Note Title 5/4/2013

HOLOGRAPHIC QUANTUM QUENCH AND CRITICAL BEHAVIOR

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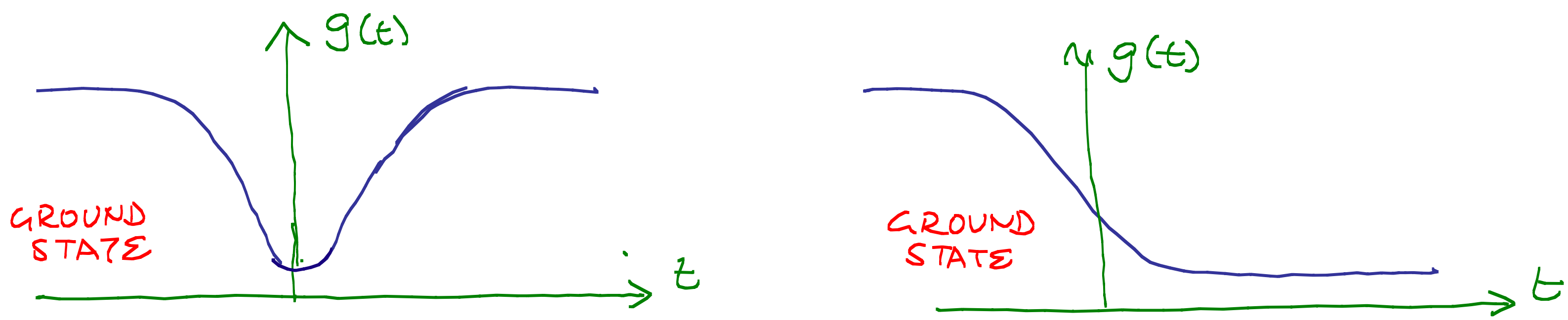
QUANTUM QUENCH AND UNIVERSALITY

QUANTUM FIELD THEORIES WITH TIME DEPENDENT COUPLINGS PROVIDE LABORATORIES TO UNDERSTAND SEVERAL PHYSICAL QUESTIONS

ONE SUCH QUESTION RELATES TO THERMALIZATION

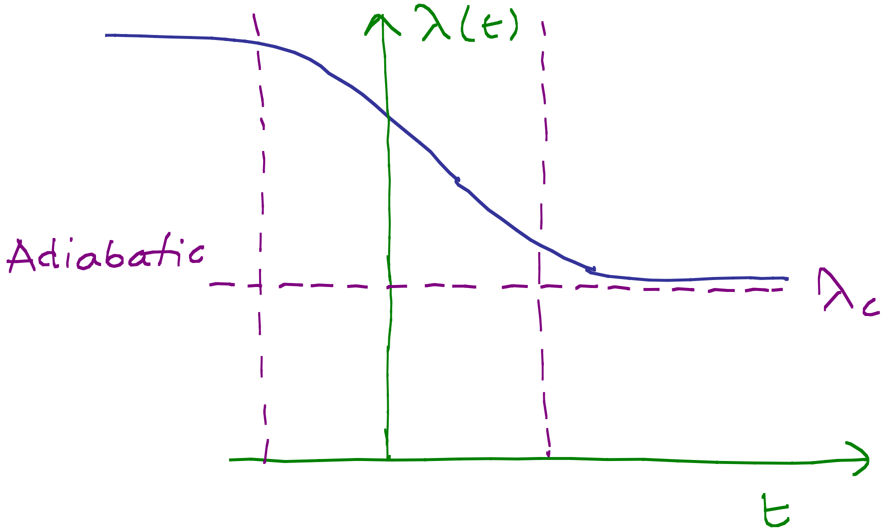
FOR A COUPLING WHICH BECOMES ASYMPTOTICALLY CONSTANT, DOES AN INITAL GROUND STATE EVOLVE INTO A STEADY STATE WHICH RESEMBLES A THERMAL STATE ?

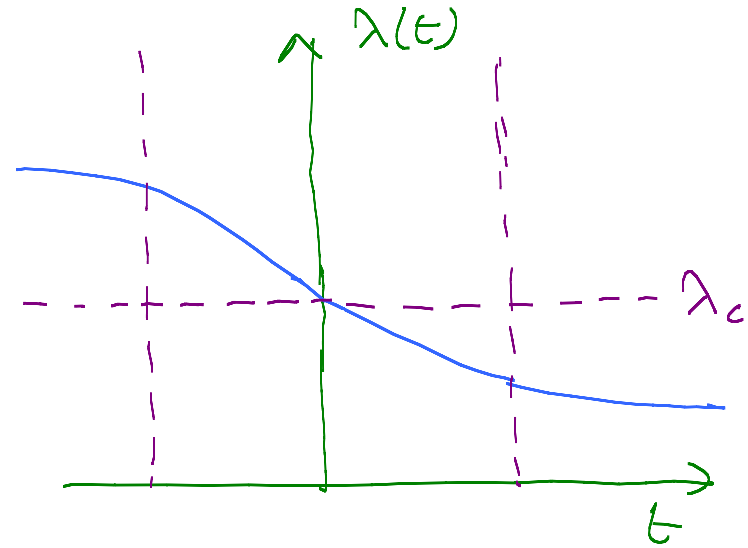


ANOTHER ISSUE RELATES TO POSSIBLE UNIVERSAL BEHAVIOR WHEN THE COUPLING CROSSES A CRITICAL POINT

PERHAPS THE BEST KNOWN ASPECT IS KIBBLE ZUREK SCALING

CONSIDER STARTING FROM A GAPPED PHASE AND APPROACH A CRITICAL POINT





GENERICALLY $\lambda(t) - \lambda_c \sim v t$

AN INITIALLY SLOW (COMPARED TO GAP) VARIATION CEASES TO BE ADIABATIC CLOSE TO λ_c

USING A RATHER DRASTIC ASSUMPTION KIBBLE-ZUREK ARGUMENT LEADS TO

$$\langle \mathcal{O} \rangle_x \sim v^{\frac{x\nu}{z\nu+1}} f\left(t v^{\frac{z\nu}{z\nu+1}}\right)$$

UNLIKE EQUILIBRIUM CRITICAL PHENOMENA THERE IS NO WELL-DEVELOPED THEORETICAL STRUCTURE LIKE RENORMALIZATION GROUP TO UNDERSTAND SUCH SCALING

NEVERTHELESS KIBBLE-ZUREK SCALING HAS BEEN FOUND IN SOME THEORETICAL MODELS

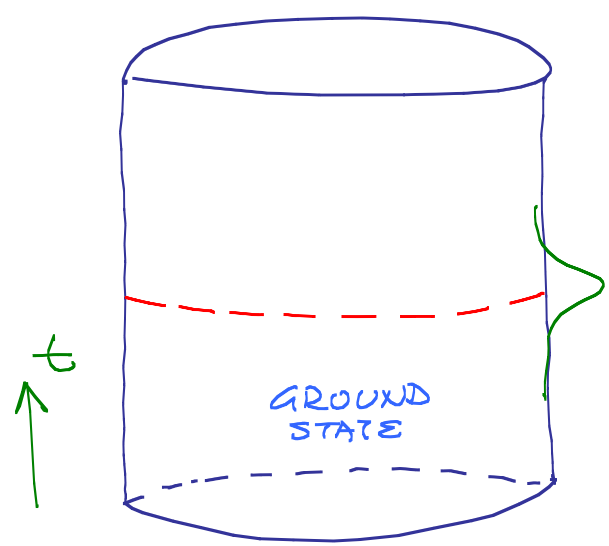
— SITUATION IN EXPERIMENTS LESS CLEAR

A DIFFERENT SET OF SCALING RELATIONS HAVE BEEN CONJECTURED FOR SUDDEN QUENCHES

QUENCH & AdS/CFT

IN AdS/CFT COUPLINGS OF THE FIELD THEORY BECOME BOUNDARY VALUES OF THE DUAL BULK FIELD

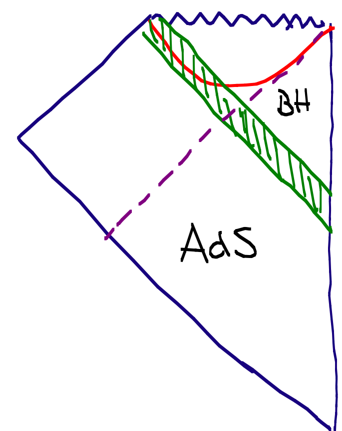
SO LONG A GRAVITY APPROXIMATION IS VALID — NEED TO SOLVE EQUATIONS OF MOTION WITH A SET OF TIME-DEPENDENT BOUNDARY CONDITIONS



THERMALIZATION

IF WE START AT T=0, PURE AdS THIS BECOMES THE PROBLEM OF BLACK HOLE FORMATION

IF ONE STARTS AT T≠0 (AdS-BH) – PROBLEM OF GROWTH OF BLACK HOLE



JANIK & PESHANSKI, SIN & SHURYAK,
CHESLER & YAFFE
BHATTACHARYA & MINWALLA
GARFINKLE & PANDO-ZAYAS
EBRAHIM & HEADRICK
ALBASH & JOHNSON
CACERES & KUNDU, GALANTE & SCHVELLINGER
BUCHEL, LEHNER & MYERS
BALASUBRAMANIAN et. al.

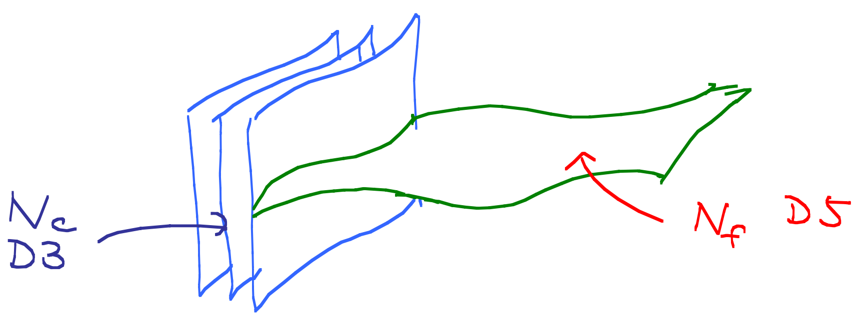
IN FIELD THEORY THIS IS THERMALIZATION

INTERESTINGLY, IN SOME SITUATIONS THERMALIZATION CAN BE SEEN EVEN IN A PROBE APPROXIMATION

IN SETUPS WHICH INCLUDE PROBE BRANES ONE FINDS — QUITE GENERICALLY — FORMATION OF APPARENT HORIZONS IN THE INDUCED METRIC ON THE BRANE

THIS LEADS TO THERMAL BEHAVIOR OF BRANE FLUCTUATIONS EVEN THOUGH THE BACKGROUND GEOMETRY REMAINS PURE AdS

(S.R.D, T. TAKAYANAGI, T. NISHIOKA)



$N_f \ll N_c$

$\boxed{\text{QUENCH DYNAMICS NEAR CRITICALITY}}$

- CONSIDER BOTTOM-UP MODELS WITH CRITICAL POINTS BOTH AT $T \neq 0$ AND AT $T = 0$

- TUNE PARAMETERS (e.g. TEMPERATURE, CHEMICAL POTENTIAL) TO THEIR CRITICAL VALUE

- TURN ON A TIME-DEPENDENT SOURCE FOR THE ORDER PARAMETER WHICH CROSSES ZERO AT SOME TIME

EXAMPLE IN MIND :

MAGNET AT $T = T_c$ IN THE PRESENCE OF A TIME DEP MAGNETIC FIELD

$\boxed{\text{A } T=0 \text{ HOLOGRAPHIC SUPERCONDUCTOR}}$

VARIATION OF A MODEL OF RYU, NISHIOKA & TAKAYANAGI

$$\boxed{S = \int d^{d+2}x \sqrt{g} \left\{ \frac{1}{2k^2}\left(R + \frac{d(d+1)}{L^2}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{\lambda}\left[|\partial_\mu \Phi - iqA_\mu \Phi|^2 - m^2|\Phi|^2 - \frac{1}{2}|\Phi|^4 \right] \right\}}$$

ONE OF THE SPATIAL DIRECTIONS θ IS COMPACT

A_t HAS A NON-ZERO BOUNDARY VALUE - DUAL FIELD THEORY HAS A CHEMICAL POTENTIAL

FOR $\Phi = 0$ THIS THEORY CAN BE IN EITHER OF TWO PHASES SEPARATED BY HAWKING-PAGE

$$\boxed{\text{AdS}_{d+2} \text{ SOLITON}}$$

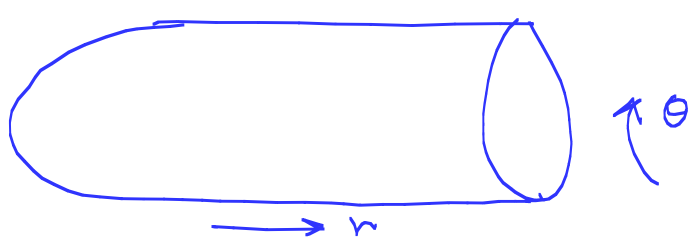
$$\boxed{ds^2 = \frac{dr^2}{r^2 f_s(r)} + r^2\left(-dt^2 + \sum_{i=1}^{d-1} dx_i^2\right) + r^2 f_s(r)\, d\theta^2}$$

$$f_s(r) = 1 - \left(\frac{r_0}{r}\right)^{d+1}$$

$$A_t = \mu$$

THIS HAS NO HORIZON — REGULARITY AT TIP $r = r_0$ REQUIRES

$$\theta \sim \theta + \frac{4\pi}{(d+1)\, r_0}$$



TEMPERATURE ARBITRARY

$$\boxed{AdS_{d+2} \text{ RN BLACK BRANE}}$$

$$\boxed{ds^2 = -r^2 f_b(r)\,dt^2 + \frac{dr^2}{r^2 f_b(r)} + r^2\left(\sum dx_i^2 + d\theta^2\right)}$$

$$f_b(r) = 1 - \left[1 + \frac{d-1}{2d}\left(\frac{\mu}{r_+}\right)^2\right]\left(\frac{r_+}{r}\right)^{d+1} + \frac{d-1}{2d}\left(\frac{\mu}{r_+}\right)^2\left(\frac{r_+}{r}\right)^{2d}$$

$$A_t = \mu\left[1 - \left(\frac{r_+}{r}\right)^{d+1}\right]$$

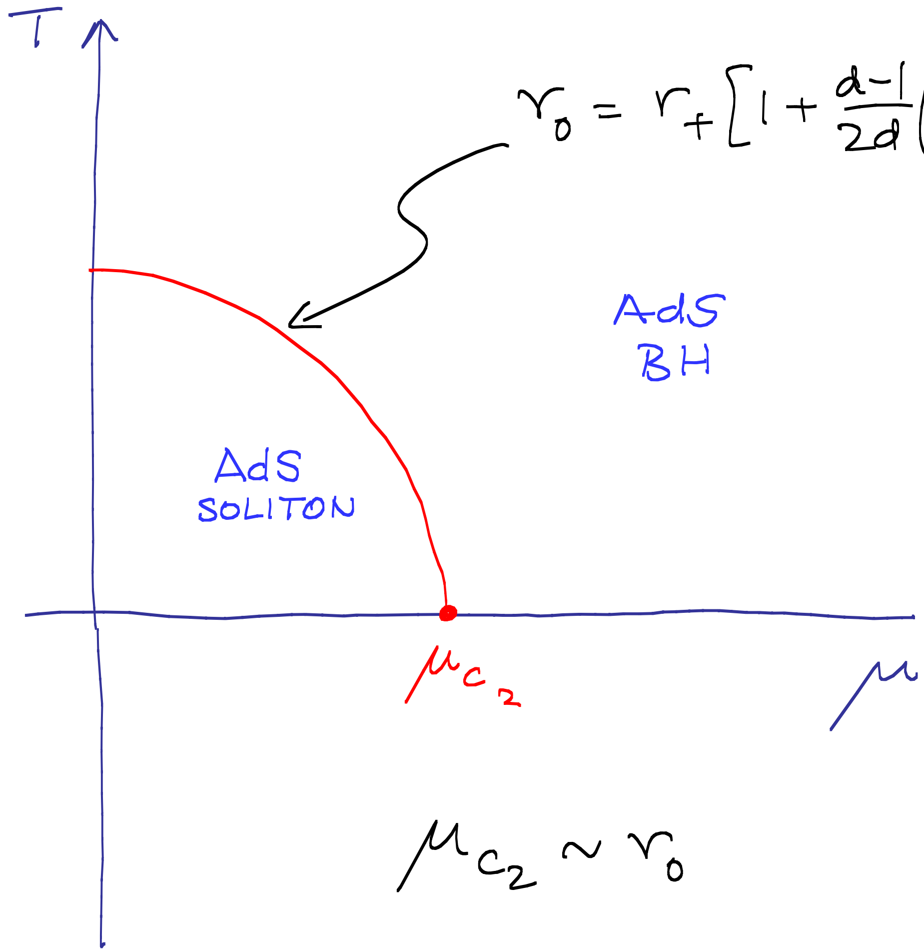
r_+ IS THE RADIUS OF OUTER HORIZON

$\rho = \mu r_+^{d+1}$ IS THE CHARGE DENSITY

$$T = \frac{r_+}{4\pi}\left[(d+1) - \frac{(d-1)^2}{2d}\left(\frac{\mu}{r_+}\right)^2\right]$$

PERIODICITY OF θ IS ARBITRARY

$$r_0 = r_+\left[1 + \frac{d-1}{2d}\left(\frac{\mu}{r_+}\right)^2\right]^{\frac{1}{d+1}}$$



$$\mu_{c_2} \sim r_0$$

PHASE STRUCTURE WITH $\Phi = 0$

SCALAR CONDENSATION

WE NOW STUDY THE EFFECT OF THE SCALAR IN A PROBE APPROXIMATION

$$\lambda \gg k^2 \qquad \lambda \gg q^2$$

IN THIS REGIME THE EFFECT OF THE SCALAR ON BOTH THE BACKGROUND GEOMETRY AND ON THE BACKGROUND GAUGE FIELD CAN BE IGNORED

CONSIDER THE SCALAR EQN IN SOLITON PHASE

$$\left[-\frac{1}{r^2}\left(\partial_t - iq\mu\right) + \frac{1}{r^3}\partial_r\left(r^5 f_s(r)\,\partial_r\right)\right]\Phi - m^2\Phi - |\Phi|^2\Phi = 0$$

$$-4 \leq m^2 \leq -3$$

ASYMPTOTICALLY $r \to \infty$

$$\Phi \to r^{-\Delta_-} J(t) \left[1 + O(1/r^2) \right] + r^{-\Delta_+} A(t) \left[1 + O(1/r^2) \right]$$

STANDARD QUANTIZATION

$J(t)$ — SOURCE

$$A(t) = \langle O \rangle$$

ALTERNATIVE QUANTIZATION $\quad J(t) \leftrightarrow A(t)$

QUESTION : IS THERE A STATIC SOLUTION Φ WHICH HAS

(1) $J = 0$

(2) REGULAR AT $r = r_0$

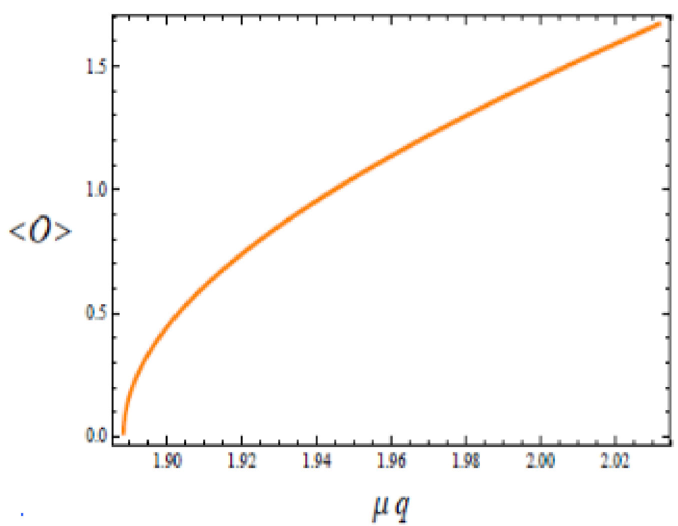
IF THERE IS A SOLUTION IT HAS LOWER ENERGY

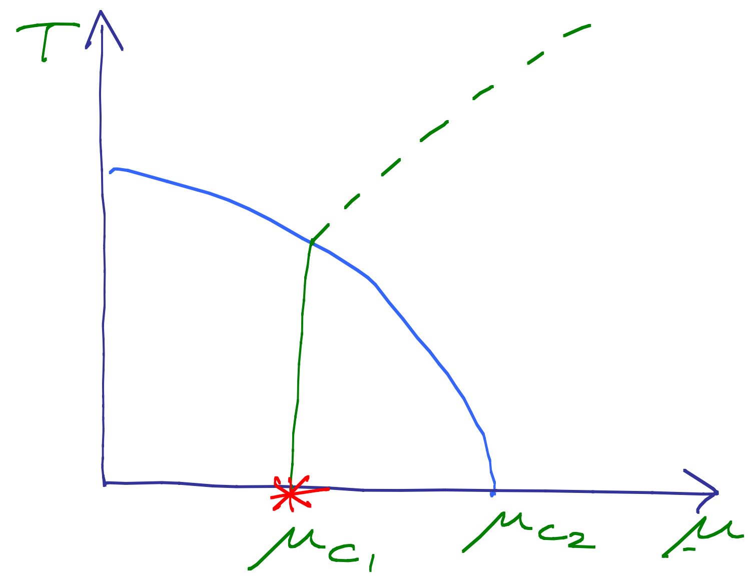
$$\frac{E}{V} = -\frac{1}{2\lambda} \int_{r_0}^{\infty} dr \sqrt{g}\, |\Phi|^4$$

A NUMERICAL SOLUTION SHOWS THAT FOR A FIXED m^2 IN THIS RANGE A SOLUTION EXISTS

$$q\mu \geq \mu_{c_1}$$

FOR LARGE ENOUGH q $\quad \mu_{c_1} < \mu_{c_2}$





REGULARITY AT THE TIP DETERMINES THE EXPONENTS — ALSO VERIFIED DIRECTLY

$$\langle \mathcal{O} \rangle \Big|_{J=0} \sim (\mu_{c_1} - \mu)^{1/2}$$

$$\frac{\partial \langle \mathcal{O} \rangle}{\partial J} \Big|_{J=0} \sim (\mu_{c_1} - \mu)^{-1}$$

$$\langle \mathcal{O} \rangle_{\mu = \mu_{c_1}} \sim |J|^{1/3}$$

THESE ARE MEAN FIELD EXPONENTS

QUENCH ACROSS THE QUANTUM CRITICAL POINT

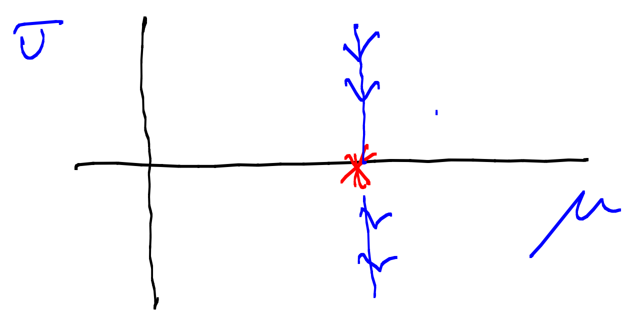
WE WANT TO STUDY THE QCP BY

- TUNING $\mu = \mu_{c_1}$
- TURNING ON A SOURCE $J(t)$

$$J(t) = J_0 \tanh(vt)$$

REMAINING IN THE PROBE LIMIT

OUR MAIN AIM IS TO LOOK FOR UNIVERSAL BEHAVIOR IN THE CRITICAL REGIME — AND TRY TO UNDERSTAND A MECHANISM BY WHICH THIS APPEARS



WE WILL DESCRIBE THE DETAILS FOR $d=3$
(AdS$_5$ SOLITON)

HOWEVER THE PHYSICS GENERALIZES FOR OTHER d

IN THE FOLLOWING WE WILL RESCALE ALL DIMENSIONAL QUANTITIES BY APPROPRIATE POWERS OF r_0

$r_0 = 1$ UNITS

DEFINE NEW COORDINATES AND FIELDS

$$\rho(r) = \int_r^\infty \frac{dx}{x^2 f_S^{1/2}(x)} = \frac{1}{r}\, {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{1}{r^4}\right)$$

$$\rho(r) \to 1/r \quad \text{AS} \quad r \to \infty$$

$$\rho(r) \to \rho_* + \sqrt{r-1} \quad \text{as} \quad r \to 1 \qquad \rho_* \sim 1.311$$

DEFINE NEW FIELD

$$\Phi(\rho) = \frac{1}{[r(\rho)]^{1/2}} \left(\frac{d\rho}{dr}\right)^{1/2} \Psi(\rho)$$

EQUATION OF MOTION NOW BECOMES

$$\boxed{\left(-\partial_t^2 + 2i\mu\partial_t\right)\Psi = \left(\mathcal{O} - \mu^2\right)\Psi + \frac{1}{r^2 f_s^{1/2}(r)}|\Psi|^2\Psi}$$

WHERE

$$\mathcal{O} \equiv -\partial_\rho^2 + V_0(\rho)$$

$$V_0(\rho) = m^2 r^2 + \frac{15r^8 - 18r^4 - 1}{4r^2(r^4-1)}$$

$\boxed{\text{ADIABATICITY LOST}}$

WE WILL CONSIDER SLOW QUENCH AT $\mu = \mu_{c_1}$

$$J(t) = J_0 \tanh(v t) \qquad v \ll 1 \quad \left(\underset{\text{UNITS}}{r_0 = 1} \right)$$

AT EARLY TIMES THE TIME EVOLUTION OF THE FIELD SHOULD BE ADIABATIC
TO LOWEST ORDER

$$\Psi^{(0)}(\rho, t) = \Psi_{\text{static}}(\rho, J(t))$$

$$\Psi_{\text{static}} \xrightarrow[r \to \infty]{} \rho^{\Delta_- - 3/2} J \left[1 + O(\rho^2)\right] + \rho^{\Delta_+ - 3/2} |J|^{1/3} \left[1 + O(\rho^2)\right]$$

CORRECTIONS TO THE LEADING SOLUTION

$$\Psi[\rho, t] = \Psi^{(0)}[\rho, \sigma(t)] + \epsilon\,\Psi^{(1)}(\rho, t) + \dots$$

REPLACE $\partial_t \to \epsilon\partial_t$ AND EQUATE ORDER BY ORDER

$\Psi^{(n)}$ SATISFIES A LINEAR ODE WITH A SOURCE WHICH DEPENDS ON TIME DERIVATIVES OF $\Psi^{(n-1)}$

TO LOWEST ORDER

$$\left[(P - \mu^2) + 3G(\rho)(\Psi^{(0)})^2\right] \mathrm{Re}\,\Psi^{(1)} = 0$$

$$\left[(P - \mu^2) + G(\rho)(\Psi^{(0)})^2\right] \mathrm{Im}\,\Psi^{(1)} = 2\mu\partial_t\Psi^{(0)}$$

HERE

$$G(\rho) = \frac{1}{r\sqrt{f_S(r)}}$$

THESE EQUATIONS CAN BE SOLVED BY CONSTRUCTING THE GREEN'S FUNCTION $\mathcal{g}(\rho, \rho')$ OF THE OPERATOR

$$\mathrm{Im}\, \Psi^{(1)}(\rho, t) = 2\mu \dot{J}(t) \int_0^{\rho_*} d\rho' \, \mathcal{g}(\rho, \rho') \frac{\partial \Psi^{(0)}}{\partial J}$$

WHERE

$$\left[(\mathcal{P} - \mu^2) + G(\rho) \left(\Psi^{(0)} \right)^2 \right] \mathcal{g}(\rho, \rho') = \delta(\rho - \rho')$$

WE WILL NOW SHOW THAT THIS LEADING CORRECTION BECOMES LARGE WHEN $J \sim 0$

THE ZERO MODE

CONSIDER THE OPERATOR

$$\mathcal{D} \equiv \mathcal{P} - \mu^2 = -\partial_\rho^2 + V_0(\rho) - \mu^2.$$

IT TURNS OUT THAT THE OPERATOR $\mathcal{P}$ HAS A POSITIVE SPECTRUM

THUS FOR SOME $\mu = \bar{\mu}$ THE OPERATOR $\mathcal{D}$ HAS A ZERO MODE

FOR $\mu > \bar{\mu}$ THE SCALAR FIELD CONDENSES

$\bar{\mu} = \mu_{c_1}$ — THE EQUILIBRIUM CRITICAL POINT

THE OPERATOR INVOLVED IN THE LOWEST ORDER ADIABATIC CORRECTION IS

$$\mathcal{P} - \mu_{c_1}^2 + G(\rho)\left(\Psi^{(0)}\right)^2$$

FOR SMALL J WE HAVE $\psi^{(0)} \sim J^{1/3}$

MAY USE PERTURBATION THEORY TO ESTIMATE

$$\operatorname{Im} \Psi^{(1)} \sim \frac{\dot{J}(t)}{J^{4/3}}$$

THUS ADIABATICITY BREAKS WHEN $|\Psi^{(1)}| \sim |\Psi^{(0)}|$

$$\dot{J}(t) \sim J^{5/3}$$

FOR $J(t) \sim v t$ NEAR $t \sim 0$

$$\boxed{t_{ad} \sim v^{-2/5}} \qquad \boxed{\mathcal{O}(t_{ad}) \sim v^{1/5}}$$

$\boxed{\text{CRITICAL DYNAMICS}}$

ONCE ADIABATICITY IS BROKEN THERE IS NO LONGER A POWER SERIES EXPANSION IN THE RATE ϑ

WE WILL NOW ARGUE THAT IN CRITICAL REGION

- THERE IS A DIFFERENT LOW ϑ EXPANSION IN FRACTIONAL POWERS OF ϑ

- IN THE LOWEST ORDER OF THIS EXPANSION THE ZERO MODE DOMINATES THE DYNAMICS

IN THE CRITICAL REGION APPROXIMATE

$$J(t) \sim J_0 \vartheta t$$

- SEPARATE OUT THE SOURCE PIECE

$$\Psi(\rho,t) = \rho^{\alpha} J(t) + \Psi_s(\rho,t)$$

- NEXT RESCALE

$$t = \nu^{-2/5}\eta \qquad \Psi_s = \nu^{1/5}\chi$$

- THE EQUATION OF MOTION BECOMES

$$\mathcal{D}\chi = \nu^{2/5}\left[2i\mu\,\partial_\eta\chi - G(\rho)\,|\chi|^2\chi - \eta\,\mathcal{D}\rho^{\alpha}\right] + O(\nu^{4/5})$$

RECALL

$$\mathcal{D} \equiv -\partial_\rho^2 + V_0(\rho) - \mu^2$$

- NOW DECOMPOSE THE FIELD IN TERMS OF EIGENMODES

$$\mathcal{D}\varphi_n(\rho) = \lambda_n \varphi_n(\rho)$$

$\varphi_0(\rho)$ IS THE ZERO MODE

$$X(\rho,\eta) = \sum_n x_n(\eta)\,\varphi_n(\eta)$$

- THE EQUATIONS NOW BECOME

$$\lambda_n x_n = \vartheta^{2/5}\Big[2i\mu(\partial_\eta x_n) - \sum_{n_1 n_2 n_3} C^n_{n_1 n_2 n_3} x^*_{n_1} x_{n_2} x_{n_3} + \mathcal{T}_n \eta\Big] + O(\vartheta^{4/5})$$

$$C^n_{n_1 n_2 n_3} = \int d\rho\, \varphi^*_n(\rho)\,\varphi_{n_3}(\rho)\,\varphi_{n_2}(\rho)\,\varphi_{n_1}(\rho)\,G(\rho)$$

$$\mathcal{T}_n = \int d\rho\, \varphi^*_n(\rho)\,\mathcal{D}\rho^\alpha J_0$$

- FOR SMALL ϑ, ALL THE MODES WITH $n \neq 0$ ARE SUPPRESSED

$$\chi_n(\eta) = \vartheta^{2/5} \xi_n(\eta) + O(\vartheta^{4/5})$$

$$\chi_0(\eta) = \xi_0(\eta) + O(\vartheta^{2/5})$$

- IN THE EQUATION FOR $\xi_0(\eta)$ THE BOUNDARY CONDITION APPEARS AS A TIME DEPENDENT SOURCE IN AN EFFECTIVE LANDAU-GINSBURG DYNAMICS

$$2i\mu \frac{\partial \xi_0}{\partial \eta} = C^0_{000} |\xi_0|^2 \xi_0 + J_0 \eta$$

THE DYNAMICS HAS $z=2$ BUT NON-DISSIPATIVE

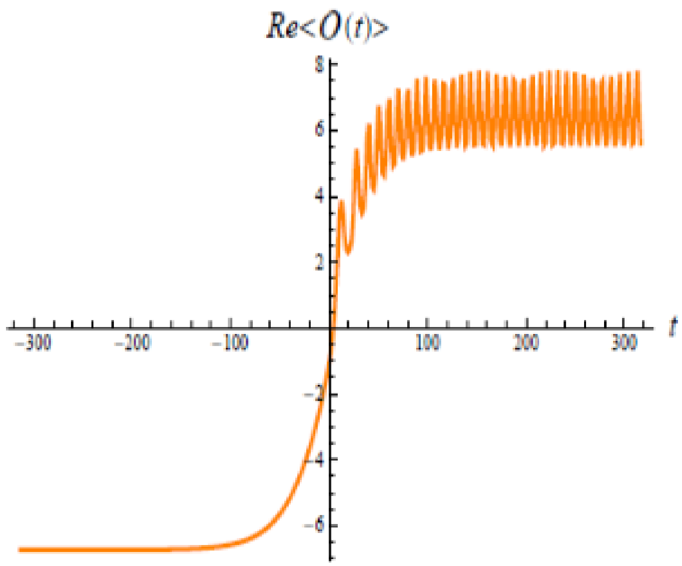
- THEFORE TO LOWEST ORDER IN v THERE IS A SCALING SOLUTION

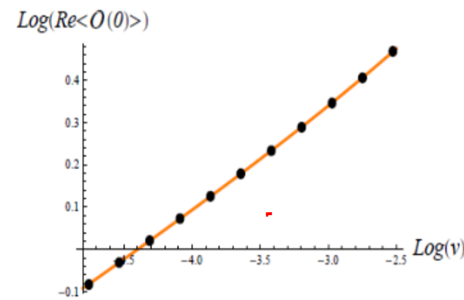
$$\Psi_S(\varrho, t; v) = v^{1/5}\, \Psi_S(\varrho, t v^{2/5}; 1)$$

THIS IMPLIES A KIBBLE-ZUREK SCALING OF THE ORDER PARAMETER

$$\langle \Theta(t, v) \rangle = v^{1/5} \langle \Theta(t v^{2/5}, 1) \rangle$$

WE CHECKED THIS BY A DIRECT NUMERICAL CALCULATION





The plot of $\log(\mathrm{Re}\langle\mathcal{O}(0)\rangle)$ vs $\log(v)$. We also plotted the closest fit

GOOD FIT WITH

$$\log\langle \mathcal{O}(0)\rangle = 0.79 + 0.206 \log v$$

$\sim 1/5$

TYPICAL BEHAVIOR OF ORDER PARAMETER

THERMAL QUENCHES

THE EMERGENCE OF SCALING SOLUTIONS IN THE CRITICAL REGION WAS IN FACT FIRST FOUND IN A SYSTEM AT $T \neq 0$ — NEUTRAL SCALAR FIELD IN THE BACKGROUND OF AdS-RN BLACK HOLE

(P. BASU & S.R.D)

THE MECHANISM WAS SIMILAR

- ZERO MODE AT CRITICAL POINT
- ADIABATICITY BREAKDOWN
- EXPANSION IN FRACTIONAL POWERS OF v
- ZERO MODE DOMINANCE

HOWEVER THIS IS A BIT MORE SUBTLE BECAUSE OF THE EXISTENCE OF A CONTINUUM SPECTRUM

THERE HAS BEEN SOME REMARKABLE RESULTS IN HOLOGRAPHIC THERMAL QUENCHES

- Bhaseen, Gauntlett, Simons, Sonner & Weisman
- Buchel & Myers

Bhaseen et al CONSIDER HOLOGRAPHIC SUPERFLUID OF THE USUAL KIND

START WITH $T < T_c$ IN ORDERED PHASE

INJECT ENERGY BY TIME DEPENDENT SOURCE

STUDY RELAXATION

DEPENDING ON THE AMPLITUDE SYSTEM RELAXES TO EITHER SUPERFLUID OR NORMAL STATE

HOWEVER IN THE FIRST CASE (RELAXATION TO CONDENSED PHASE) THEY FIND TWO DIFFERENT BEHAVIOR

- DAMPED OSCILLATIONS
- OVERDAMPED

SEPARATED BY A DYNAMICAL TRANSITION

SIMILAR BEHAVIOR HAS BEEN CONJECTURED FOR USUAL BCS SUPERCONDUCTORS

THE DYNAMICS OF THE ORDER PARAMETER HAS
$z = 2$ — BUT DISSIPATIVE

EXPECTED — BECAUSE OF THE PRESENCE OF A BLACK HOLE HORIZON

WHEN $T = 0$ (EXTREMAL BLACK HOLE) THE EQUILIBRIUM TRANSITION BECOMES BKT TYPE
—
WE HAVE NOT BEEN ABLE TO GET ANY ANALYTIC HANDLE ON THIS

NON-MEAN FIELD TRANSITIONS

- THE KIND OF EQUILIBRIUM TRANSITIONS WE HAVE DISCUSSED SO FAR ARE STANDARD MEAN FIELD TRANSITIONS — THIS IS THE MOST COMMON SITUATION FOR HOLOGRAPHIC CRITICAL POINTS

- IT WOULD BE REALLY INTERESTING TO SEE IF THE SAME STORY PERSISTS FOR EXAMPLES OF NON-MEAN-FIELD THEORY TRANSITIONS WHICH ARE KNOWN

- IN A CLASS OF SUCH MODELS WE ARE FINDING THAT THE SAME ROUTE TO UNIVERSAL CRITICAL DYNAMICS HOLDS — BUT $z = 1$

(P. Basu, D. Das, S.R.D & K. Sengupta — in progress)

THANK YOU.