Multi-particle production in proton-nucleus collisions in the CGC framework

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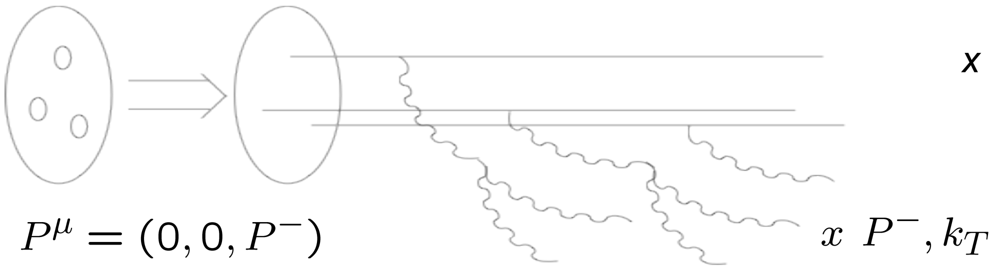
- Parton saturation and non-linear evolution in QCD
- The Color Glass Condensate (CGC) framework
- Single hadron production and dipole evolution
- Di-hadron production and quadrupole evolution
- Multi-particle production in the large-Nc limit

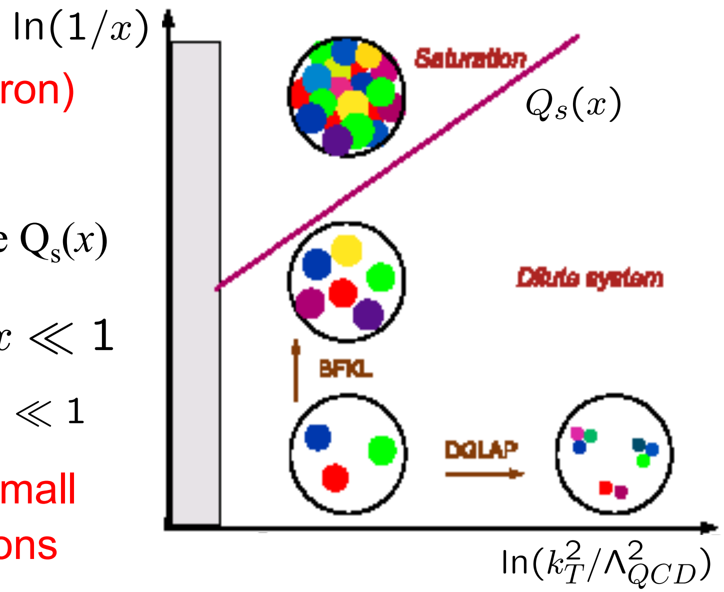
Map of parton evolution in QCD



x : parton longitudinal momentum fraction

k_T : parton transverse momentum

the distribution of partons as a function of x and k_T :



QCD linear evolutions: $k_T \gg Q_s$

DGLAP evolution to larger k_T (and a more dilute hadron)
BFKL evolution to smaller x (and denser hadron)

dilute/dense separation characterized by the saturation scale $Q_s(x)$

QCD non-linear evolution: $k_T \sim Q_s$ meaning $x \ll 1$

this regime is non-linear yet weakly coupled: $\alpha_s(Q_s^2) \ll 1$

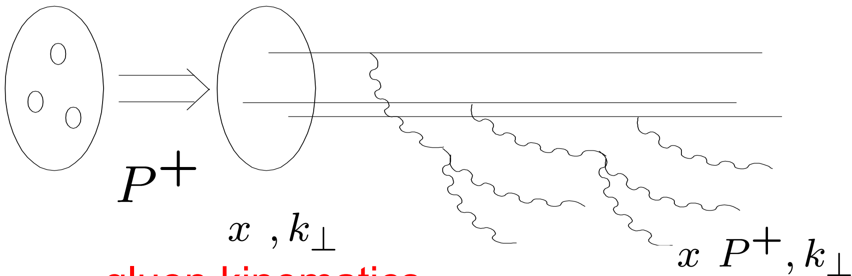
collinear factorization does not apply when x is too small and the hadron has become a dense system of partons

$$\sigma_{DIS}(x_{Bj}, Q^2) = \sum_{partons\ a} \int_{x_{Bj}}^{1} dx\ \varphi_{a/p}(x, Q^2)\hat{\sigma}_a(x_{Bj}/x, Q^2) + O\left(Q_0^2/Q^2\right)$$

parton density — partonic cross-section — higher twist $\rightarrow \dfrac{(A/x)^{1/3}}{Q^2}$

The saturation momentum

- gluon recombination in the hadronic wave function



gluon kinematics

$\rho \sim \frac{xf(x,k_\perp^2)}{\pi R^2}$ gluon density per unit area
it grows with decreasing *x*

$\sigma_{rec} \sim \alpha_s / k_\perp^2$ recombination cross-section

recombinations important when $\rho \; \sigma_{rec} > 1$

the saturation regime: for $k_\perp^2 < Q_s^2$ with $Q_s^2 = \frac{\alpha_s x f(x,Q_s^2)}{\pi R^2}$ $\qquad \alpha_s(Q_s^2) \ll 1$

for a given value of $k_\perp^2$, the saturation regime in a nuclear wave function extends to a higher value of *x* compared to a hadronic wave function

- the CGC: an effective theory to describe the saturation regime

the idea in the CGC is to take into account saturation via strong classical fields $\mathcal{A} \sim 1/g_s$

McLerran and Venugopalan (1994)

lifetime of the fluctuations in the wave function ~ $xP^+/k_\perp^2$ $\Rightarrow$
- high-x partons ≡ static sources ρ
- low-x partons ≡ dynamical fields $\mathcal{A}$

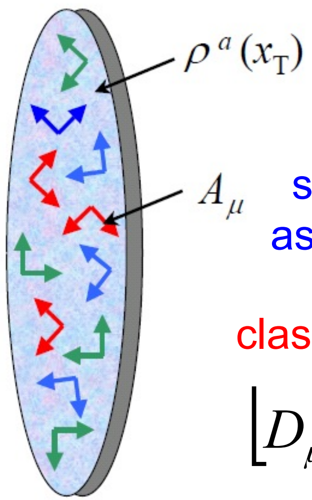
The Color Glass Condensate

- the CGC wave function

$$|\text{hadron}\rangle = |qqq\rangle + |qqqg\rangle + \ldots + |qqq\ldots\ldots ggggg\rangle \quad \Rightarrow \quad |\text{hadron}\rangle = \int D\rho\, \Phi_x[\rho]\,|\rho\rangle \equiv |\text{CGC}\rangle$$

CGC wave function

separation between
the long-lived high-x partons
and the short-lived low-x gluons



valence partons
as static random
color source

small x gluons
as radiation field

classical Yang-Mills equations

$$\left[D_\mu, F^{\mu\nu}\right]_c = \delta^{\nu-}\rho_c(z^+, \mathbf{z})$$

$$\alpha_c(z^+, \mathbf{z}) = A_c^-(z^+, \mathbf{z}, z^- = 0) \sim 1/g_s$$

$$-\nabla^2 \alpha_c\,(z^+, \mathbf{z}) = \rho_c(z^+, \mathbf{z})$$

in the $A^+=0$ gauge

from $|\Phi_x[\alpha]|^2$, one can obtain
the unintegrated gluon distribution,
as well as any n-parton distributions

- the small-x evolution

the evolution of $|\Phi_x[\alpha]|^2$ with *x* is a renormalization-group equation

the solution gives $Q_s^2(x, A) \sim A^{1/3} x^{-0.3}$

Jalilian-Marian, Iancu, McLerran,
Weigert, Leonidov, Kovner (1997-2002)

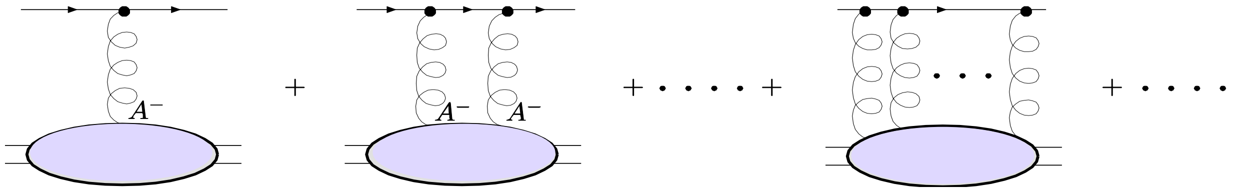
Scattering off the CGC

- this is described by Wilson lines

scattering of a quark:

$$W_F[\alpha](\mathbf{x}) = \mathcal{P}\exp\left\{ig_s\int dx^+ T^c\alpha_c(x^+,\mathbf{x})\right\}$$

α dependence kept implicit in the following



in the CGC framework, any cross-section is determined by colorless combinations of Wilson lines $S[\alpha]$, averaged over the CGC wave function $\langle S\rangle_x = \int D\alpha\ |\Phi_x[\alpha]|^2 S[\alpha]$

- the 2-point function or dipole amplitude

the $q\bar{q}$ dipole scattering amplitude:

$\langle T_{q\bar{q}}(\mathbf{x},\mathbf{y})\rangle_x$ or $\langle T_{q\bar{q}}(\mathbf{r},\mathbf{b})\rangle_x$

$$T_{q\bar{q}}(\mathbf{x},\mathbf{y}) = 1 - \frac{1}{N_c} Tr(W_F^+(\mathbf{y}) W_F(\mathbf{x}))$$

x : quark transverse coordinate

y : antiquark transverse coordinate

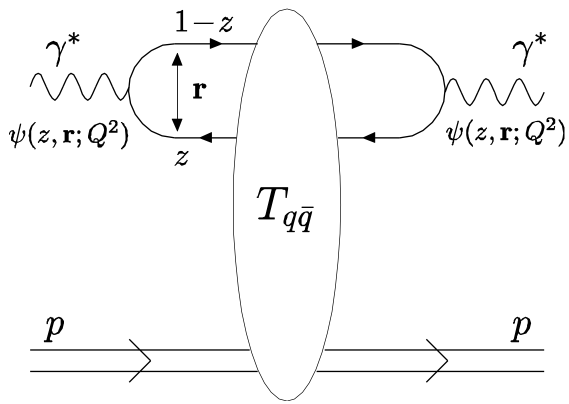
this is the most common Wilson-line average

Single hadron production and dipole evolution

The dipole scattering amplitude

a fundamental quantity to study high-energy scattering in QCD

- deep inelastic scattering at small x :



$$\sigma_{T,L}^{\gamma^* p\to X} = 2\int d^2r\, dz\, |\psi_{T,L}(z,\mathbf{r};Q^2)|^2 \int d^2b\, T_{q\bar{q}}(\mathbf{r},\mathbf{b},x_B)$$

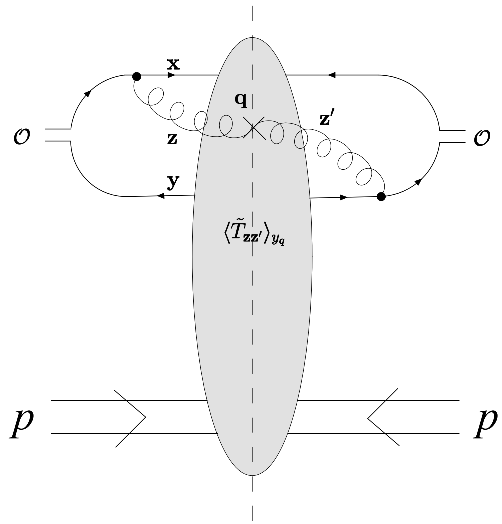
overlap of $\gamma^* \to q\bar{q}$ splitting functions

r = dipole size

dipole-hadron cross-section computed in the CGC

resums powers of $g_S A$ and powers of $\alpha_S \ln(1/x_B)$

- particle production at forward rapidities:



$$q^2 \frac{d\sigma}{d^2q\, d^2b} \propto \int \frac{d^2r}{(2\pi)^2}\, e^{-i\mathbf{q}\cdot\mathbf{r}} [1 - T_{gg}(\mathbf{r},\mathbf{b},x)]$$

r = z-z'

dipole-hadron scattering amplitude (adjoint or fundamental)

FT of dipole amplitude ≡ unintegrated gluon distribution

The Balitsky-Kovchegov equation

- for impact-parameter independent solutions $T_{q\bar{q}}(\mathbf{r},\mathbf{b},x) \equiv \mathcal{N}(x,r)$

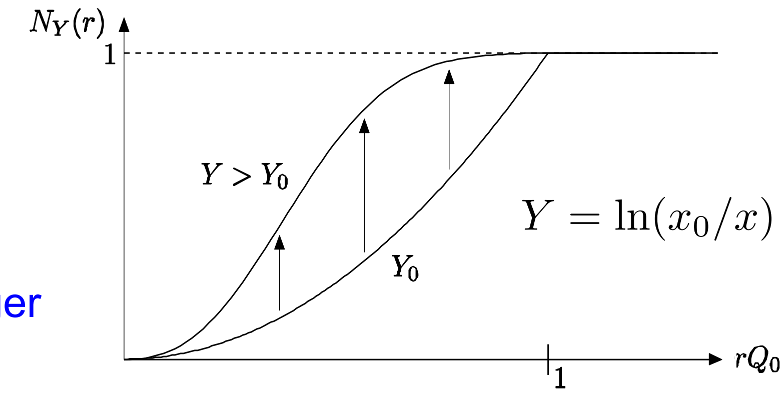
$$\frac{\partial \mathcal{N}(x,r)}{\partial \ln(x_0/x)} = \bar{\alpha} \int \frac{d^2 r_1}{2\pi} \frac{r^2}{r_1^2 r_2^2} \underbrace{[\mathcal{N}(x,r_1) + \mathcal{N}(x,r_2) - \mathcal{N}(x,r)}_{\text{linear evolution : BFKL}} - \underbrace{\mathcal{N}(x,r_1)\mathcal{N}(x,r_2)]}_{\text{saturation}}$$

$$r_2 = |\mathbf{r} - \mathbf{r}_1|$$

- solutions: qualitative behavior

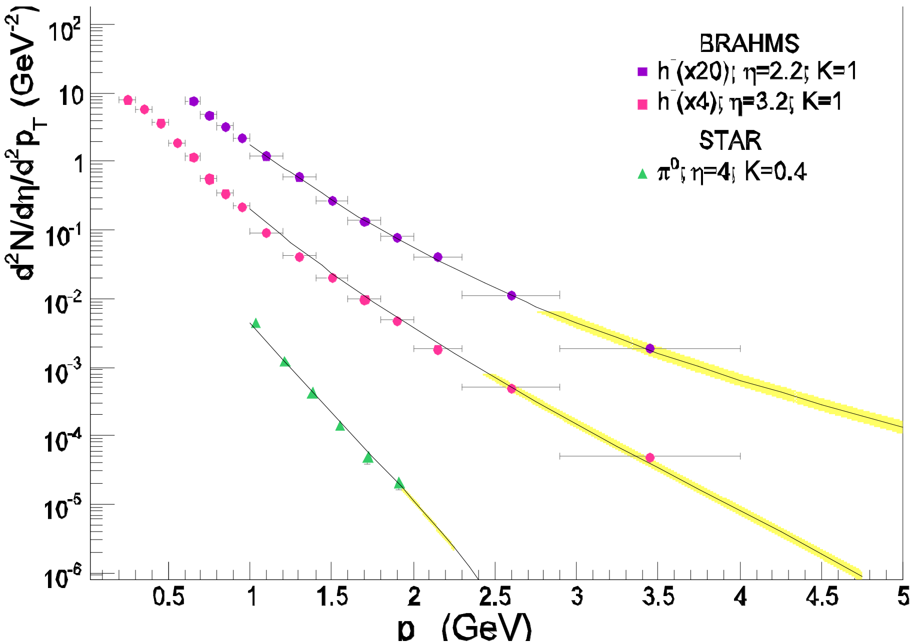
at large *x*, $\mathcal{N}$ is small, and the quadratic term can be neglected, the equation reduces then to the linear BFKL equation and $\mathcal{N}$ rises exponentially with decreasing x

as $\mathcal{N}$ gets close to 1 (the stable fixed point of the equation), the non-linear term becomes important, and $d\mathcal{N}/dY \to 0$, $\mathcal{N}$ saturates at 1



with increasing *Y*, the unitarization scale get bigger

rcBK description of d+Au data



Albacete and C.M. (2010)

the shapes and normalizations are well reproduced, except the π^0 normalization

the speed of the *x* evolution and of the p_T decrease are predicted

this fixes the two parameters of the theory:

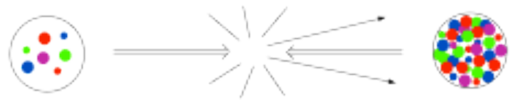
- the value of *x* at which one starts to trust (and therefore use) the CGC description
- and the saturation scale at that value of *x* $Q_s^2(x_0) = 0.4\ \text{GeV}^2$ $x_0 = 0.02$

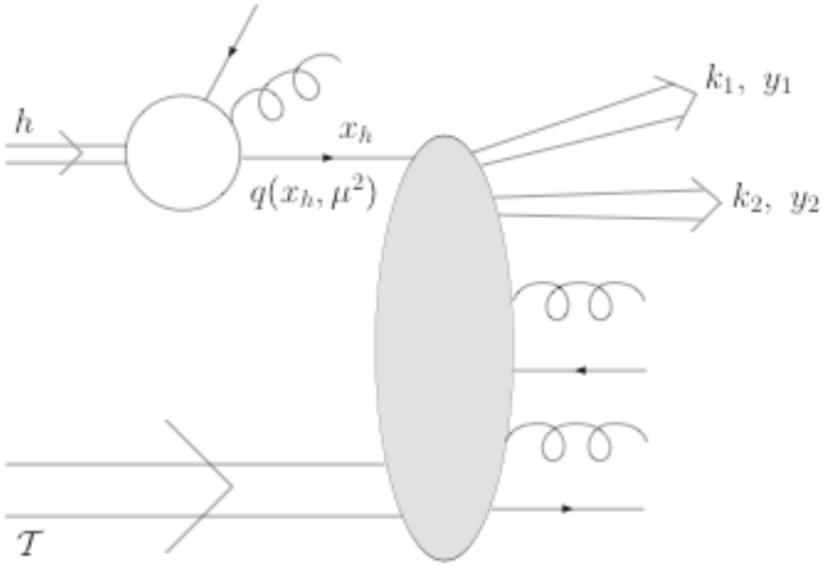
Note: in forward particle production in p+p collisions at RHIC (where NLO DGLAP fails), using this formalism to describe the (small-x) proton also works

Di-hadron production and quadrupole evolution

Forward di-hadron production

in p+A type collisions $x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}} << 1$





CM (2007)

the saturation regime is better probed compared to single particle production

$$\frac{d\sigma^{dAu\to h_1 h_2 X}}{d^2k_1 dy_1 d^2k_2 dy_2}$$

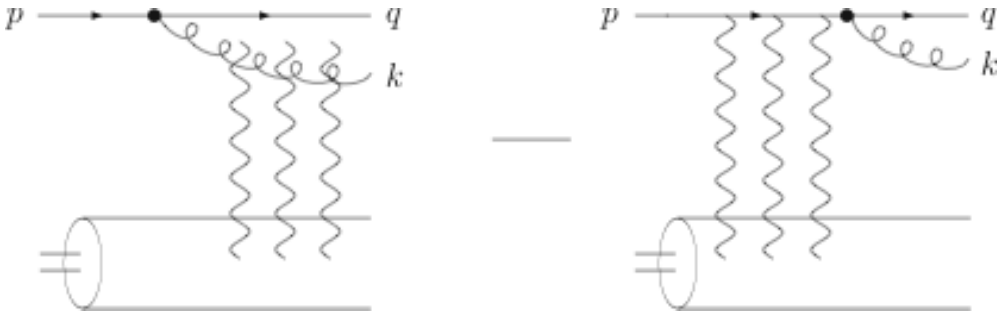
is sensitive to multi-parton distributions, and not only to the gluon distribution

the CGC cannot be described by a single gluon distribution

$$\frac{d\sigma^{dAu\to h_1 h_2 X}}{d^2k_1 dy_1 d^2k_2 dy_2}$$ involves 2-, 4- and 6- point functions

(no k_T factorization: 4- and 6- point functions)

The two-particle spectrum



b: quark in the amplitude
x: gluon in the amplitude
b': quark in the conj. amplitude
x': gluon in the conj. amplitude

Fourier transform $k_\perp$ and $q_\perp$ into transverse coordinates

collinear factorization of quark density in deuteron

$$\frac{d\sigma^{dAu\to qgX}}{d^2k_\perp dy_k d^2q_\perp dy_q} = \alpha_S C_F N_c \, x_d q(x_d,\mu^2) \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2b}{(2\pi)^2}\frac{d^2b'}{(2\pi)^2} \, e^{ik_\perp.(\mathbf{x}'-\mathbf{x})} e^{iq_\perp.(\mathbf{b}'-\mathbf{b})}$$

$$\left|\Phi^{q\to qg}(z,\mathbf{x}-\mathbf{b},\mathbf{x}'-\mathbf{b}')\right|^2 \Big\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]$$

$$-S^{(3)}_{\bar{q}gq}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_A] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A] \Big\}$$

pQCD $q \to qg$ wavefunction

interaction with target nucleus

$$z = \frac{|k_\perp| e^{y_k}}{|k_\perp| e^{y_k} + |q_\perp| e^{y_q}}$$

n-point functions that resums the powers of $g_S A$ and the powers of $\alpha_S \ln(1/x_A)$

2- 4- and 6-point functions

the scattering off the CGC is expressed through the following correlators of Wilson lines:

if the gluon is emitted before the interaction, four partons scatter off the CGC

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(W_F(\mathbf{b})W_F^\dagger(\mathbf{b}')T^d T^c\right)\left[W_A(\mathbf{x})W_A^\dagger(\mathbf{x}')\right]^{cd}\right\rangle_{x_A}$$

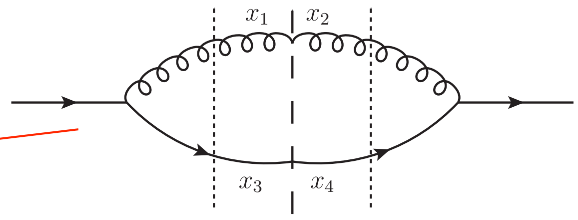
if the gluon is emitted after the interaction, only the quarks interact with the CGC

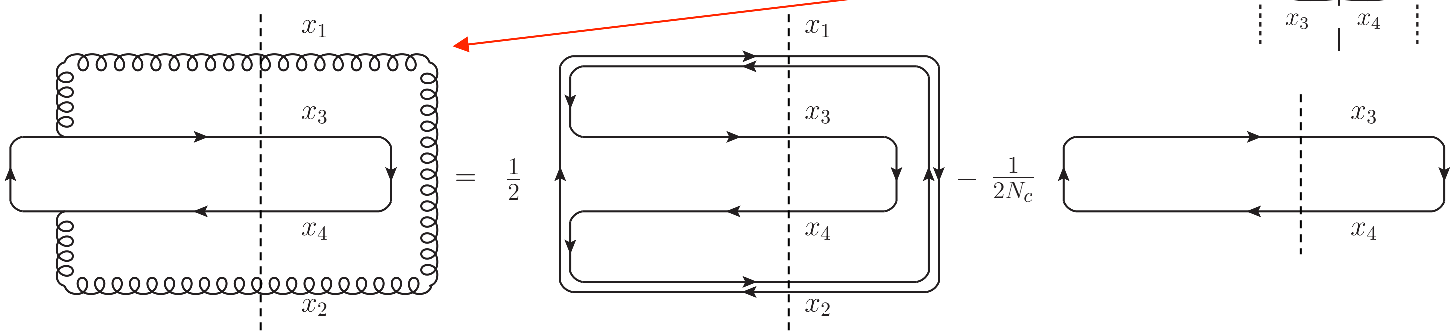
$$S^{(2)}_{q\bar{q}}(\mathbf{b},\mathbf{b}';x_A) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(W_F(\mathbf{b})W_F^\dagger(\mathbf{b}')\right)\right\rangle_{x_A}$$

interference terms, the gluon interacts in the amplitude only (or c.c. amplitude only)

$$S^{(3)}_{qg\bar{q}}(\mathbf{b},\mathbf{x},\mathbf{b}';x_A) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(W_F^\dagger(\mathbf{b}')T^c W_F(\mathbf{b})T^d\right)W_A^{cd}(\mathbf{x})\right\rangle_{x_A}$$

in the large Nc limit, everything is expressed in terms of dipoles and quadrupoles





Dealing with the 4-point function

- in the large-Nc limit, the cross section is obtained from

$$S^{(4)} = \frac{1}{N_c}\left\langle \mathrm{Tr}(W_{\mathbf{x}} W^\dagger_{\mathbf{y}} W_{\mathbf{u}} W^\dagger_{\mathbf{v}})\right\rangle_{x_A} \text{ and } S^{(2)} = \frac{1}{N_c}\left\langle \mathrm{Tr}(W_{\mathbf{x}} W^\dagger_{\mathbf{y}})\right\rangle_{x_A}$$

the 2-point function is fully constrained
by e+A DIS and d+Au single hadron data

- in principle the 4-point function should be obtained from an evolution equation (equivalent to JIMWLK + large Nc)

Jalilian-Marian and Kovchegov (2005)

- in practice one uses an approximation that allows to express $S^{(4)}$ as a (non linear) function of $S^{(2)}$ C.M. (2007)

this approximation misses some leading-Nc terms Dumitru and Jalilian-Marian (2010)

they may become dominant for $|k_\perp + q_\perp| \ll |k_\perp|,\ |q_\perp|$ Dominguez, Xiao and Yuan (2010)

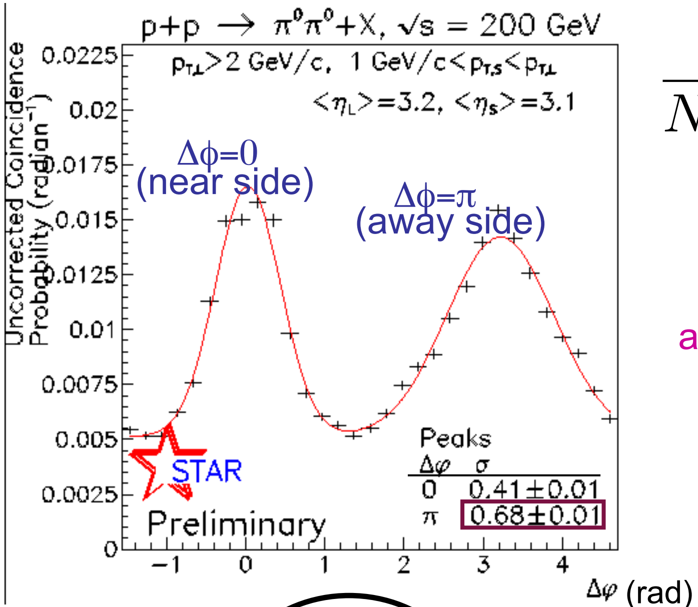
- recent results: 4-point function obtained from a numerical solution of the JIMWLK equation Dumitru, Jalilian-Marian, Lappi, Schenke and Venugopalan (2011)

the so-called dipole approximation used in the calculation show ~10% deviations

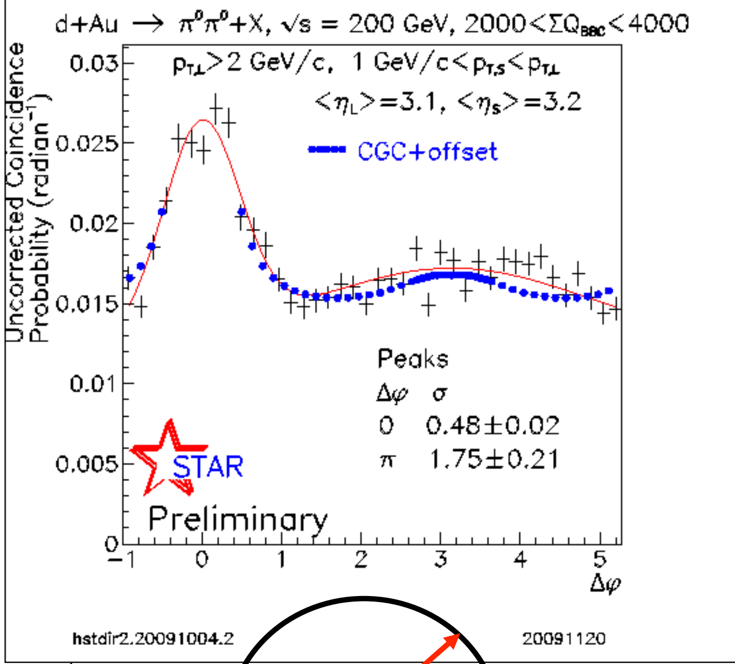
Di-hadron angular correlations

comparisons between d+Au → h_1 h_2 X (or p+Au → h_1 h_2 X) and p+p → h_1 h_2 X

p+p collisions

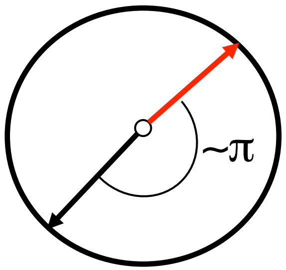


central d+Au collisions

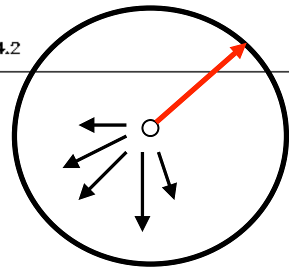


$$\frac{1}{N_{trig}} \frac{dN_{pair}}{d\Delta\varphi}$$

Albacete and CM (2010)



$$x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}} << 1$$

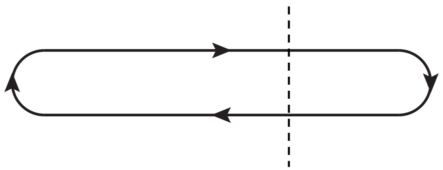


however, when $y_1 \sim y_2 \sim 0$ (and therefore $x_A \sim 0.03$), the p+p and d+Au curves are almost identical

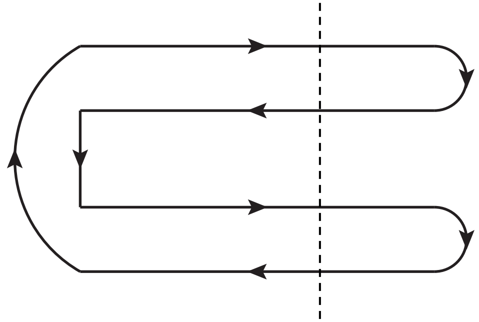
Multi-particle production in the large-Nc limit

From n to n+1 particles produced

- assume the cross section for the production of n partons is made of only dipoles and quadrupoles



(a)



(b)

- then one can show that the cross section for the production of n+1 partons is also made of only dipoles and quadrupoles

gluon line added between two different objects

does not increase the total number of color traces (loops in the diagramatic representation)
therefore these contributions will be subleading in Nc (they do involve higher-point functions)

gluon line added within same object

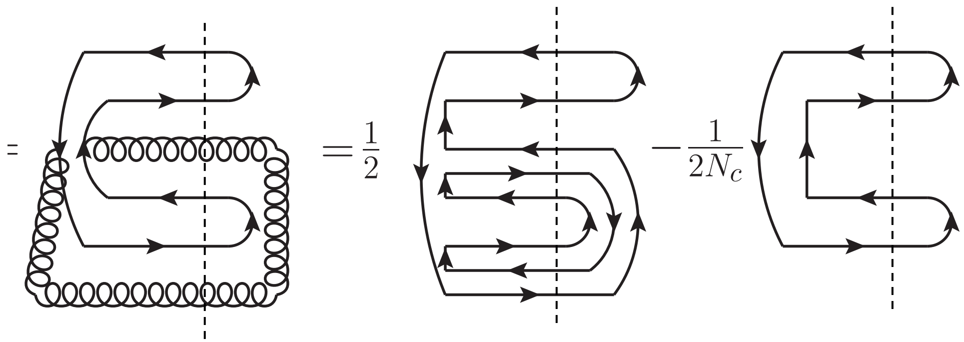
Dominguez, CM, Stasto and Xiao (2013)

this will provide the leading-Nc terms

From n to n+1 particles produced

- gluon line added within the same objects

if the extra gluon interacts both in the amplitude and in the conjugate amplitude
this adds a quadrupole to the expression for the n-parton cross section



if the extra gluon interacts only in the amplitude (or in the conjugate amplitude)
this adds a dipole to the expression for the n-parton cross section

if the extra gluon interacts in neither the amplitude or the conjugate amplitude
this splits a quadrupole into two dipoles (or adds a Nc factor to a dipole)

- other case to consider

  q-qbar pair added to n-1 partons

Conclusions

- Non-linear small-x evolution well established for dipoles
  - cornerstone: the Balitsky-Kovchegov equation
  - running-coupling corrections necessary for phenomenology

- The non-linear evolution of the quadrupole
  - brought to our attention by recent di-hadron correlation measurements
  - so far the phenomenology relies on (too many?) approximations

- Large-Nc limit: dipoles and quadrupoles sufficient for multi-particle production in pA collisions
  - worth to put some effort to accurately solve the quadrupole evolution
  - need data to constrain the initial condition (e.g. p+Pb at the LHC)