ANOMALOUS DIMENSIONS, POMERON INTERCEPT AND ADS/CFT

Chung-I Tan, Brown University
Non-Perturbative QCD Workshop
Paris, June 10-13, 2013

Brower, Polchinski, Strassler, Tan (BPST) The Pomeron and Gauge/String Duality (2006)

Richard Brower, Marko Djurić, Ina Sarcević and Chung-I Tan: *String-Gauge Dual Description of DIS and Small-x*, 10.1007/JHEP 11(2010)051, arXiv:1007.2259

R. Brower, M. Costa, M. Djuric, T. Raben, and C-I Tan: *"Conformal Pomeron and Odderon Intercepts at Strong Coupling"* (to appear.)

Outline

- QCD High Energy Scattering with AdS/CFT -- Universality
- Consequence of Conformal Invariance
    - DIS at low-x -- Unification
        - DGLAP (large Q) vs BFKL (small x)
        - OPE (Anomalous Dimensions) and Conformal Pomeron
    - Conformal Pomeron and Odderon Intercepts in strong coupling
- Saturation, Confinement, etc., and DIS
- Summary and Outlook

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Executive Summary:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Establishing "Pomeron" in QCD non-perturbatively,

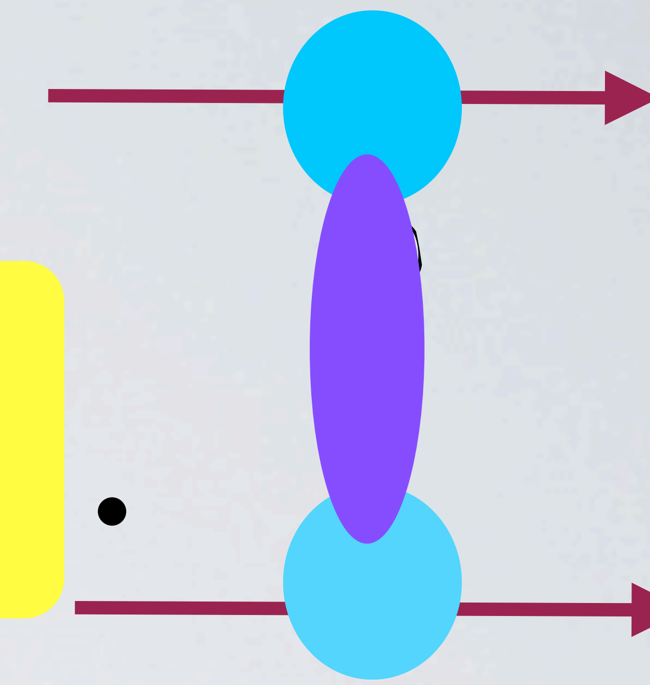
- Unification of Soft and Hard Physics in High Energy Collision

- New phenomenology based on "Large Pomeron intercept", e.g., DIS at small-x: (DGLAP vs Pomeron), DVCS, Central Diffractive Higgs Production, etc.

ADS BUILDING BLOCKS BLOCKS

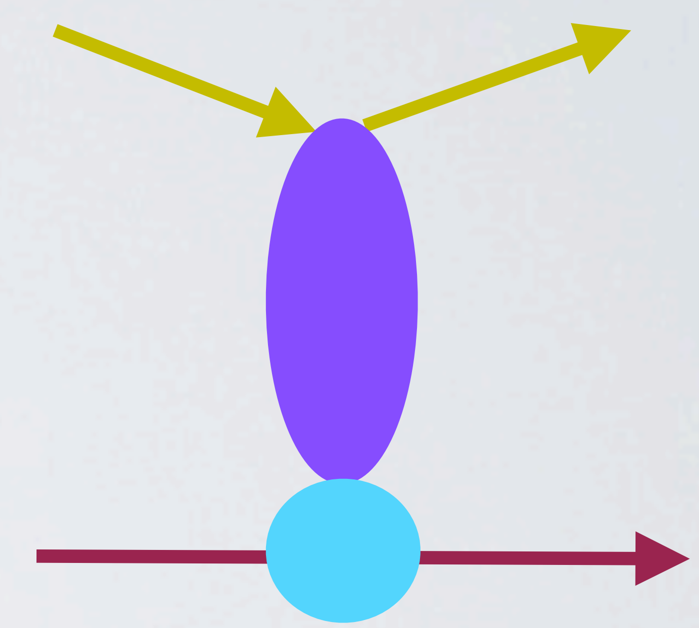
For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24}\,.$$



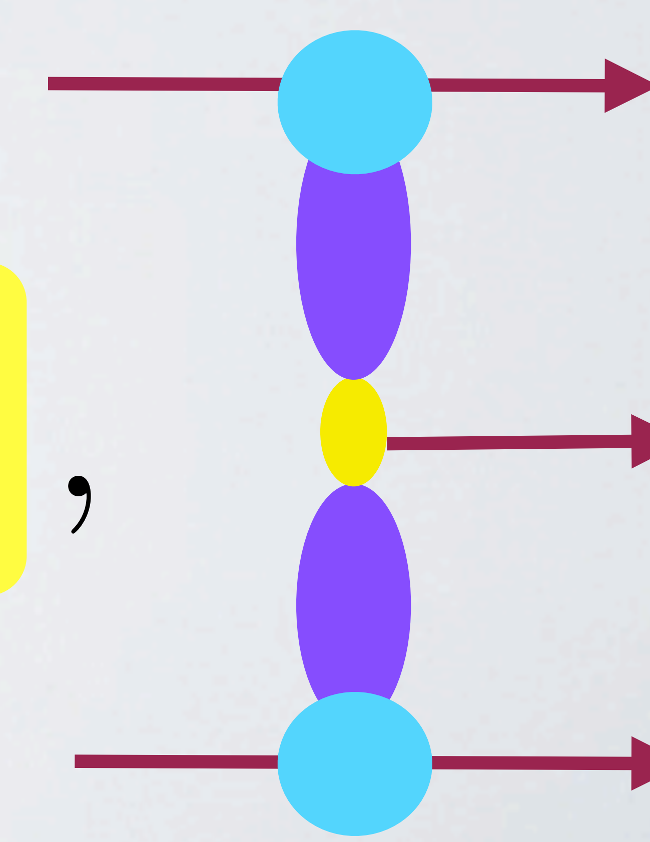
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}'\; e^{i\mathbf{q}_\perp\cdot(\mathbf{x}-\mathbf{x}')}\; \Phi_{13}(z)\; \mathcal{K}(s,\mathbf{x}-\mathbf{x}',z,z')\; \Phi_{24}(z')$$

$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$



For 2-to-3

$$A(s,s_1,s_2,t_1,t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24}\,,$$



BASIC BUILDING BLOCK

- Elastic Vertex: 

- Pomeron/Graviton Propagator: 

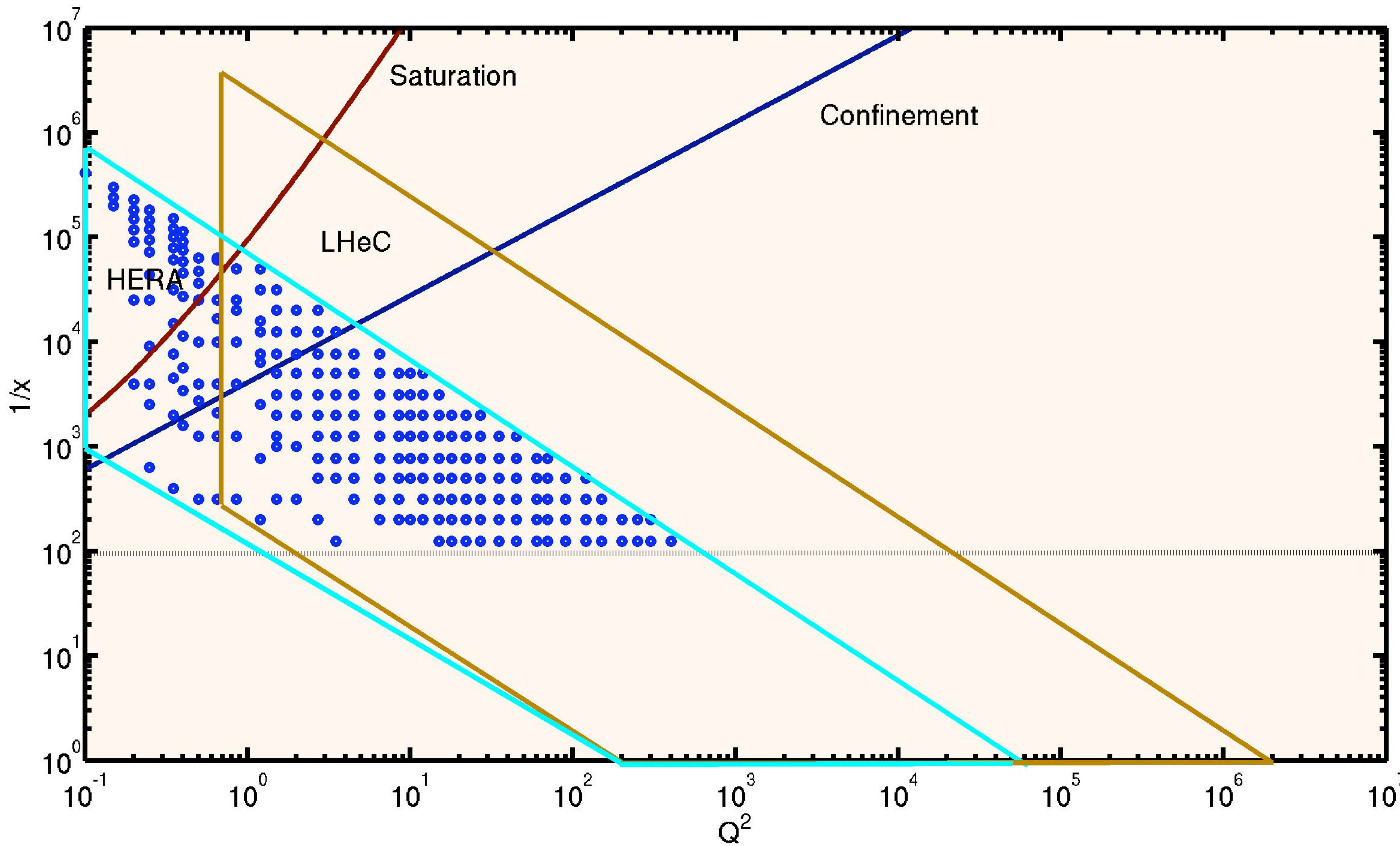
$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right)\widehat{s}^j\, G_j(z,x^\perp,z',x'^\perp;j)$$

conformal:
$$G_j(z,x^\perp,z',x'^\perp) = \frac{1}{4\pi z z'}\frac{e^{(2-\Delta(j))\xi}}{\sinh \xi}\ ,$$

$$\Delta(j) = 2+\sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

confinement: $G_j(z,x^\perp,z',x'^\perp;j) \longrightarrow$ discrete sum

HERA vs LHeC region: dots are H1-ZEUS small-x data points



I. Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar\psi(x)\psi(x), \;\; \bar\psi(x) D_\mu \psi(x)$

$S(x) = TrF_{\mu\nu}^2(x), \;\; O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x), \;\; etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar\psi \not{D} \psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x), \; a(x), \; etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

$\mathcal{N} = 4$ SYM Scattering at High Energy

$$\langle e^{\int d^4x \phi_i(x)\mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string}\left[\phi_i(x,z)|_{z\sim 0} \to \phi_i(x)\right]$$

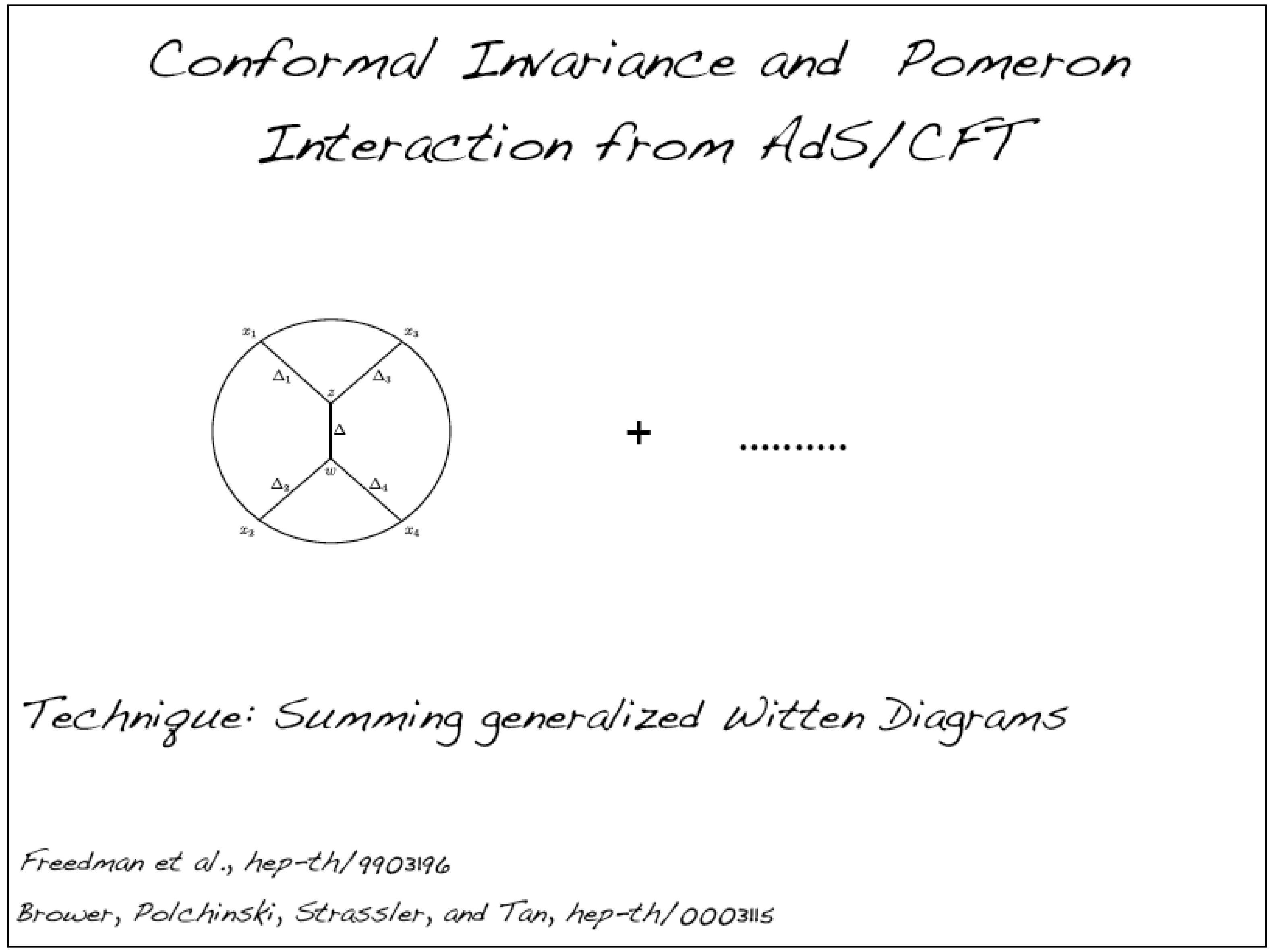
Bulk Degrees of Freedom from type-IIB Supergravity on AdS_5:

- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

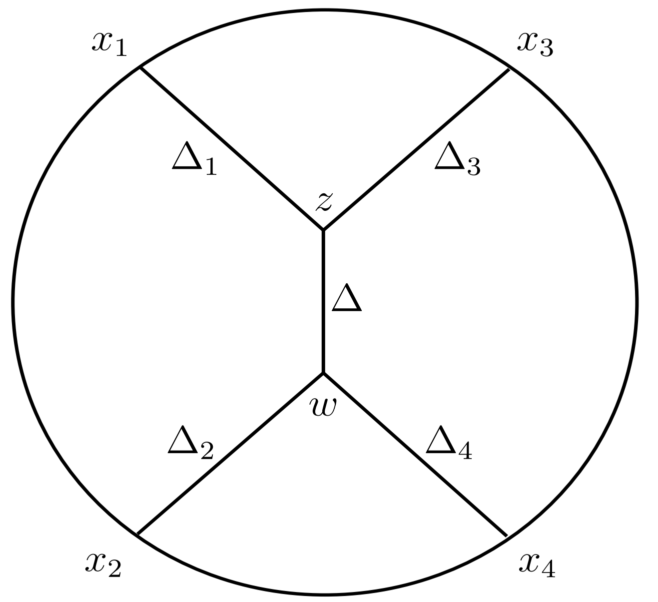
Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS



- Draw all "Witten-Feynman" Diagrams in AdS5,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$



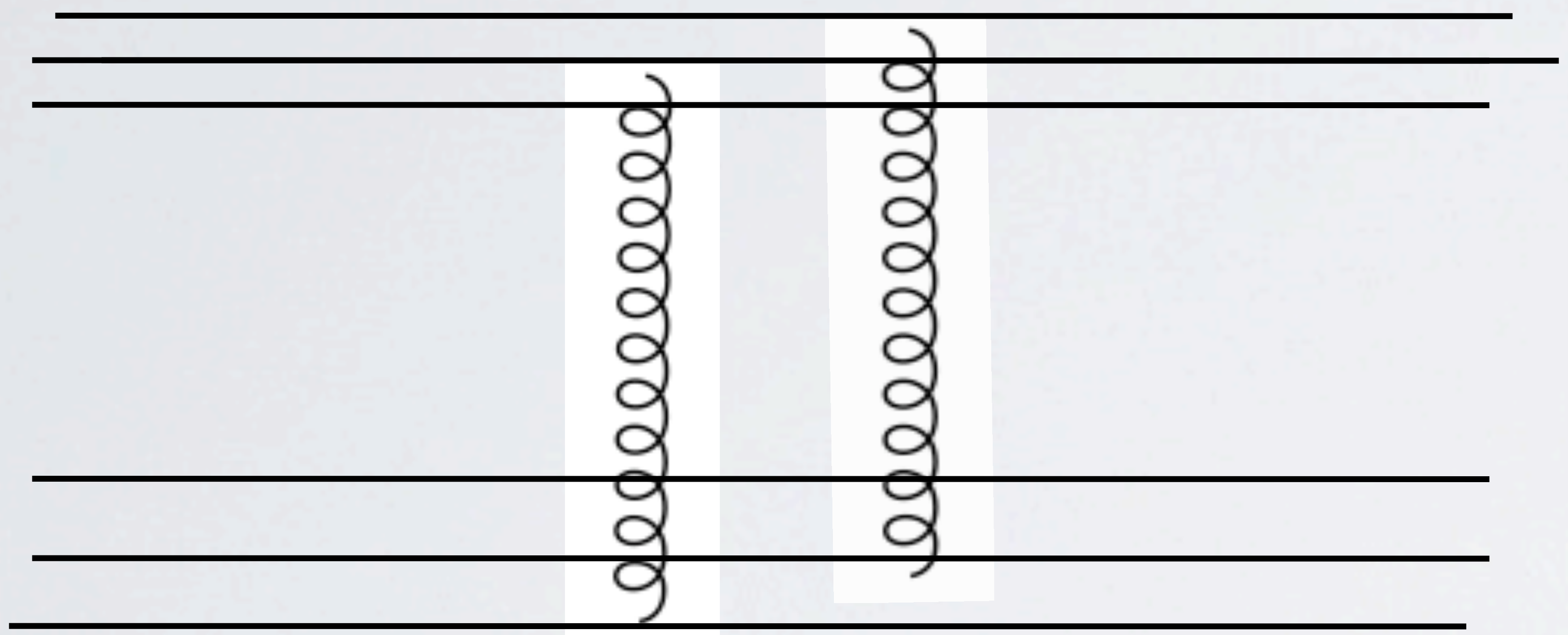
One Graviton Exchange at High Energy

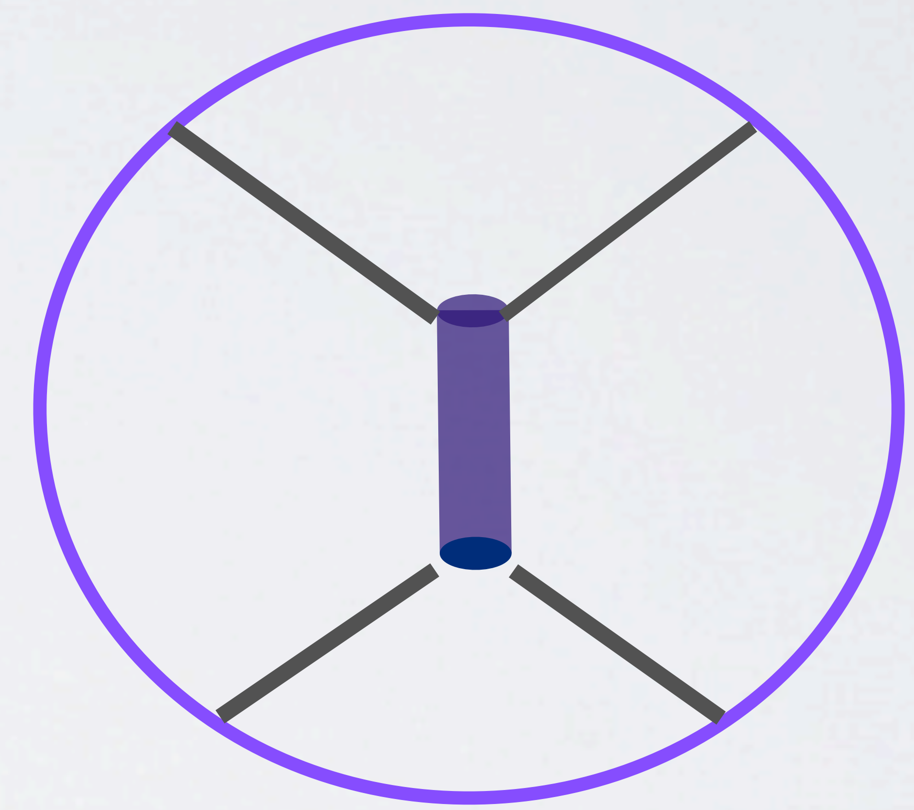
$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

WHAT IS THE BARE POMERON ? LEADING 1/N TERM CYLINDER EXCHANGE

WEAK: TWO GLUON <=> STRONG: ADS GRAVITON





$$J = 2$$

$$J_{cut} = 1 + 1 - 1 = 1$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \Big(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \Big)$$

F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253

Holographic Approach to QCD

- Spin-2 leads to too fast a rise for cross sections
- Need to consider $\lambda \equiv g^2 N_c$ finite
- Graviton (Pomeron) becomes j-Plane singularity at

$$\boxed{j_0 : \ 2 \to 2 - 2/\sqrt{\lambda}}$$

- Comfinement: Particles and Regge trajectories

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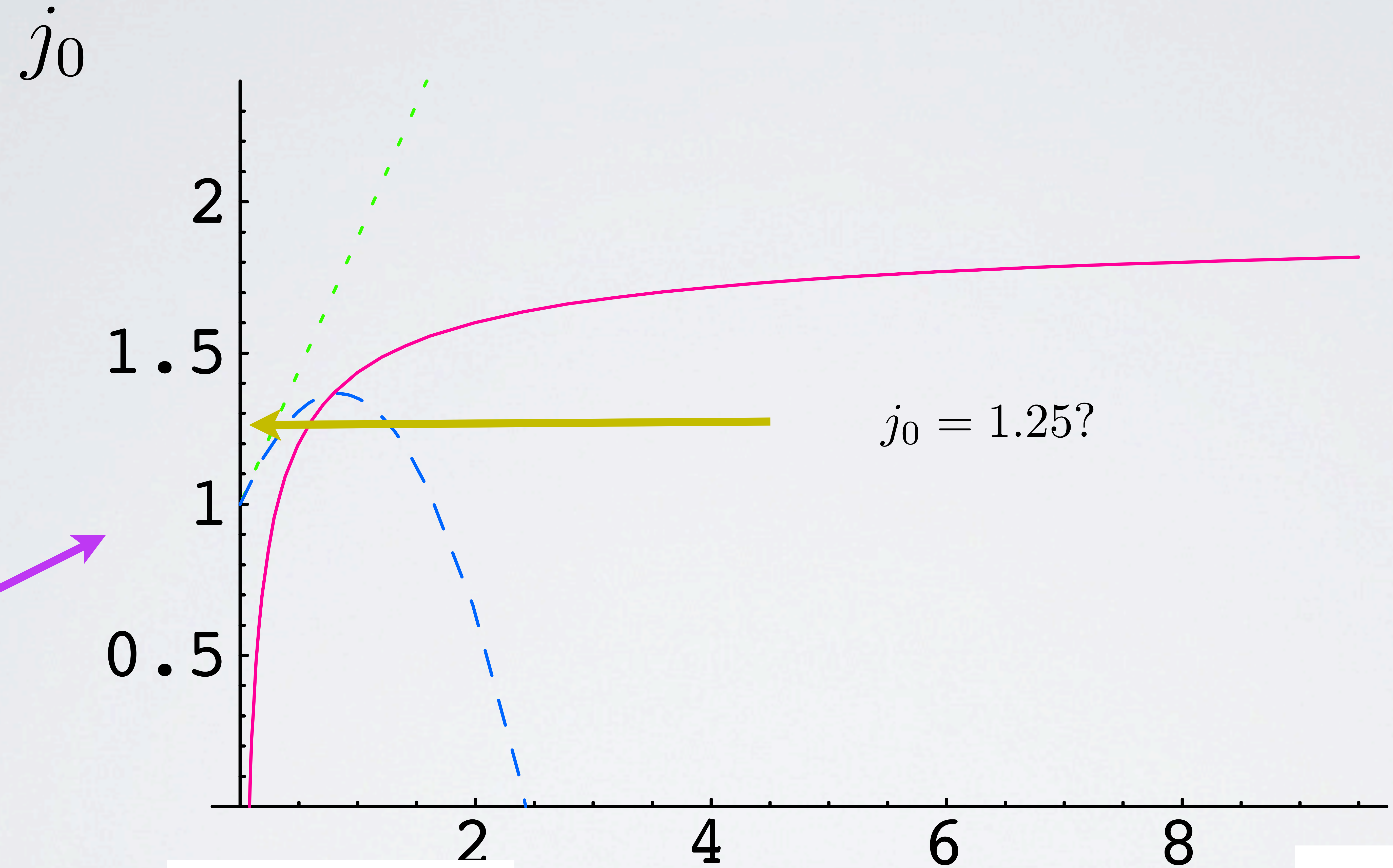
$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



$j_0 = 1$

Two Gluon

BFKL

$j_0 = 1 + \ln(2) g^2 N_c / \pi^2$

QCD?

BPST

$j_0 = 2 - 2/\sqrt{g^2 N_c}$

$j_0 = 2$

Graviton

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II: Pomeron in the conformal Limit, OPE, and Anomalous Dimensions

$$G_{mn} = g^0_{mn} + h_{mn}$$

Massless modes of a closed string theory:

Need to keep higher string modes

As CFT, equivalence to OPE in strong coupling: using AdS

CFT correlate function – coordinate representation

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \sim \cdots\cdots \mathcal{A}(u,v)$$

OPE:

$$\mathcal{A}(u,v) = \sum_{\Delta,J} a(\Delta,J) G_{(\Delta,J)}(u,v)$$

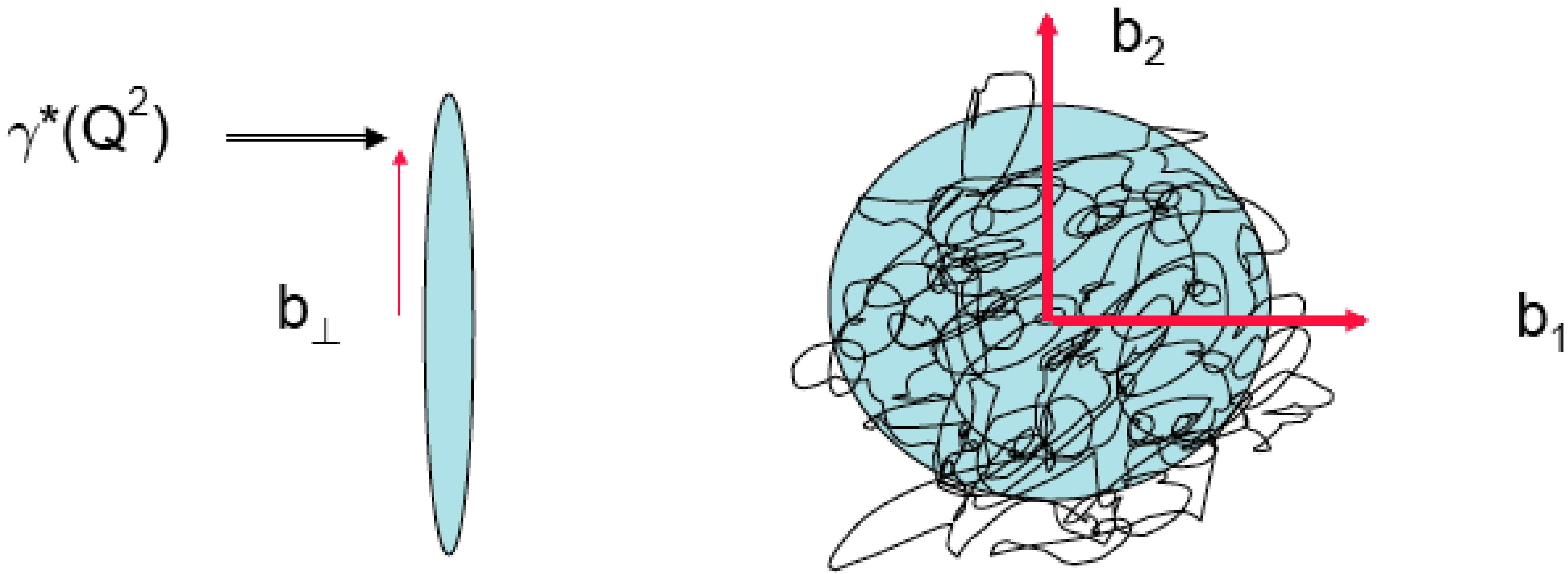
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}$$

Minkowski Limit: $u \longrightarrow 0, \qquad v \longrightarrow 1 + O(\sqrt{u})$

Dynamics $\boxed{\Delta(J,\lambda)\ , \quad J = 0,1,2,\cdots}$ $\quad \lambda = g^2 N_c$

QCD: EMERGENCE OF 5-DIM: ADS

"Fifth" co-ordinate is size z / z' of proj/target



5 kinematical Parameters:

2-d Longitudinal $p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$

2-d Transverse space: $x'_\perp - x_\perp = b_\perp$

1-d Resolution: $z = 1/Q$ (or $z' = 1/Q'$)

Full $O(4,2)$ Conformal Group

15 generators: $P_\mu, M_{\mu\nu}, D, K_\mu$

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

Longitudinal Boost: $SO(1,1)$

Maximal commuting subgroup: $SO(3,1)$

6 generators $\quad iD \pm M_{12}, P_1 \pm iP_2, K_1 \mp iK_2$

CFT correlate function – coordinate representation

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \sim \cdots\cdots \mathcal{A}(u,v)$$

OPE: $$\mathcal{A}(u,v) = \sum_{\Delta,J} a(\Delta,J) G_{(\Delta,J)}(u,v)$$

Amplitude – in mixed representation

$$SO(4,2) = SO(1,1) \times SO(3,1)$$

$$A(s,b) = \int_{-i\infty}^{i\infty} \frac{d\Delta}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dj}{2\pi i}\ \mathcal{A}(\Delta,j)\ \widetilde{s}^{\,j}\ \mathcal{Y}_{(\Delta,j)}(\vec{b})$$

Dynamics $$\boxed{\mathcal{A}(\Delta,j) \sim \frac{1}{\Delta - \Delta(j,\lambda)}}$$

$AdS/CFT ===> \quad Symmetry \leftrightarrow Isometry$

Full $O(4,2)$ Conformal Group as Isometries of AdS_5 Space

$$ds^2 = \frac{-dx^+ dx^- + (dx_\perp)^2 + dz^2}{z^2}$$

$SO(4,2) = SO(1,1) \times SO(3,1)$

$S(3,1) \simeq SL(2,C)$: "Mobius group" -
as Isometries of the Euclidean (transverse) AdS_3 Space

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as Isometries of the Euclidean (transverse) AdS_3 Space

$$A(s,b) = \int dz \int dz' \int_{-i\infty}^{i\infty} \frac{d\Delta}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dj}{2\pi} \; \mathcal{A}(\Delta, j, z, z') \; \widetilde{s}^{\,j} \; \mathcal{Y}_\Delta(L_{(b,z,z')})$$

$$A(s,b) = \int dz \int dz' \int_{-i\infty}^{i\infty} \frac{d\Delta}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dj}{2\pi} \; \mathcal{A}(\Delta, j, z, z') \; \widetilde{s}^{\,j} \; \mathcal{Y}_\Delta(L_{(b,z,z')})$$

AdS/CFT:

$$\boxed{\mathcal{A}(\Delta, j, z, z') = \Phi_1(z)\Phi_2(z)\Phi_3(z')\Phi_4(z') \times \frac{1+e^{-i\pi j}}{\sin \pi j} \times \mathcal{A}(\Delta, j)}$$

Dynamics:

$$\boxed{\mathcal{A}(\Delta, j) \sim \frac{1}{\Delta - \Delta(j,\lambda)}}$$

$$A(s,b) = \int dz dz' \, \Pi \Phi_i \sum_{j=0,2,\cdots} \beta(j) \, \widetilde{s}^{\,j} \; \mathcal{Y}_{\Delta(j)}(L_{(z,z',b)})$$

Anomalous Dimension: $\gamma(j,\lambda) \equiv \Delta(j,\lambda) - j - 2$

In the limit $\lambda \to \infty$, only $j = 2$ survives.

$$\mathcal{A}(\Delta, j) \sim \frac{1}{\Delta - \Delta(j, \lambda)}$$

String Theoretic Approach (BPST):

$$OPE ==> Pomeron\ \ Vertex\ \ Operator$$

$$(L_0 - 1)V_P = (\bar{L}_0 - 1)V_P = 0$$

- Brower, Polchinski, Strassler, and Tan: "The Pomeron and Gauge/String Duality," hep-th/063115

Strong Coupling Pomeron Propagator--Conformal Limit

- Use J-dependent Dimension

$$\Delta : \quad 4 \to \Delta(J) = 2 + [2\sqrt{\lambda}(J - J_0)]^{1/2} = 2 + \sqrt{j}$$

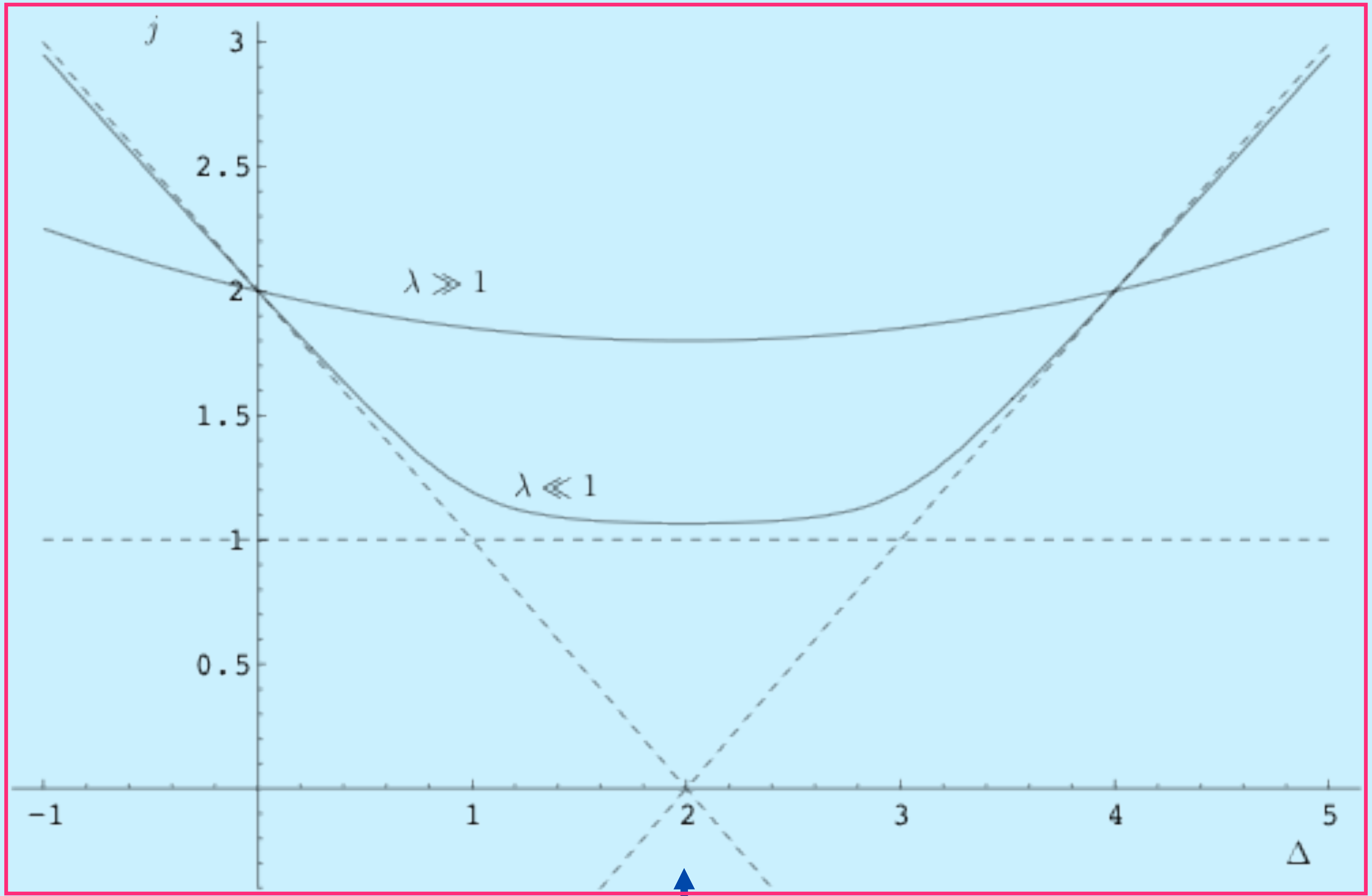
- BFKL-cut:

$$J_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

- Double-Mellin representation:

$$A(s, b - b', z, z') = \int \frac{dj}{2\pi i} \int \frac{d\Delta}{2\pi i} \cdots \frac{s^j e^{\xi \Delta}}{j - j_0 - \mathcal{D}(\Delta - 2)^2}$$

$\mathcal{N} = 4$ SYM Leading Twist $\Delta(J)$ vs J: Anomalous Dimensions



λ = 0 DGLAP (DIS moments)

$Tr[F_{+\mu} D_+^{j-2} F_+^\mu]$

(0,2) $T_{\mu\nu}$ $\gamma = 0$

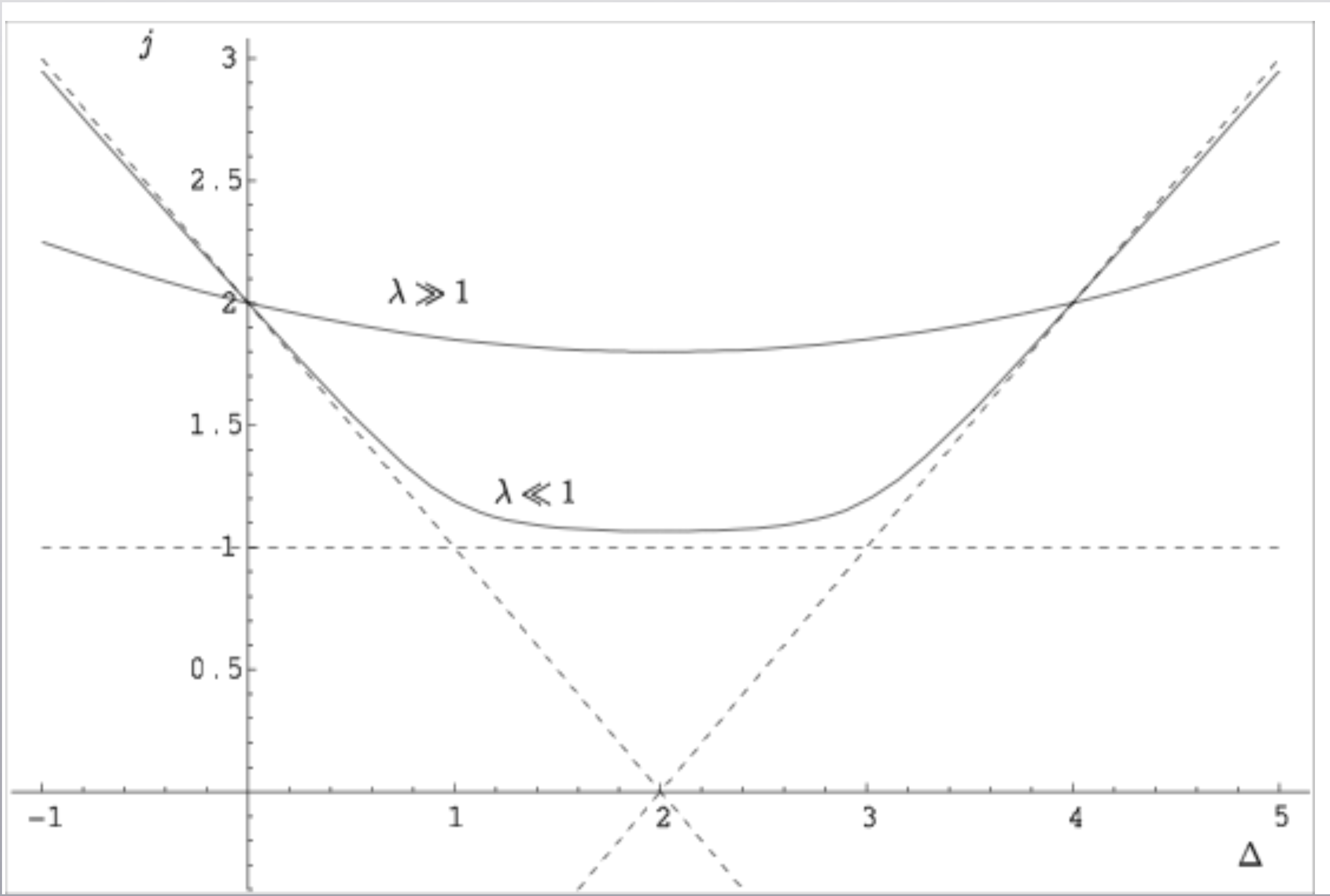
λ = 0, BFKL

$\lambda = g^2 N = 0$

j = j_0 @ min Δ

ANOMALOUS DIMENSIONS:

$$\gamma(j,\lambda) = \Delta(j,\lambda) - j - 2$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Energy-Momentum Conservation built-in automatically.

Holographic Approach to QCD

- Need to consider $\lambda \equiv g^2 N_c$ finite
- Graviton (Pomeron) becomes j-Plane singularity at

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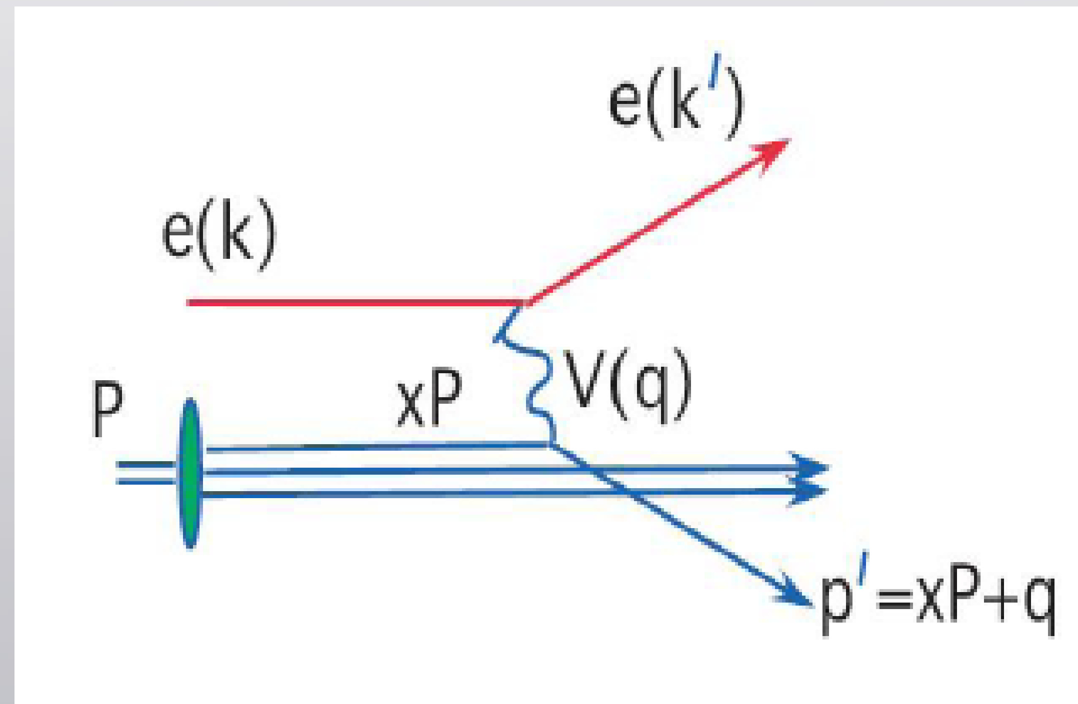
III. Deep Inelastic Scattering (DIS) at small-x

$Deep\ Inelastic\ Scattering\ (DIS)$



$$F_2(x, Q2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

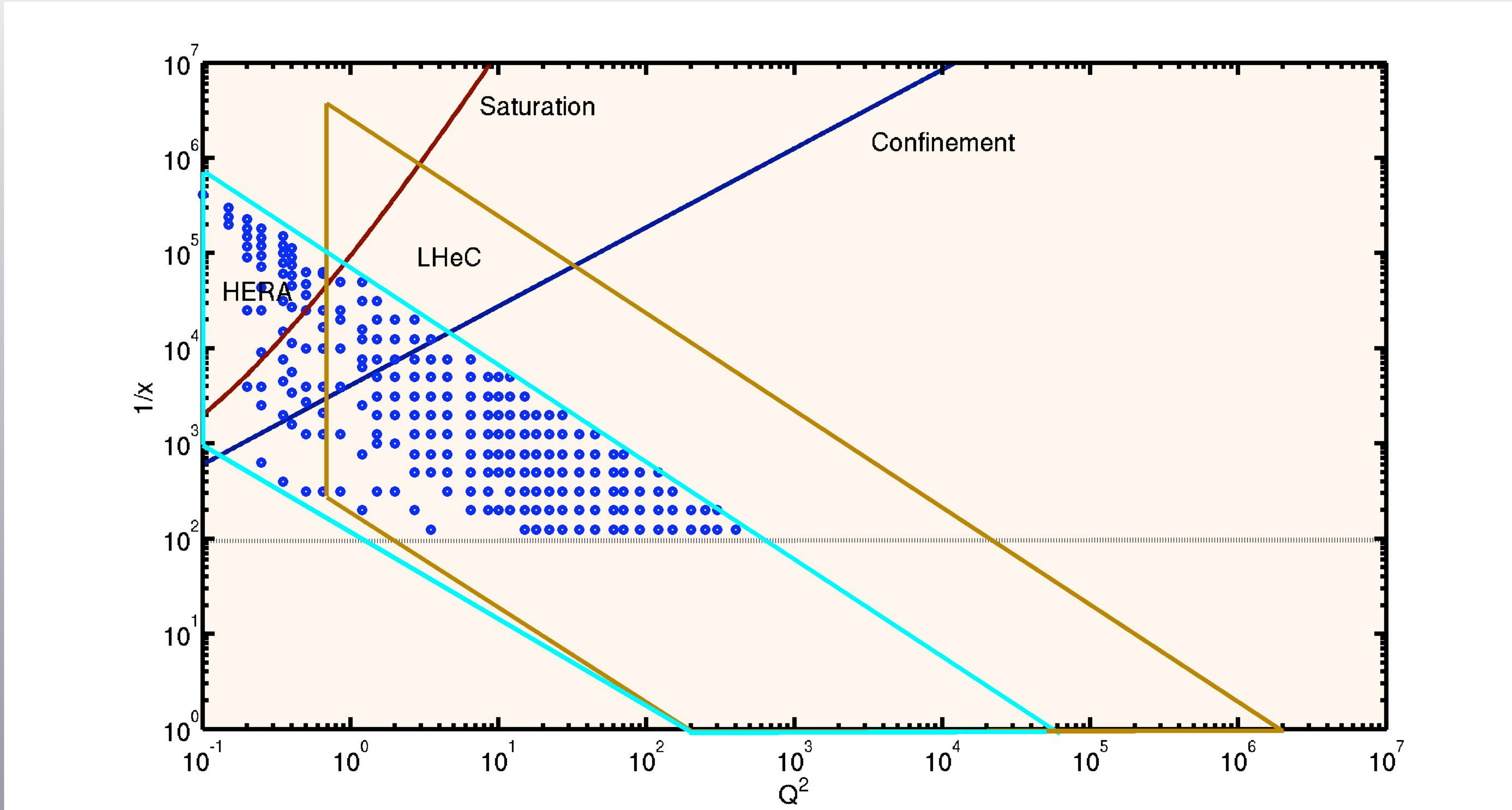
$$x \equiv \frac{Q^2}{s}$$

$Small\ x:\ \frac{Q^2}{s} \to 0$

$Optical\ Theorem$

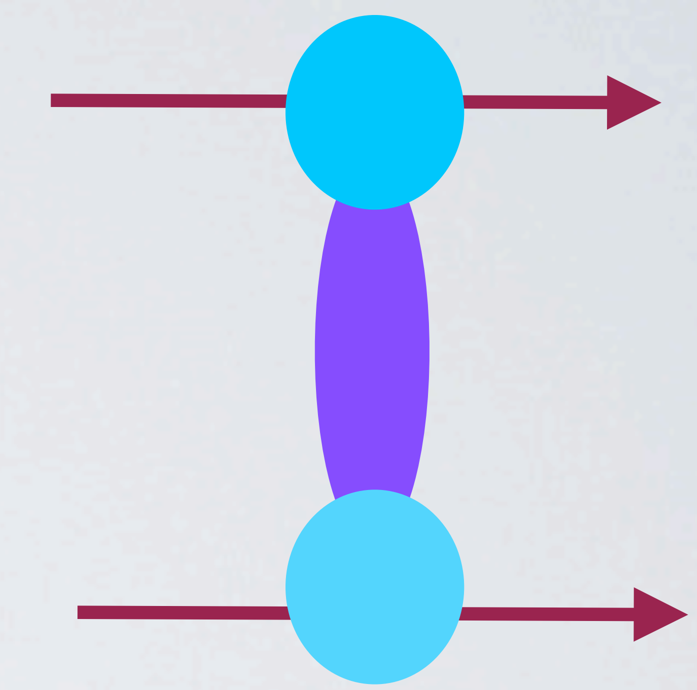
$$\sigma_{total}(s, Q^2) = (1/s)\mathrm{Im}\ A(s, t = 0; Q^2)$$

HERA vs LHeC region: dots are H1-ZEUS small-x data points



ELASTIC VS DIS ADS BUILDING BLOCKS

$$A(s, x_\perp - x'_\perp) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \Phi_{12}(z) G(s, x_\perp - x'_\perp, z, z') \Phi_{34}(z')$$

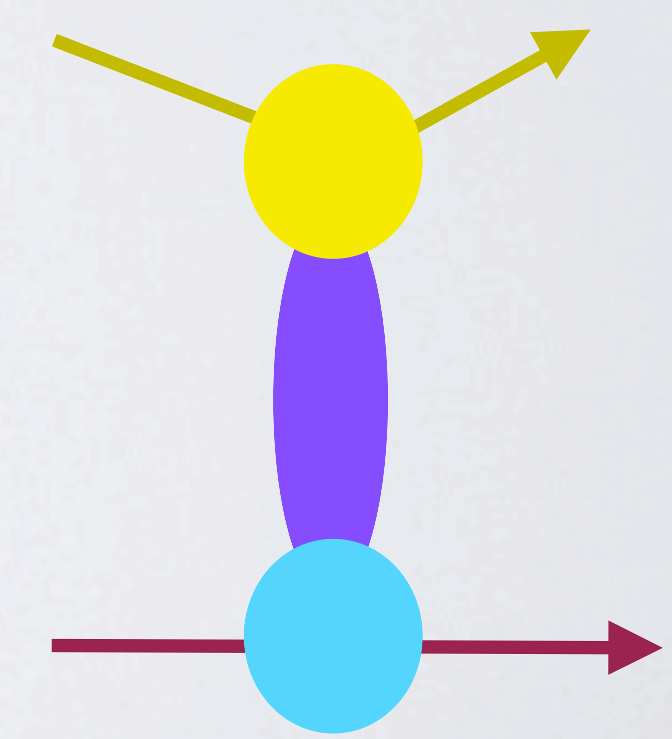


$$\sigma_T(s) = \frac{1}{s} Im A(s, 0)$$

---

for $\quad F_2(x, Q)$

$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z, Q) = \frac{1}{z}[Qz)^4 (K_0^2(Qz) + K_1^2(Qz)]$$



$$d^3\mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

DIS in String Theory
continued

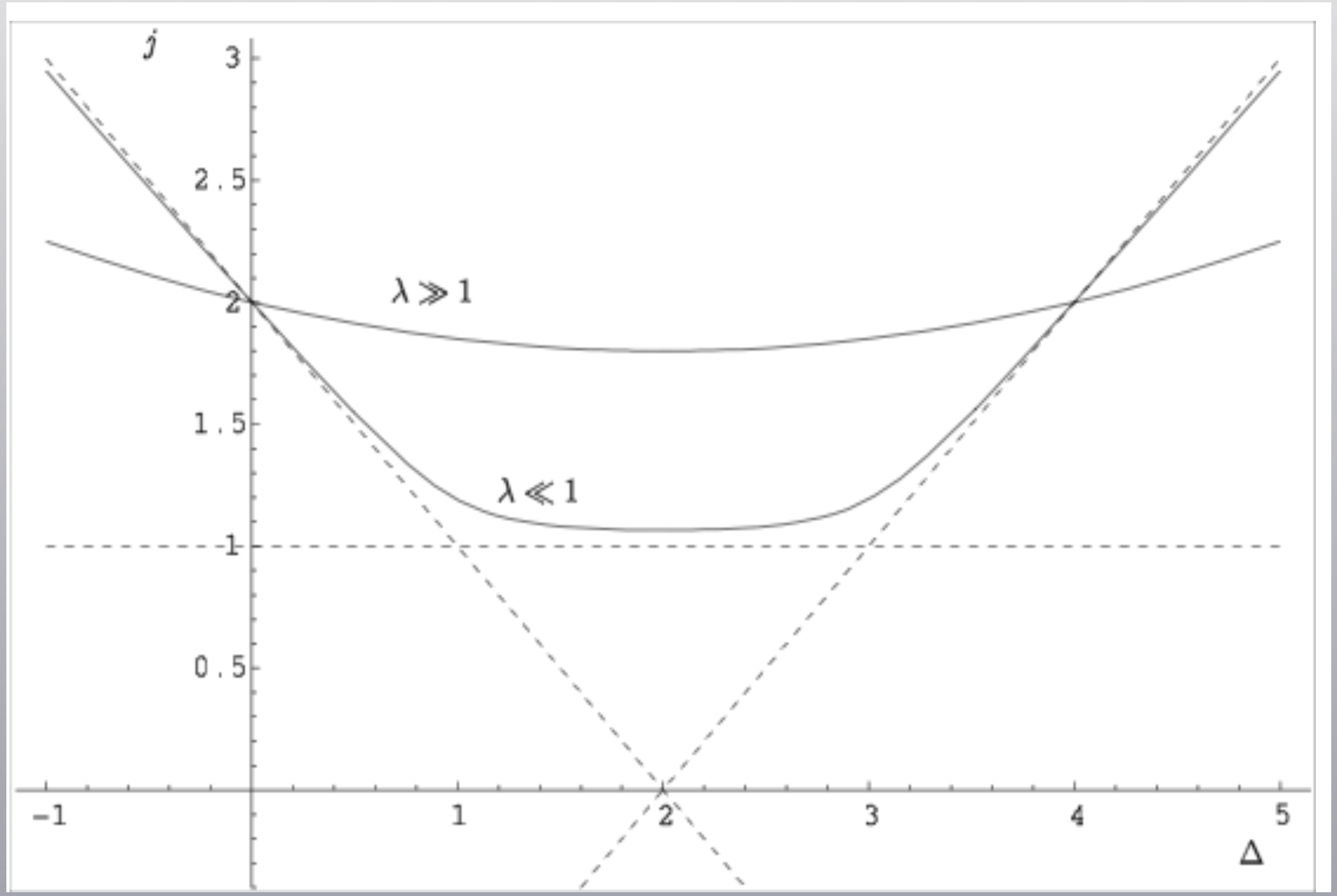
$F_2(x, Q^2)$ from AdS/CFT

$$F_2 = c \frac{Q}{Q'} \frac{(Q_0^2 \frac{Q}{Q'} \frac{1}{x})^{1-\rho}}{\sqrt{\log(Q_0^2 \frac{Q}{Q'} \frac{1}{x})}} \exp(-\frac{\log^2(\frac{Q}{Q'})}{\rho \log(Q_0^2 \frac{Q}{Q'} \frac{1}{x})})$$

- This is the expression we will use later for comparing to data. Let's make a few comments about this function.
- c is a dimensionless normalization constant. I have grouped here all the constants the multiply F_2, including the coupling constant that comes from χ, and only appears as product together with normalization.
- At any Q^2 fixed, we see that at small x the term $(\frac{1}{x})^{(1-\rho)}$ dominates. This leads to a violation of the Froissart bound.

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2} F_2(x, Q^2) \rightarrow Q^{-\gamma_n}$$



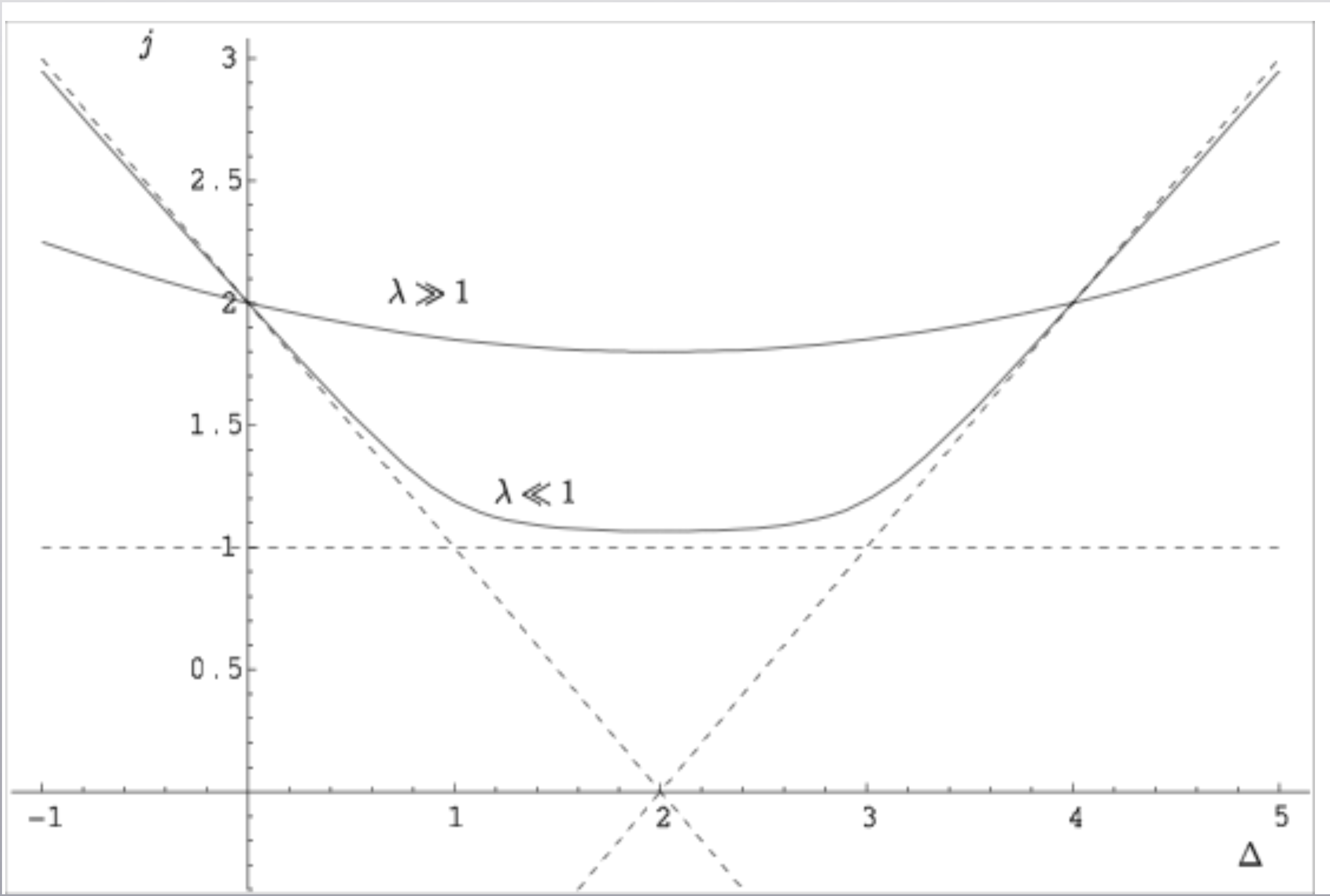
$\gamma_2 = 0$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

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IV: More on Pomeron and Odderon in the conformal Limit

Massless modes of a closed string theory:

metric tensor, $G_{mn} = g^0_{mn} + h_{mn}$

Kolb-Ramond anti-sym. tensor, $b_{mn} = -b_{nm}$

dilaton, etc. $\phi, \chi, \cdots$

Gauge/String Duality: Conformal Limit

- C=+1: Pomeron <===> Graviton

$$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda) .$$

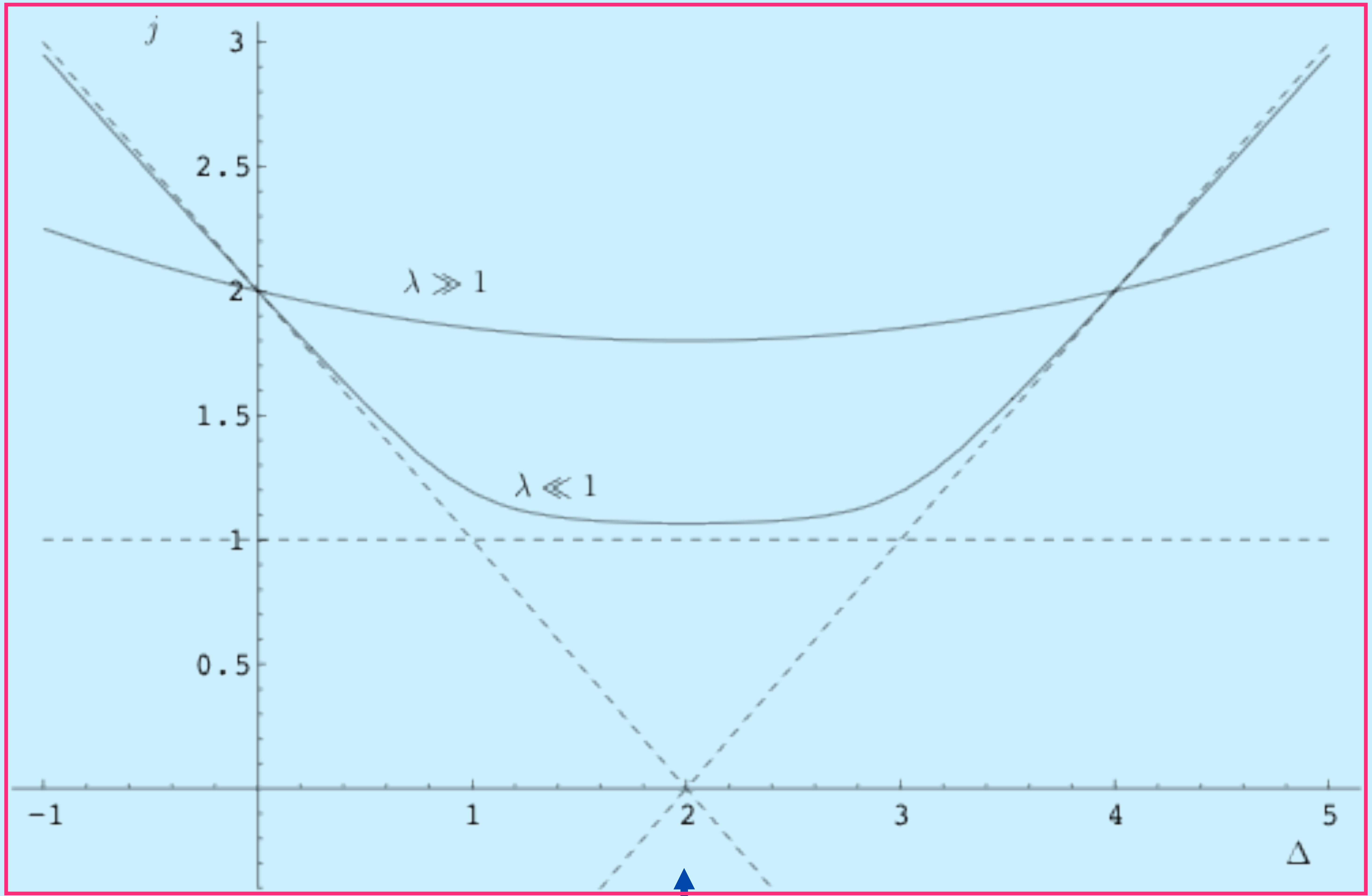
- C=-1: Odderon <===> Kalb-Ramond Field

$$j_0^{(-)} = 1 - m_{AdS}^2/2\sqrt{\lambda} + O(1/\lambda) .$$

	Weak Coupling	Strong Coupling
$C=+1$	$j_0^{(+)} = 1 + (\ln 2)\,\lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C=-1$	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\,\lambda/\pi + O(\lambda^2)$ $j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ $j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

Table 1: Pomeron and Odderon intercepts at weak and strong coupling.

$\mathcal{N} = 4$ SYM Leading Twist $\Delta(J)$ vs J: Anomalous Dimensions



$\lambda = 0$ DGLAP (DIS moments)

$Tr[F_{+\mu} D_+^{j-2} F_+^{\mu}]$

(0,2) $T_{\mu\nu}$ $\gamma = 0$

$\lambda = 0$, BFKL

$\lambda = g^2 N = 0$

j = j_0 @ min Δ

ANOMALOUS DIMENSION:

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)} \qquad \gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

$$\gamma_2 = 0$$

Energy-Momentum Conservation built-in automatically.

Connection to Spin Chain in $\mathcal{N} = 4$ YM:

$$tr\ D^S\ Z^\tau \qquad \widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

$$\tau = 2\,, \quad \widetilde{\Delta}(S) = \Delta(S+2) - 2$$

$$tr F_{\mu\nu} D_\nu \cdots D_{\nu'} F_{\nu'\mu'}$$

$$a_1(2,\lambda) = 2\sqrt{\lambda} - 1 + O(1/\sqrt{\lambda})$$

$$a_2(2,\lambda) = 3/2 + O(1/\sqrt{\lambda})$$

B.Basso, 1109.3154v2

$$S = 0 \to \mathrm{BPS} \qquad \widetilde{\Delta}(S)^2 \simeq 4 + 2\sqrt{\lambda}\ S$$

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau, \lambda)S + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_P = 2 - \frac{2}{\lambda^{1/2}}$$

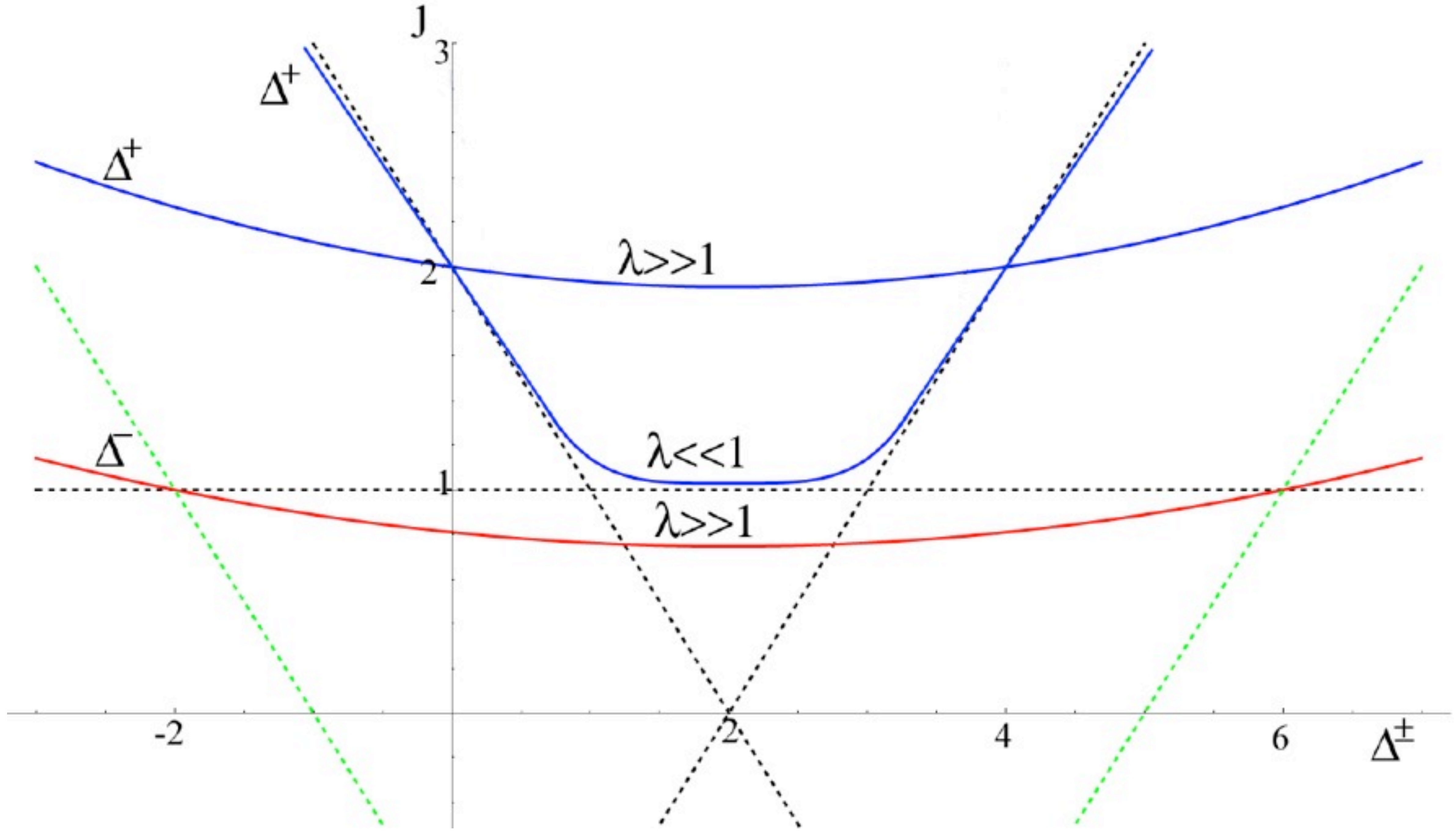
Brower, Polchinski, Strassler, Tan

$$S = J - 2$$

$$\widetilde{\Delta} = \Delta - 2$$

J vs Delta Curves

$$\Delta^{(\pm)}(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j - j_0^{(\pm)})}$$



POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_P = 2 - \frac{2}{\lambda^{1/2}}$$

Brower, Polchinski, Strassler, Tan

ODDERON

Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}}$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}}$$

Brower, Djuric, Tan

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_P = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \cdots$$

Brower, Polchinski, Strassler, Tan

Costa, Goncalves, Penedones (1209.4355)
Kotikov, Lipatov (1301.0882)

ODDERON

Kotikov, Lipatov, et al.

Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} -$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$$

Brower, Djuric, Tan

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_P = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \cdots$$

Brower, Polchinski, Strassler, Tan

Costa, Goncalves, Penedones (1209.4355)

Kotikov, Lipatov (1301.0882)

ODDERON

Kotikov, Lipatov, et al.

Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \cdots\cdots$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}}$$

Brower, Djuric, Tan

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_P = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \cdots$$

Brower, Polchinski, Strassler, Tan

Costa, Goncalves, Penedones (1209.4355)

Kotikov, Lipatov (1301.0882)

ODDERON

Kotikov, Lipatov, et al.

Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \cdots\cdots$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} - \frac{0}{\lambda} - \frac{0}{\lambda^{3/2}} - \cdots\cdots$$

Brower, Costa, Djuric, Raben, Tan (to appear shortly.)

Outline

- QCD High Energy Scattering with AdS/CFT -- Universality
- Consequence of Conformal Invariance
    - DIS at low-x -- Unification
        - DGLAP (large Q) vs BFKL (small x)
        - OPE (Anomalous Dimensions) and Conformal Pomeron
    - Conformal Pomeron and Odderon Intercepts in strong coupling
- Saturation, Confinement, etc. and DIS
- Summary and Outlook

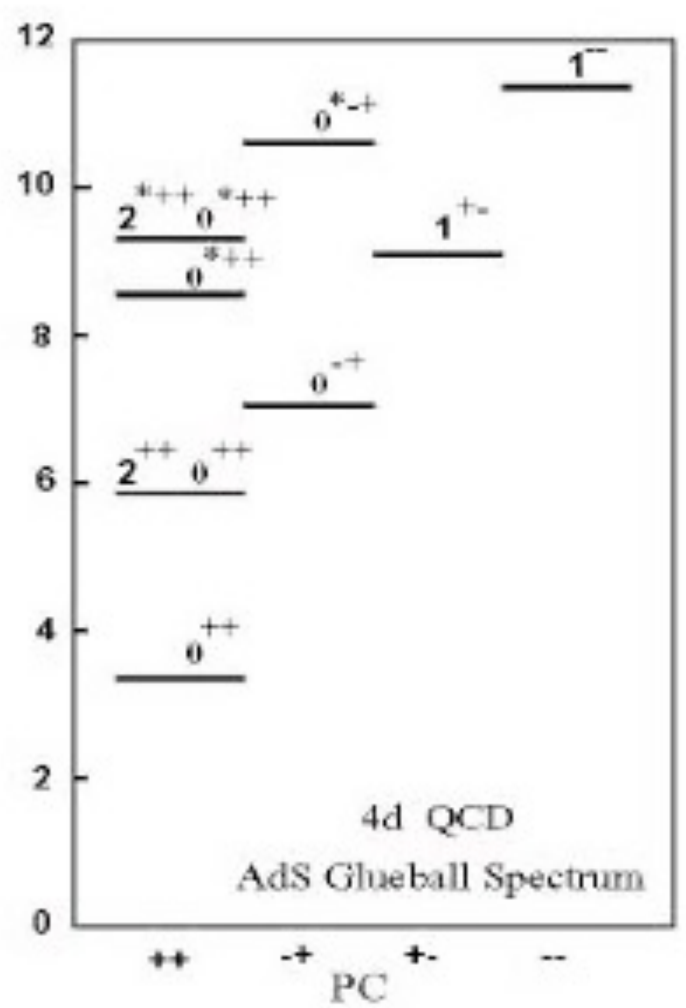
IV. Deep Inelastic Scattering (DIS) at small-x:

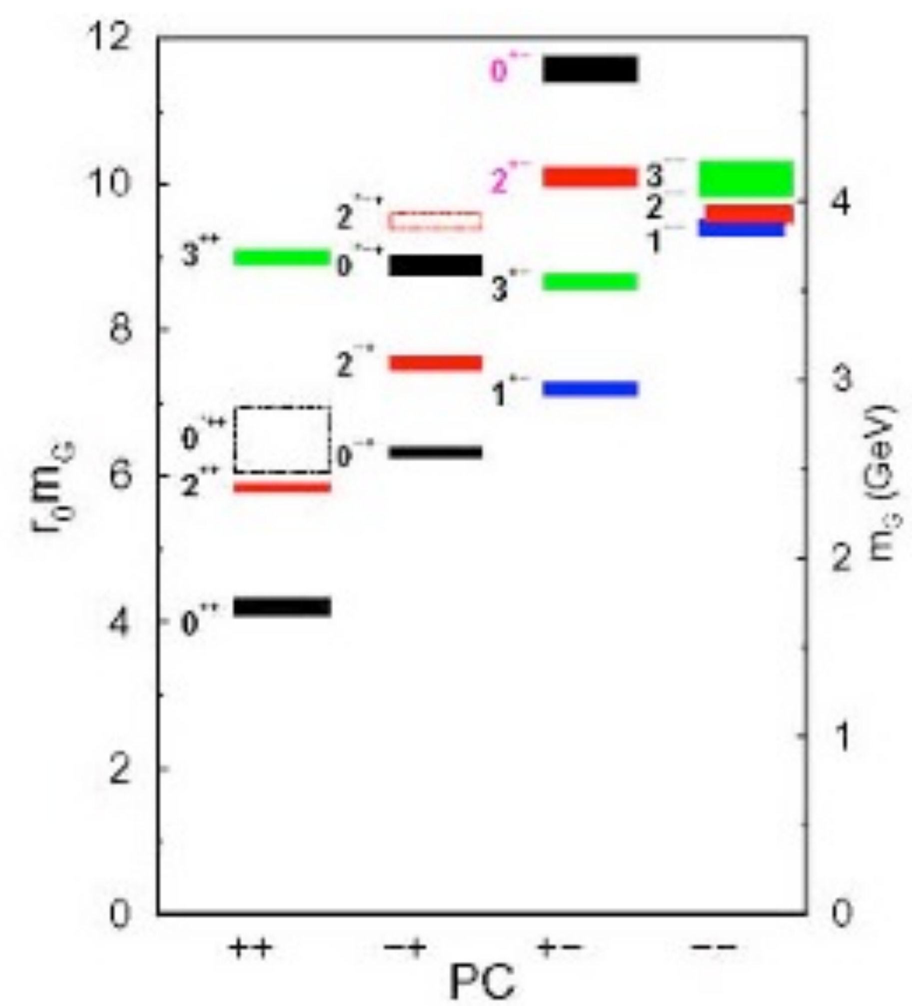
Confinement ?

Satuation ?

Confinement Deformation: Glueball Spectrum

$(\lambda = \infty)$





Four-Dimensional Mass:

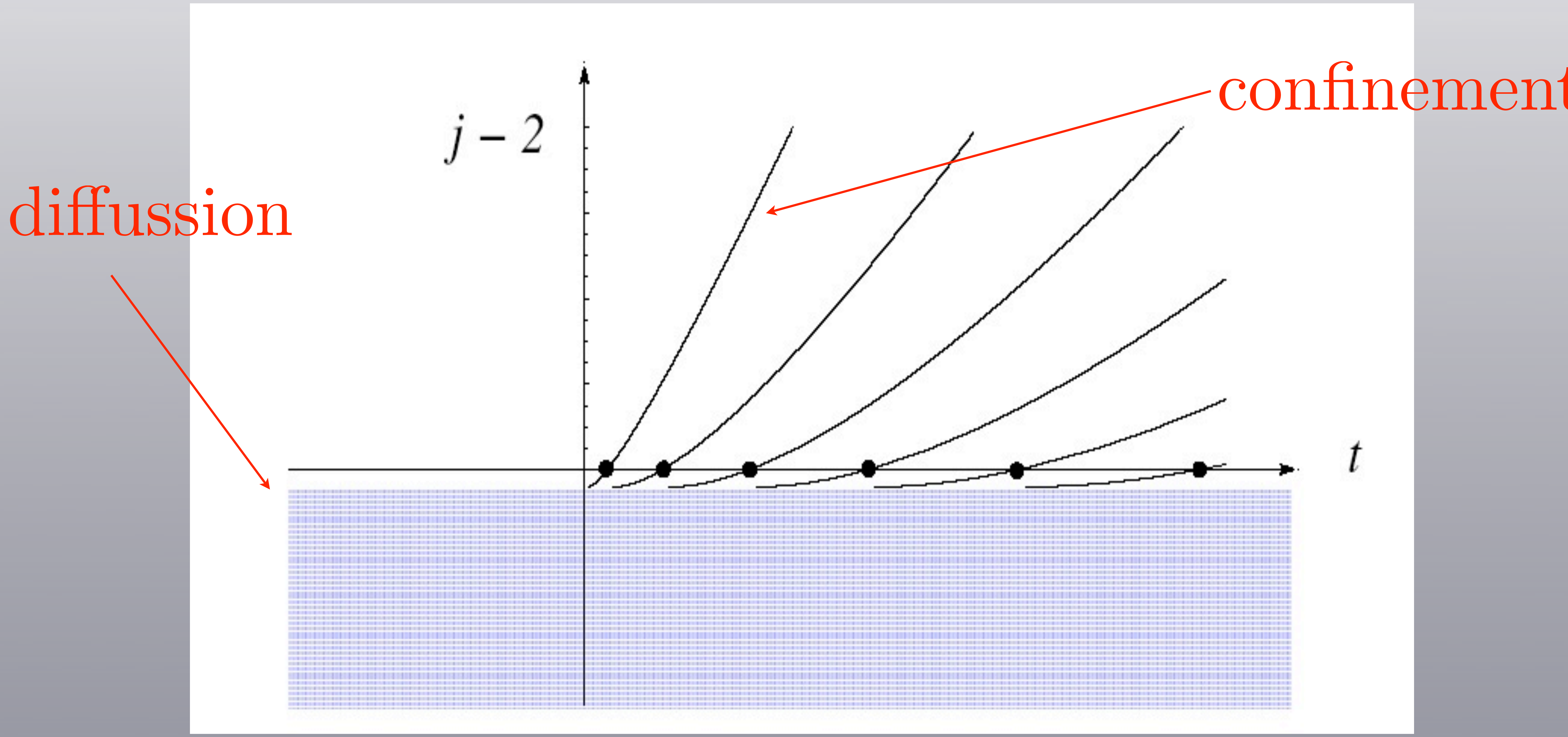
$$E^2 = (p_1^2 + p_2^2 + p_3^2) + M^2$$

5-Dim Massless Mode:

$$0 = E^2 - (p_1^2 + p_2^2 + p_3^2 + p_r^2)$$

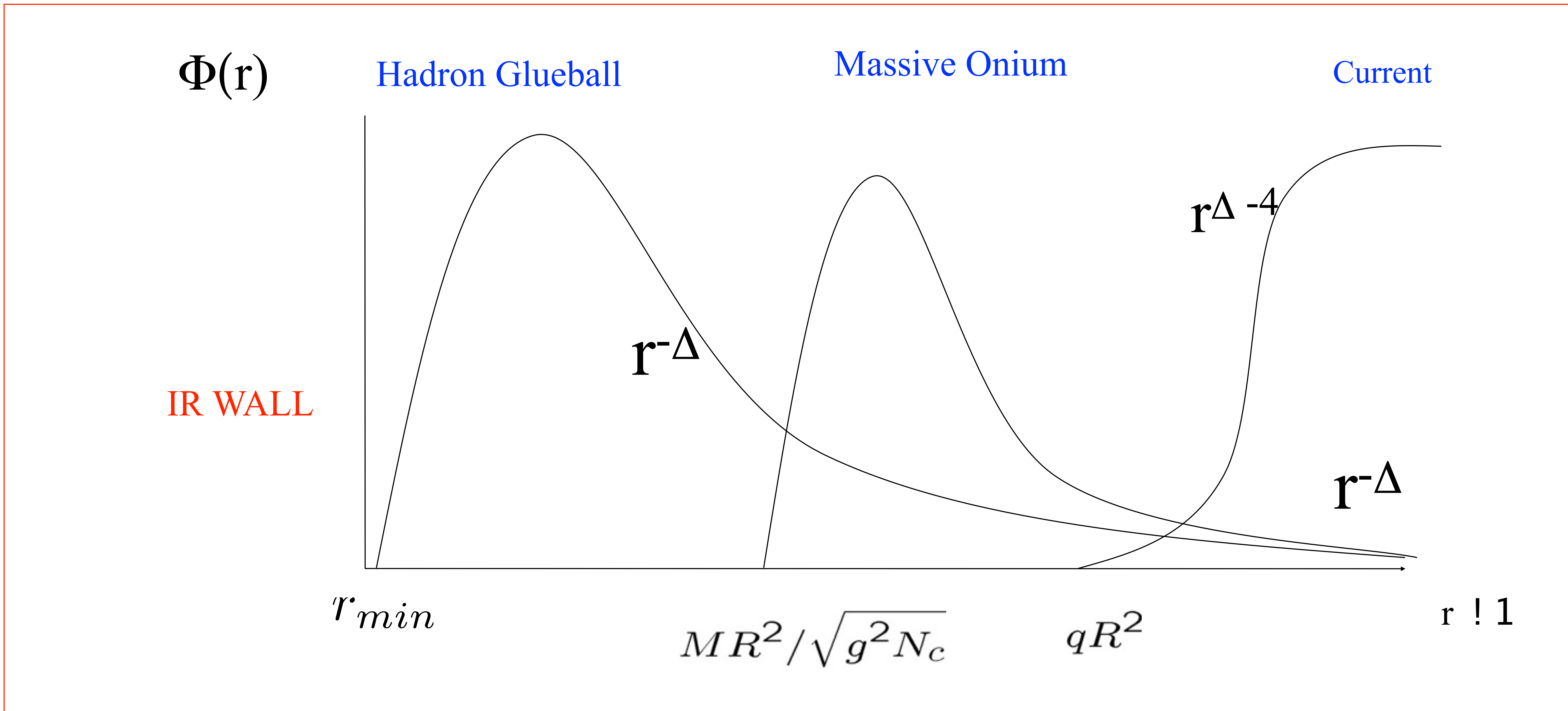
Unified Hard (conformal) and Soft (confining) Pomeron

At finite λ, due to Confinement in AdS, *at* $t > 0$
aymptotical linear Regge trajectories



- Universality and Holographic:

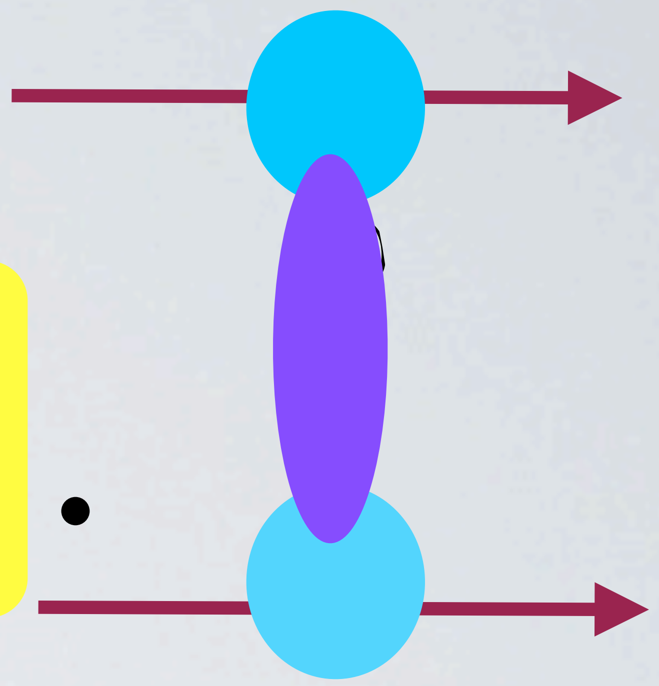
By choosing wave functions, Φ, can treat
DIS, Higgs Production, Proton-Proton, etc., on equal footing.



ADS BUILDING BLOCKS BLOCKS

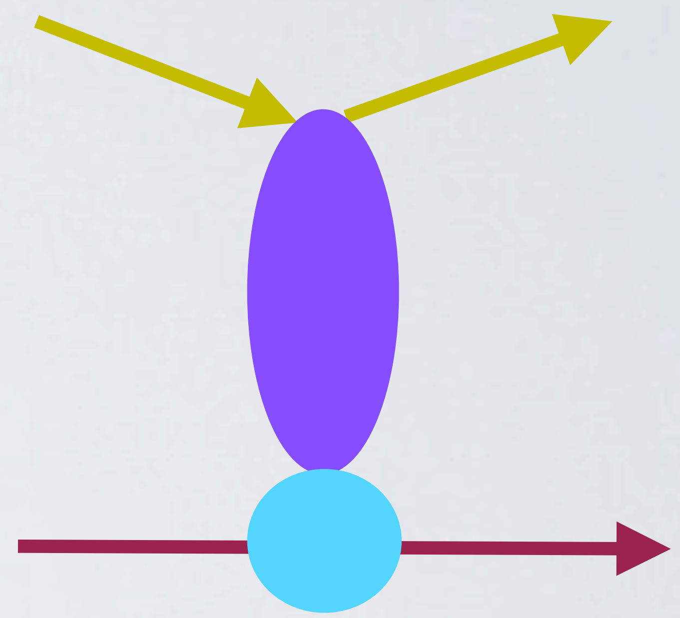
For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24} \, .$$



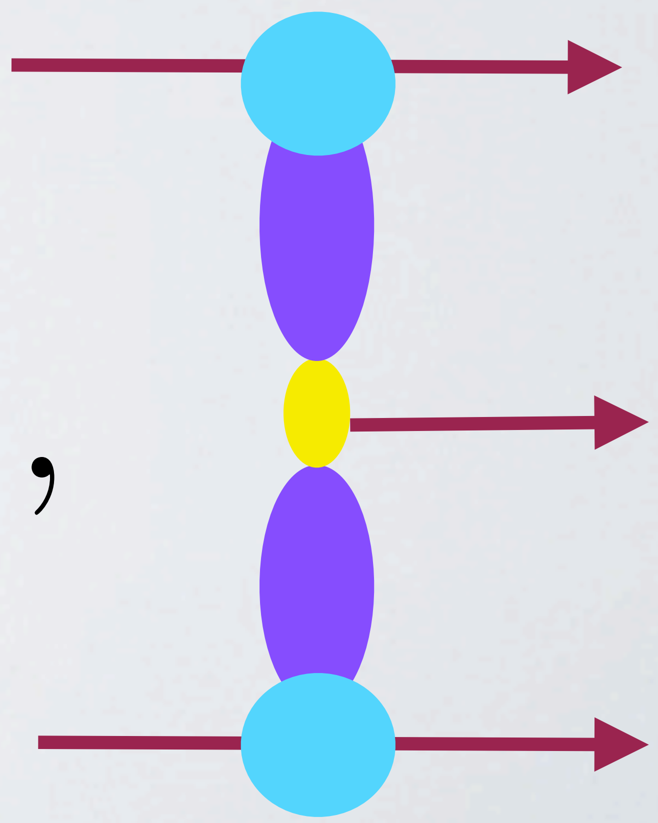
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \; e^{i\mathbf{q}_\perp \cdot (\mathbf{x}-\mathbf{x}')} \; \Phi_{13}(z) \, \mathcal{K}(s, \mathbf{x}-\mathbf{x}', z, z') \; \Phi_{24}(z')$$

$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

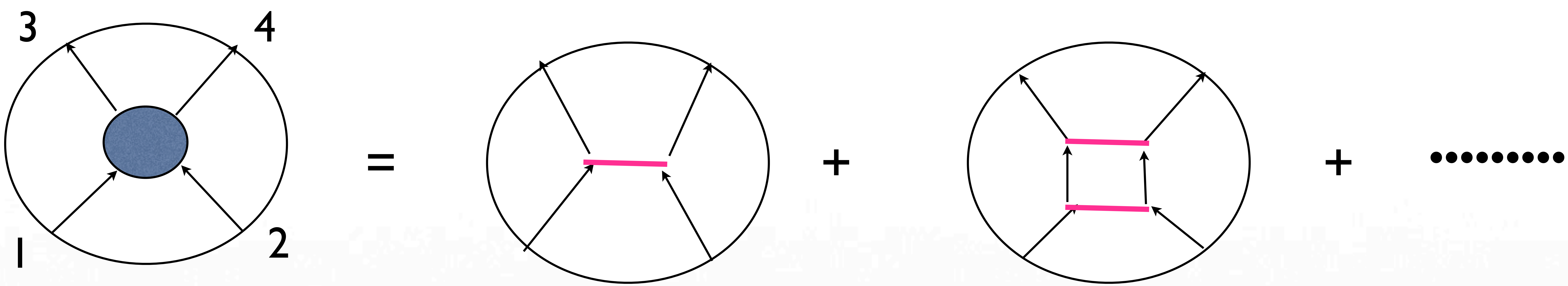


For 2-to-3

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24} \, ,$$



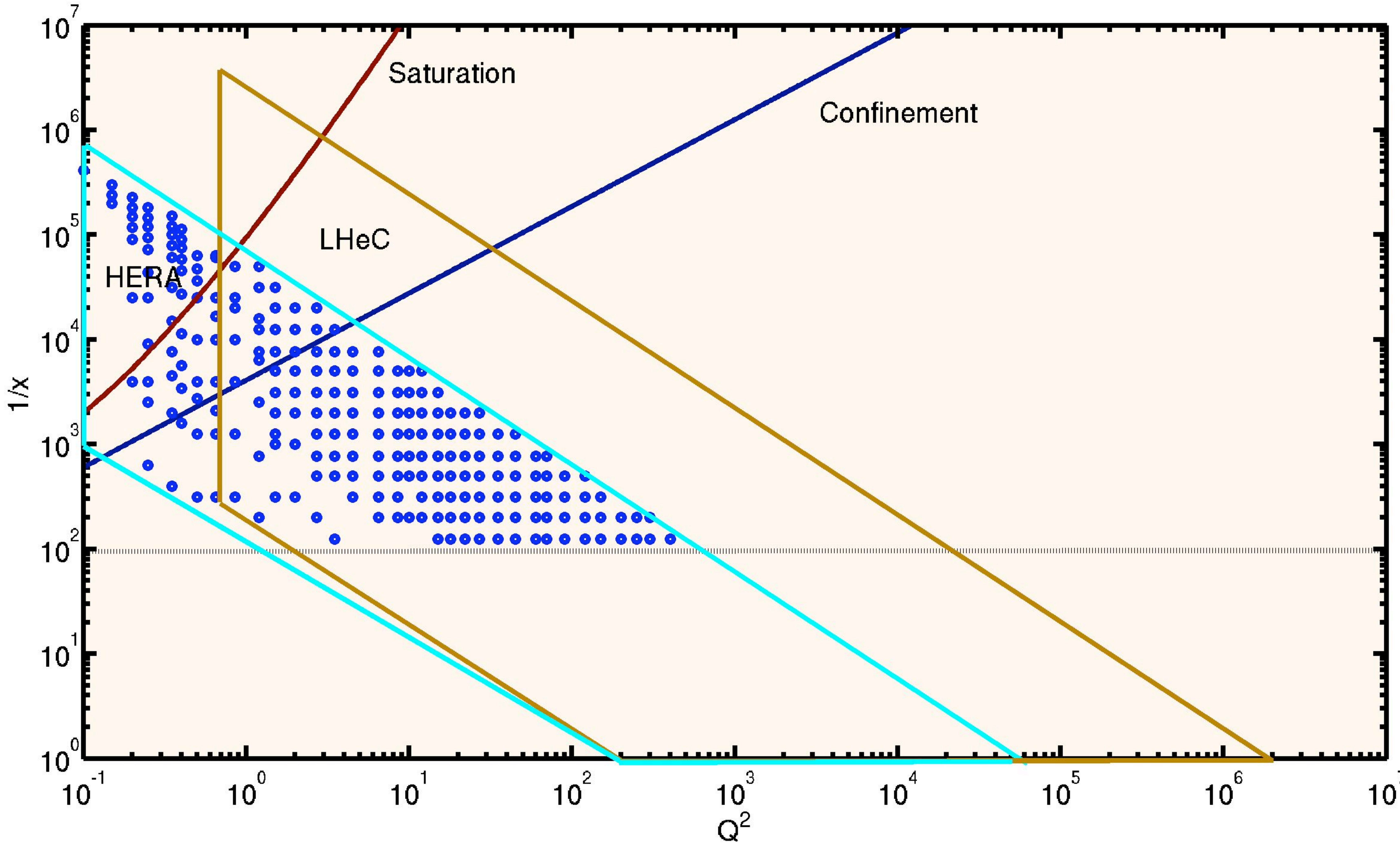
Higher Orders Witten Diagrams:



$s \to \infty,\ t = -q_\perp^2 < 0$

$$A_4(s,t) \simeq \int d^2b\, e^{-i\mathbf{b}\cdot\mathbf{q}_\perp} \int d\mu(z) \int d\mu(z')$$

$$\times \quad \phi_1(z,\mathbf{b})\phi_3(z,\mathbf{b})\mathcal{K}(s,\mathbf{b}-\mathbf{b}',z,z')\phi_2(z',\mathbf{b}')\phi_4(z',\mathbf{b}')$$


Saturation
Confinement
LHeC
HERA
1/x
Q^2
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7

DIS in String Theory

The Hard-wall Model continued

We will take over the structure function formula we had before, and just replace the Pomeron exchange kernel with the new version.

$F_2(x, Q^2)$ from hard-wall AdS/CFT

$$F_2 = c\frac{Q}{Q'}\frac{(Q_0^2\frac{Q}{Q'}\frac{1}{x})^{1-\rho}}{\sqrt{\log(Q_0^2\frac{Q}{Q'}\frac{1}{x})}}(\exp(-\frac{\log^2(\frac{Q}{Q'})}{\rho\log(Q_0^2\frac{Q}{Q'}\frac{1}{x})}) + \mathcal{F}\exp(-\frac{\log(\frac{Q_0^2}{QQ'})^2}{\rho\log(Q_0^2\frac{Q}{Q'}\frac{1}{x})}))$$

we see that this part is the same as before, while this part is new. The function $\mathcal{F}$ is given by

$$\mathcal{F}(x, Q, Q') = 1 - 4\sqrt{\pi\log(Q_0^2\frac{Q}{Q'}\frac{1}{x})}e^{\eta^2}erfc(\eta)$$

where

$$\eta = \frac{\log(\frac{x}{Q^2}(Q_0^2\frac{Q}{Q'}\frac{1}{x})^{1-\rho})}{\sqrt{\rho\log(Q_0^2\frac{Q}{Q'}\frac{1}{x})}}$$

- # Eikonal Sum: derived both via Cheng-Wu or by Shock-wave method

$$A_{2\to 2}(s,t) \simeq -2is \int d^2b\, e^{-ib^\perp q_\perp} \int dz dz' P_{13}(z) P_{24}(z') \left[e^{i\chi(s,b^\perp,z,z')} - 1\right]$$

$$P_{13}(z) = (z/R)^2 \sqrt{g(z)}\Phi_1(z)\Phi_3(z) \qquad P_{24}(z) = (z'/R)^2 \sqrt{g(z')}\Phi_2(z')\Phi_4(z')$$

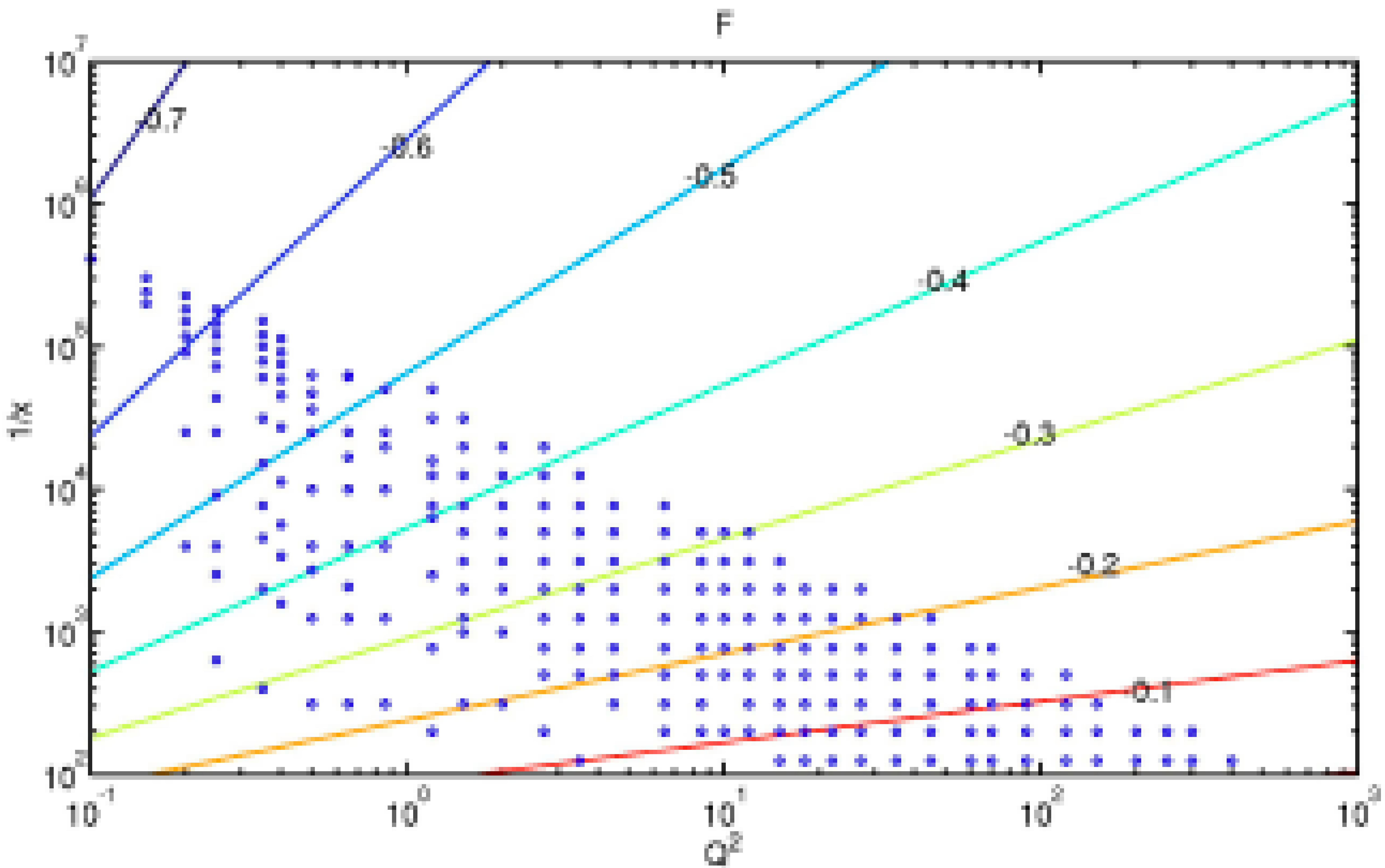
transverse AdS_3 space !!

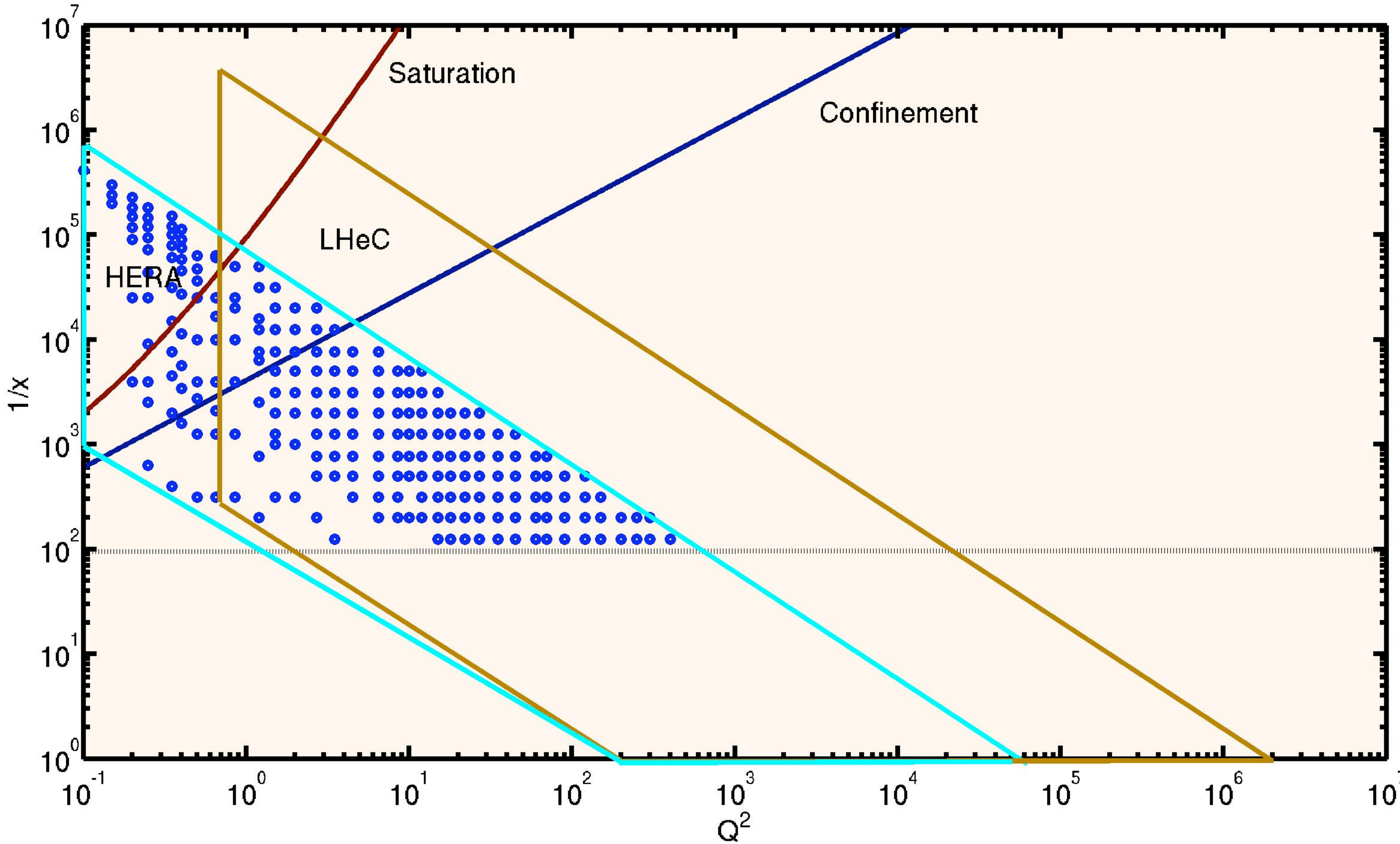
$$\chi(s, x^\perp - x'^\perp, z, z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s, x^\perp - x'^\perp, z, z')$$

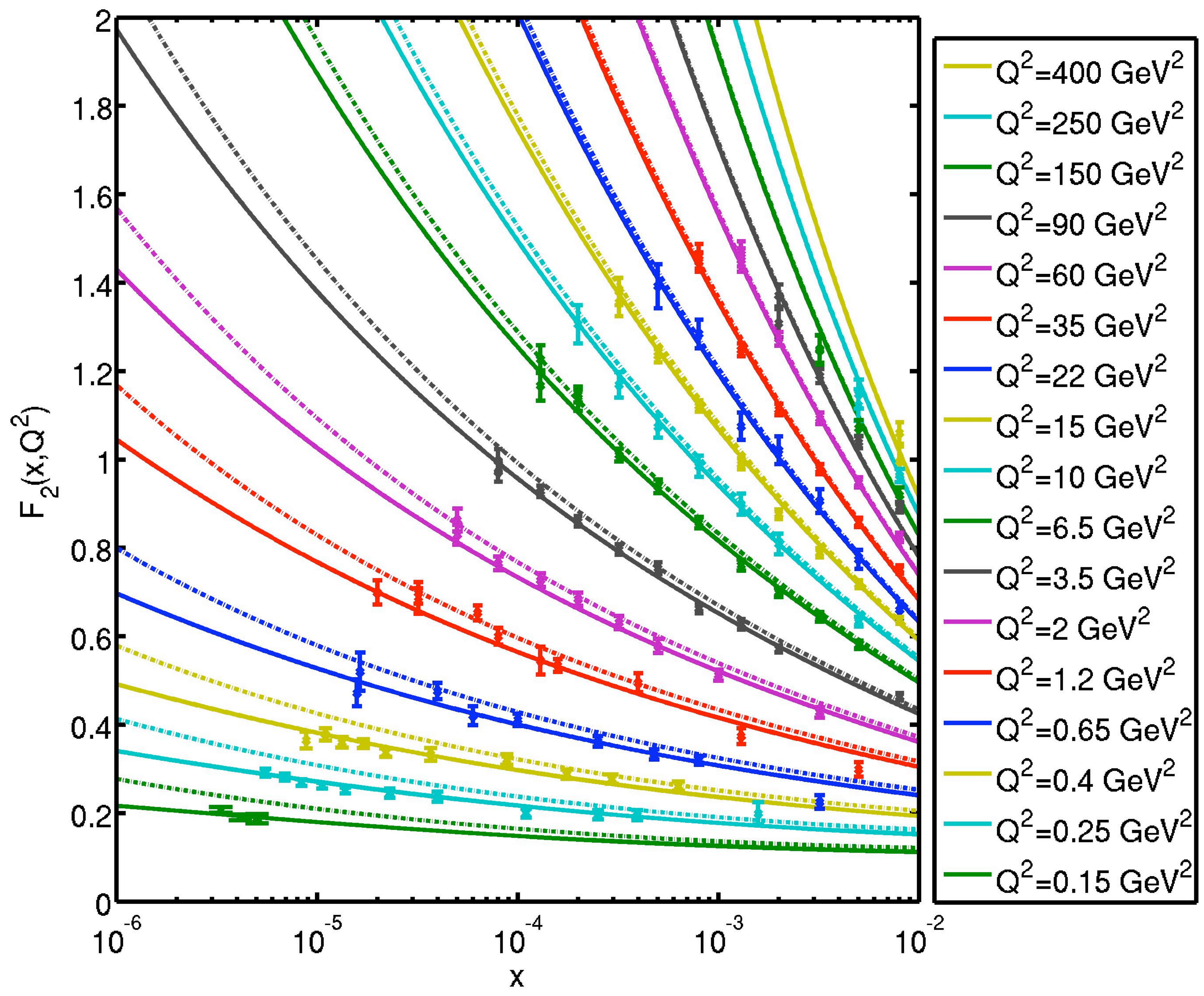
- Saturation:

$$\chi(s, x^\perp - x'^\perp, z, z') = O(1)$$

- Universality: e.g., Choose Φ_1 and Φ_3 for DIS.


F
1/x
Q^2
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
10^7
10^6
10^5
10^4
10^3
10^2
10^-1
10^0
10^1
10^2
10^3

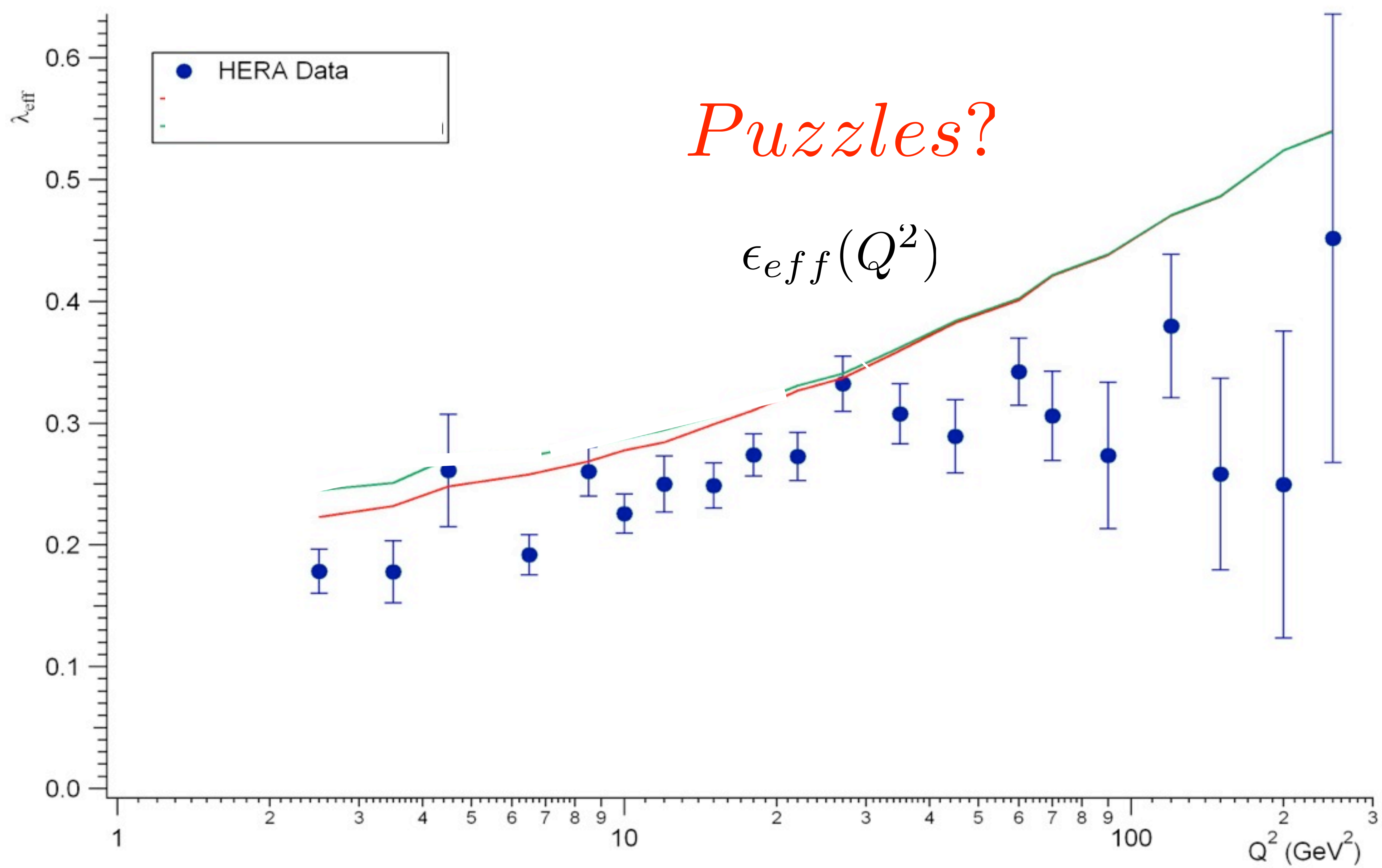

Saturation
Confinement
LHeC
HERA
1/x
Q^2
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1



F_2(x,Q^2)
x
Q^2=400 GeV^2
Q^2=250 GeV^2
Q^2=150 GeV^2
Q^2=90 GeV^2
Q^2=60 GeV^2
Q^2=35 GeV^2
Q^2=22 GeV^2
Q^2=15 GeV^2
Q^2=10 GeV^2
Q^2=6.5 GeV^2
Q^2=3.5 GeV^2
Q^2=2 GeV^2
Q^2=1.2 GeV^2
Q^2=0.65 GeV^2
Q^2=0.4 GeV^2
Q^2=0.25 GeV^2
Q^2=0.15 GeV^2

Effective Pomeron Intercept from HERA data:

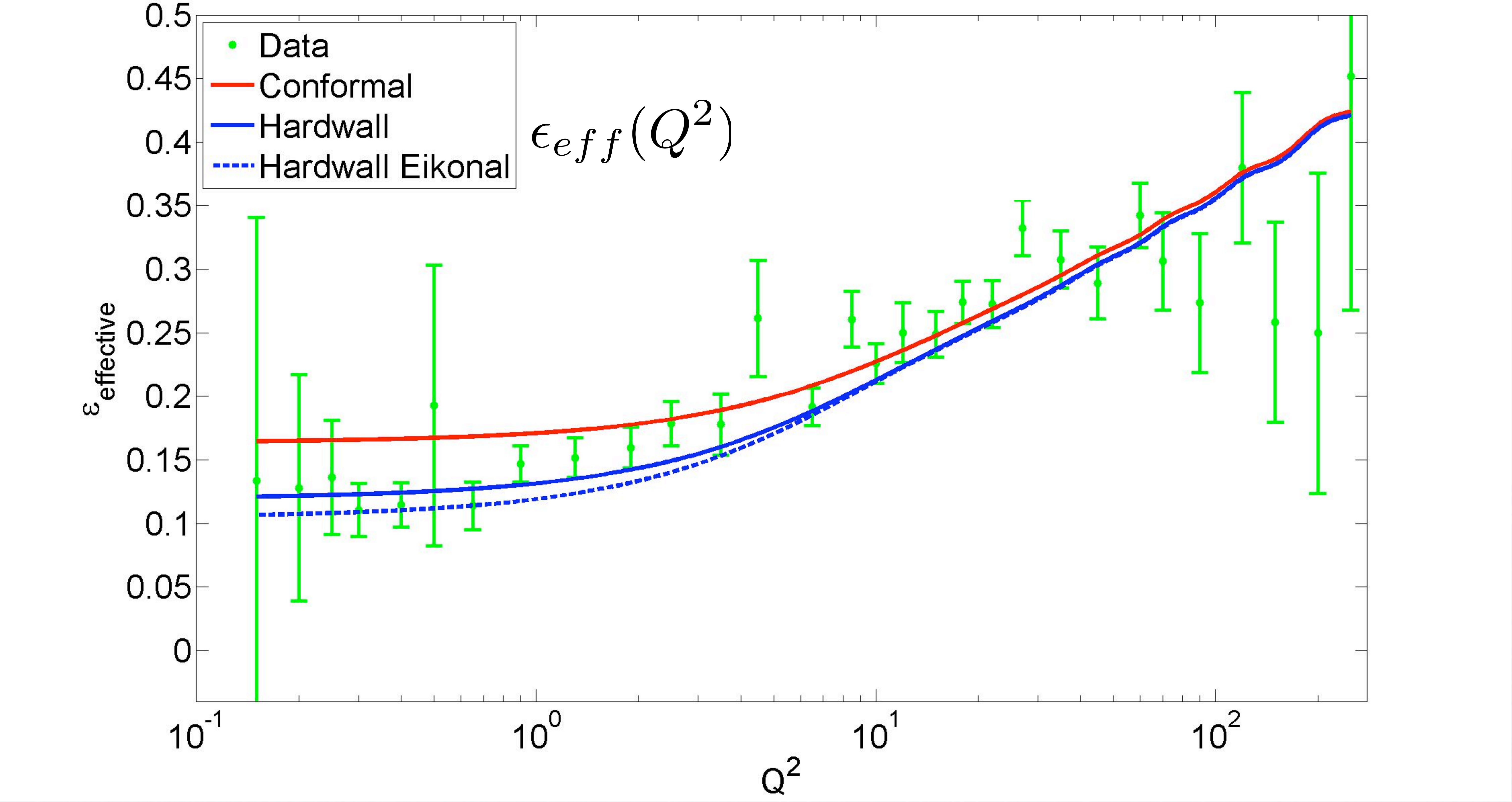
$$F_2 \simeq C(Q^2)\ x^{-\epsilon_{eff}}$$



Questions on HERA DIS small-x data:

- Why $\alpha_{eff} = 1 + \epsilon_{eff}(Q^2)$?
- Confinement? (Perturbative vs. Non-perturbative?)
- Saturation? (evolution vs. non-linear evolution?)

$$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effecti}}$$



Scattering in Conformal Limit:

Use the condition: $\chi(s, x^\perp - x'^\perp, z, z') = O(1)$

Elastic Ring: $b_{\rm diff} \sim \sqrt{zz'}\; (zz's/N^2)^{1/6}$

No Froissart $\boxed{\sigma_{total} \sim s^{1/3}}$

Inner Absorptive Disc:

$$b_{\rm black} \sim \sqrt{zz'}\; \frac{(zz's)^{(j_0-1)/2}}{\lambda^{1/4}N} \qquad b_{\rm black} \sim \sqrt{zz'}\; \left(\frac{(zz's)^{j_0-1}}{\lambda^{1/4}N}\right)^{1/\sqrt{2\sqrt{\lambda}}(j_0-1)}$$

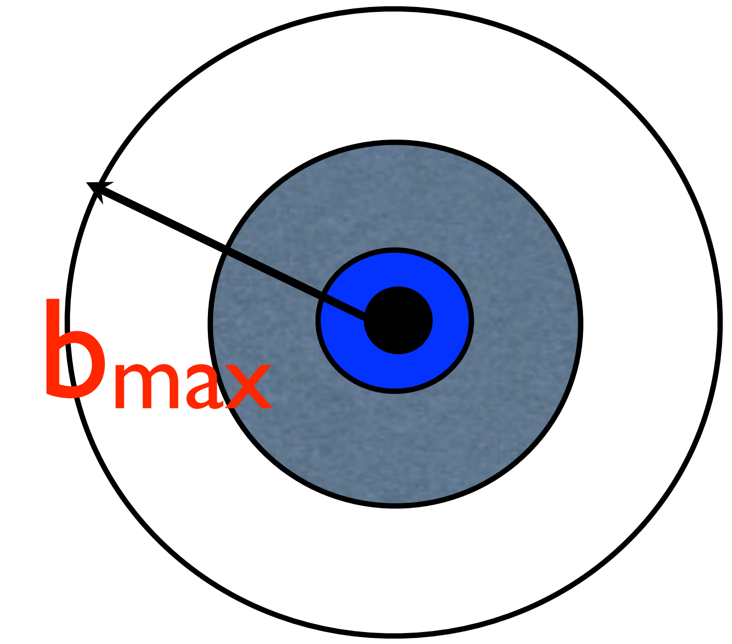
Inner Core: “black hole” production ?

Saturation of Froissart Bound

- The Confinement deformation gives an exponential cutoff for $b > b_{max} \sim c \log(s/s_0)$,
- Coefficient $c \sim 1/m_0$, m_0 being the mass of lightest tensor glueball.
- Froissart is respected and saturated.

$$\Delta b \sim \log(s/s_0)$$

Disk picture



b_{max} determined by confinement.

VI. Beyond Pomeron

- Sum over all Pomeron graph (string perturbative, $1/N^2$)
- Eikonal summation in AdS_3
- Constraints from Conformal Invariance, Unitarity, Analyticity, Confinement, Universality, etc.
- Froissart Bound?
- "non-perturbative" (e.g., blackhole production)

VIII. Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences, DIS at small-x, Diffractive Higgs production at LHC, etc.

(STRONG) RUNNING