Deconfinement phase transition in SU(3)/Z3 QCD (adj) via the gauge theory/affine XY-model duality

MOHAMED ANBER
UNIVERSITY OF TORONTO

12TH WORKSHOP ON NON-PERTURBATIVE QCD

M.A., Erich Poppitz, Mithat Unsal arXiv:1112.6389
M.A., Scott Collier, Erich Poppitz arXiv: 1211.2824

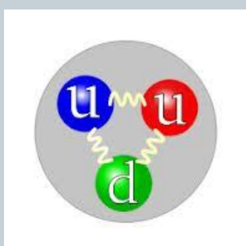
LHC

- LHC exciting news?!!

Motivation

- spontaneous breaking of chiral symmetry (via confinement) → the visible mass in the universe

- Confinement is the mechanism for holding quarks inside nucleons



$$V = \sigma R$$

Confinement is Hard!

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First Clay Mathematics Institute Millennium Prize Announced

Prize for Resolution of the Poincaré Conjecture Awarded to Dr. Grigoriy Perelman

March 18, 2010. The Clay Mathematics Institute (CMI) announces today that Dr. Grigoriy Perelman of St. Petersburg, Russia, is the recipient of the Millennium Prize for resolution of the Poincaré conjecture. The citation for the award reads:

The Clay Mathematics Institute hereby awards the Millennium Prize for resolution of the Poincaré conjecture to Grigoriy Perelman.

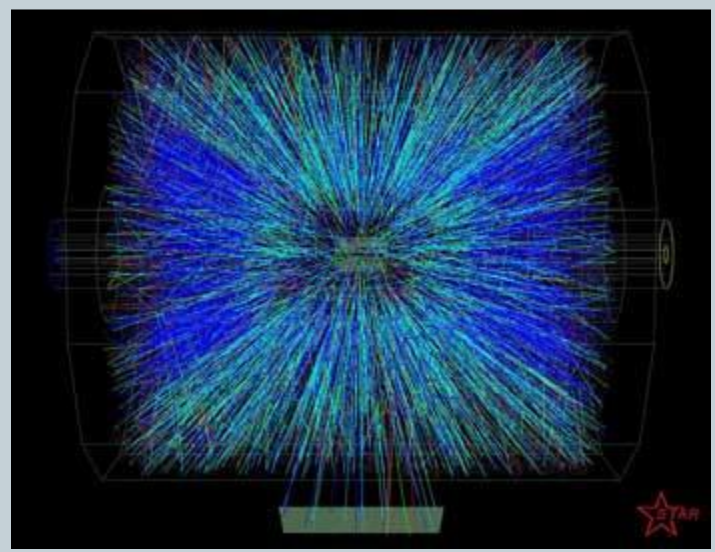
- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

- Rules
- Millennium Meeting Videos

Follwing the decision of the Scientific Advisory Board, the Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to the solution of each problem.

Motivation

- As we increase the temperature, deconfinement happens
- Quark-gluon plasma: a new state of matter
- Novel phenomena, e.g. chiral magnetic effect

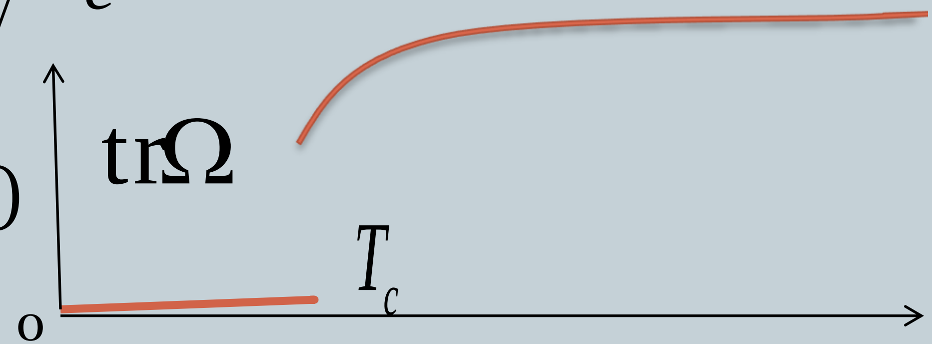


Phase transition order parameters

- Studying phase transition ⟶ order parameter (magnetization $\langle |m| \rangle$)
- Order parameter $\Omega = \exp\left[i \oint_s A_0 dx_0 \right]$

Thermal circle: compact time $T = \frac{1}{\text{circumference}}$

- Confined phase $\langle \mathrm{tr}[\Omega] \rangle = 0, \mathrm{T} < \mathrm{T_c}$
- Deconfined phase $\langle \mathrm{tr}[\Omega] \rangle \neq 0, \mathrm{T} > \mathrm{T_c}$
- The physics is that $\langle \mathrm{tr}[\Omega] \rangle \sim e^{-F/T}$
- This is attributed to Z_N center symmetry in SU(N)

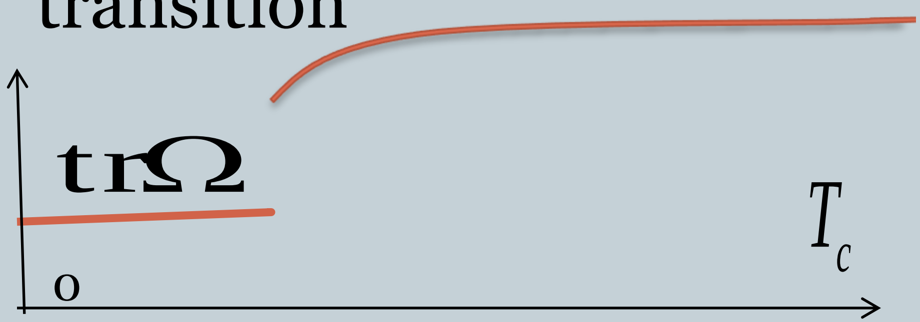


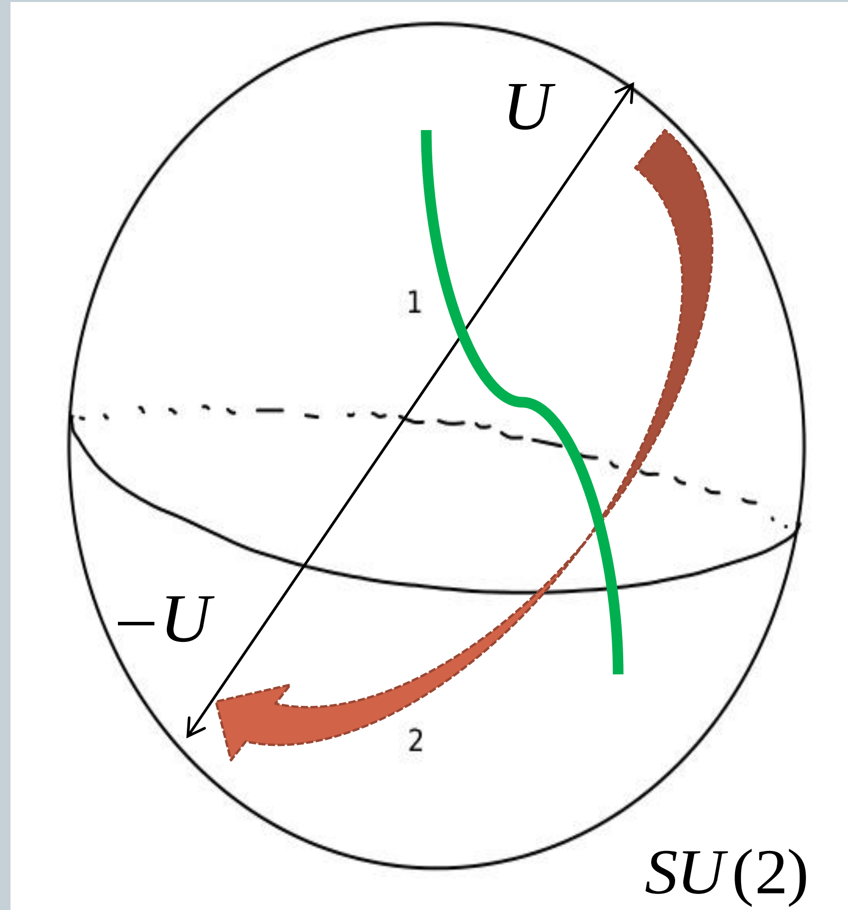
Phase transition order parameters

- Divide by the center

$$SU(N)/Z_N$$

- Still $\mathrm{tr}\Omega$ jumps at the transition





$$\pi_1\left(SU(N)/Z_N\right)=Z_N$$

Topological group

Gauge theories on $R^{2,1}\times S^1$

- Lattice QCD is in excellent agreement with this picture
- QCD is a strongly coupled system, not analytically tractable
- Ways AdS/CFT
- One needs a simpler theory that is under complete control, yet resembles the original theory
- A promising setup is Yang-Mills on $R^{2,1}\times S^1$ → Spatial circle
- These ideas started in the 1990 in supersymmetry

Gauge theories on $R^{2,1} \times S^1$

- Pros:
- Perform reliable semi-classical calculations
- Test the rules of different symmetries (center, topological, chiral, etc.)
- Disentangle different physical phenomena (e.g. confinement & chiral symmetry breaking)
- Mapping to lower-dimensional condensed matter systems (simulations, or using analogue systems to test our guage theories)
- FUN!!!!

Gauge theories on $R^{2,1}\times S^1$

❖ Cons:

- Not the real world
- Large L limit is not under control

Gauge theories on $R^{2,1} \times S^1$

- Ultimate goal
  - Cook up models that contain the same ingredients of realistic theories (adjoint & fundamental fermions, magnetic field, the vacuum angle, etc)
  - Compare the results with existing experiments (either real or full 4-D lattice experiments)
  - Make predictions and propose further experiments

Gauge theories on $R^{2,1} \times S^1$

- Today's meal: deconfining phase transition in SU(3)/Z_3 QCD with adjoint fermions

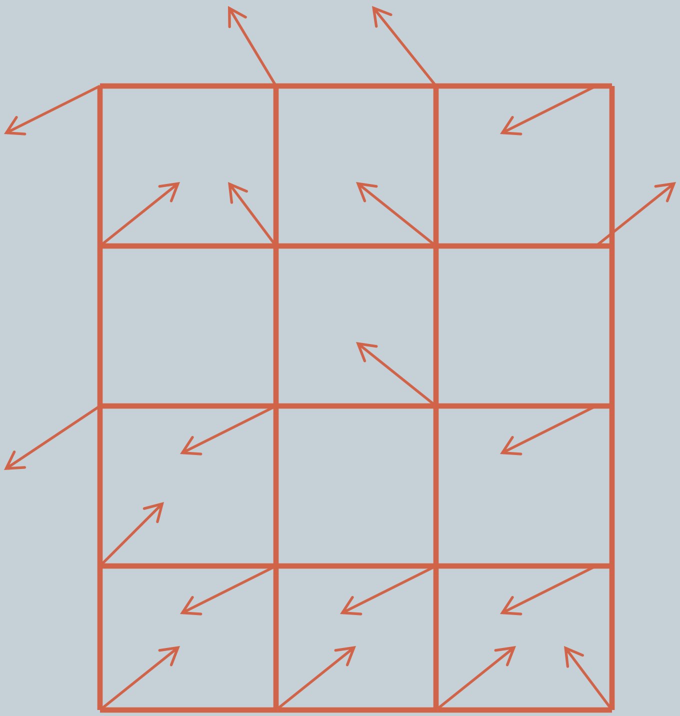
$\langle \mathrm{tr}[\Omega] \rangle$

- Lattice experiments for SU(3) model were conducted in 4-D : first order transition   Karsch and Lutgemeir 1998

Gauge theories on $R^{2,1} \times S^1$

- Deconfining phase transition of SU(N) (adj) gauge theories with one compact dimension via gauge theory/affine XY model duality M.A., E. Poppitz, M. Unsal 2011, M.A., S.C., E. Poppitz 2012

$$H = \sum_{\mu,x} J \cos\left(\vec{A}.\left(\vec{\vartheta}_x - \vec{\vartheta}_{x+\mu}\right)\right) + \sum_x \kappa \cos\left(\vec{B}.\vec{\vartheta}_x\right)$$



- SU(2) & SU(2)/Z2 using RG (second order transition)
- SU(3)/Z3 using Monte Carlo

Outline:

- Formulation of QCD adj on $R^{2,1}\times S^1$, pertubative and non-perturbative effects at T=0 and Coulomb gas
- QCD adj at finite temperature, partition function
- Mapping to XY spin-models
- Monte Carlo Simulations
- Conclusion and future directions

QCD adj on $R^{2,1}\times S^1$, Formulation

$$S=\int_{R^{1,2}\times S^1}\frac{1}{g^2}\mathrm{tr}\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}+2i\mathrm{tr}\,\overline{\lambda_I}\,\overline{\sigma}^{\mu}D_\mu\lambda_I\right] \quad SU(n_f)\times U(1)$$

Flavor symmetry

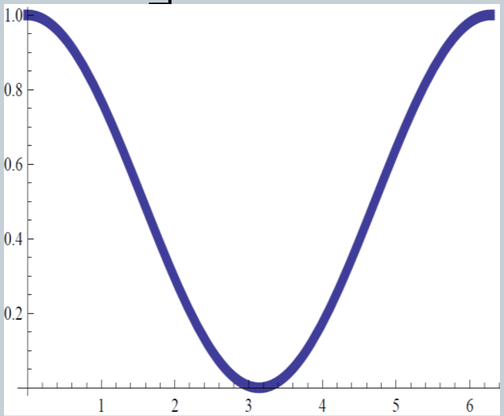
n_f Adjoint fermions with periodic boundary conditions along the S^1 circle

$$\overbrace{\Rightarrow}^{\text{small } S^1}\int_{R^{1,2}}\frac{L}{g^2}\mathrm{tr}\left[-\frac{1}{2}F_{ij}F^{ij}+\left(D_iA_4\right)^2-\frac{g^2}{2}V_{eff}(A_4)\right]$$

One-loop effect

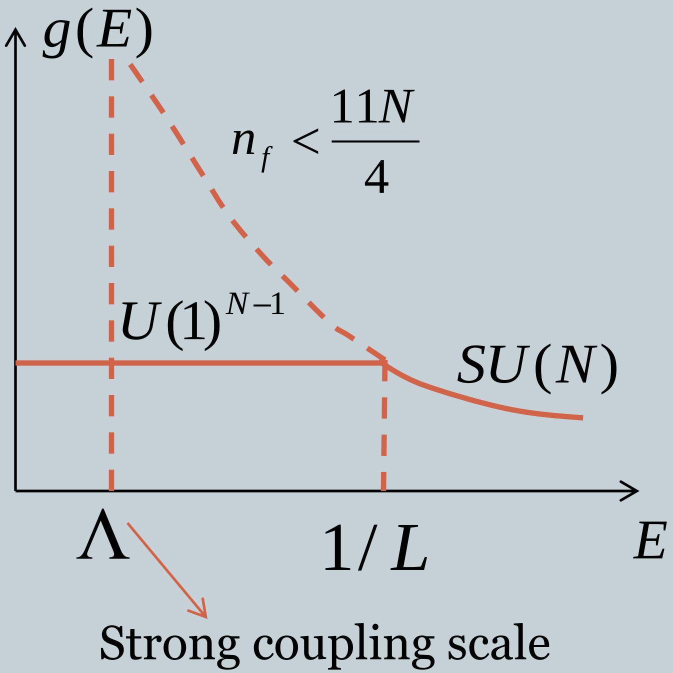
Compact scalar

Z_N symmetry



QCD adj on $R^{2,1}\times S^1$, perturbative treatment

- The theory abelianizes $SU(N)\rightarrow U(1)^{N-1}$
- at small S^1 the gauge coupling is small
- The theory is effectively 3-D



$$S=\int_{R^{1,2}}\frac{1}{2L}\left(\frac{g}{4\pi}\right)^2\left(\partial\vec{\sigma}\right)^2+i\overline{\lambda}_I\,\overline{\sigma_\mu}\,\partial^\mu\lambda_I+O\left(\frac{1}{L}\right)$$

$O\left(\frac{1}{L}\right)$: W and heavy charged fermions

$(\partial\vec{\sigma})^2$: $N-1$ components of dual photons

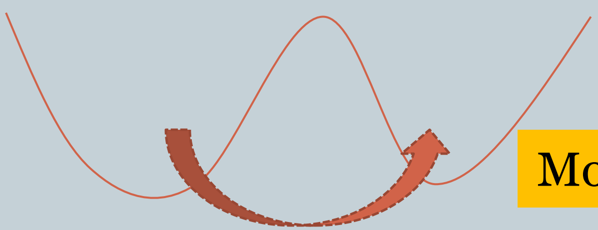
λ_I: massless fermions

Non-perturbative objects

- More interesting story to tell: non-perturbative effects (Polyakov model)
- Feynman path integral

$$Z_{\text{Euclid}} = \sum_{\text{paths}} e^{-S_E}$$

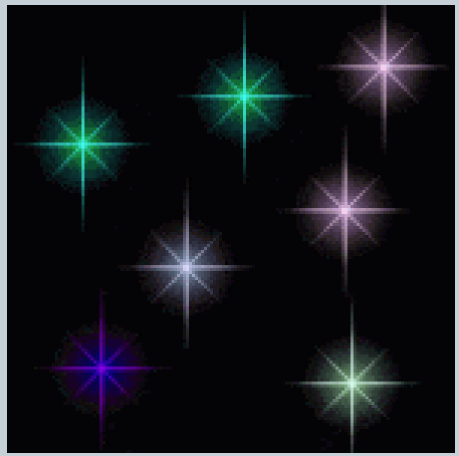
Perturbative +non-perturbative (instantons)



Monopole-instantons

Non-perturbative objects

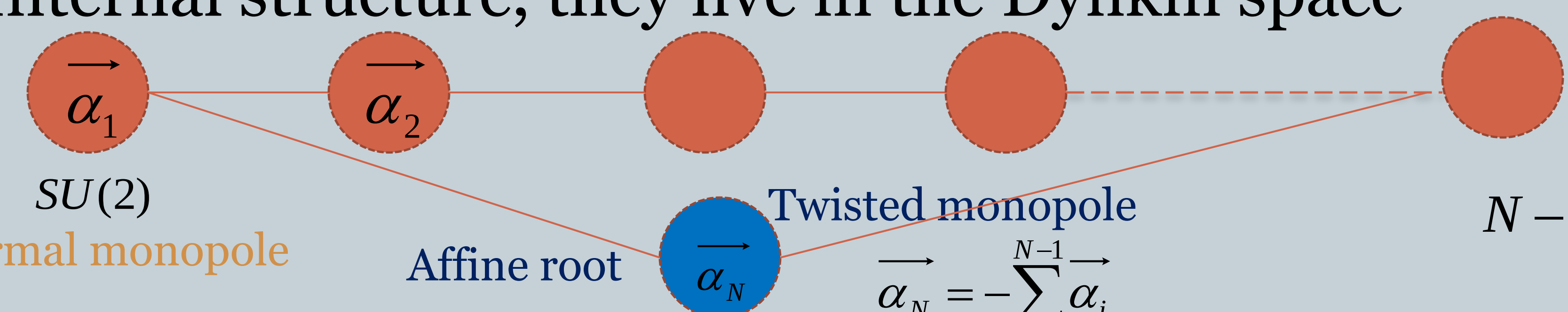
- These instantons are localized in space and time



probability $= e^{-S_0/2}$

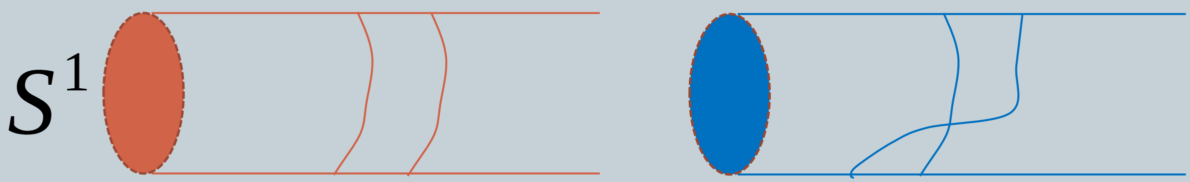
$$S_0 = \frac{8\pi^2}{g^2}$$

- Internal structure, they live in the Dynkin space



Non-perturbative objects

- Twisted monopoles were discovered by Kraan and Baal and (calarons), and Lee and Lu (D-branes) (1998)



$1/L$ ⟷

Normal | Twisted

$\vec{\alpha}_i$	$-\vec{\alpha}_i$	$\vec{\alpha}_N$	$-\vec{\alpha}_N$
NORM $\frac{1}{N}$	$\overline{\text{NORM}}$ - $\frac{1}{N}$	TWIST $\frac{1}{N}$	$\overline{\text{TWIST}}$ - $\frac{1}{N}$
$e^{-S_0/2}e^{i\vec{\alpha}_i.\vec{\sigma}}$	$e^{-S_0/2}e^{-i\vec{\alpha}_i.\vec{\sigma}}$	$e^{-S_0/2}e^{i\vec{\alpha}_N.\vec{\sigma}}$	$e^{-S_0/2}e^{-i\vec{\alpha}_N.\vec{\sigma}}$

Non-perturbative objects

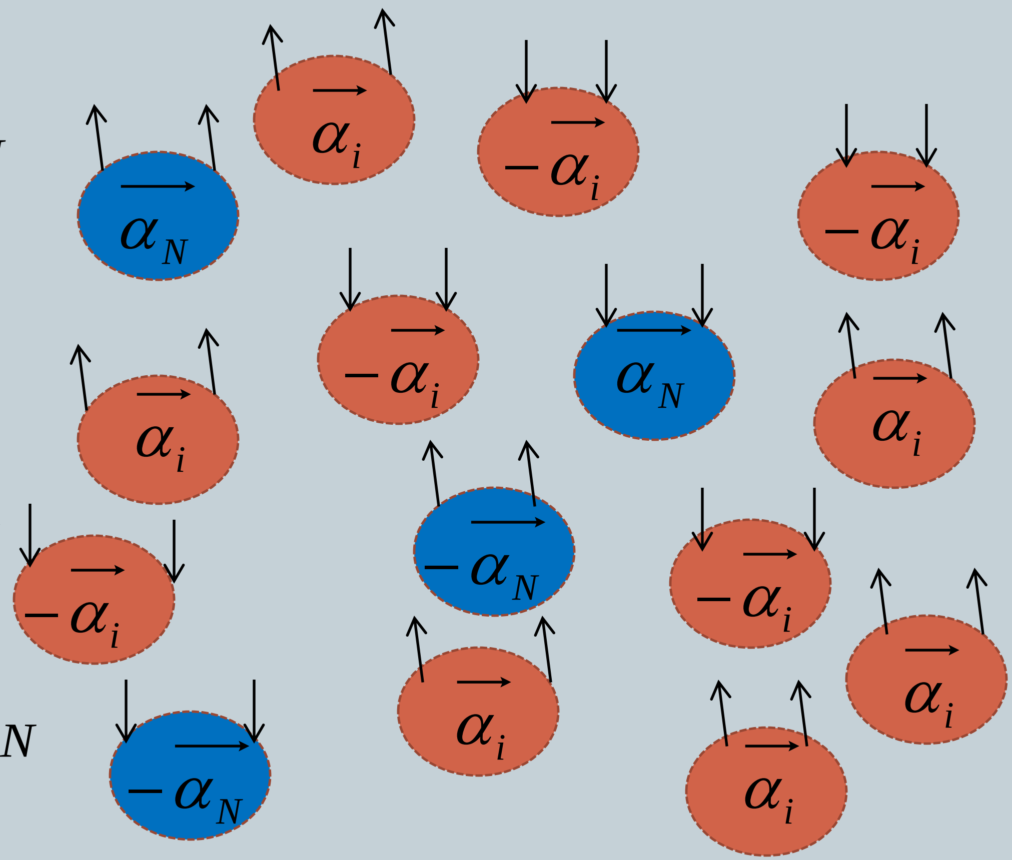
- But we have additional <u>adjoint fermions</u>

$$e^{-S_0/2}e^{i\vec{\alpha}_N\cdot\vec{\sigma}}\left(\det_{IJ}\bar{\lambda}_I\bar{\lambda}_J\right)^N$$

$$e^{-S_0/2}e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\left(\det_{IJ}\lambda_I\lambda_J\right)^N$$

$$e^{-S_0/2}e^{-i\vec{\alpha}_i\cdot\vec{\sigma}}\left(\det_{IJ}\bar{\lambda}_I\bar{\lambda}_J\right)^N$$

$$e^{-S_0/2}e^{-i\vec{\alpha}_N\cdot\vec{\sigma}}\left(\det_{IJ}\lambda_I\lambda_J\right)^N$$



Non-perturbative objects

- If nothing else

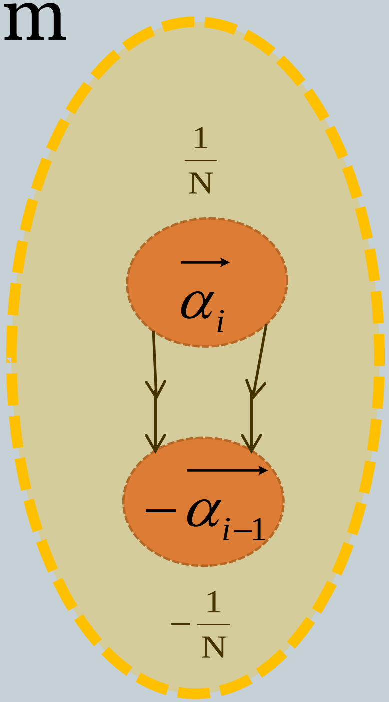
$$S=\int_{R^3}\frac{1}{2L}\left(\frac{g}{2\pi}\right)^2\left(\partial\vec{\sigma}\right)^2+i\overline{\lambda_I}\,\overline{\sigma^\mu}\partial_\mu\lambda_I+be^{-S_0/2}\mathrm{e}^{\mathrm{i}\vec{\alpha}.\vec{\sigma}(x)}\left(\det{}_{\mathrm{IJ}}\lambda_I\lambda_J\right)^N+\mathrm{h.c.}$$

- No confinement!
- However, for $n_f=1$ the theory is supersymmetric.
- Supersymmetric theories on $R^3\times S^1$ confine. (Khoze et al 1999)
- Solution by Unsal 2007, mechanism is transcendent beyond SUSY

Molecular objects (Bions)

Bions can form in the vacuum

$$\overrightarrow{\alpha_i}.\overrightarrow{\alpha_j} = \delta_{i,j} - \frac{1}{2}\delta_{i,j-1} - \frac{1}{2}\delta_{i,j+1}$$

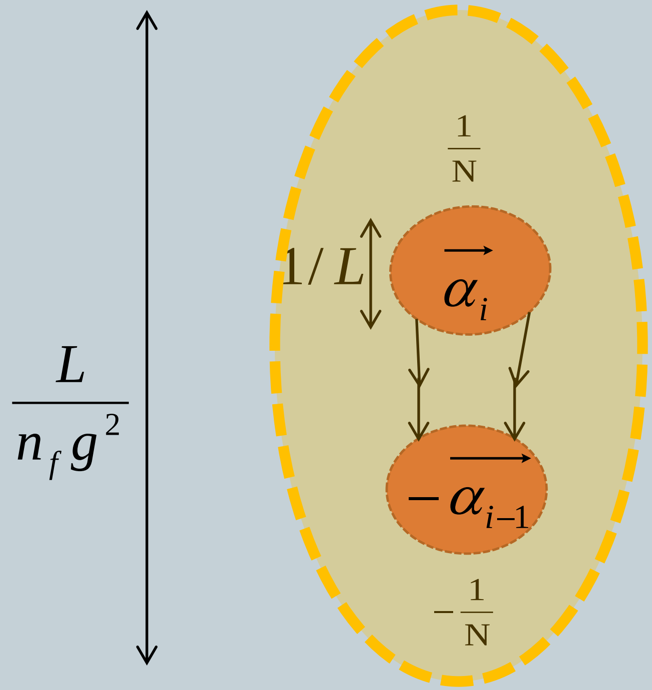


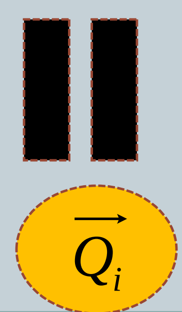
$$\text{bion}(\overrightarrow{\alpha_i} - \overrightarrow{\alpha_{i-1}})$$

Molecular objects (Bions)

- Forces and scales

$$V = \underbrace{\frac{2\pi L}{g^2}\frac{1}{r}}_{\text{Coulomb}} + \underbrace{4n_f \log\left(\frac{r}{L}\right)}_{\text{fermions hopping}}$$

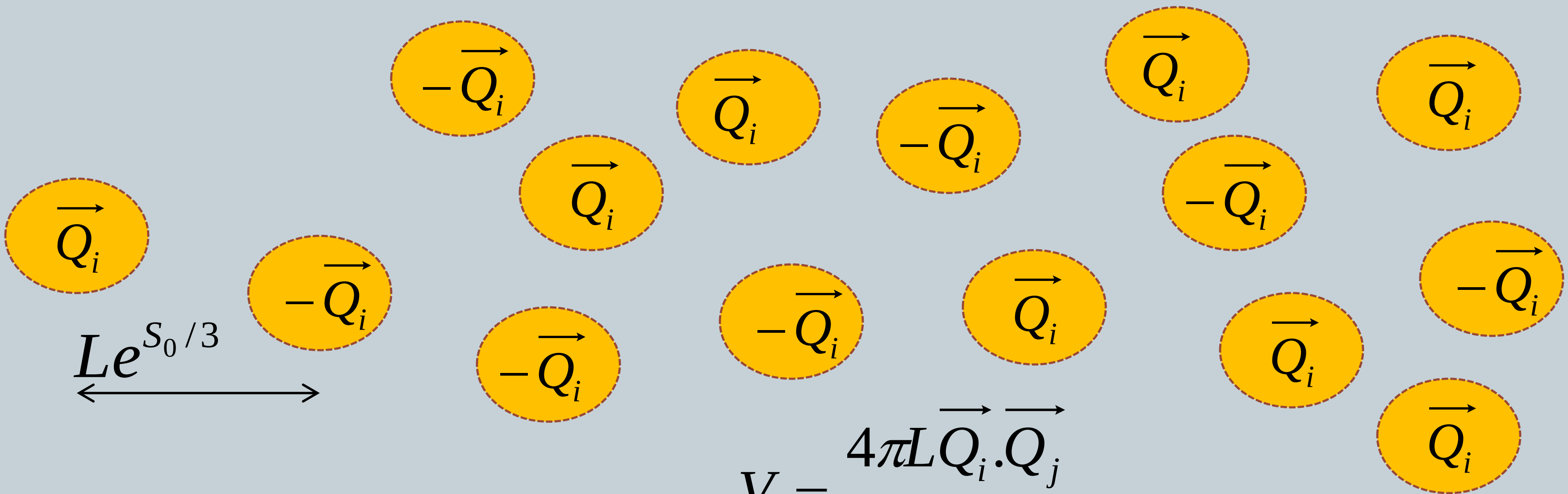




$$e^{-S_0} e^{i\vec{Q}_i.\vec{\sigma}(x)}, \vec{Q}_i = \vec{\alpha}_i - \vec{\alpha}_{i-1}$$

QCD adj: Confinement

Bions proliferate in the vacuum: 3-D Coulomb gas



$$V = \frac{4\pi L \vec{Q}_i . \vec{Q}_j}{g^2 r}$$

$$S = \int_{R^3} \frac{1}{2L}\left(\frac{g}{4\pi}\right)^2 \left(\partial\vec{\sigma}\right)^2 + \underbrace{\sum_{i=1}^{N} c e^{-S_0} \cos\left(\vec{Q}_i . \vec{\sigma}\right)}_{\text{mass term}} + \text{fermions}$$

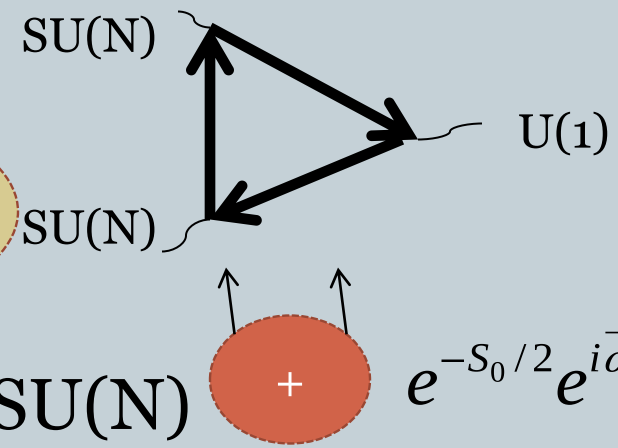
Polyakov 1977

symmetries

- The U(1) chiral is anomalous. This symmetry breaks to Z_{2Nn_f}

$\bar{\lambda}_I \bar{\sigma}^{\mu} D_{\mu} \lambda_I$

$SU(n_f) \times U(1)$



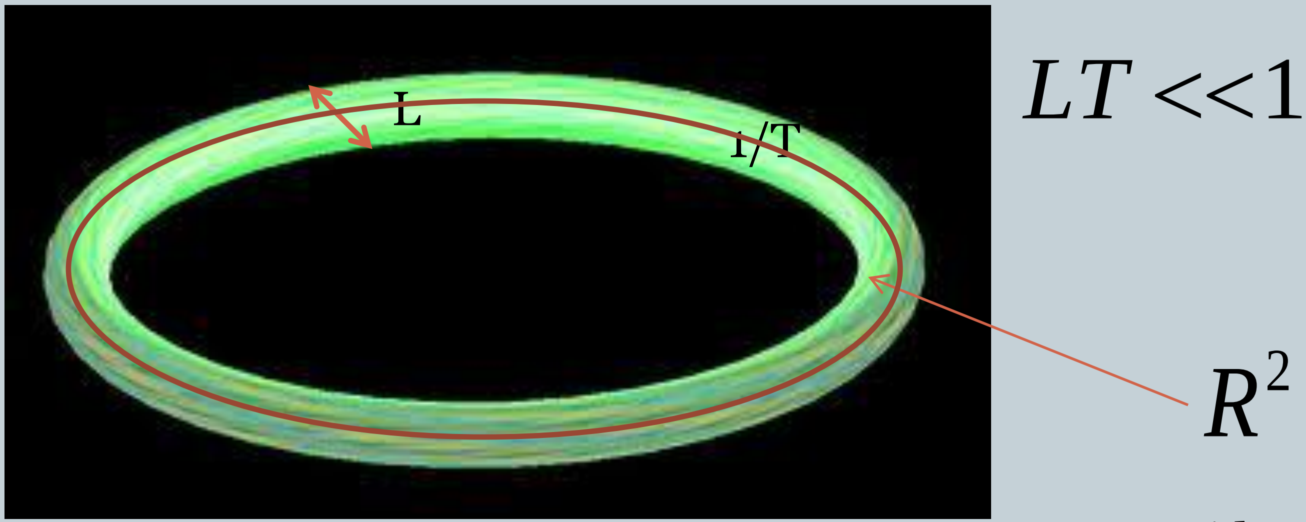
- 't Hooft vertex for SU(N) $e^{-S_0/2} e^{i\vec{\alpha}_i . \vec{\sigma}(x)} \left(\det_{IJ} \lambda_I \lambda_J\right)^N$

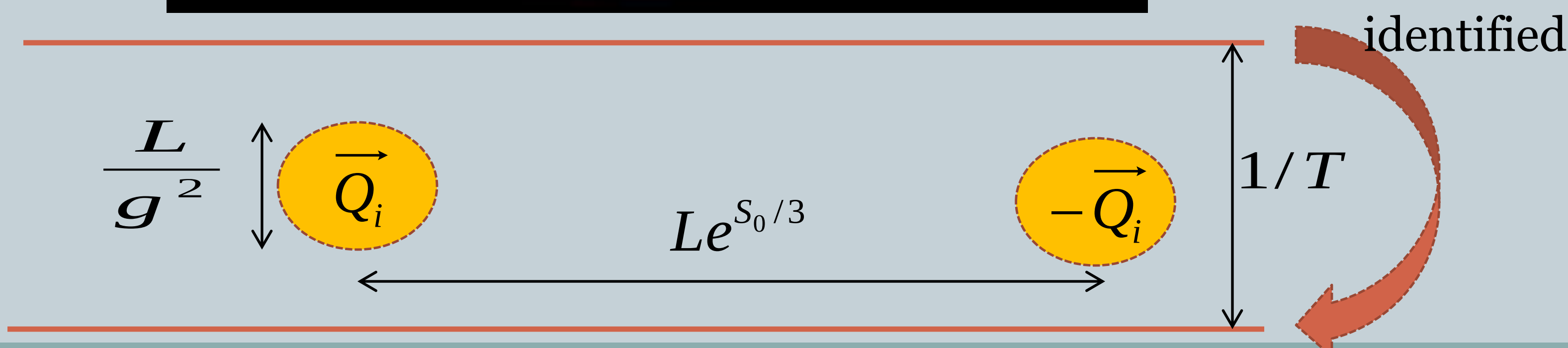
- Relevant symmetries for confinement/deconfinement

$$\underbrace{Z_N}_{\text{discrete chiral}} \times \underbrace{Z_N}_{\text{center for } SU(N)\text{, or topoolgical for } SU(N)/Z_N}$$

QCD adj at finite temperature

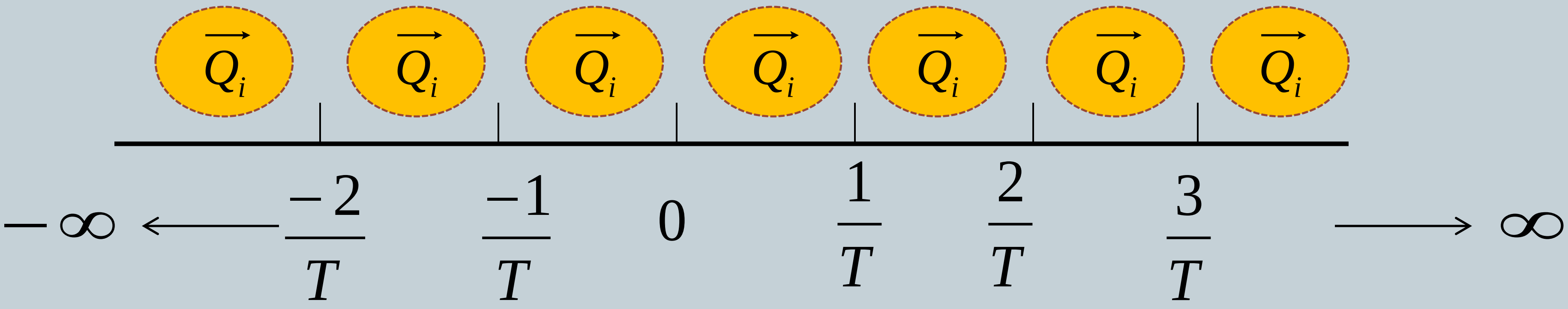
- At finite temperature we compactify the time direction





QCD adj at finite temperature

- For $\frac{L}{g^2} << \frac{1}{T} << Le^{S_0/3}$ we can sum up images



$$\frac{\vec{Q}_i . \vec{Q}_j}{g^2 r} \rightarrow \frac{\vec{Q}_i . \vec{Q}_j}{g^2} \log r$$

QCD adj at finite temperature

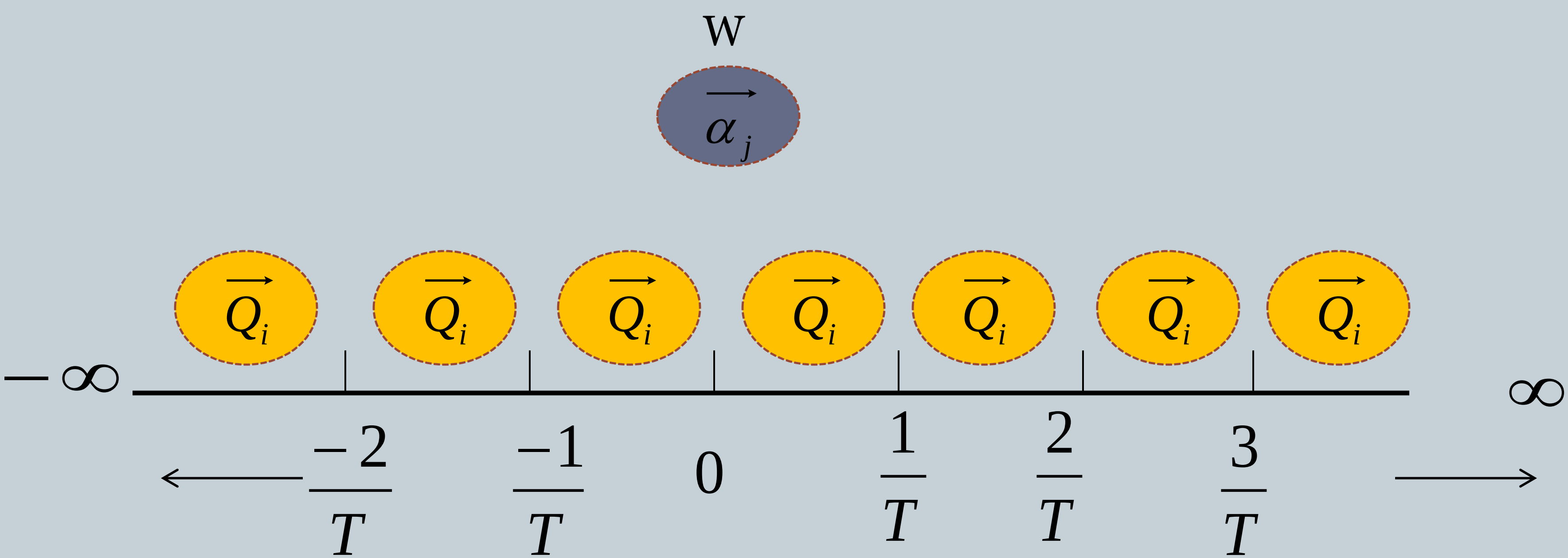
- Story has an other twist!
- At finite temperature, the W's are important

$$\text{density} \propto e^{-m_W/T} \left(\text{electric fugacity}\right)$$

$$\frac{g^2}{2\pi LT}\vec{\alpha}_i.\vec{\alpha}_j \log r$$

QCD adj at finite temperature

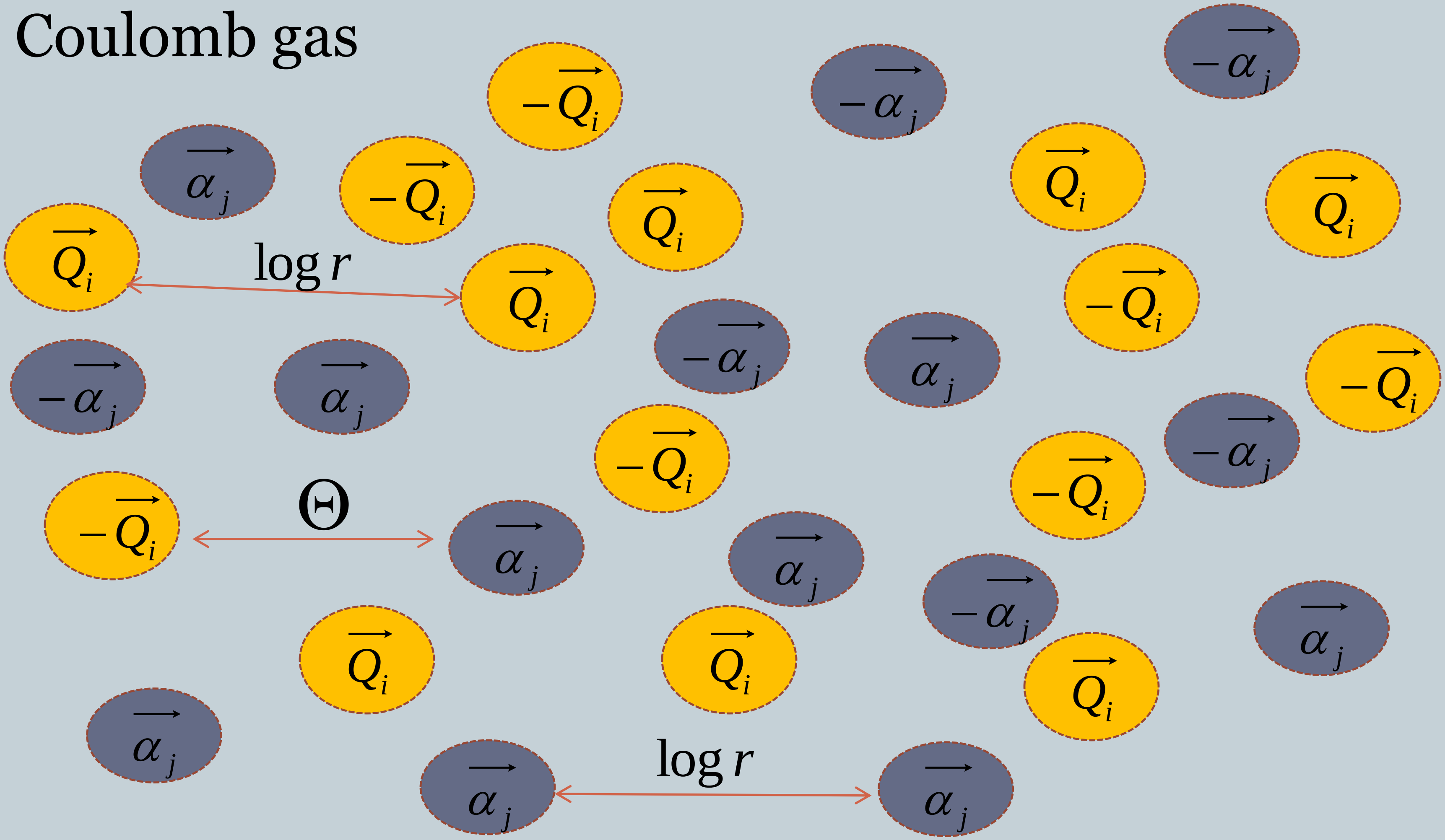
- Add to that the Aharonov-Bohm effect



$$\oint_{\text{time circle}} dx^0 A^0{}_{\text{bion}} = 2\vec{\alpha}_j . \vec{Q}_i \, \Theta\left(\vec{x}_W - \vec{x}_{\text{bion}}\right)$$

QCD adj at finite temperature

- Coulomb gas



QCD adj at finite temperature

- The correct partition function

$$Z = \sum_{\vec{Q}_i, N_{bion-}, N_{bion+}, N_{W+}, N_{W-}} \frac{\xi_{bion}^{N^i_{bion+} + N^i_{bion-}}}{N_{bion+}! N_{bion-}!} \frac{\xi_W^{N^i_{W+} + N^i_{W-}}}{N_{W+}! N_{W-}!} \prod_{a,i} \int d^2 R^i_a \prod_{Ai} \int d^2 R^i_A$$

$$\exp\left[\sum_{i,j a,b,A,B} \underbrace{\frac{8\pi L T}{g^2}}_{4/\kappa} \vec{Q}_i . \vec{Q}_j \log\left|\vec{R}^i_a - \vec{R}^j_b\right| + \underbrace{\frac{g^2}{2\pi L T}}_{\kappa} \vec{\alpha}_i . \vec{\alpha}_j \log\left|\vec{R}^i_A - \vec{R}^j_B\right| + 2i\vec{\alpha}_j . \vec{Q}_i \Theta\left(\vec{R}^i_a - \vec{R}^j_A\right) \right]$$

For $SU(3)$ $\quad \vec{Q}_i . \vec{Q}_j = 3\vec{\alpha}_i . \vec{\alpha}_j$

Potential sign problem for simulations

duality $\kappa \to \frac{12}{\kappa}, \xi_W \to \xi_{bion}$

Strong coupling at the self-dual point

Mapping QCD adj to spin models

- Cook again, but now with “different” looking ingredient

$$e^{2i\vec{\vartheta}(\vec{x}).\vec{\alpha}}$$

$$e^{2i\vec{v}.\vec{\vartheta}(\vec{x})}$$

Spin model

$$\left\langle e^{2i\vec{v}.\vartheta(\vec{x})} e^{-2i\vec{v}.\vartheta(\vec{0})} \right\rangle$$

Spin model

- For SU(2), there is an exact solution via RG analysis (M.A. , E. Poppitz, M. Unsal)
- However, the RGEs break down for SU(3) and so we must turn to simulations on the lattice
- The spin model dual to $SU(3)/\mathbb{Z}_3\mathrm{QCD}(\mathrm{adj})$ is the theory of two coupled XY-spins:

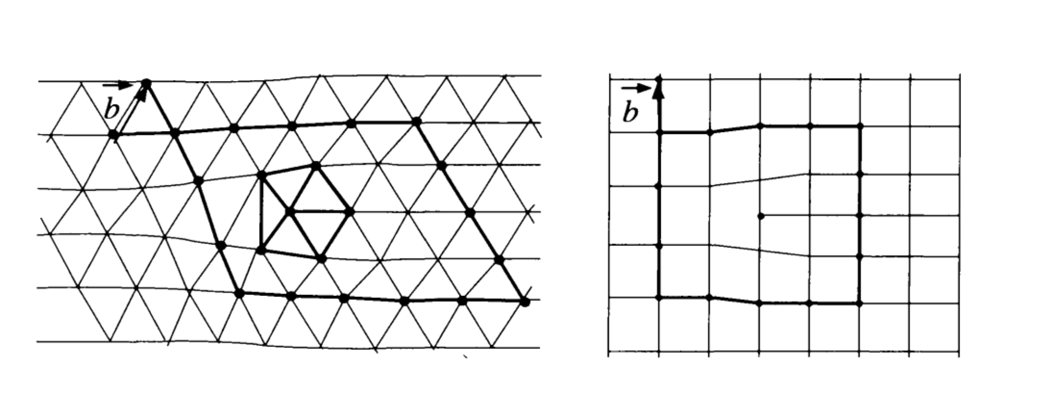
$$\vec{\theta}_x = (\theta_x^1, \theta_x^2) \equiv \vec{\theta}_x + 2\pi\vec{\alpha}_1 \equiv \vec{\theta}_x + 2\pi\vec{\alpha}_2$$

Spin model

- The spin model is defined by a lattice partition function with:

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \sum_{i=1}^{N_c=3} \frac{\kappa}{4\pi} \cos 2\vec{\nu}_i \cdot (\vec{\theta}_{x+\hat{\mu}} - \vec{\theta}_x) + \sum_{x} \sum_{i=1}^{N_c=3} \tilde{y} \cos 2(\vec{\alpha}_i - \vec{\alpha}_{i-1}) \cdot \vec{\theta}_x$$

- Kinetic term: similar to a model used to describe melting of a 2d crystal on a triangular lattice (Nelson, 1977)

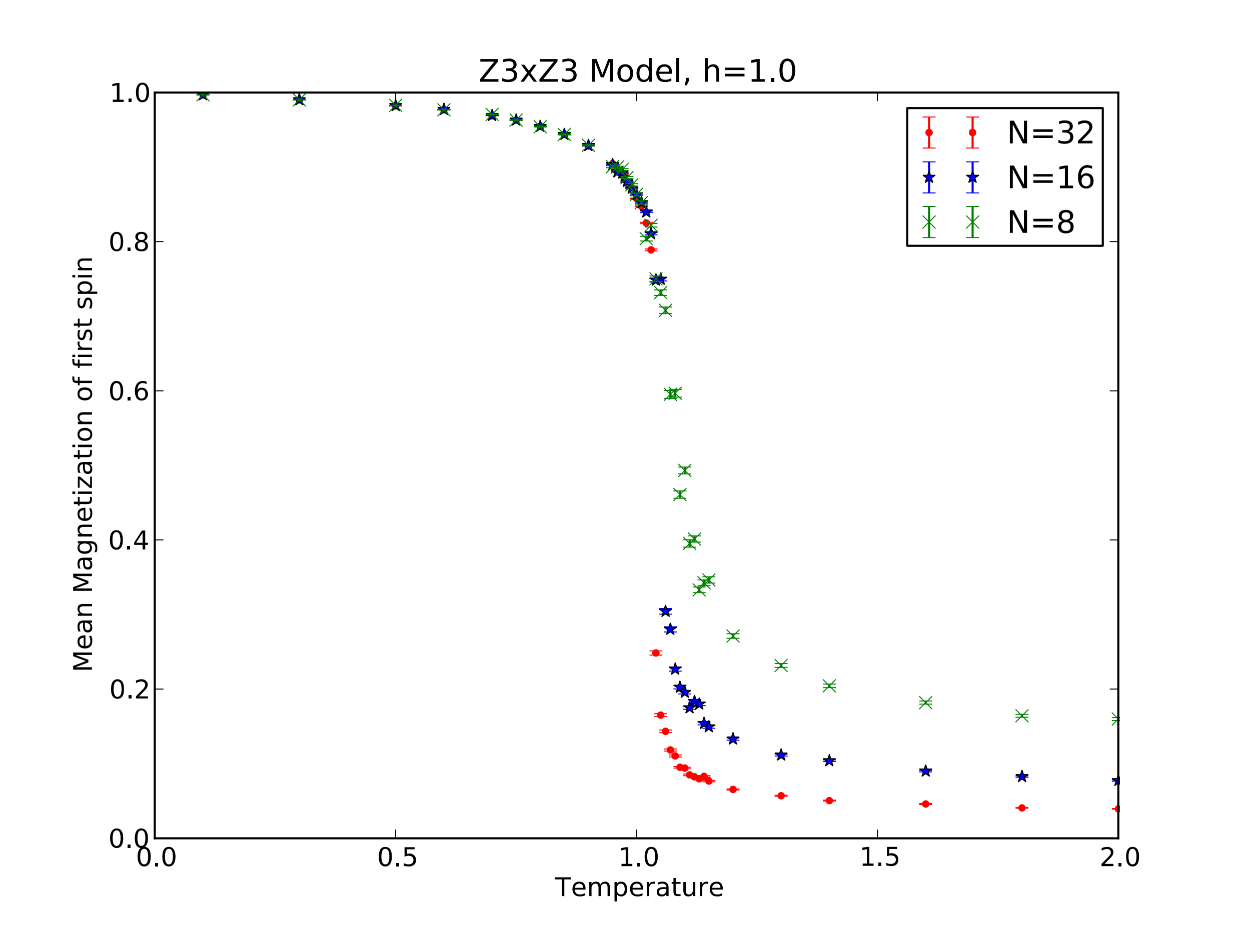


(Kardar, Statistical Physics of Fields)

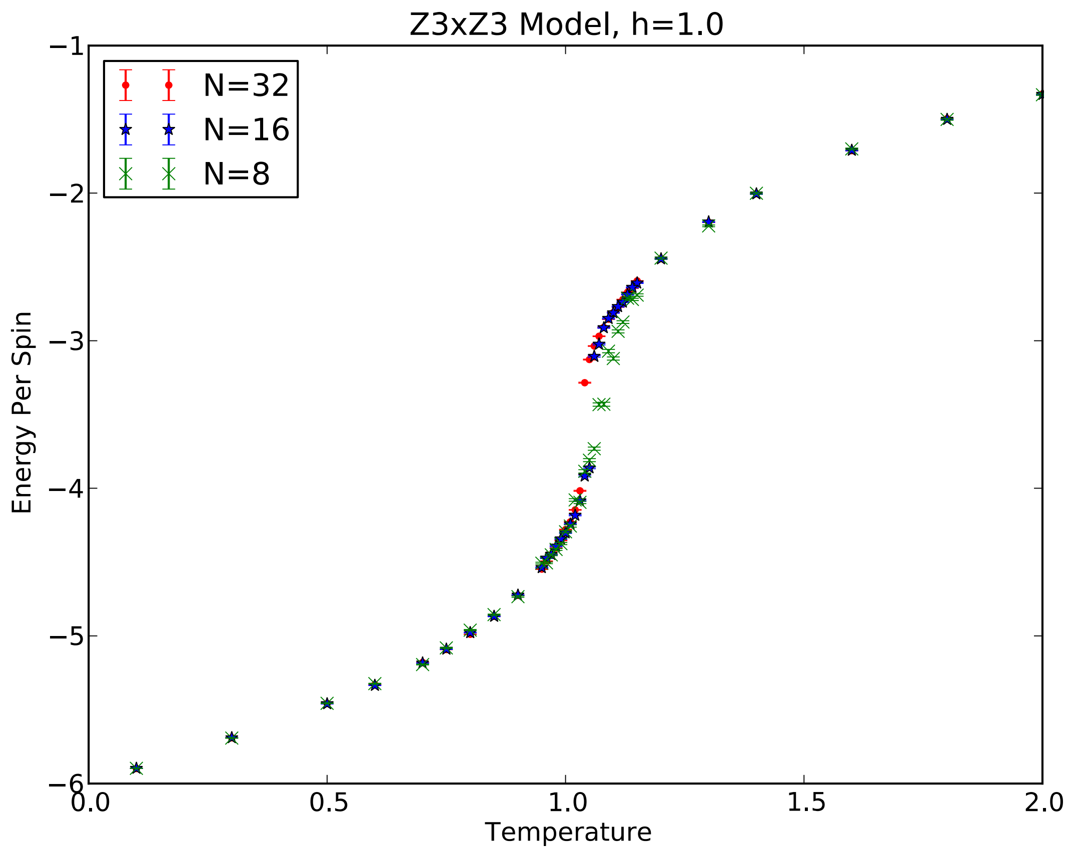
Spin model

- SU(3) interpretation of spin model:
- Fluctuations in $\{\vec{\theta}_x\}$: duals of two massless photons sourced by magnetic bions
- Vortices $\frac{1}{2\pi}\oint d\vec{l}.\nabla\vartheta = \pm 1$ describe electric excitations in theory (W-bosons) excited at T > 0
- Exact $U(1)\times U(1)$ symmetry (corresponding to two dual photons) is broken by potential "external field" term to $\mathbb{Z}_3^t \times \mathbb{Z}_3^{d\chi}$ symmetry

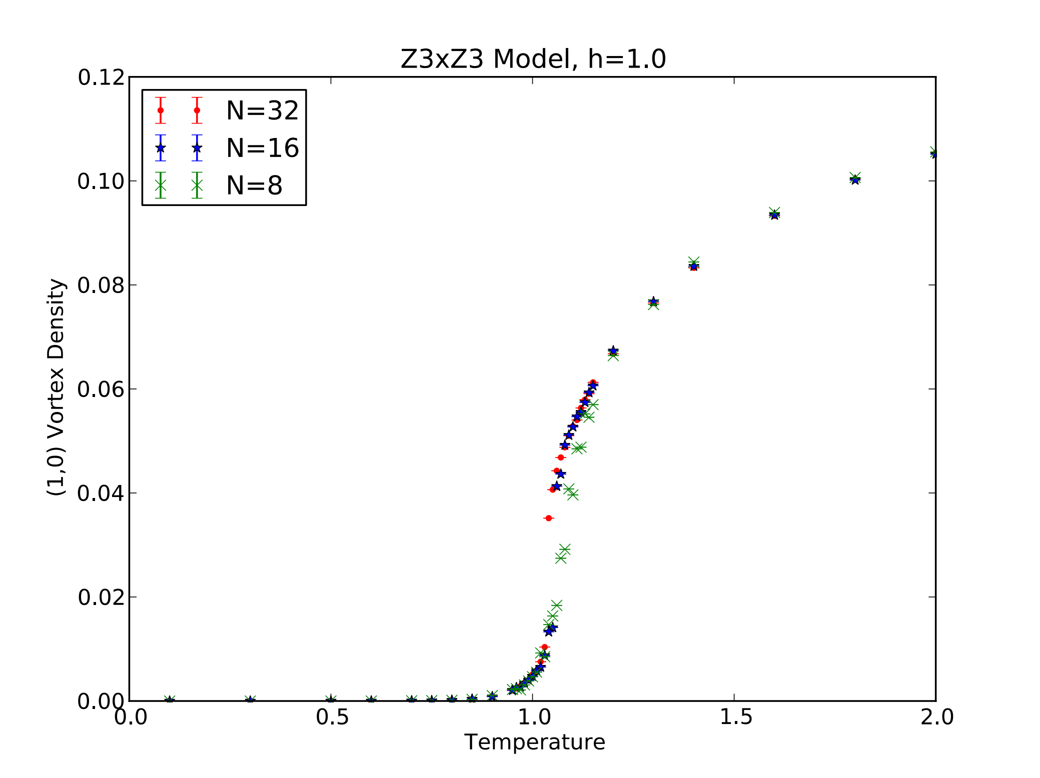
Computational results: order parameter



Computational results: energy per spin

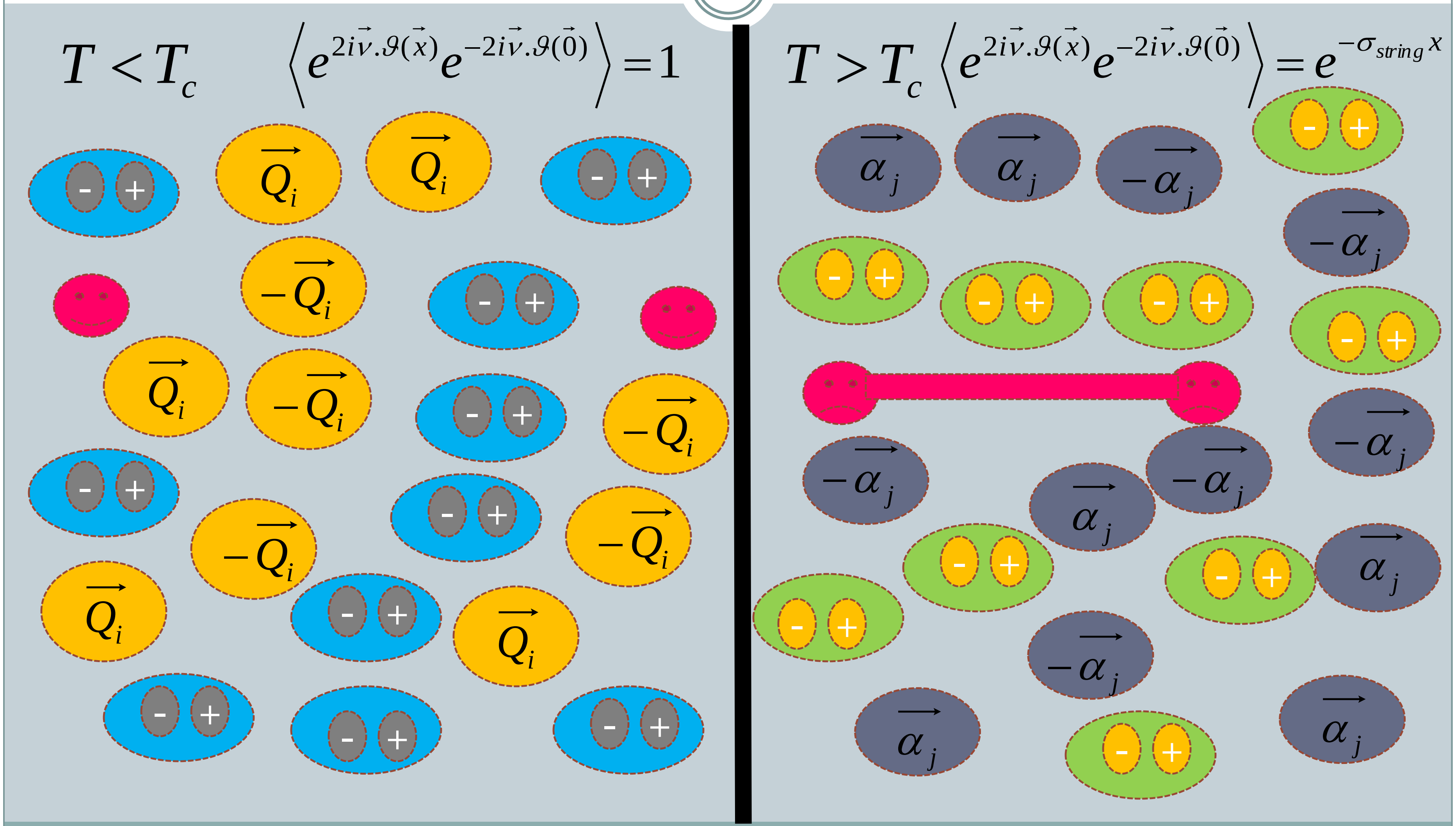


Computational results: vortex density

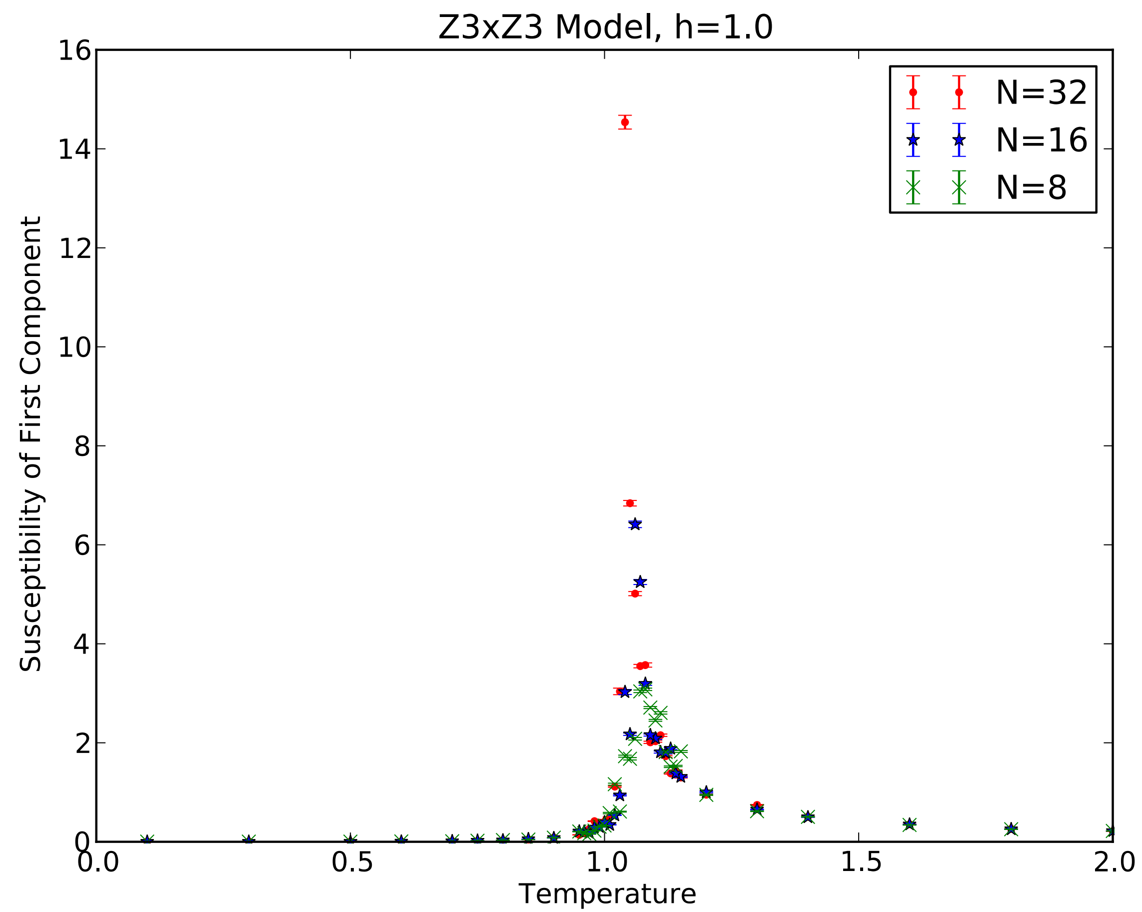


- Recall: vortices in the spin model are dual to liberated W-bosons in the gauge theory

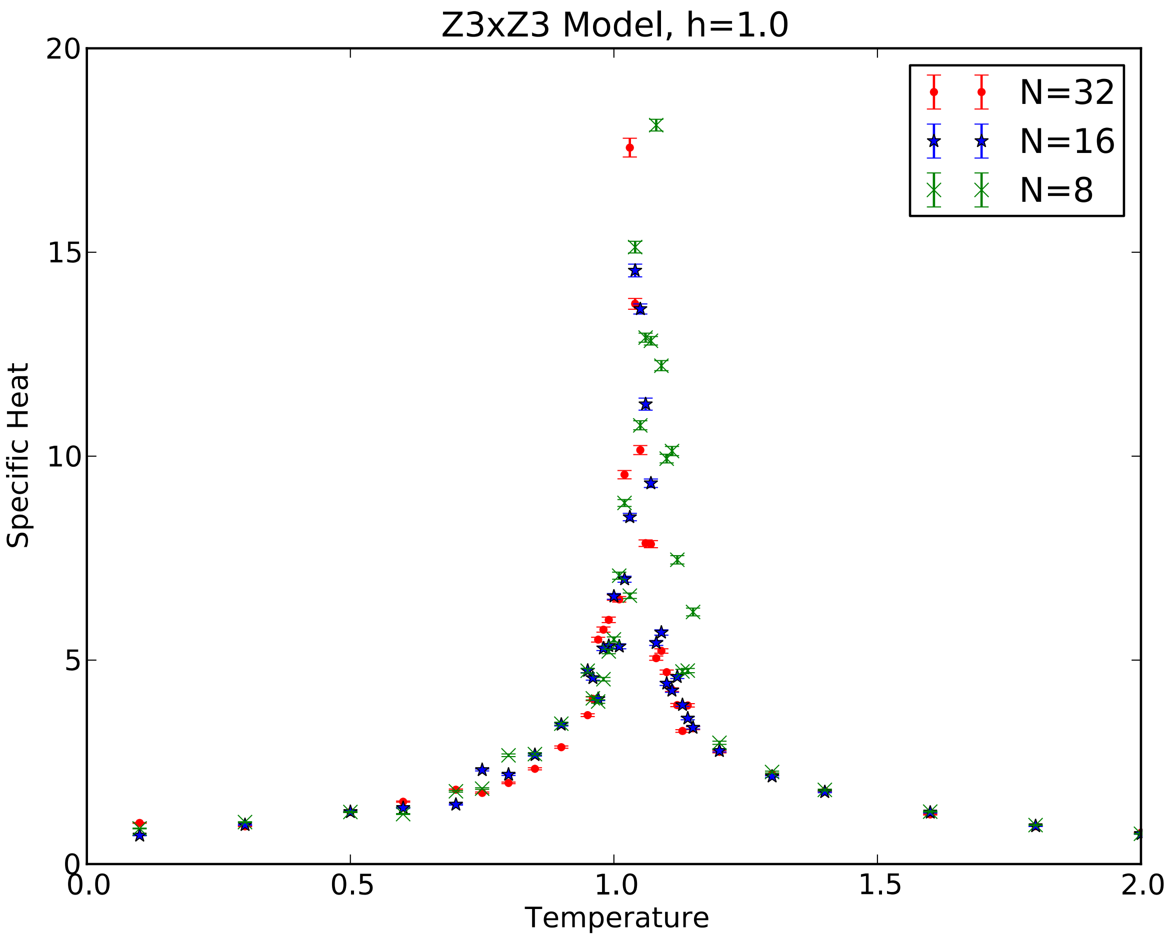
Picture for QCD adj



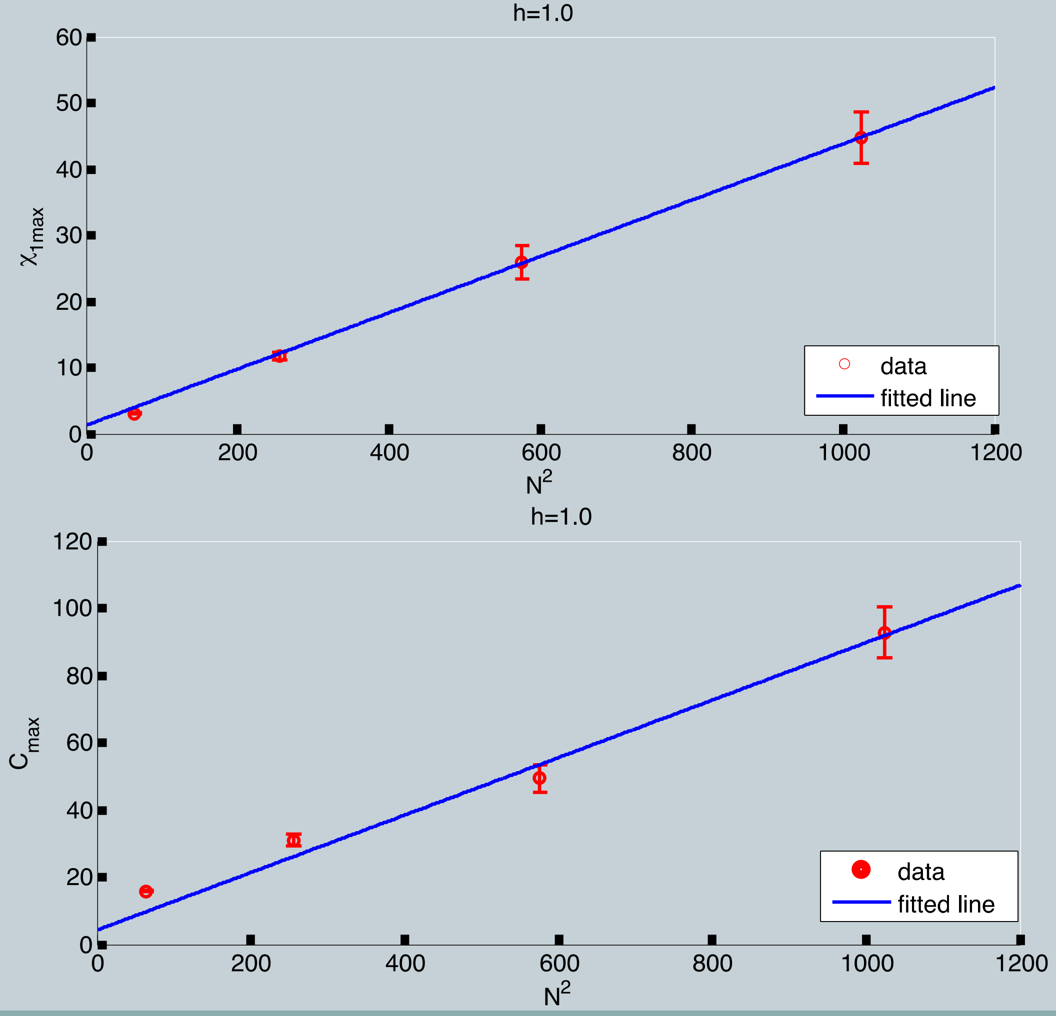
Computational results: magnetic susceptibility



Computational results: specific heat



Finite-size scaling

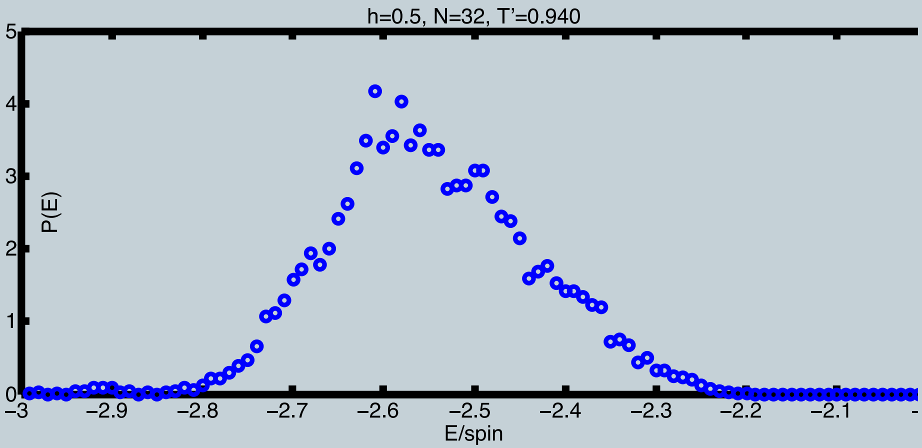


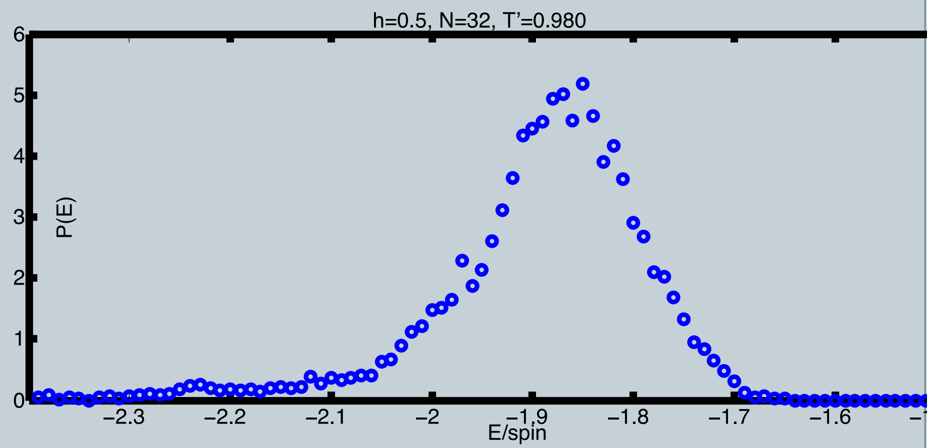
Finite-size scaling

- Our findings appear consistent with a first-order phase transition!
- Computational finite-size scaling provides a necessary counterpoint to the suggestion of (unconvincing, due to the onset of strong coupling) renormalization group analyses that the self-dual point is a fixed point

Energy probability distribution

- Further corroboration for the observed first-order phase transition: phase coexistence



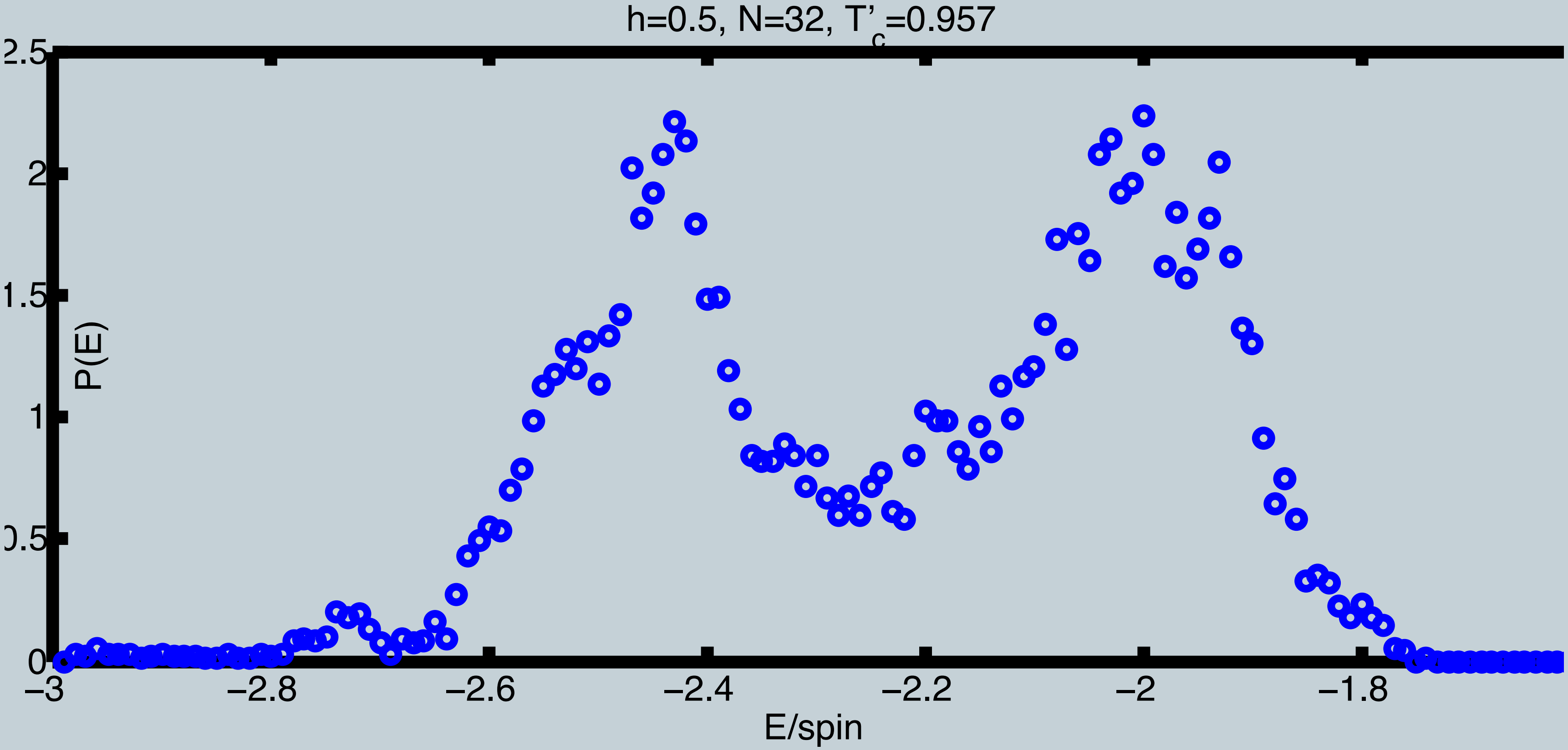


Below critical temperature

Above critical temperature

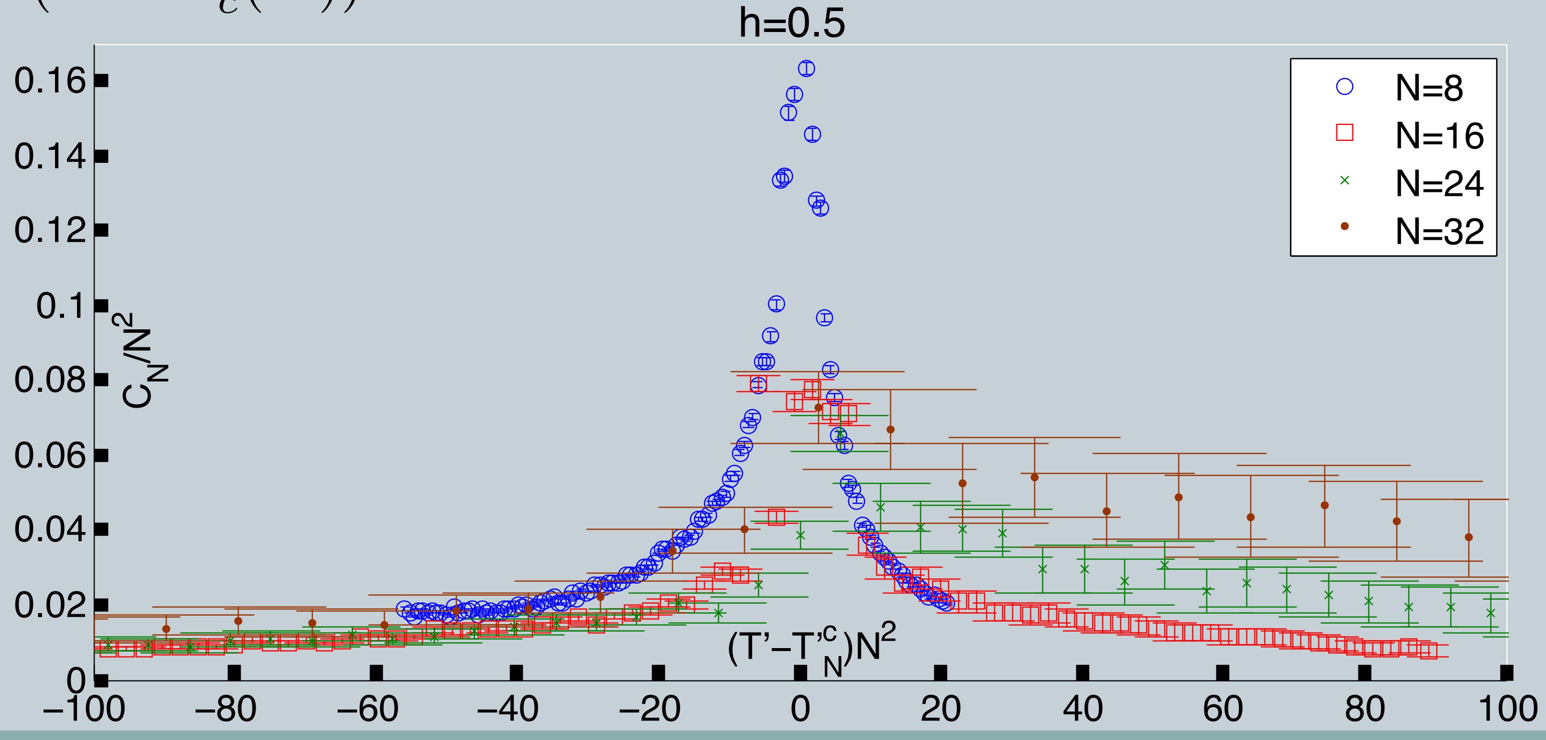
Energy probability distribution

- Phase coexistence at the critical temperature:



Finite-size scaling

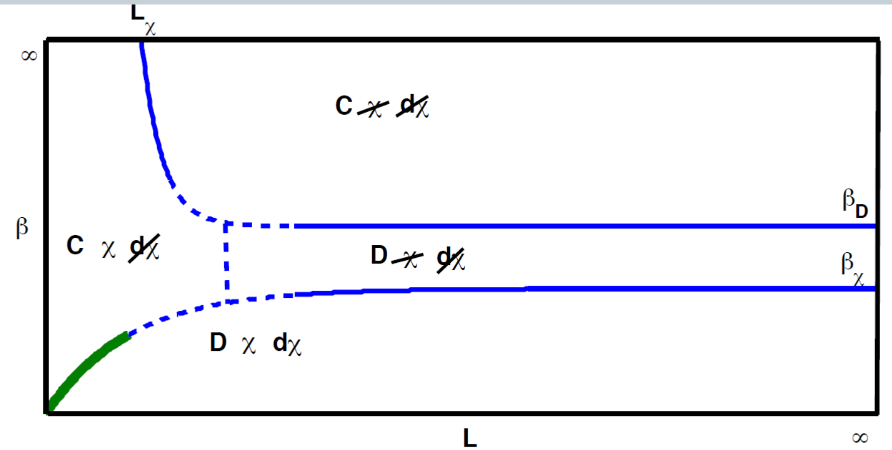
- Final check:
  - It has been shown that for sufficiently large volumes and in the critical region $\frac{C(N,T')}{N^2}$ is expected to be a universal function of $(T' - T'_c(N))N^2$

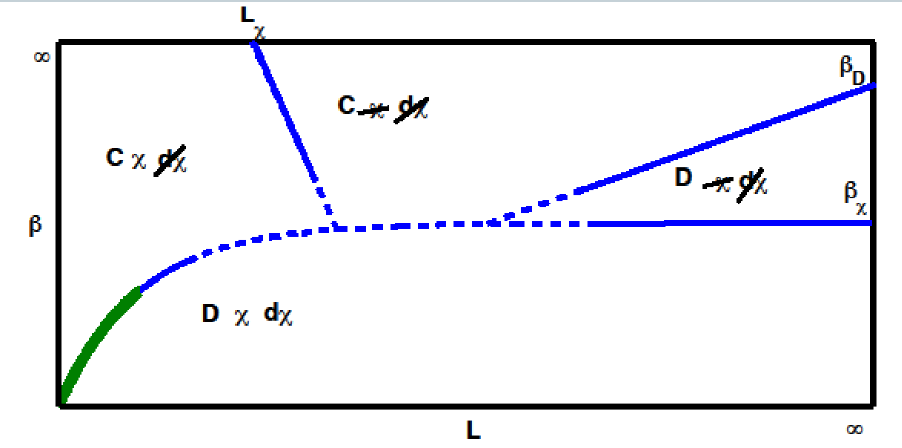


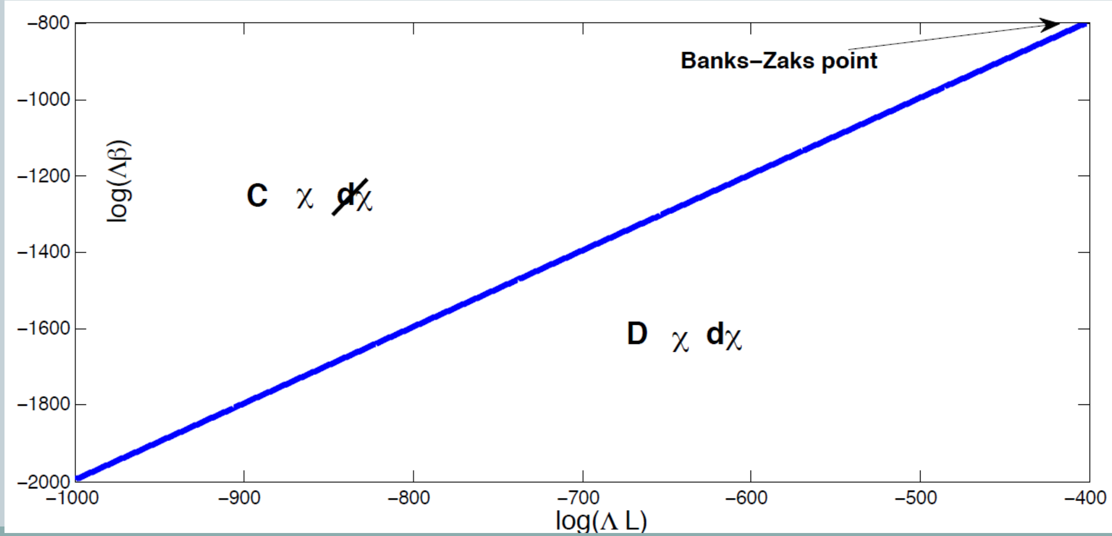
Conclusion

- Study of the phase transition in the XY-spin model dual to QCD adj theory on $R^{2,1} \times S^1$
- The phase transition is first order for SU(3)/Z_3 using Monte carlo simulations
- This agrees with what was found for the deconfinement transition in SU(3) 4-D QCD(adj) (with $n_f = 4$)
- One would also want to study other effects, like adding fundamental fermions and turning on a background field
- Work along these lines is in progress

Speculations and further studies







$n_f = 5$