_i\})\rangle$$

Spontaneous collapse amplification mechanism

N identical particles

collapse operator: $\hat{C} = \sum_{i=1}^{N} \hat{x}_i$ acting on $|\Psi(\{x_i\})\rangle = |\Psi_{\mathrm{CM}}(R)\rangle \otimes |\Psi_{\mathrm{rel}}(\{r_i\})\rangle$

$$\mathrm{d}|\Psi_{\mathrm{rel}}(\{r_i\})\rangle = \left\{ \left[-i\hat{H}_{\mathrm{rel}} - \frac{\gamma}{2} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)^2 \right] \mathrm{d}t + \sqrt{\gamma} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)\, \mathrm{d}W_t^{(i)} \right\} |\Psi_{\mathrm{rel}}(\{r_i\})\rangle$$

usual quantum behavior

Spontaneous collapse amplification mechanism

N identical particles

collapse operator: $\hat{C} = \sum_{i=1}^{N} \hat{x}_i$ acting on $|\Psi(\{x_i\})\rangle = |\Psi_{\text{CM}}(R)\rangle \otimes |\Psi_{\text{rel}}(\{r_i\})\rangle$

$$\mathrm{d}|\Psi_{\text{rel}}(\{r_i\})\rangle = \left\{ \left[-i\hat{H}_{\text{rel}} - \frac{\gamma}{2} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)^2 \right] \mathrm{d}t + \sqrt{\gamma} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)\, \mathrm{d}W_t^{(i)} \right\} |\Psi_{\text{rel}}(\{r_i\})\rangle$$

usual quantum behavior

$$\mathrm{d}|\Psi_{\text{CM}}(R)\rangle = \left\{ \left[-i\hat{H}_{\text{CM}} - \frac{N\gamma}{2} \left(\hat{R} - \langle \hat{R} \rangle \right)^2 \right] \mathrm{d}t + \sqrt{N\gamma} \left(\hat{R} - \langle \hat{R} \rangle \right) \mathrm{d}W_t \right\} |\Psi_{\text{CM}}(R)\rangle$$

Spontaneous collapse amplification mechanism

N identical particles

collapse operator: $\hat{C} = \sum_{i=1}^{N} \hat{x}_i$ acting on $|\Psi(\{x_i\})\rangle = |\Psi_{\text{CM}}(R)\rangle \otimes |\Psi_{\text{rel}}(\{r_i\})\rangle$

$$\mathrm{d}|\Psi_{\text{rel}}(\{r_i\})\rangle = \left\{ \left[-i\hat{H}_{\text{rel}} - \frac{\gamma}{2} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)^2 \right] \mathrm{d}t + \sqrt{\gamma} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)\, \mathrm{d}W_t^{(i)} \right\} |\Psi_{\text{rel}}(\{r_i\})\rangle$$

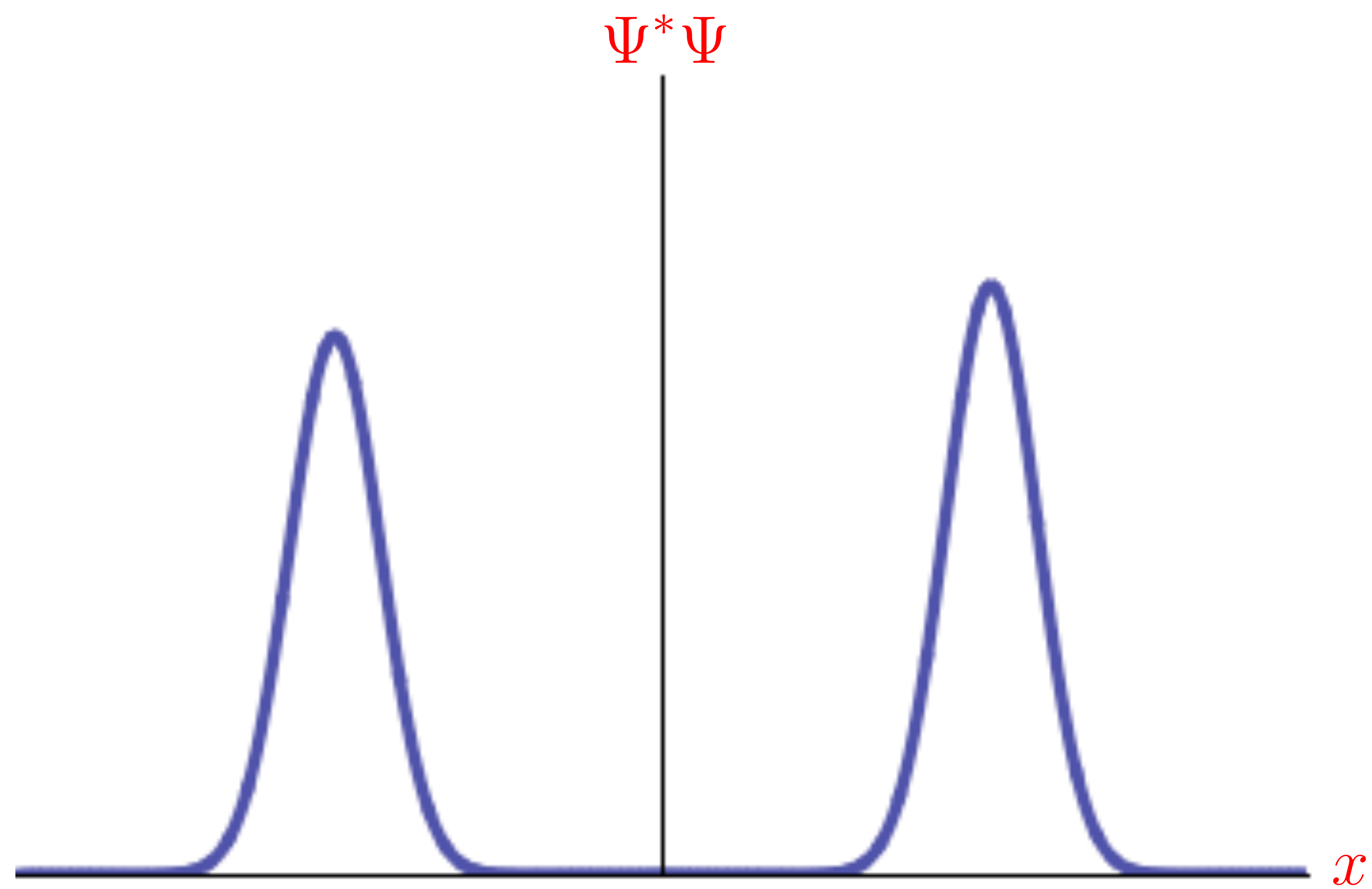
usual quantum behavior

$$\mathrm{d}|\Psi_{\text{CM}}(R)\rangle = \left\{ \left[-i\hat{H}_{\text{CM}} - \frac{N\gamma}{2} \left(\hat{R} - \langle \hat{R} \rangle\right)^2 \right] \mathrm{d}t + \sqrt{N\gamma} \left(\hat{R} - \langle \hat{R} \rangle\right) \mathrm{d}W_t \right\} |\Psi_{\text{CM}}(R)\rangle$$

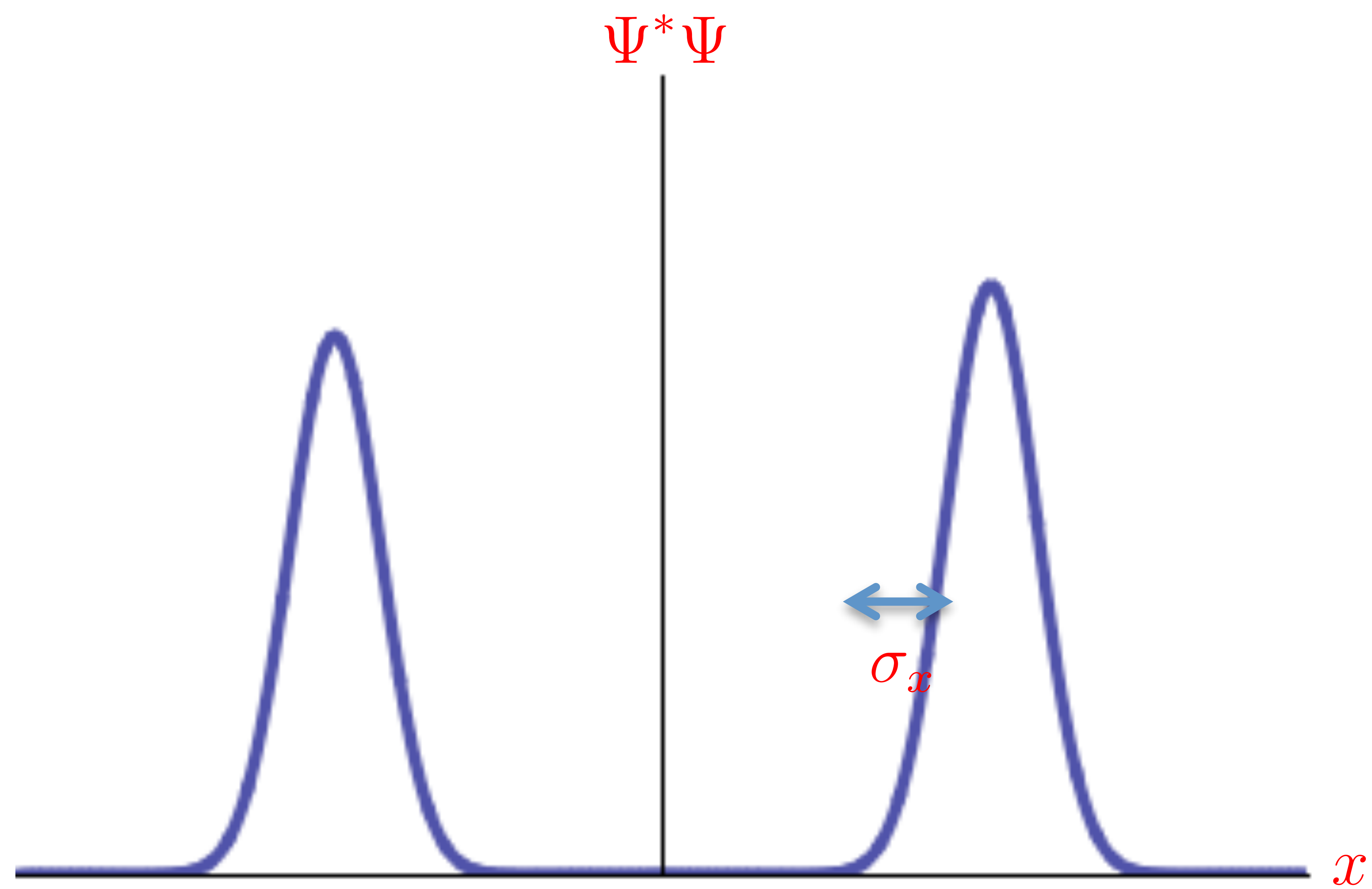
macro objectification

$\Psi^*\Psi$

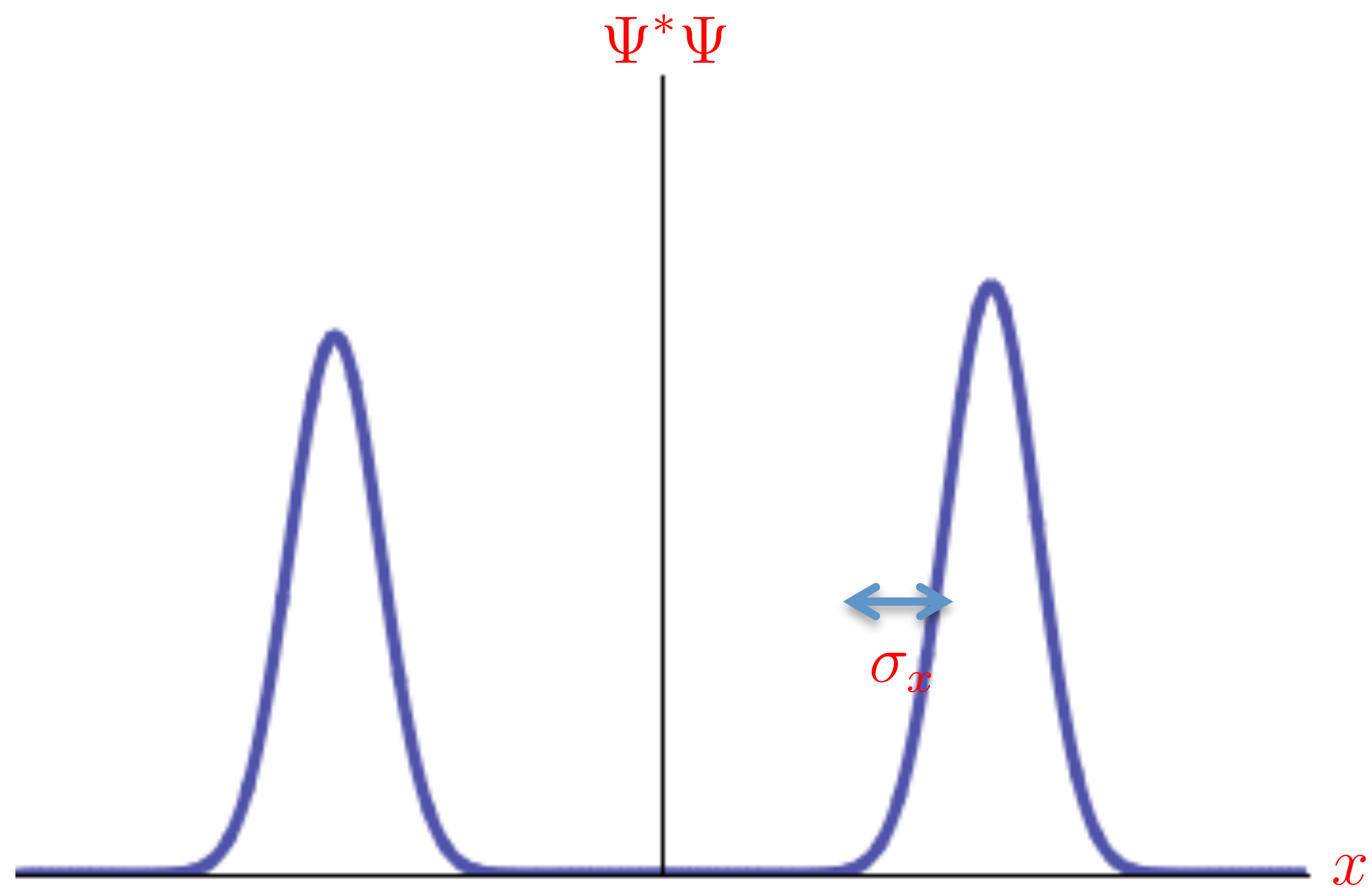
x


$\Psi^*\Psi$
x

Animations provided by V. Vennin... thx!

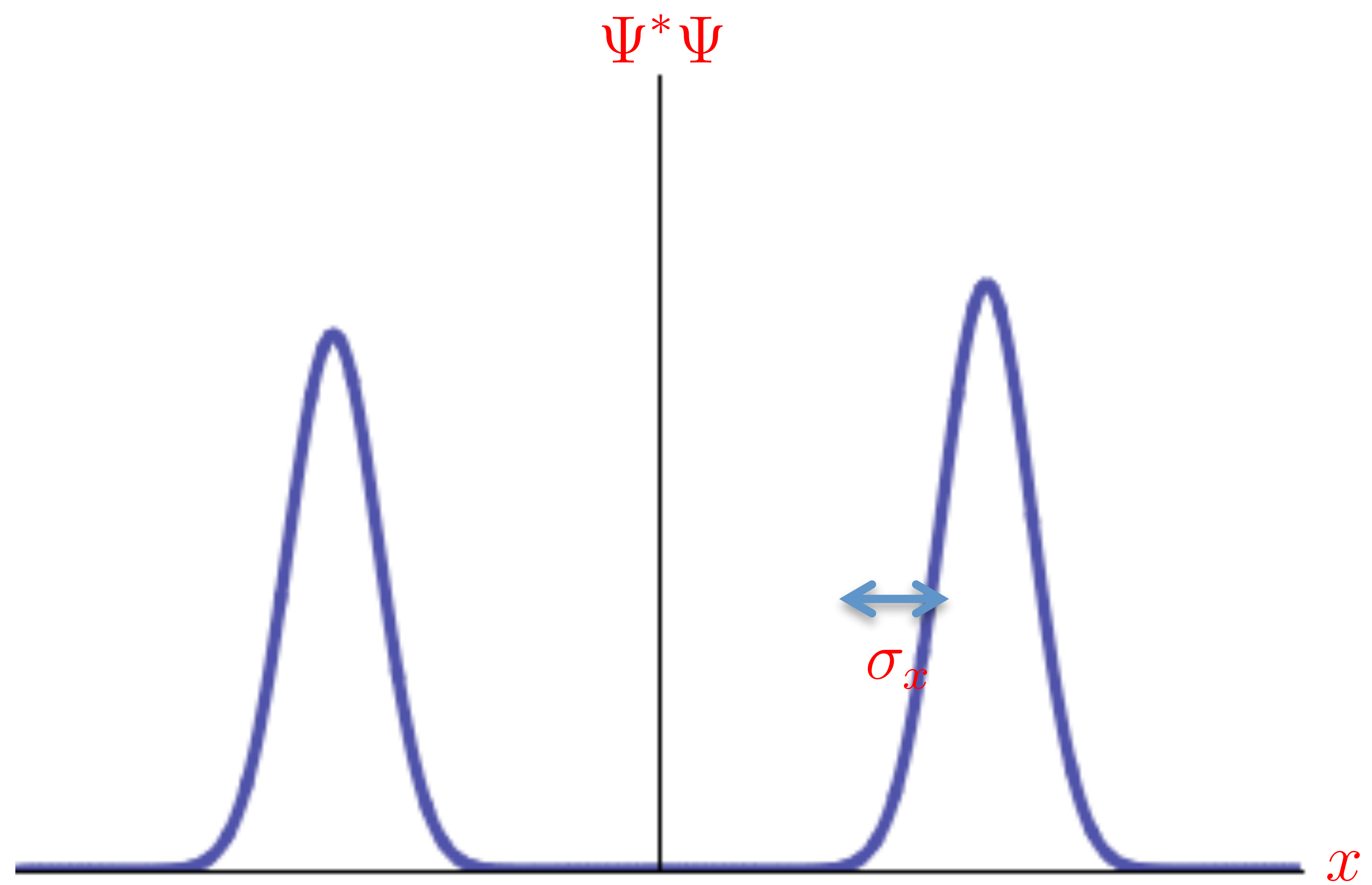


Animations provided by V. Vennin... thx!



Amplification mechanism $\Longrightarrow$ $\gamma \propto N$ (number of particles)

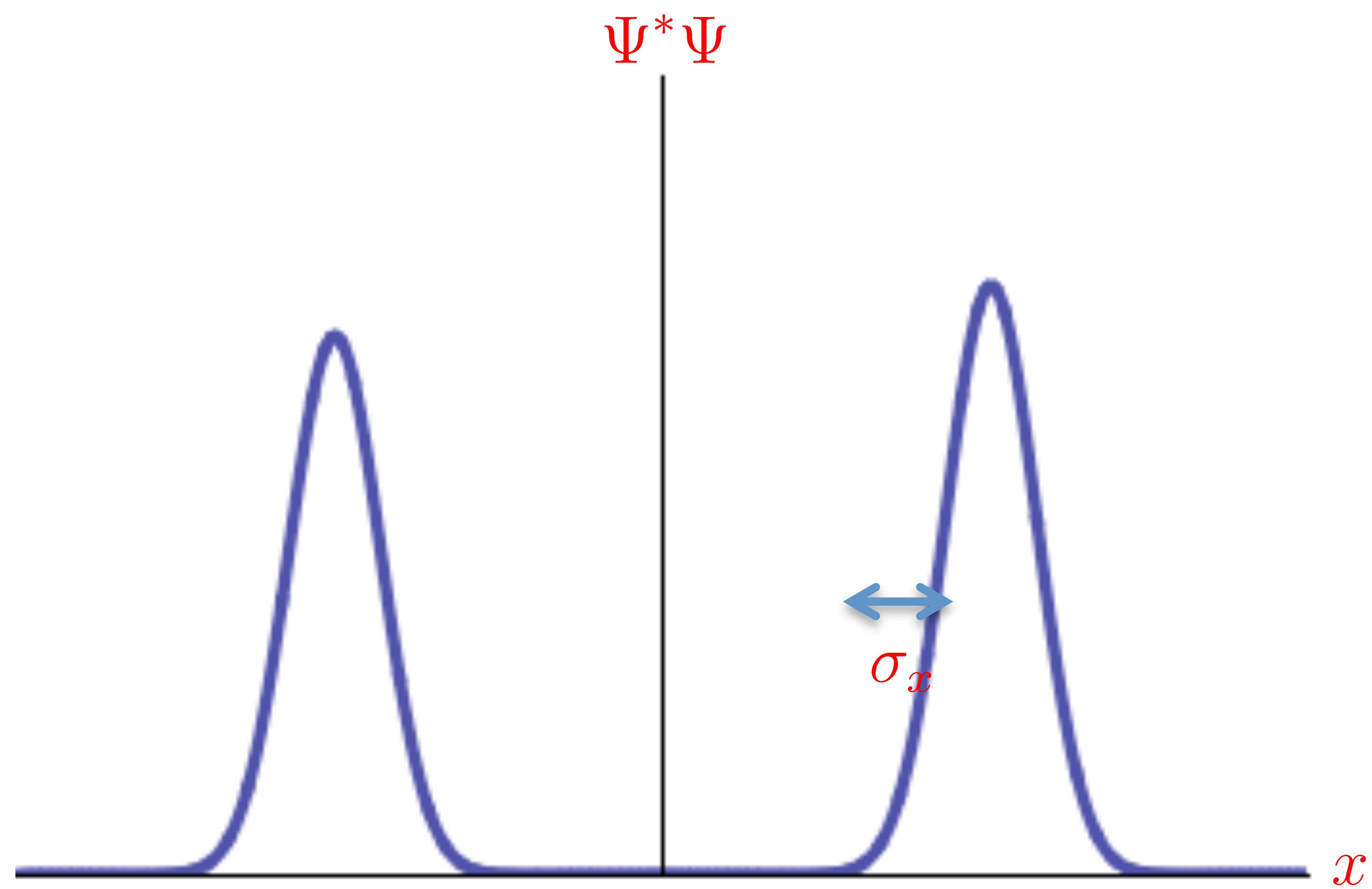
Animations provided by V. Vennin... thx!



Amplification mechanism $\Longrightarrow$ $\gamma \propto N$ (number of particles)

$$\sigma_x(\infty) = \left(\frac{\hbar}{4m\gamma}\right)^{\frac{1}{4}}$$

Animations provided by V. Vennin... thx!

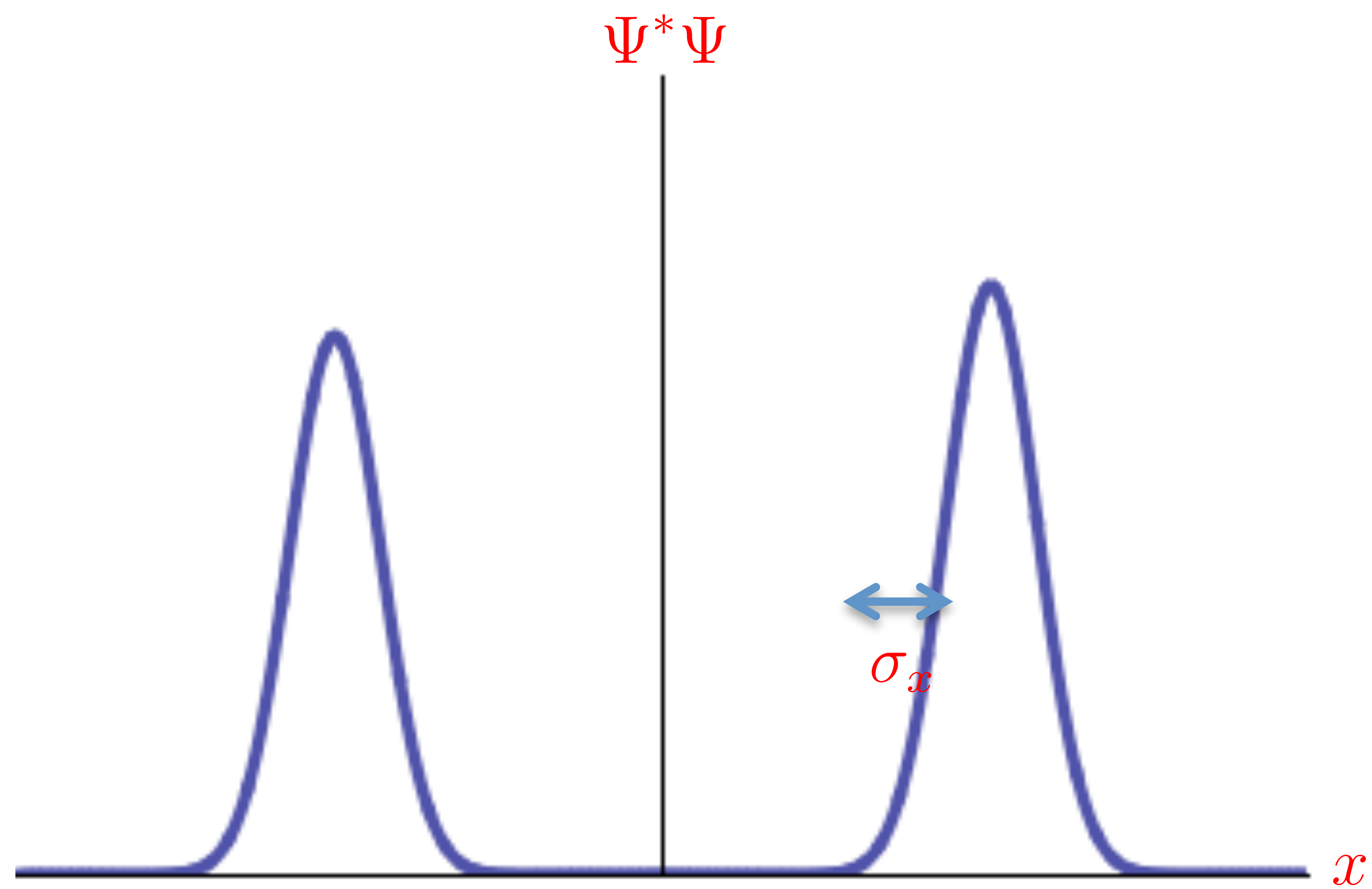


Amplification mechanism $\Longrightarrow$ $\gamma \propto N$ (number of particles)

$$\sigma_x(\infty) = \left(\frac{\hbar}{4m\gamma}\right)^{\frac{1}{4}}$$

→ 4.7 cm for a proton

Animations provided by V. Vennin... thx!



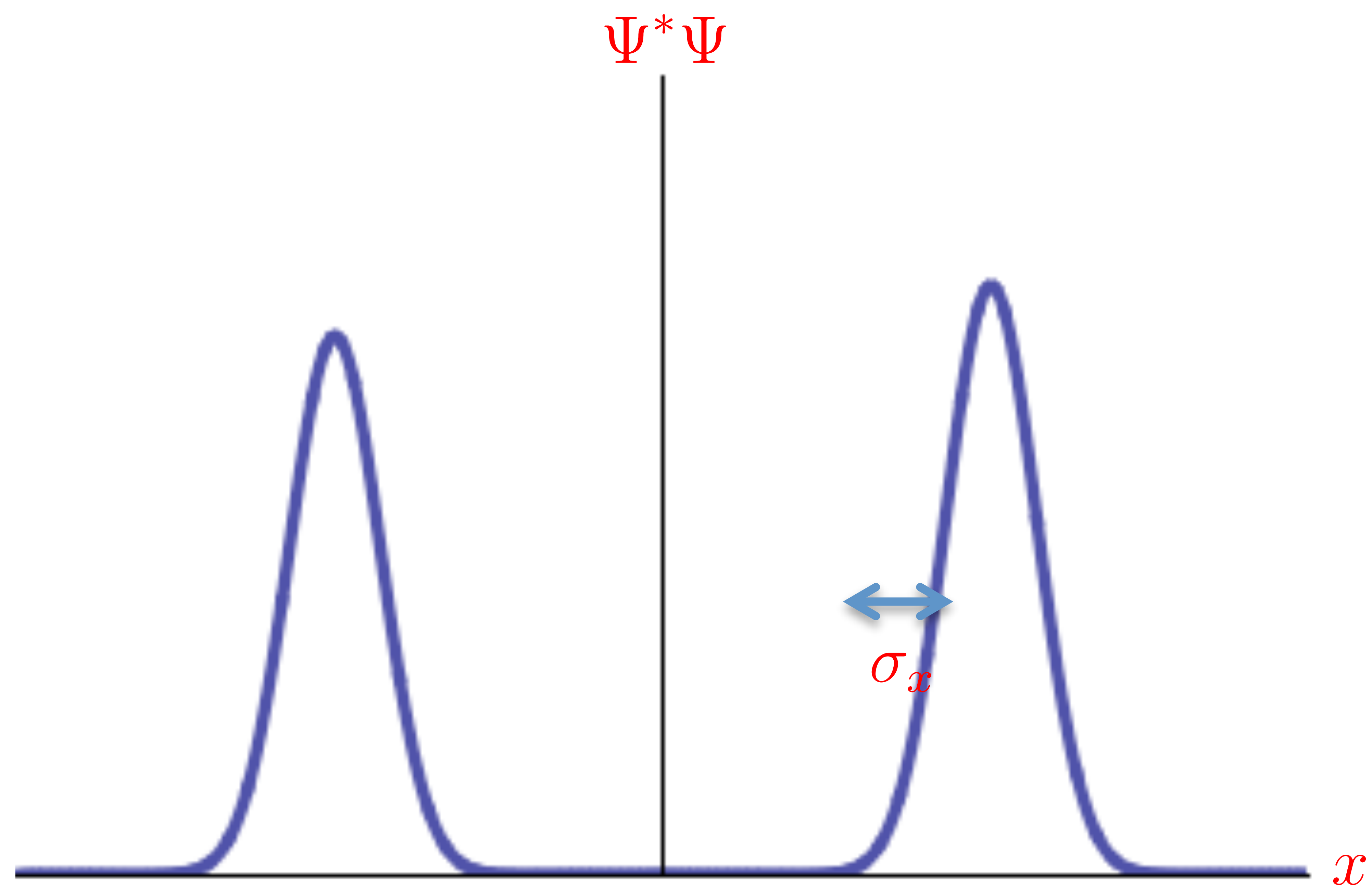
Amplification mechanism $\Longrightarrow$ $\gamma \propto N$ (number of particles)

$$\sigma_x(\infty) = \left(\frac{\hbar}{4m\gamma}\right)^{\frac{1}{4}}$$

$4.7\ \mathrm{cm}$ for a proton

$4.6 \times 10^{-14}\ \mathrm{m}$ for 1g object

Animations provided by V. Vennin... thx!



Amplification mechanism $\Longrightarrow$ $\gamma \propto N$ (number of particles)

$$\sigma_x(\infty) = \left(\frac{\hbar}{4m\gamma}\right)^{\frac{1}{4}}$$

→ 4.7 cm for a proton

→ 4.6×10^{-14} m for 1g object

→ 5.9×10^{-28} m for the Earth

Constraints:
(falsifiable theory!)

- Atomic energy levels
- Nuclear energy levels
- Diffraction Experiments
- Proton Decay
- Spontaneous Xray emission
- Spontaneous IGM warming
- Dissociation of cosmic H
- Decay of supercurrents
- Latent image formation
- Thermalized spectral distorsions
- Neutrino and kaon oscillations

Constraints:

(falsifiable theory!)

Cosmological perturbations: different test by orders of magnitude!

Constraints:

(falsifiable theory!)

Cosmological perturbations: different test by orders of magnitude!

↓

Measurement problem exacerbated

Classicalization of Cosmological Perturbations

Predictions of the theory: Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

Classicalization of Cosmological Perturbations

Predictions of the theory:

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

Usually in a lab:
repeat the experiment

Ensemble average over experiments ⟷ Quantum average

Classicalization of Cosmological Perturbations

Predictions of the theory:

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

Usually in a lab:
repeat the experiment

Ensemble average over experiments ⟷ Quantum average

Here one has a single experiment (a single universe)

→ Ergodicity

Spatial average over directions in the sky ⟷ Quantum average

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations

$$\frac{\Delta T}{T} \propto v$$

From quantum to classical cosmological perturbations?

Classical temperature fluctuations

$$\frac{\Delta T}{T} \propto v$$

From quantum to classical cosmological perturbations?

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v}$$

From quantum to classical cosmological perturbations?

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v}$$

second order perturbed Einstein action $\quad {}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4x \left[(v')^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2\right]$

From quantum to classical cosmological perturbations?

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v}$$

second order perturbed Einstein action $${}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4x \left[(v')^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2\right]$$

$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$
slow-roll parameter

From quantum to classical cosmological perturbations?

Classical temperature fluctuations promoted to quantum operators

$$\widehat{\frac{\Delta T}{T}} \propto \hat{v}$$

variable-mass scalar fields in Minkowski spacetime

second order perturbed Einstein action $\quad {}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4x \left[(v')^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2 \right]$

$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$

slow-roll parameter

From quantum to classical cosmological perturbations?

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v}$$

variable-mass scalar fields in Minkowski spacetime

second order perturbed Einstein action $${}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4 x \left[(v')^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2\right]$$

$$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$$

slow-roll parameter

+ Fourier transform $$v(\eta, \boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, v_{\boldsymbol{k}}(\eta) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\Rightarrow \quad {}^{(2)}\delta S = \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{k} \left\{ v'_{\boldsymbol{k}} v^{*\prime}_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[\frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} - k^2\right]\right\}$$

Lagrangian formulation...

Hamiltonian

$\omega^2(\eta, \boldsymbol{k})$

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

$$\Psi\left[v(\eta, \boldsymbol{x})\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}, v_{\boldsymbol{k}}^{\mathrm{I}}\right) = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^{\mathrm{R}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}\right) \Psi_{\boldsymbol{k}}^{\mathrm{I}}\left(v_{\boldsymbol{k}}^{\mathrm{I}}\right)$$

$$i\frac{\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}}{\partial\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} \Psi_{\boldsymbol{k}}^{\mathrm{R,I}}$$

$$\hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} = -\frac{1}{2}\frac{\partial^2}{\partial\left(v_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2} + \frac{1}{2}\omega^2(\eta, \boldsymbol{k})\left(\hat{v}_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2$$

Gaussian state solution $\Psi(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\Re\mathrm{e}\,\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}) = \int \frac{\mathrm{d}x}{2\pi^2} \Psi^*\left(v_{\boldsymbol{k}} - \frac{x}{2}\right) \mathrm{e}^{-ip_{\boldsymbol{k}}x} \Psi\left(v_{\boldsymbol{k}} + \frac{x}{2}\right)$

large squeezing limit ⇒ $W \propto \delta\left(p_{\boldsymbol{k}} + k \tan\phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$

Stochastic distribution of classical processes

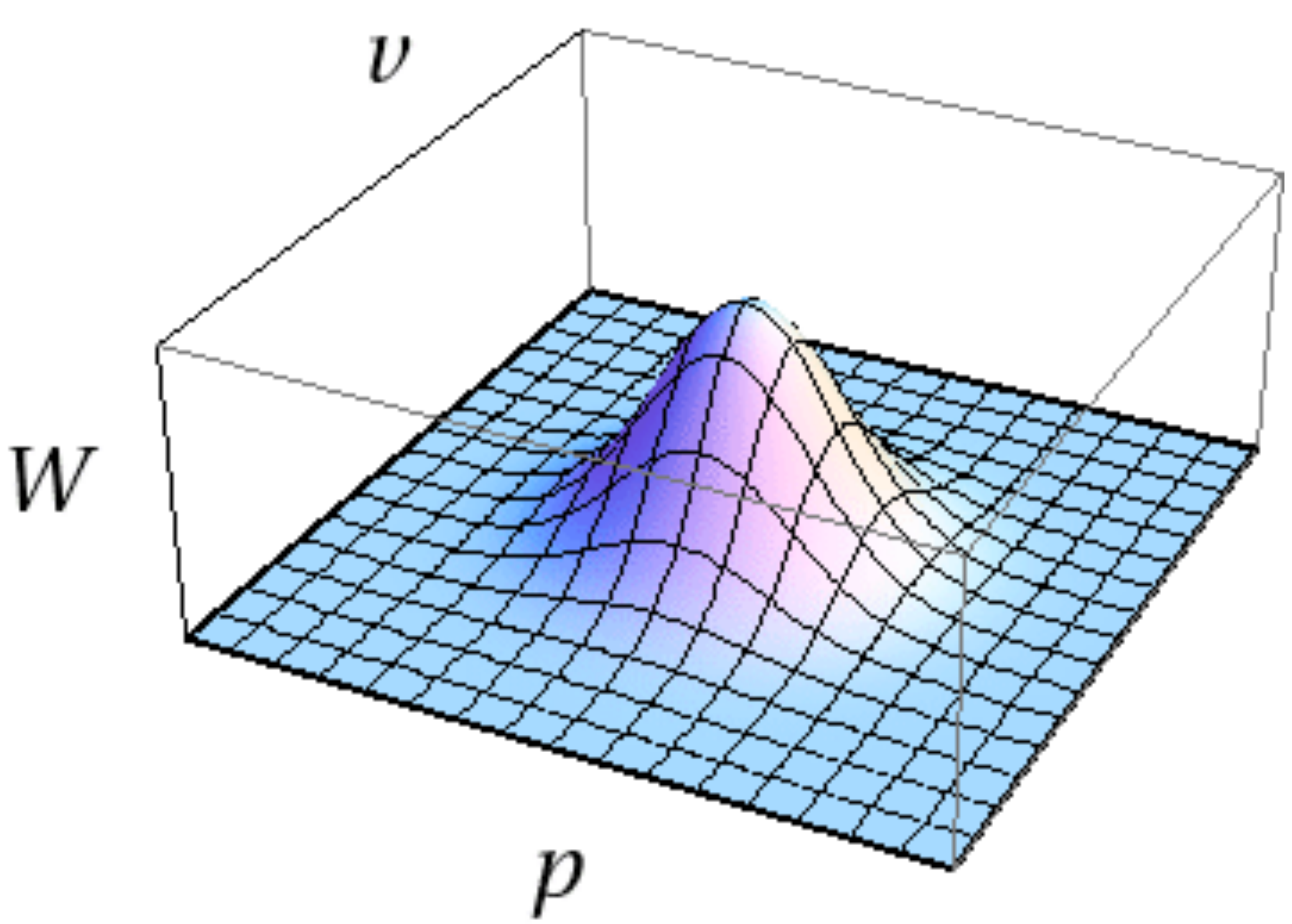
realization, spatial direction

Ergodicity $\left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\xi} \simeq \left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right)=\left[\frac{2 \Re \mathrm{e}\, \Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1 / 4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^{2}}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right)=\int \frac{\mathrm{d} x}{2 \pi^{2}} \Psi^{*}\left(v_{\boldsymbol{k}}-\frac{x}{2}\right) \mathrm{e}^{-i p_{\boldsymbol{k}} x} \Psi\left(v_{\boldsymbol{k}}+\frac{x}{2}\right)$

large squeezing limit ➡ $W \propto \delta\left(p_{\boldsymbol{k}}+k \tan \phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

Ergodicity

realization, spatial direction

$$\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\xi} \simeq\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\boldsymbol{e}}$$

Animations provided by V. Vennin... thx!

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

and

$$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$

$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

$$a(\eta) = \ell_0(-\eta)^{1+\beta}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with
$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

and
$$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$
$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

Power-law inflation example

$$a(\eta) = \ell_0(-\eta)^{1+\beta}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

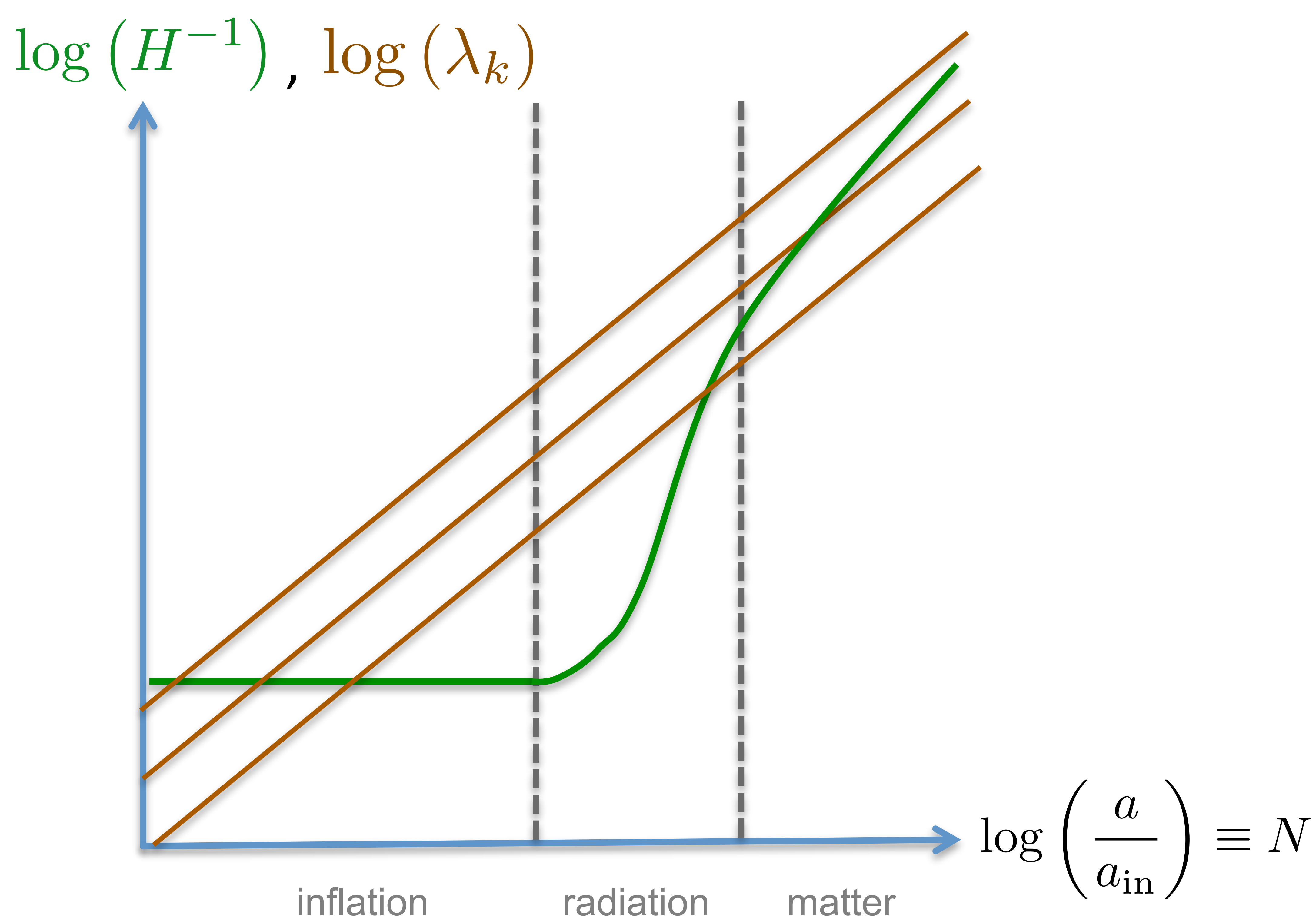
with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k}, \eta)\hat{v}_{\boldsymbol{k}}^2$$

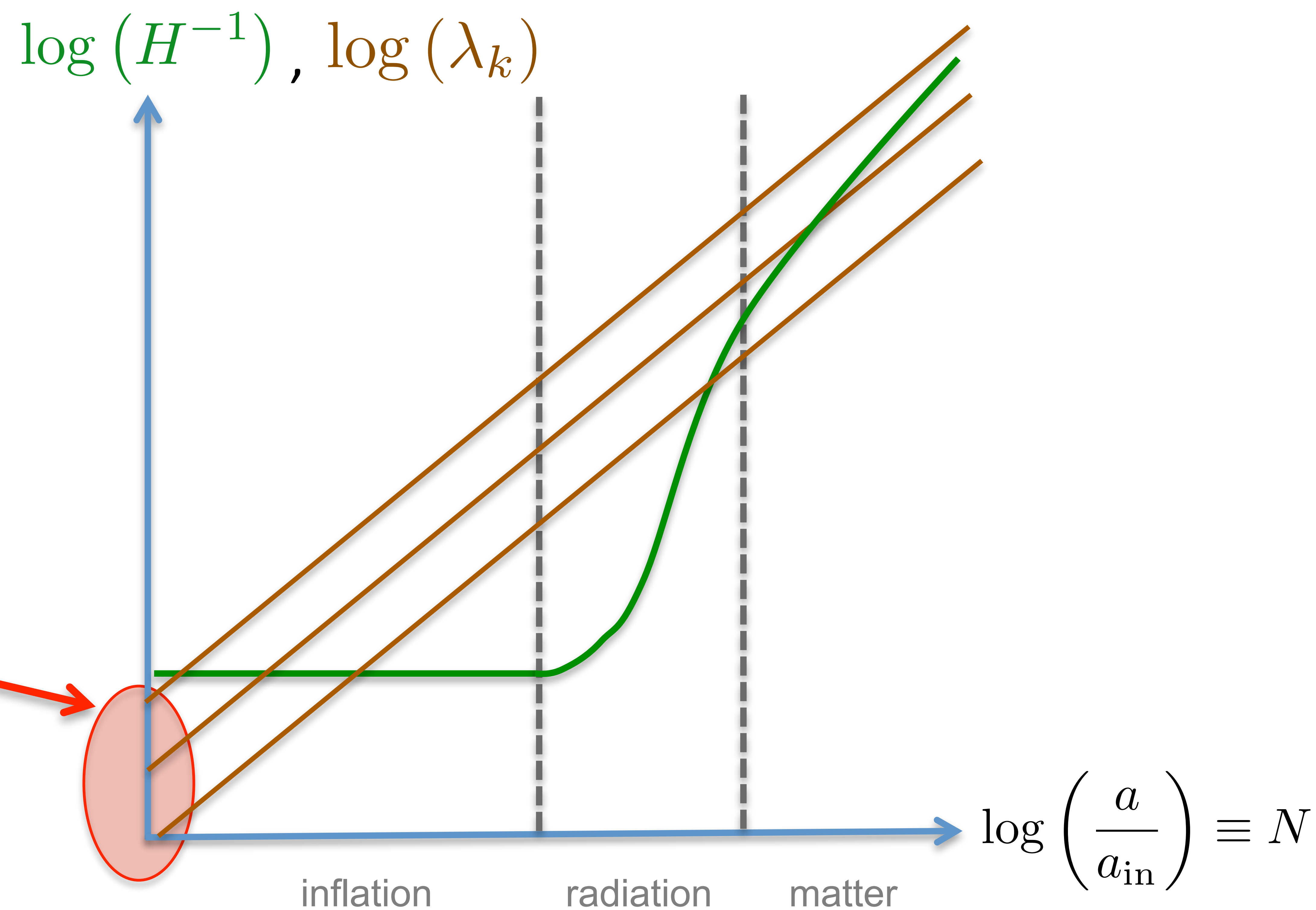
$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k})$$

$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}$$

$$f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k}, \eta) f_{\boldsymbol{k}} = 0$$



$\log\left(H^{-1}\right)$, $\log\left(\lambda_k\right)$
$\log\left(\frac{a}{a_{\rm in}}\right) \equiv N$
inflation
radiation
matter

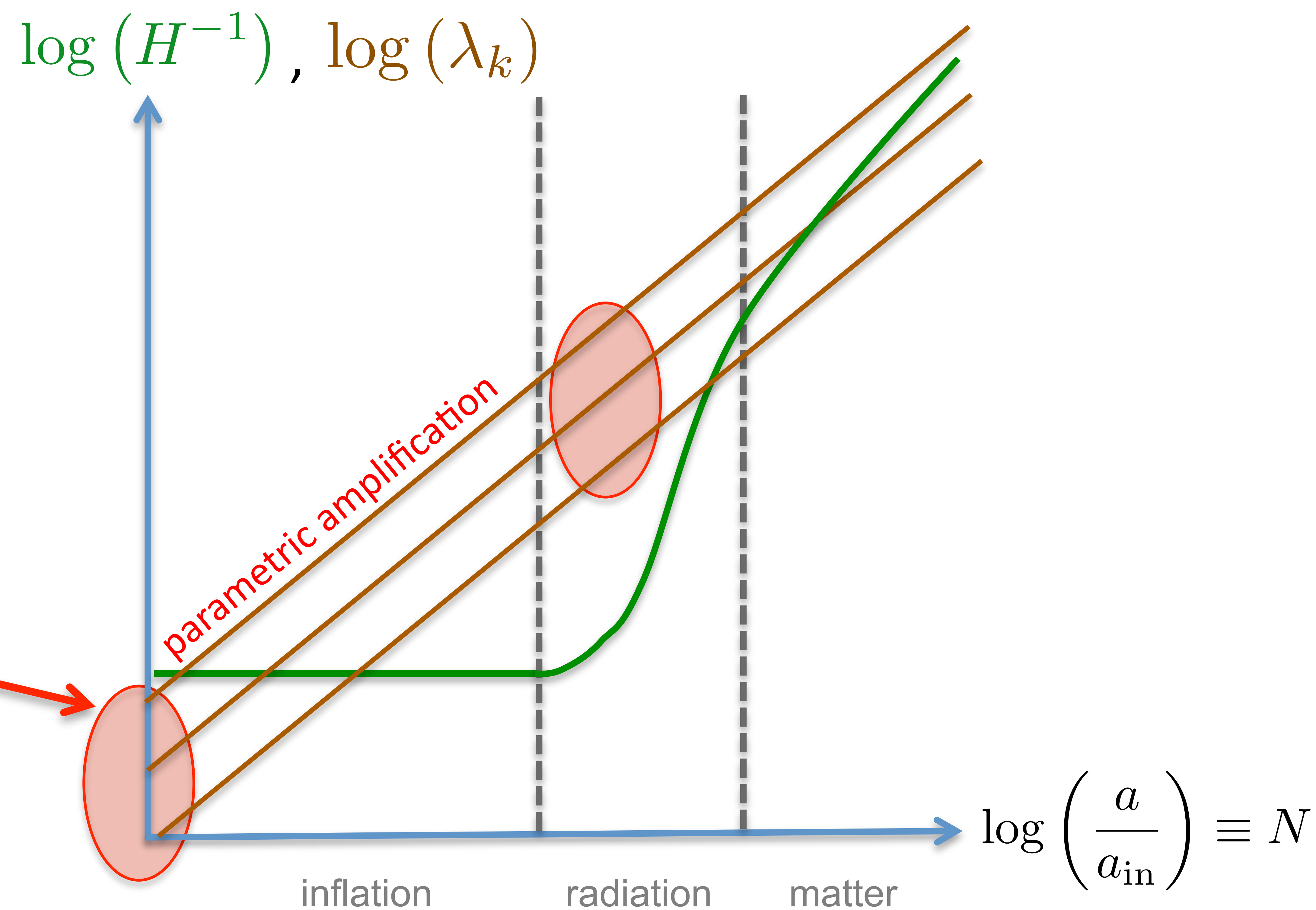


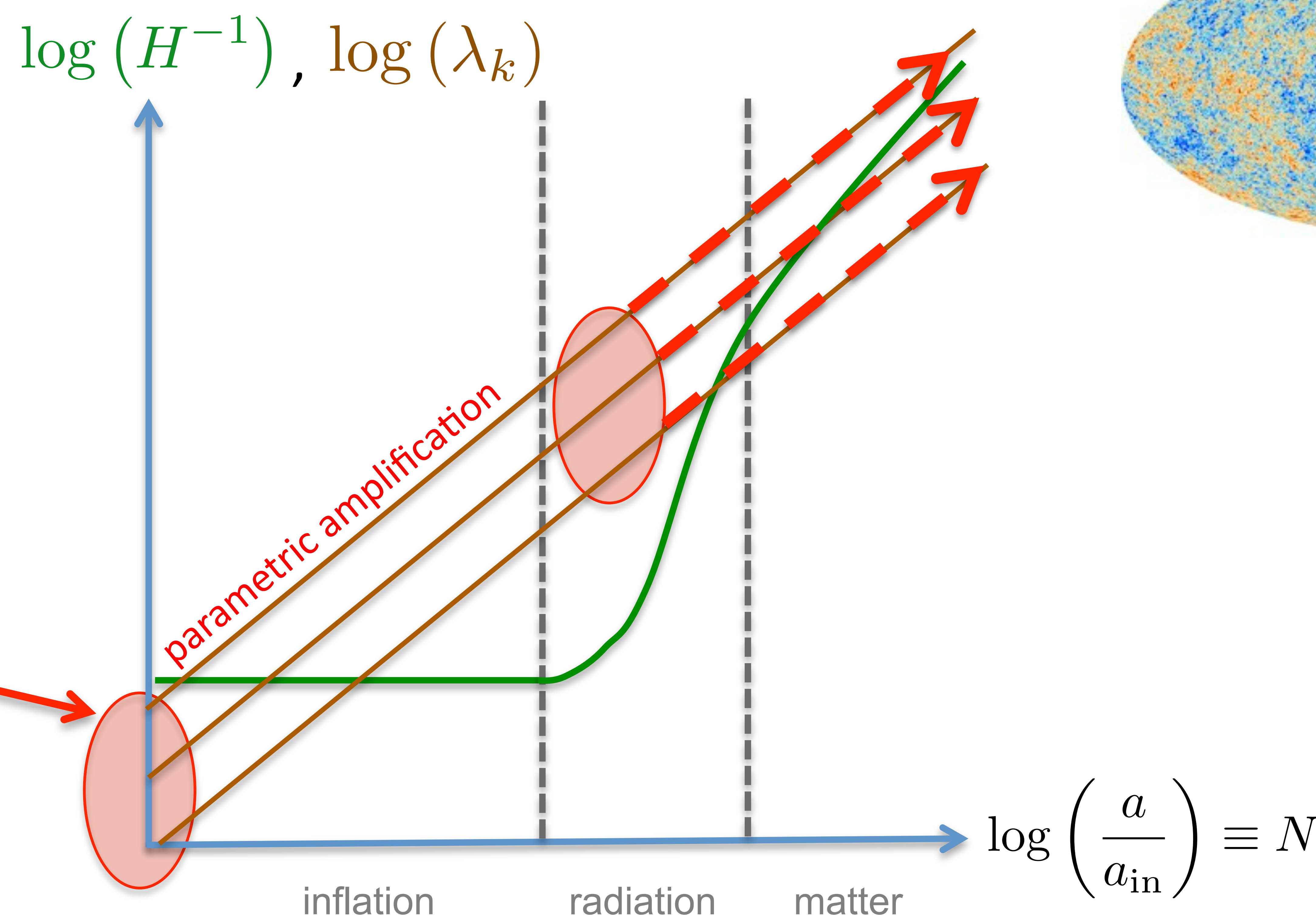
Harmonic oscillator fundamental state

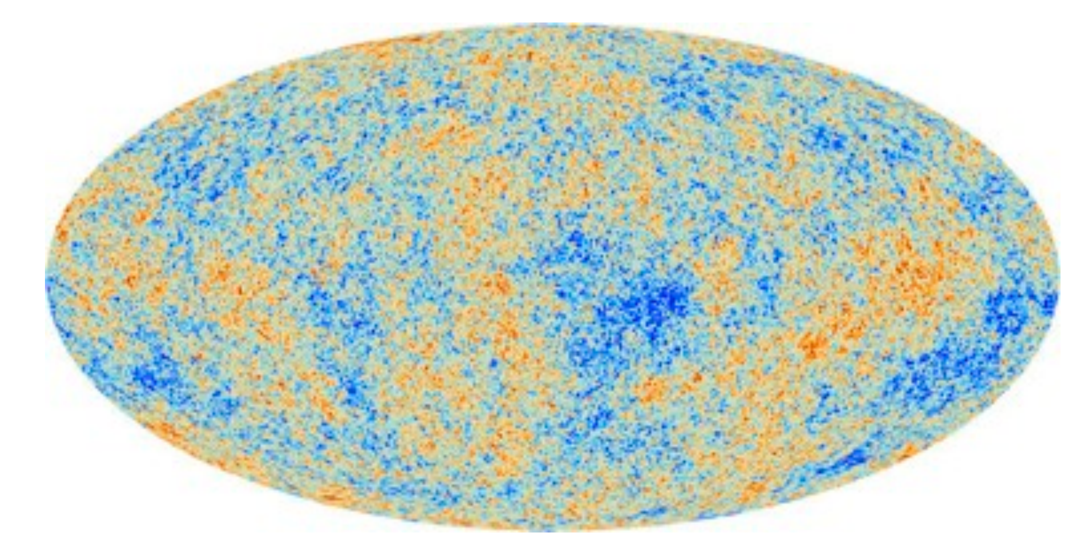
$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$

Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$







Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \frac{a^2}{a'} \underset{\beta\sim-2}{\simeq} \ell_0$

wavelength $\lambda = \frac{a}{k} \underset{\beta\sim-2}{\simeq} \frac{\ell_0}{-k\eta}$

Sub-Hubble regime

$\lambda \ll H^{-1}$

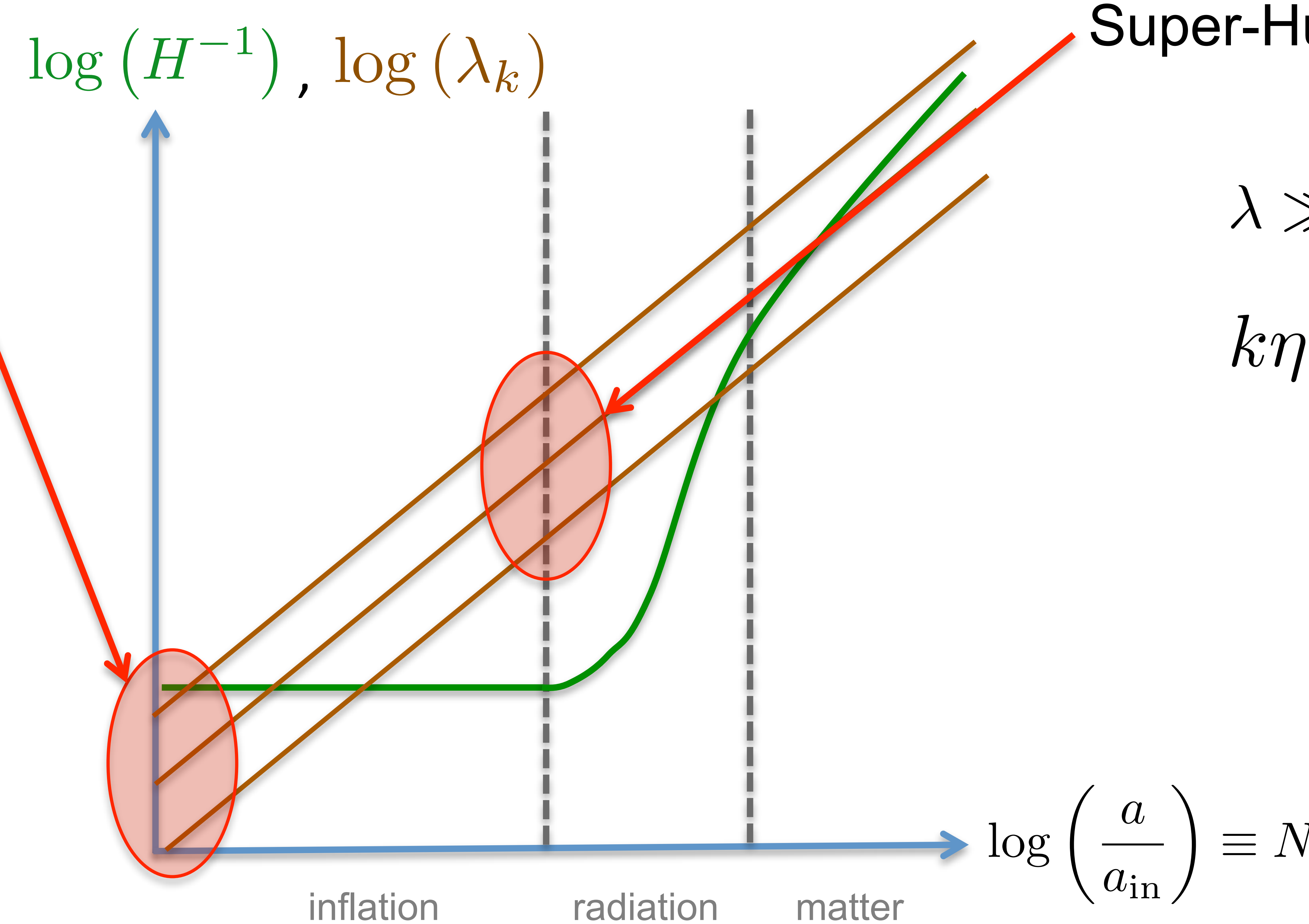
$k\eta \to -\infty$

$\omega \simeq k$

harmonic oscillator

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum



Super-Hubble regime

$\lambda \gg H^{-1}$

$k\eta \to 0^-$

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

Primordial Power Spectrum

Standard case

$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0}$ with $\omega^2(\boldsymbol{k},\eta) = k^2 - \dfrac{\beta(\beta+1)}{\eta^2}$ and $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

Uniquely determines $f_{\boldsymbol{k}}$ → ($\Omega_{\boldsymbol{k}} = -\dfrac{i}{2}\dfrac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}$) → $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation($k\eta \to 0^-$), this gives $P_v(k) = \dfrac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

and eventually $P_\zeta(k) = \dfrac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_{\mathrm{S}}-1}$

with $n_{\mathrm{S}} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

Planck: $1 - n_{\mathrm{s}} = 0.0389 \pm 0.0054$

Primordial Power Spectrum

Standard case

$$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0} \quad \text{with} \quad \omega^2(\boldsymbol{k},\eta) = k^2 - \frac{\beta(\beta+1)}{\eta^2} \quad \text{and} \quad f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$$

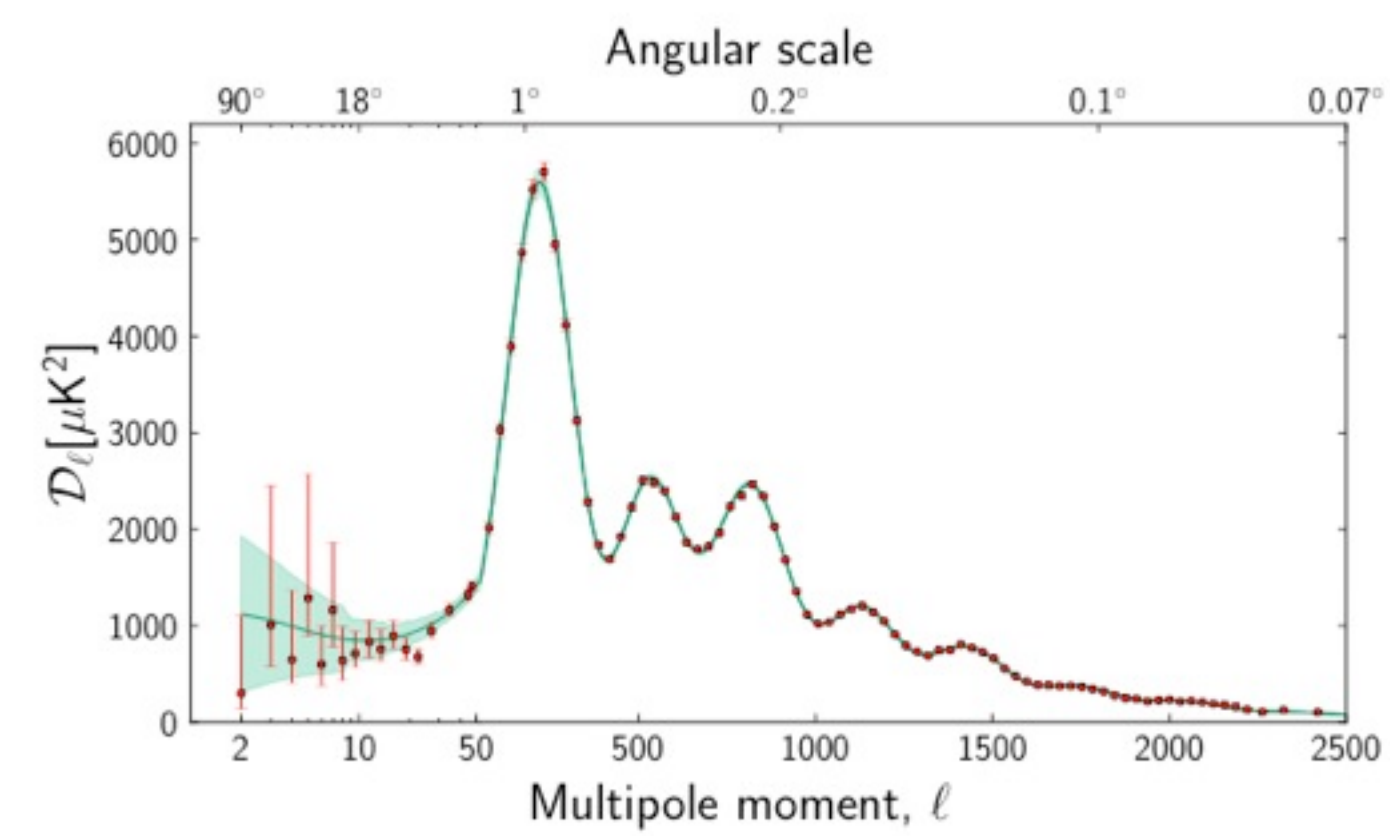
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation($k\eta \to 0^-$), this gives $P_v(k) = \frac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

and eventually $P_\zeta(k) = \frac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_\mathrm{S}-1}$

with $n_\mathrm{S} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

Planck: $1 - n_\mathrm{s} = 0.0389 \pm 0.0054$



Primordial Power Spectrum

Standard case

$$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0} \quad \text{with} \quad \omega^2(\boldsymbol{k},\eta) = k^2 - \frac{\beta(\beta+1)}{\eta^2} \quad \text{and} \quad f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$$

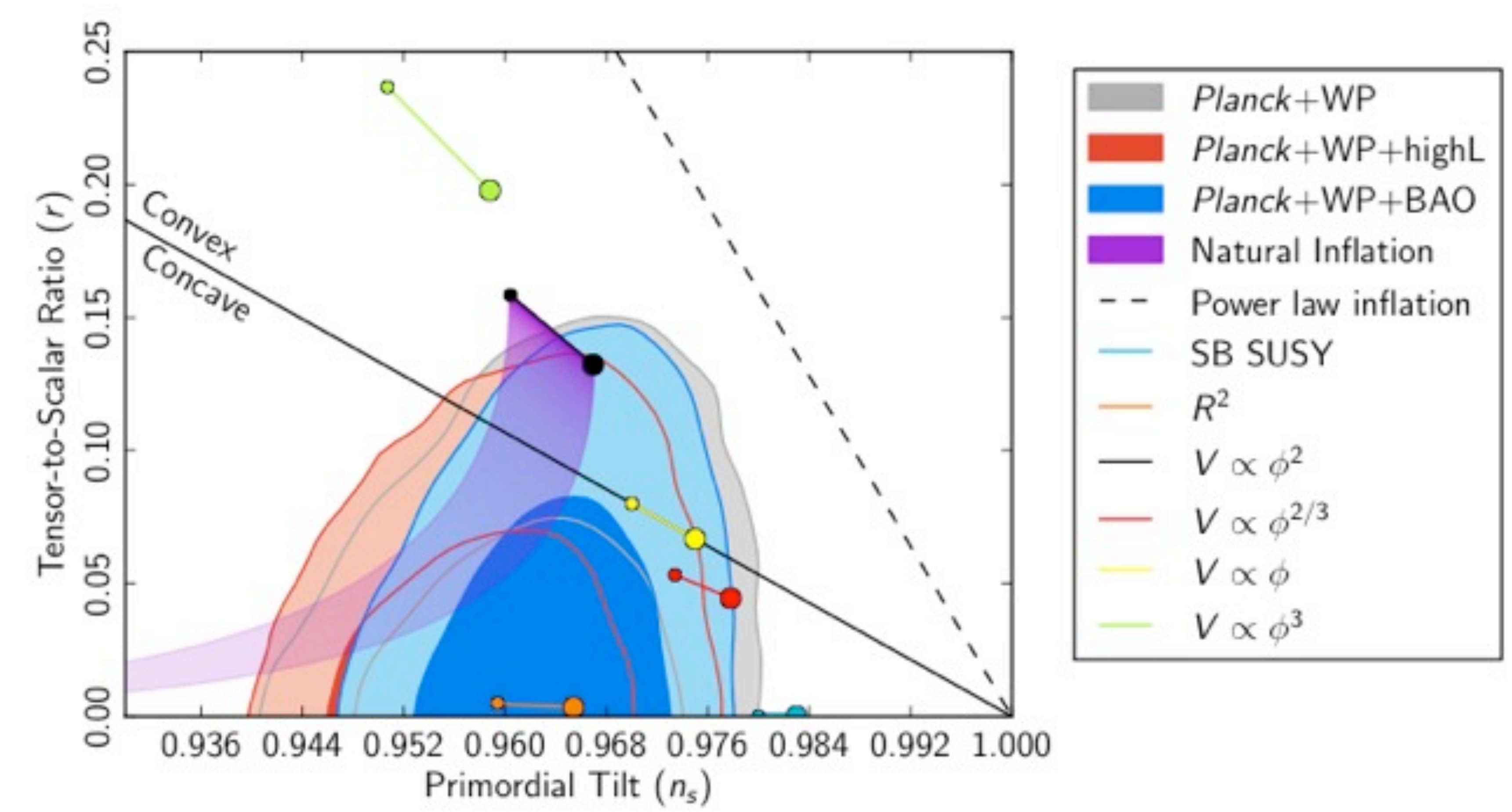
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation($k\eta \to 0^-$), this gives $P_v(k) = \frac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

and eventually $P_\zeta(k) = \frac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_{\mathrm{S}}-1}$

with $n_{\mathrm{S}} = 2\beta + 5 \underset{\beta\sim-2}{\simeq} 1$

Planck: $1 - n_{\mathrm{s}} = 0.0389 \pm 0.0054$



Primordial Power Spectrum

Modified Theory

Modified Schrödinger equation

Extended Gaussian wave function

$$\Psi_{\boldsymbol{k}}\left(\eta, v_{\boldsymbol{k}}\right)=\left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4}\exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}\left(\eta\right)\left[v_{\boldsymbol{k}}-\bar{v}_{\boldsymbol{k}}\left(\eta\right)\right]^2+i\sigma_{\boldsymbol{k}}(\eta)+i\chi_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}}-i\Im\mathrm{m}\Omega_{\boldsymbol{k}}(\eta)\left(v_{\boldsymbol{k}}\right)^2\right\}$$

Modified equation of motion

$$\Omega_{\boldsymbol{k}}'=-2i\Omega_{\boldsymbol{k}}^2+\frac{i}{2}\omega^2(\eta,\boldsymbol{k})+\gamma \quad \xrightarrow{\Omega_{\boldsymbol{k}}=-\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}} \quad f_{\boldsymbol{k}}''+\left[\omega^2(\eta,k)-2i\gamma\right]f_{\boldsymbol{k}}=0$$

Primordial Power Spectrum

Modified Theory

Modified Schrödinger equation

$$\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle = -i\hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi\rangle\mathrm{d}\eta + \sqrt{\gamma}\left(\hat{C}_{\boldsymbol{k}} - \langle\hat{C}_{\boldsymbol{k}}\rangle\right)\mathrm{d}W_\eta\,|\Psi_{\boldsymbol{k}}\rangle - \frac{\gamma}{2}\left(\hat{C}_{\boldsymbol{k}} - \langle\hat{C}_{\boldsymbol{k}}\rangle\right)^2\mathrm{d}\eta\,|\Psi_{\boldsymbol{k}}\rangle$$

Extended Gaussian wave function

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4}\exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}(\eta)\right]^2 + i\sigma_{\boldsymbol{k}}(\eta) + i\chi_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}} - i\Im\mathrm{m}\Omega_{\boldsymbol{k}}(\eta)\left(v_{\boldsymbol{k}}\right)^2\right\}$$

Modified equation of motion

$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k}) + \gamma$$

$$\xrightarrow{\;\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}\;}$$

$$f_{\boldsymbol{k}}'' + \left[\omega^2(\eta, k) - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Primordial Power Spectrum

Modified Theory

Modified Schrödinger equation

$$\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle = -i\hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi\rangle\mathrm{d}\eta + \sqrt{\gamma}\left(\hat{v}_{\boldsymbol{k}} - \langle\hat{v}_{\boldsymbol{k}}\rangle\right)\mathrm{d}W_\eta\,|\Psi_{\boldsymbol{k}}\rangle - \frac{\gamma}{2}\left(\hat{v}_{\boldsymbol{k}} - \langle\hat{v}_{\boldsymbol{k}}\rangle\right)^2\mathrm{d}\eta\,|\Psi_{\boldsymbol{k}}\rangle$$

Extended Gaussian wave function

$$\Psi_{\boldsymbol{k}}\left(\eta, v_{\boldsymbol{k}}\right) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4}\exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}\left(\eta\right)\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}\left(\eta\right)\right]^2 + i\sigma_{\boldsymbol{k}}(\eta) + i\chi_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}} - i\Im\mathrm{m}\Omega_{\boldsymbol{k}}(\eta)\left(v_{\boldsymbol{k}}\right)^2\right\}$$

Modified equation of motion

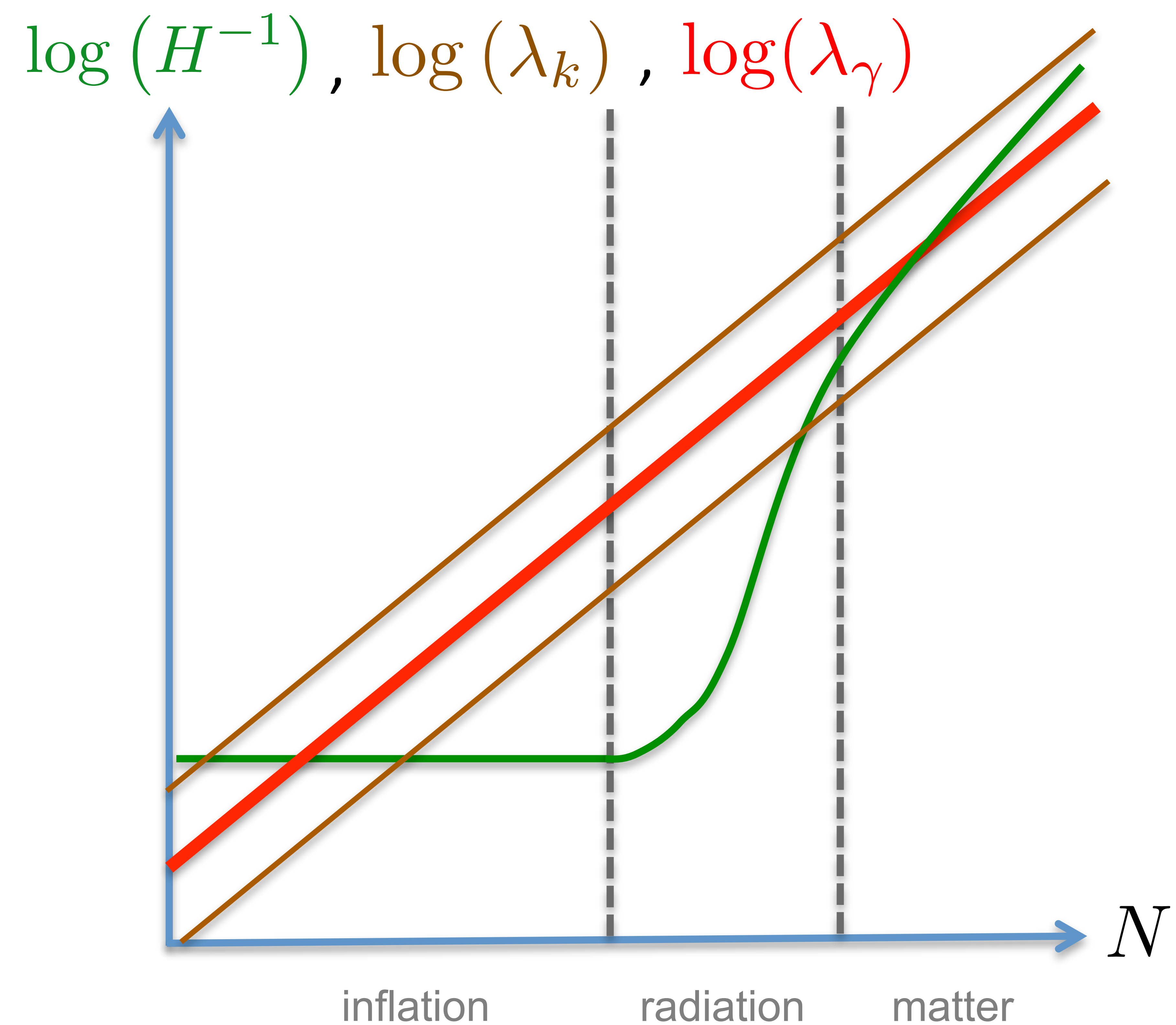
$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k}) + \gamma \quad \xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}} \quad f_{\boldsymbol{k}}'' + \left[\omega^2(\eta, k) - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

$$\omega^2(\eta, k)$$



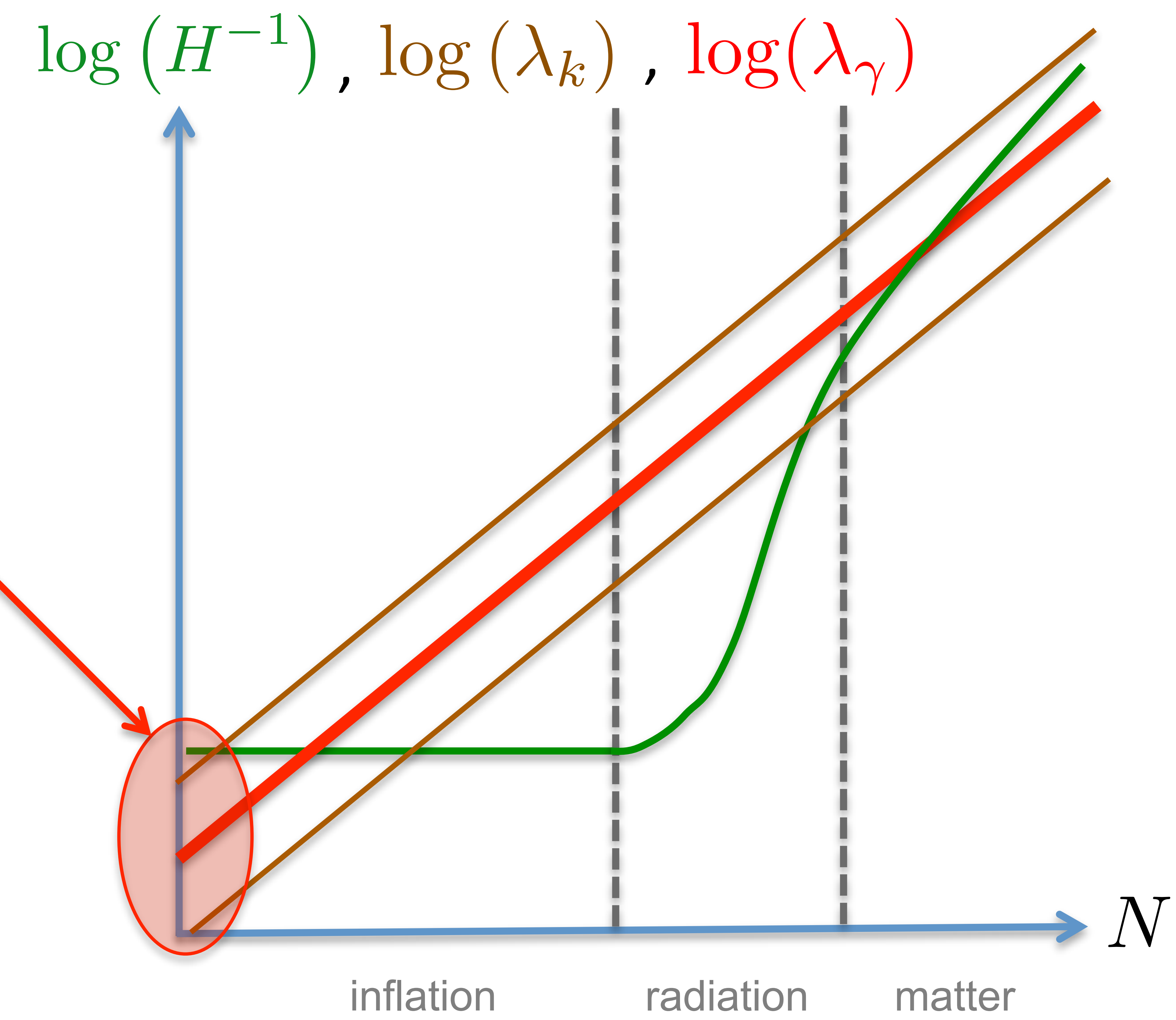
Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Ability to fix Bunch Davis vacuum as an initial condition ?

$\omega^2(\eta, k)$



Primordial Power Spectrum

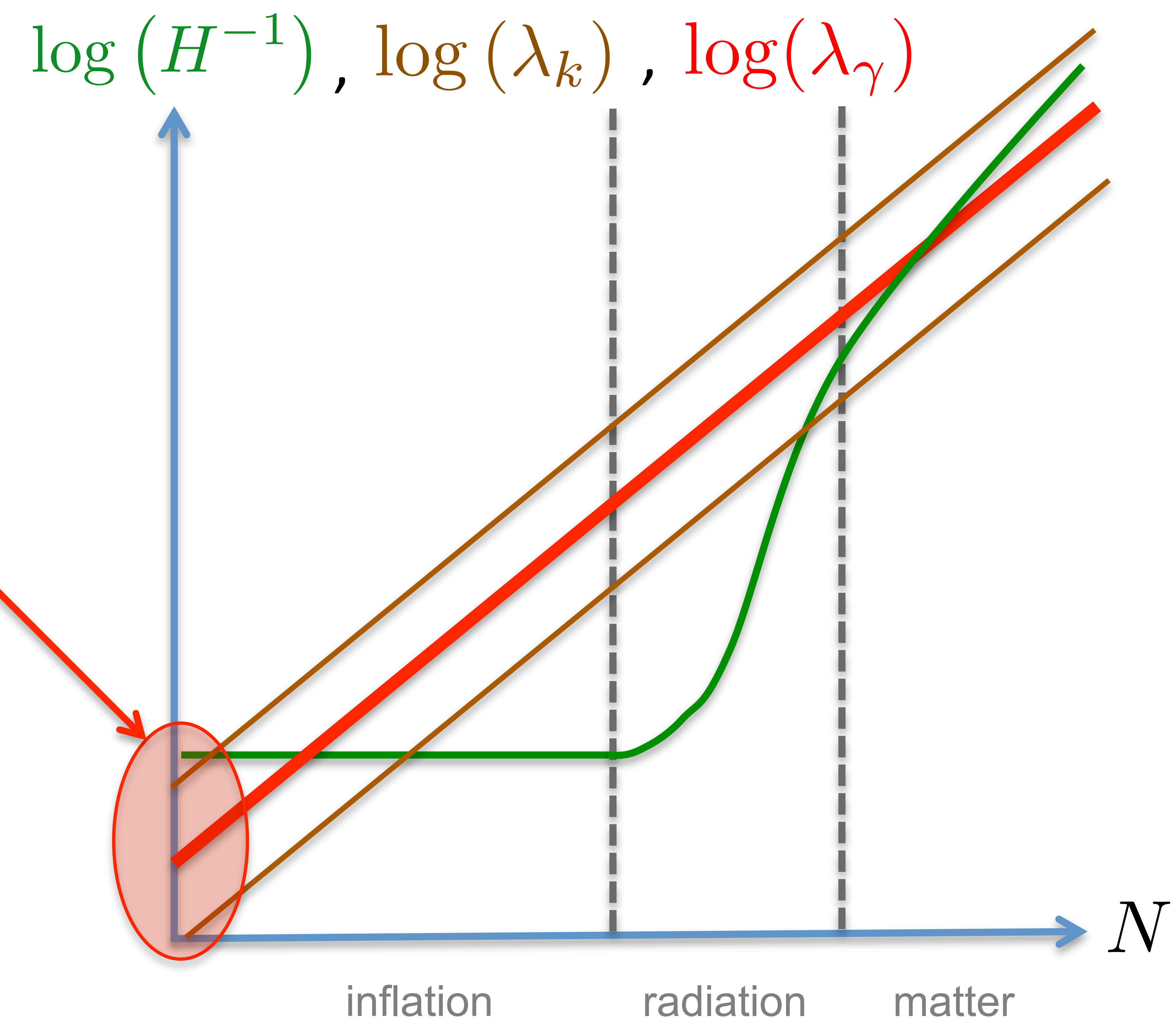
Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Ability to fix Bunch Davis vacuum as an initial condition ?

$$\omega^2(\eta, k) \underset{k\eta \to -\infty}{\longrightarrow} k^2 - 2i\gamma$$

$$f_{\boldsymbol{k}} \underset{k\eta \to -\infty}{=} A_{\boldsymbol{k}} \mathrm{e}^{i\omega\eta} + B_{\boldsymbol{k}} \mathrm{e}^{-i\omega\eta}$$



Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

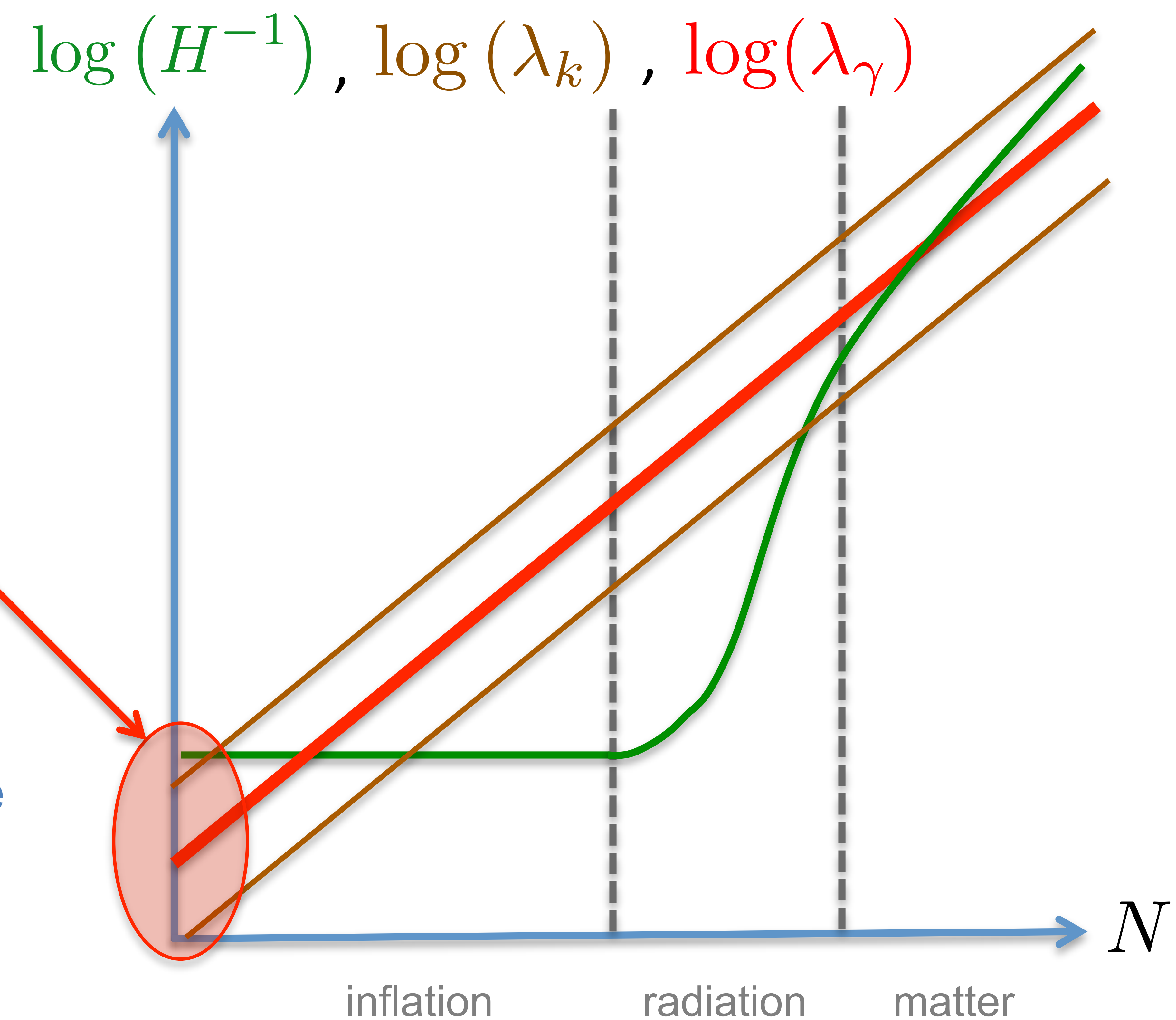
Ability to fix Bunch Davis vacuum as an initial condition ?

$$\omega^2(\eta, k) \underset{k\eta\to-\infty}{\longrightarrow} k^2 - 2i\gamma$$

$$f_{\boldsymbol{k}} \underset{k\eta\to-\infty}{=} A_{\boldsymbol{k}} e^{i\omega\eta} + B_{\boldsymbol{k}} e^{-i\omega\eta}$$

leads to non normalizable wavefunction

$(\Re e\, \Omega_{\boldsymbol{k}} < 0)$



Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Ability to fix Bunch Davis vacuum as an initial condition ?

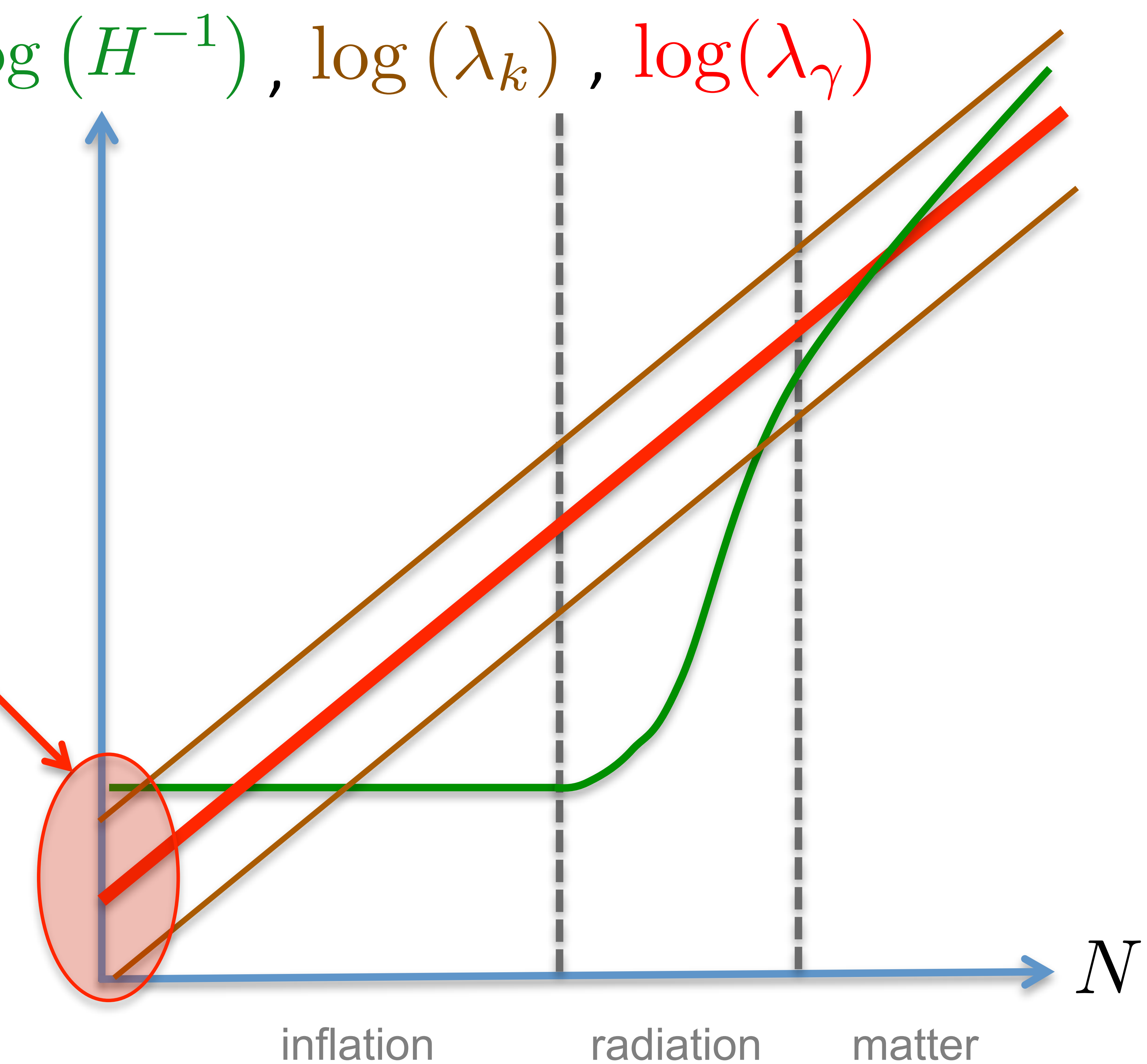
$$\omega^2(\eta, k) \underset{k\eta\to-\infty}{\longrightarrow} k^2 - 2i\gamma$$

$$f_{\boldsymbol{k}} \underset{k\eta\to-\infty}{=} A_{\boldsymbol{k}} e^{i\omega\eta} + B_{\boldsymbol{k}} e^{-i\omega\eta}$$

Bunch Davis vacuum

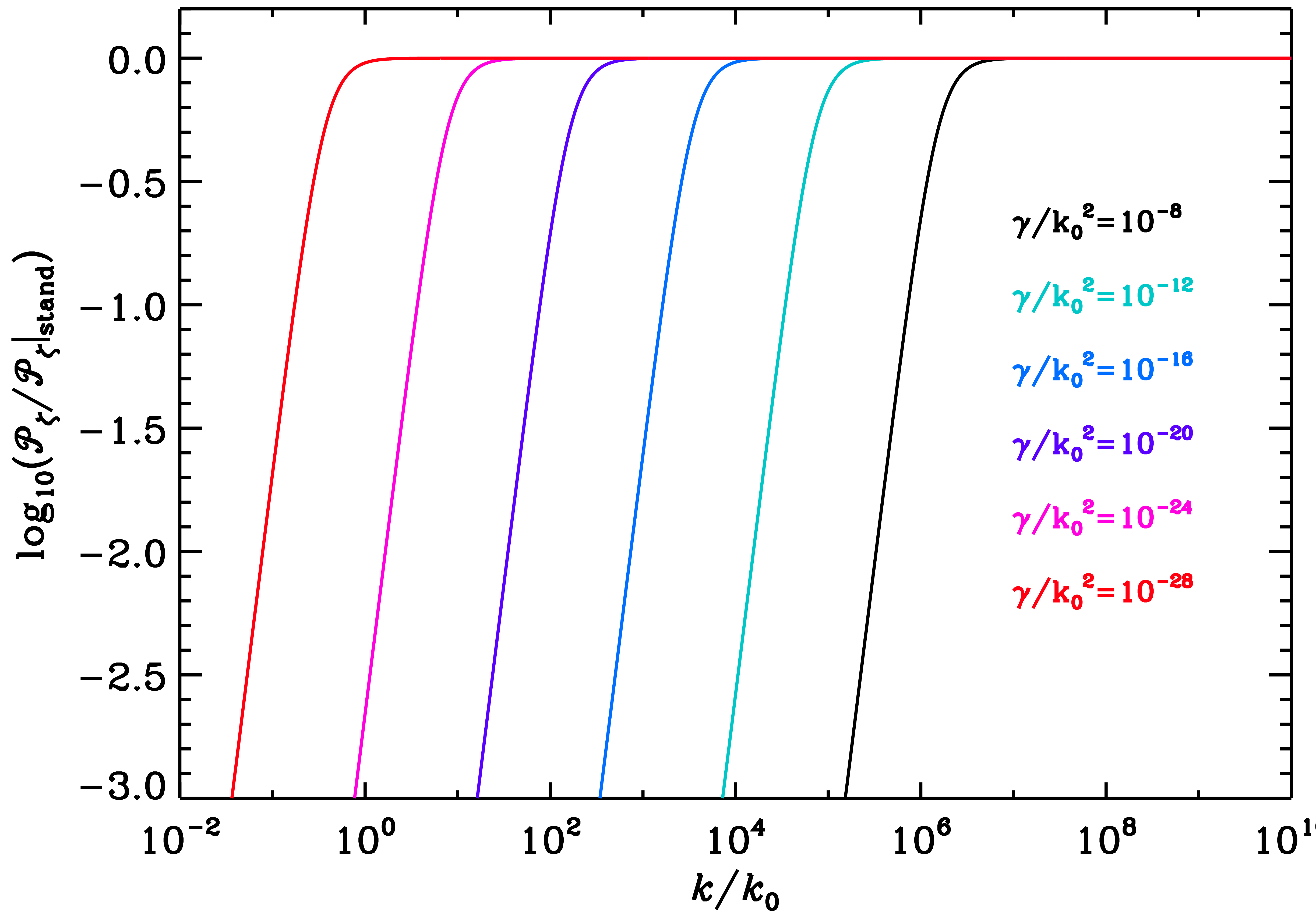
leads to non normalizable wavefunction

$(\Re e\, \Omega_{\boldsymbol{k}} < 0)$



Primordial Power Spectrum

Modified Theory

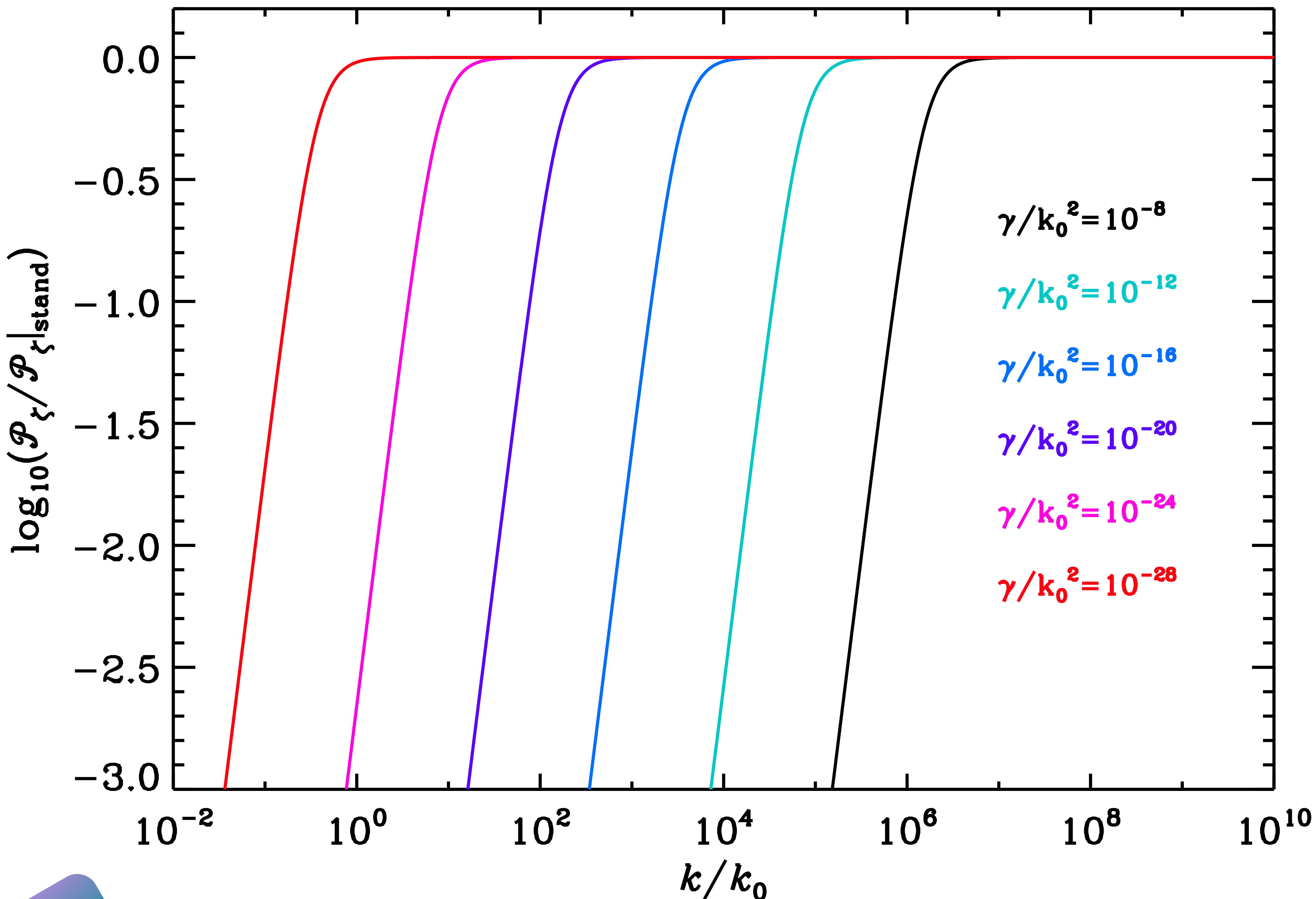


$$k < k_{\rm break} : n_{\rm S} = 4$$

$$k > k_{\rm break} : n_{\rm S} = 2\beta + 5 \simeq 1$$

Primordial Power Spectrum

Modified Theory

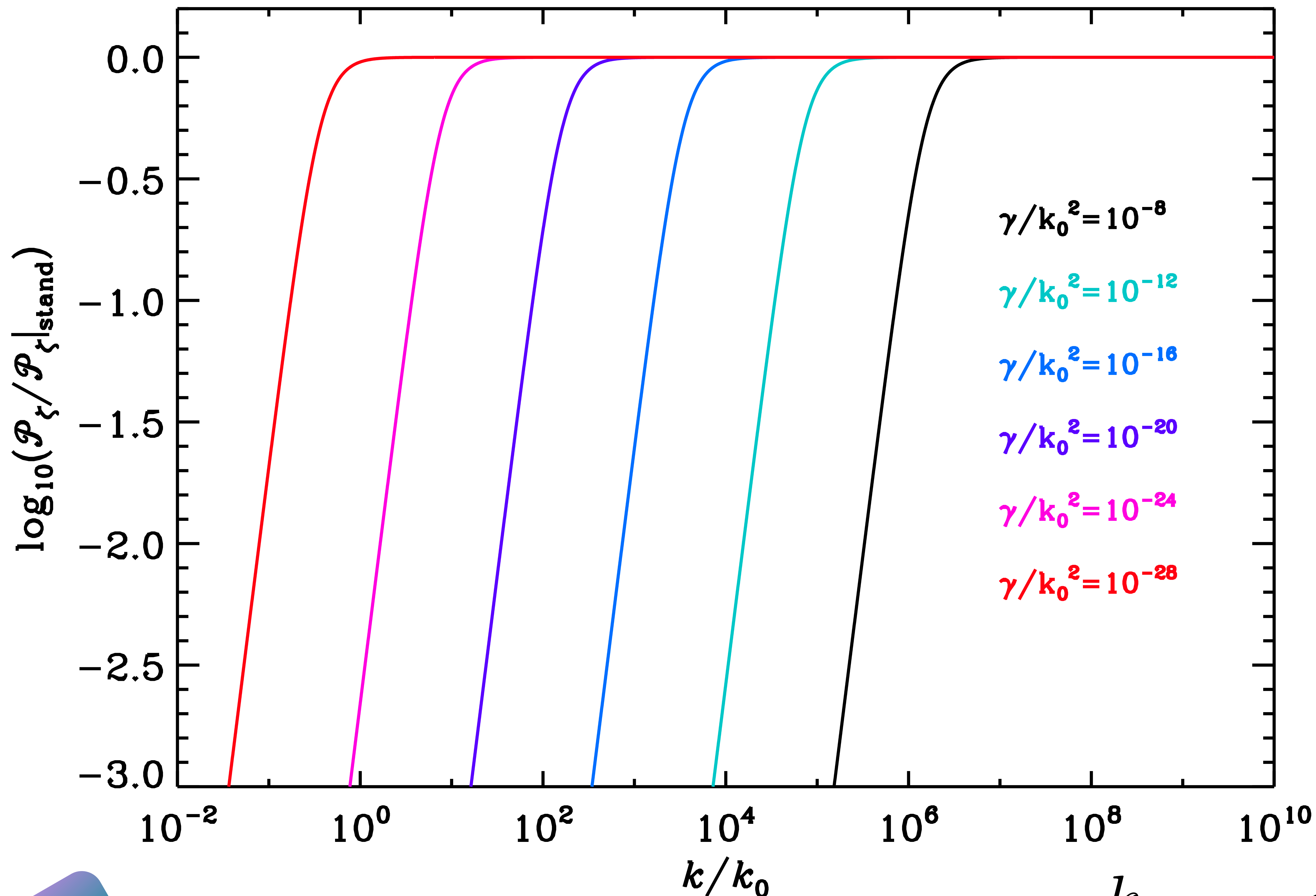


$k < k_{\rm br}$ … $= 4$

$$k > k_{\rm break} : n_{\rm S} = 2\beta + 5 \simeq 1$$

Primordial Power Spectrum

Modified Theory



$k < k_{\mathrm{bre}}$ ... $= 4$

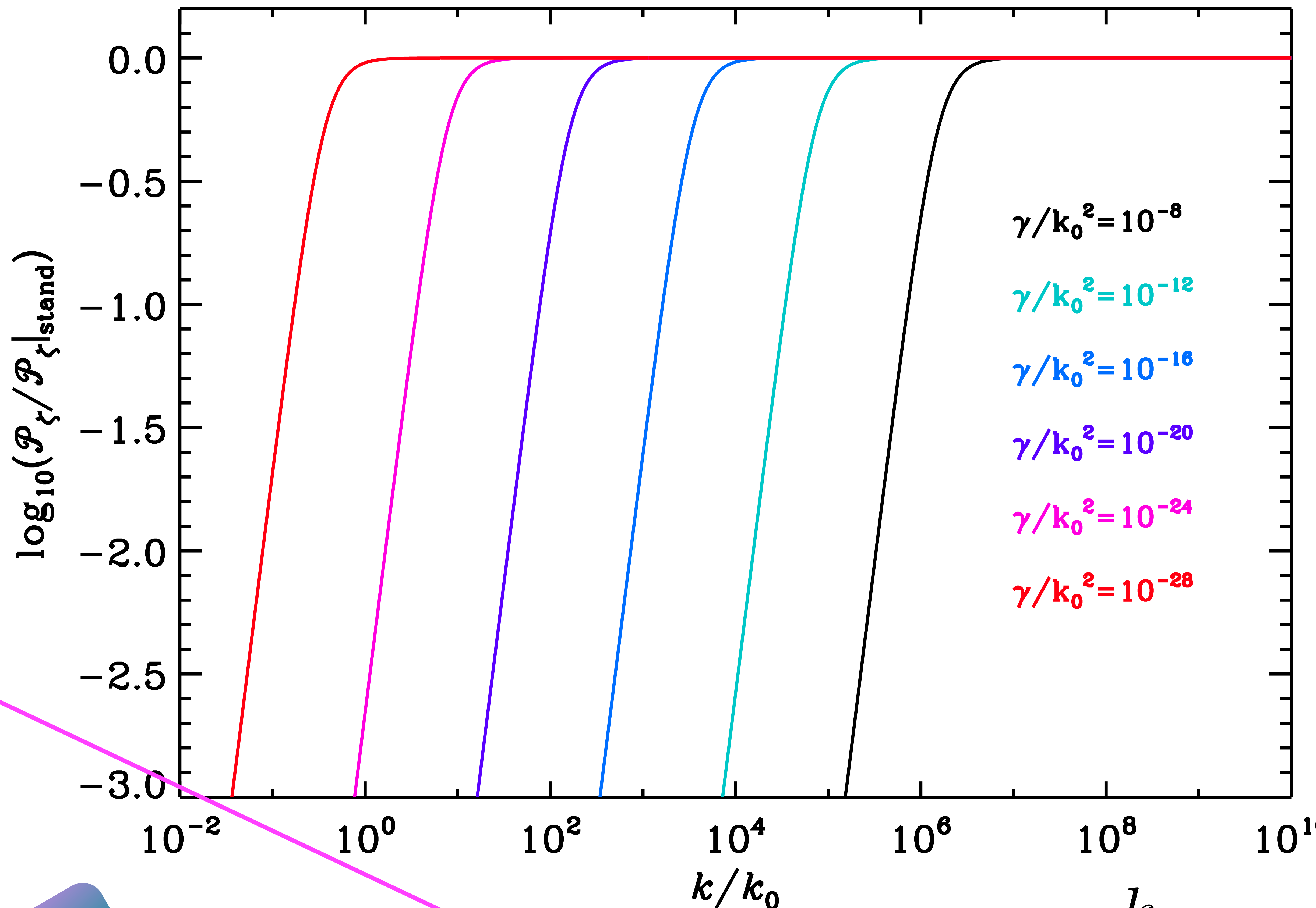
$k > k_{\mathrm{break}} : n_{\mathrm{S}} = 2\beta + 5 \simeq 1$

$k_{\mathrm{break}} < k_0$

$$\frac{\gamma}{k_0^2} \ll \mathrm{e}^{-\Delta N_*} \simeq 10^{-28}$$

Primordial Power Spectrum

Modified Theory



comoving Hubble wavenumber now

$k_{\rm break} < k_0$

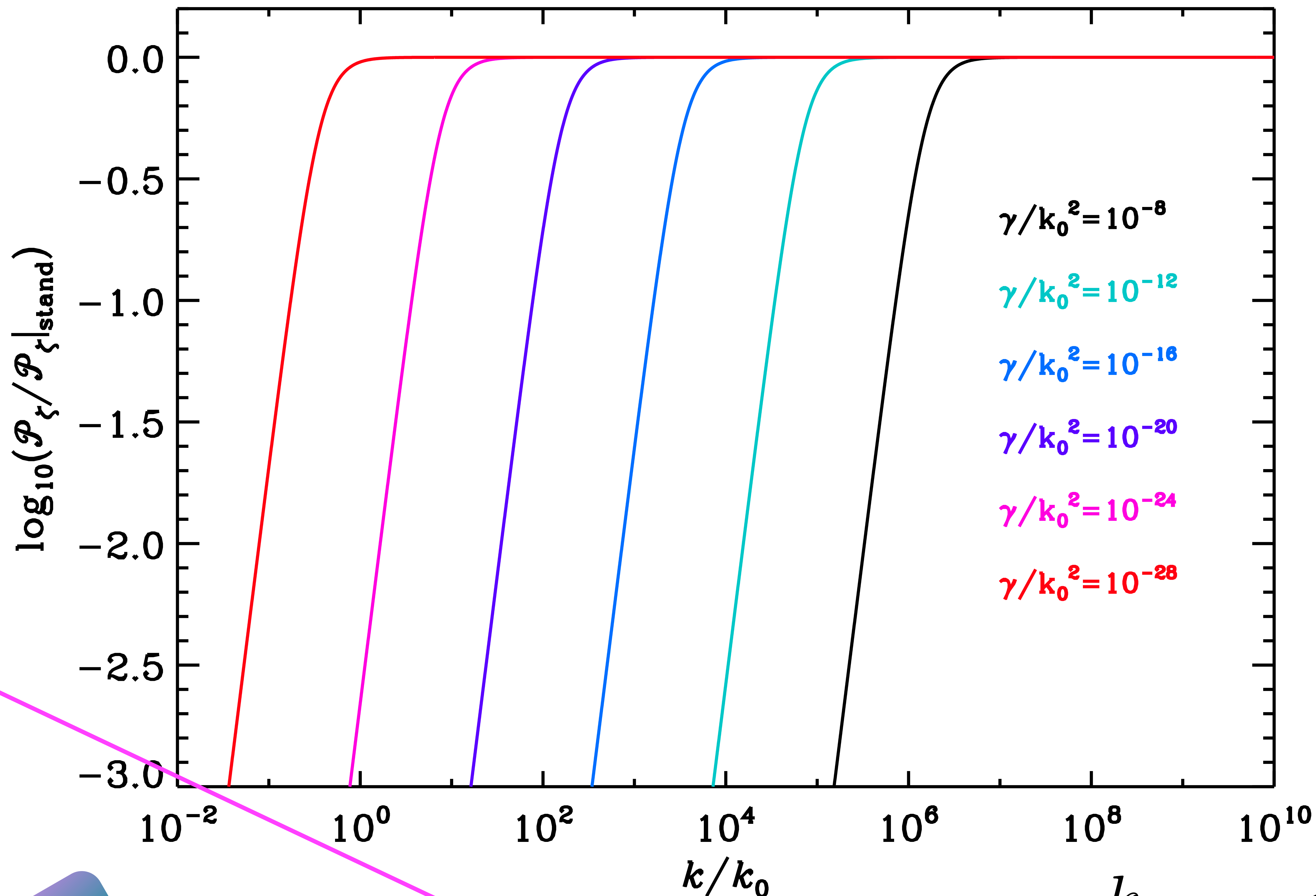
$$\frac{\gamma}{k_0^2} \ll \mathrm{e}^{-\Delta N_*} \simeq 10^{-28}$$

$k < k_{\rm br}$ ... $= 4$

$k > k_{\rm break} : n_{\rm S} = 2\beta + 5 \simeq 1$

Primordial Power Spectrum

Modified Theory



comoving Hubble wavenumber now

$$\ell_\gamma \gg 10^{13}\ell_{\rm H}$$

$$k_{\rm break} < k_0$$

$$\frac{\gamma}{k_0^2} \ll e^{-\Delta N_*} \simeq 10^{-28}$$

$k < k_{\rm bre}$ ... $= 4$

$$k > k_{\rm break} : n_{\rm S} = 2\beta + 5 \simeq 1$$

Conclusions

Conclusions

Quantum measurement problem very severe in cosmology

Conclusions

Quantum measurement problem very severe in cosmology

Two possible extensions of QM can be used

(Born rule not set by hand)

Conclusions

Quantum measurement problem very severe in cosmology

dBB ontology

Two possible extensions of QM can be used

(Born rule not set by hand)

Conclusions

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

dBB ontology

Two possible extensions of QM can be used
(Born rule not set by hand)

Conclusions

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

dBB ontology

Two possible extensions of QM can be used
(Born rule not set by hand)

Spontaneous collapse

Conclusions

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

dBB ontology

Two possible extensions of QM can be used
(Born rule not set by hand)

Spontaneous collapse

Constraint on γ

- collapse time
- final spread

Conclusions

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

Two possible extensions of QM can be used
(Born rule not set by hand)

- dBB ontology
- Spontaneous collapse

Constraint on γ

- collapse time ✓
- final spread

Conclusions

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

Two possible extensions of QM can be used
(Born rule not set by hand)

- dBB ontology
- Spontaneous collapse

Constraint on γ

- collapse time ✓
- final spread 

Conclusions

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

Two possible extensions of QM can be used
(Born rule not set by hand)

- dBB ontology
- Spontaneous collapse

Constraint on γ

- collapse time ✓
- final spread

Plenty of new effects awaiting to be discovered/understood...