Chrial and deconfiniement transitions at non-zero temperature and QGP

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- Symmetries of QCD at $T>0$: chiral and deconfinement transitions
- Universal aspects of the chiral transition and the transition temperature
- Deconfinement: color screening
- Deconfinement: Equation of State
- Deconfinement: fluctuations of conserved charges, the role of strangeness

Calculations with improved staggered quark action:

⇒ Highly Improved Staggered Quark (HISQ) action (HotQCD, BNL-Bielefeld)

⇒ Stout (Budapest-Wuppertal)

Progress using Domain Wall Fermions: effective restoration of $U_A(1)$ symmetry

For a recent review see P.P. J.Phys. G39 (2012) 093002

,12th Workshop on Non-Perturbative Quarntum Chromodynamics, IAP, Paris, June 10-14, 2013

Symmetries of QCD at T>0

- Chiral symmetry : $m_{u,d} \ll \Lambda$

  $SU_A(2)$ symmetry $\psi \to e^{i\phi T^a \gamma_5 \psi}$ $\psi_{L,R} \to e^{i\phi_{L,R} T^a} \psi_{L,R}$

  $\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_L \psi_R \rangle + \langle \bar{\psi}_R \psi_L \rangle \neq 0$ | $\langle \bar{\psi}\psi \rangle = 0$

  $U_A(1)$ is borken by anomaly

  restored

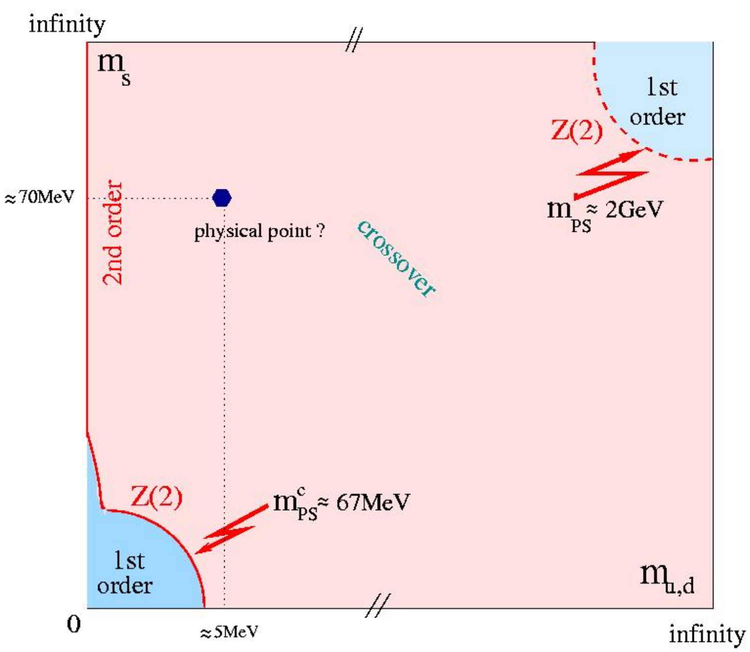
- Center (Z3) symmetry : invariance under global gauge transformation

$$A_\mu(0, \mathbf{x}) = e^{i2\pi N/3} A_\mu(1/T, \mathbf{x}), \;\; N = 1, 2, 3$$

Exact symmetry for infinitely heavy quarks $\langle L \rangle = 0$ | $\langle L \rangle \neq 0$

Polyakov loop : $L = \mathrm{tr}\mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})}$

broken



LQCD calculations with staggered quarks suggest crossover, e.g. Aoki et al, Nature 443 (2006) 675

Evidence for 2nd order transition in the chiral limit
⇒ universal properties of QCD transition:

$SU_A(2) \sim O(4)$
relation to spin models

$U_A(1)$ restoration ?

Center symmetry does not seem to play any role in QCD

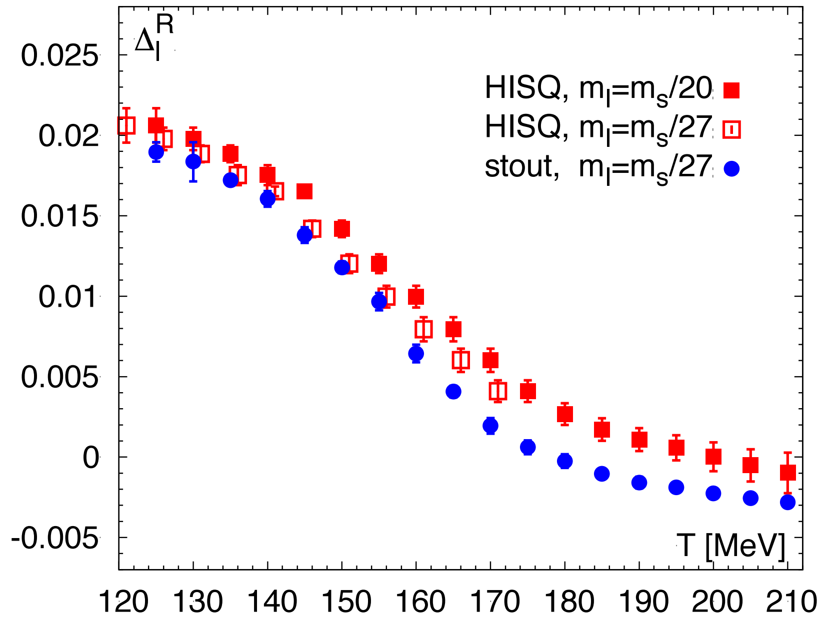
The temperature dependence of chiral condensate

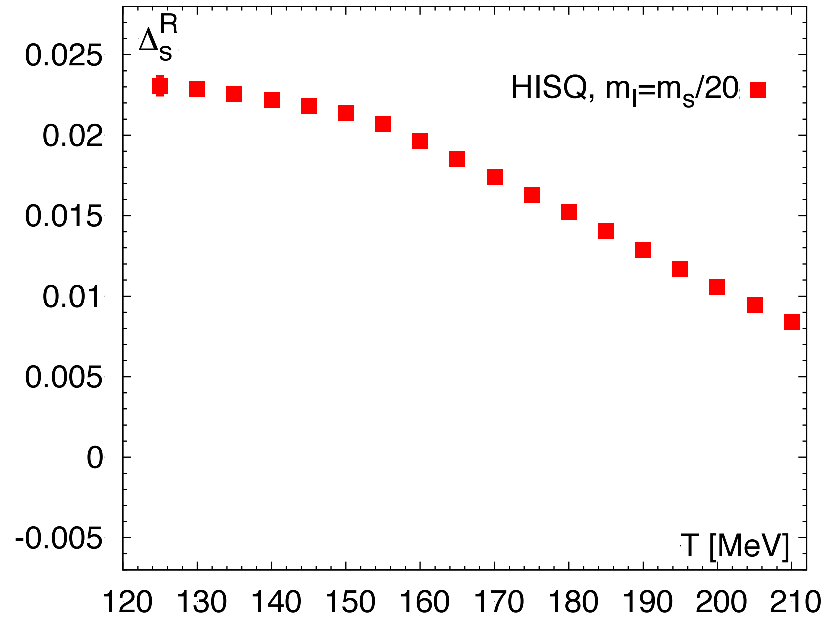
Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar{\psi}\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar{\psi}\psi\rangle_{q,T} - \langle\bar{\psi}\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

with our choice : $d = \langle\bar{\psi}\psi\rangle_{m_q=0}^{T=0}$

HotQCD : Phys. Rev. D85 (2012) 054503;
Bazavov, PP, RRD 87(2013) 094505





• after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results

• strange quark condensate does not show a rapid change at the chiral crossover => strange quark do not play a role in the chiral transition

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l) + f_s(t,h),\ t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa\frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

governed by universal *O(4)* scaling $\quad M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\ \ z = t/h^{1/\beta\delta}$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities) :

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta-1} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \qquad \chi_{t,t} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial t^2} \sim |t|^{-\alpha}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$T_{m,l} \quad = \quad T_{t,l} \quad = \quad T_{t,t} \quad = \quad \boldsymbol{T_c^0}$$

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1} f_\chi(z) + reg.\right)$$

universal scaling function has a peak at $z=z_p$

⇓

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0\frac{z_p}{z_0}H^{1/(\beta\delta)} + \ldots$$

Caveat : staggered fermions O(2)
$m_l \to 0,\ a > 0,$
proper limit $a \to 0$, before $m_l \to 0$

O(N) scaling and the transition temperature

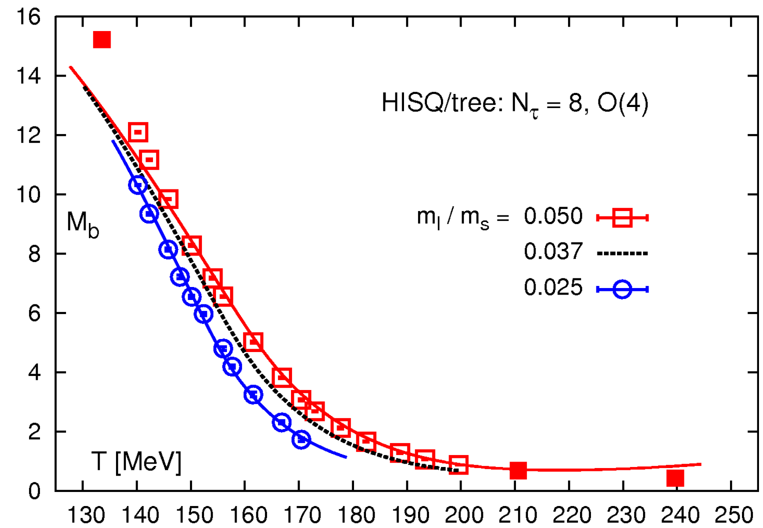
The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

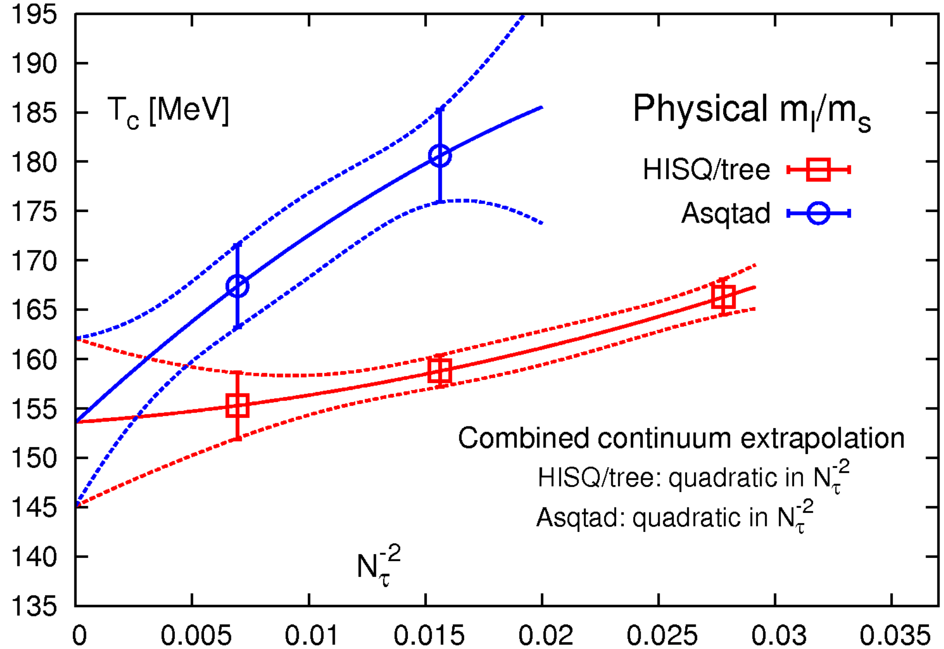
$$M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

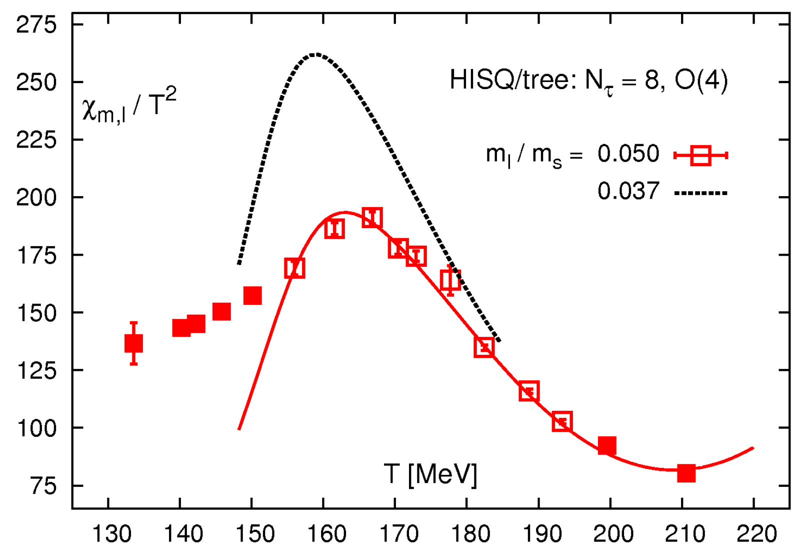
$$f_{reg}(T,H) = (a_1(T-T_c^0) + a_2(T-T_c^0)^2 + b_1)H$$

6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1

$$T_c = (154 \pm 8 \pm 1(scale))\text{MeV}$$



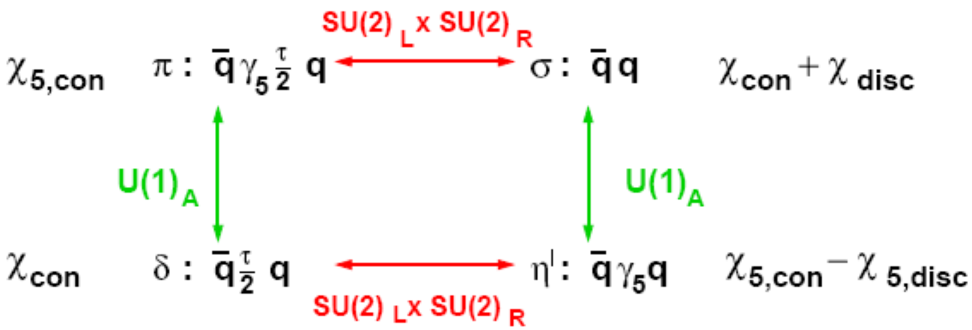




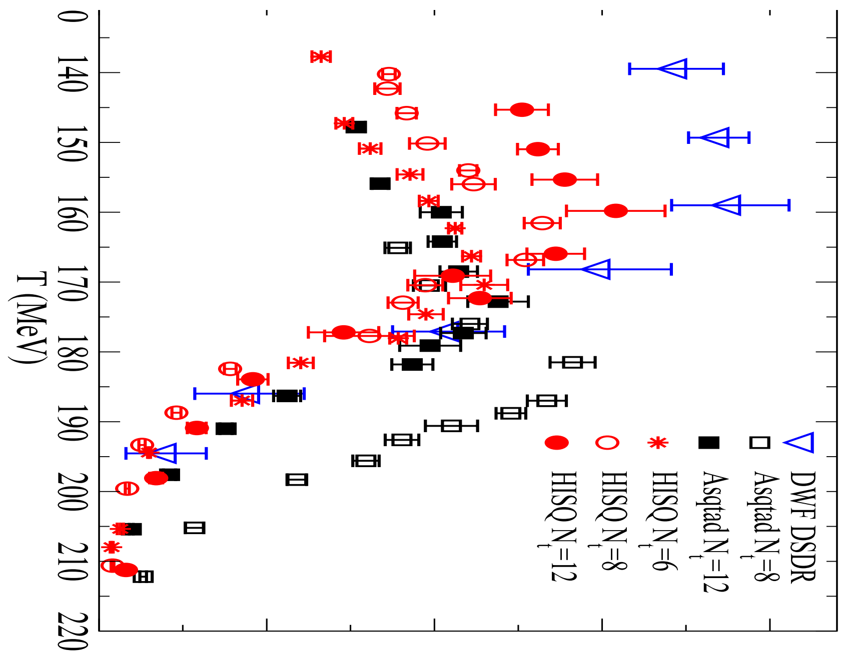
Domain wall Fermions and $U_A(1)$ symmetry restoration

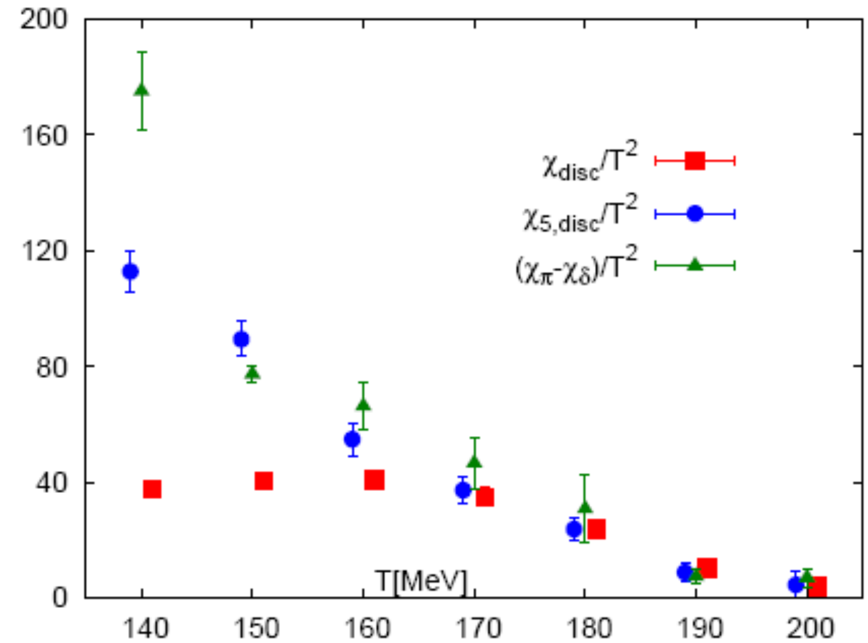
Domain Wall Fermions, Bazavov et al (HotQCD), PRD86 (2012) 094503

$$\chi_i = \int d^4x G_i(x)$$



chiral:	axial:
$\chi_\pi = \chi_\delta + \chi_{\rm disc}$	$\chi_\pi = \chi_\delta$
$\chi_\delta = \chi_\pi - \chi_{5,\rm disc}$	$\chi_\delta + \chi_{\rm disc} = \chi_\pi - \chi_{5,\rm disc}$
$\chi_{\rm disc} = \chi_{5,\rm disc}$	$\chi_{\rm disc} = -\chi_{5,\rm disc}$





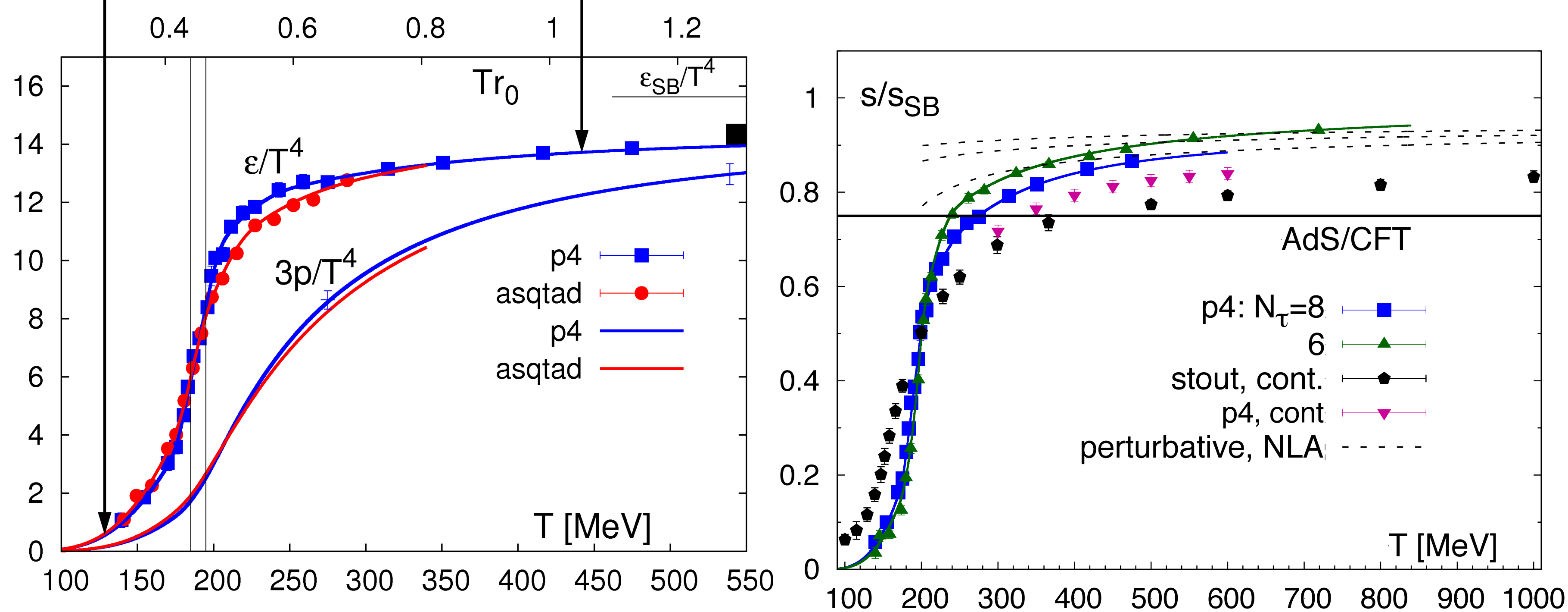
Peak position roughly agrees with previous staggered results

axial symmetry is till broken at *T=200* MeV !

Equation of state

meson gas = 3 light d.o.f.

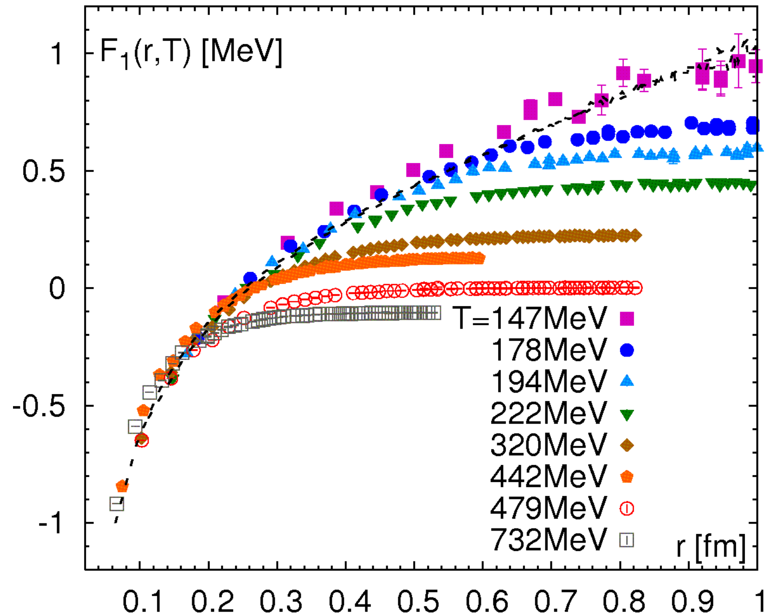
free gas of quarks and gluons = 18 quark+18 anti-quarks +16 gluons =52 mass-less d.o.f

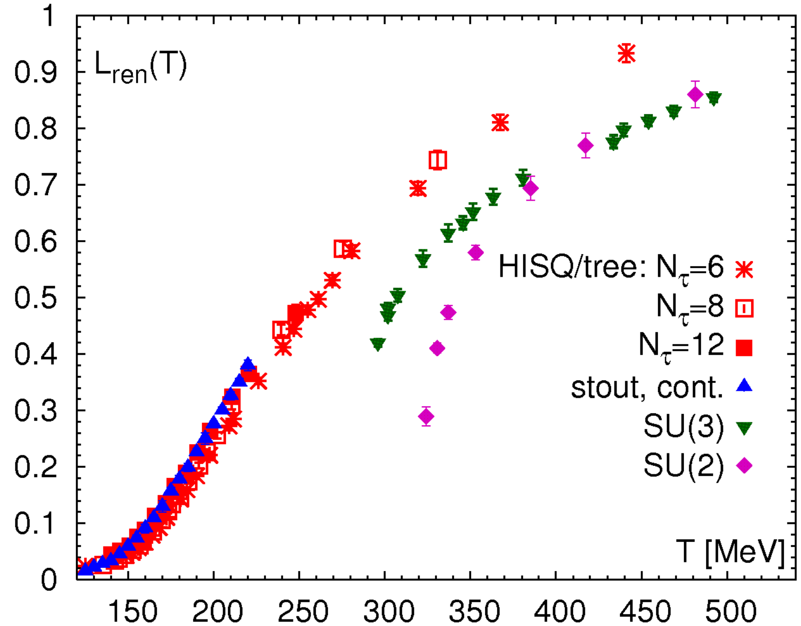


Bazavov et al (HotQCD), PRD 80 (09) 14504

- rapid change in the number of degrees of freedom at $T=160-200$MeV: deconfinement
- deviation from ideal gas limit is about 10-20% at high T consistent with the perturbative result
- discrepancies between stout and p4 (asqtad) calculations
- energy density at the chiral transition temperature $\varepsilon(T_c=154MeV)=240$ MeV/fm^3 :

Deconfinement and color screening





free energy of static quark anti-quark pair shows Debye screening at high temperatures

Pure glue ≠ QCD !

$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \Rightarrow L_{ren} = \exp(-F_Q(T)/T)$$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T),\ m_D \sim T$$

$$F_Q(T) \simeq \Lambda_{QCD} - C_F \alpha_s m_D$$

→ infinite in the pure glue theory or large in the “hadronic” phase ~600MeV

→ Decreases in the deconfined phase

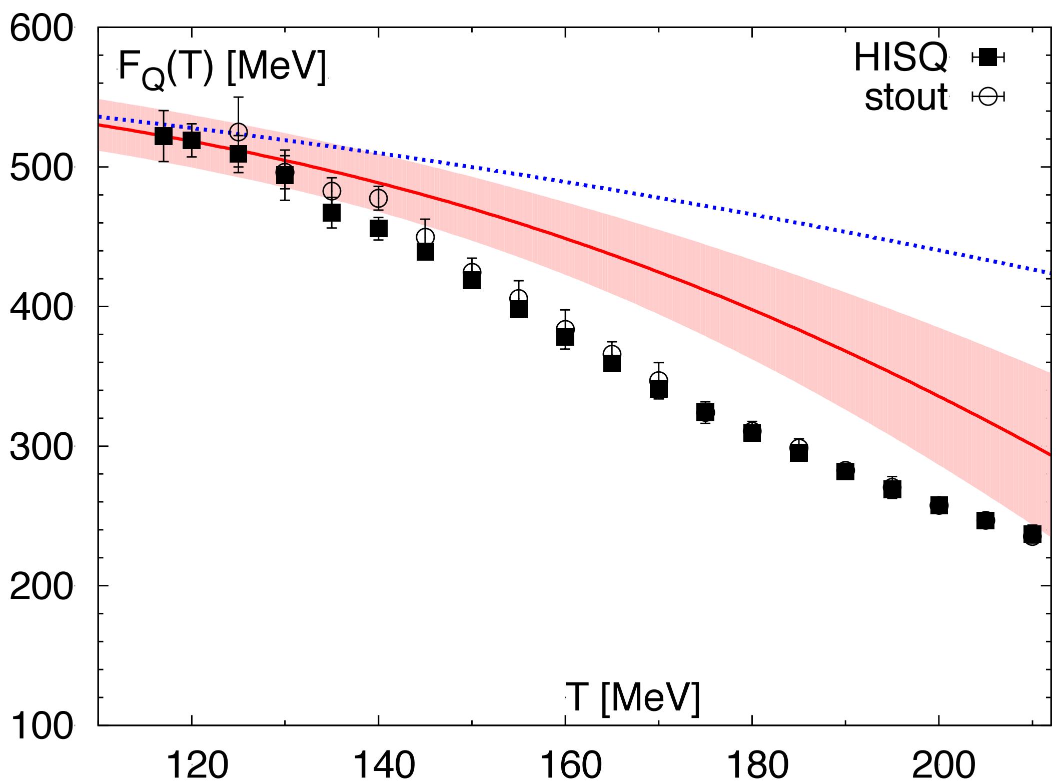
melting of bound states of heavy quarks => quarkonium suppression at RHIC: $r_{bound} > 1/m_D$

Polyakov and gas of static-light hadrons

$$Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_n^{Q\bar{Q}}(r \to \infty)/T)$$

Energies of static-light mesons: $E_n^{Q\bar{Q}}(r \to \infty) = M_n - m_Q$

Free energy of an isolated static quark: $F_Q(T) = -\frac{1}{2}(T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T))$



Megias, Arriola, Salcedo,
PRL 109 (12) 151601

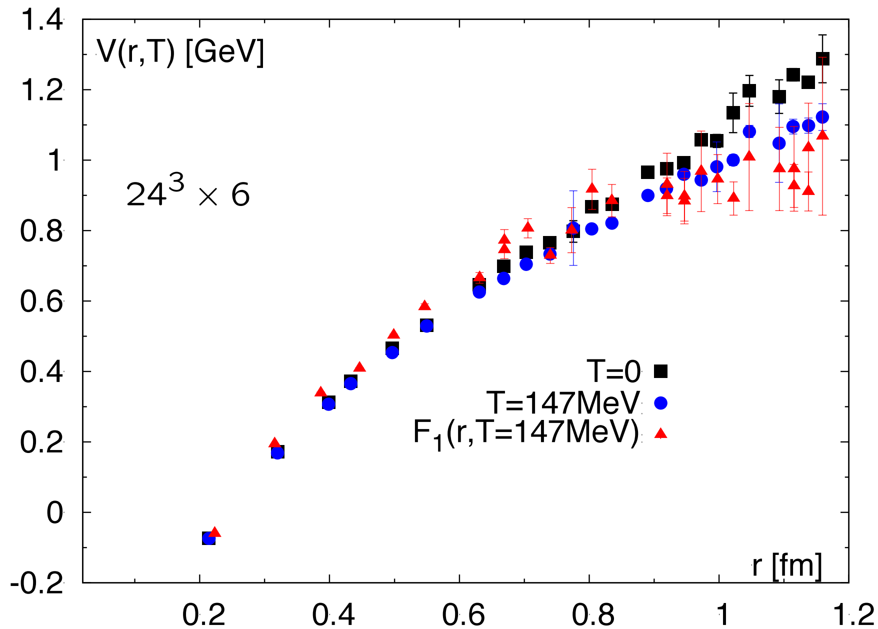
Bazavov, PP, PRD 87 (2013) 094505

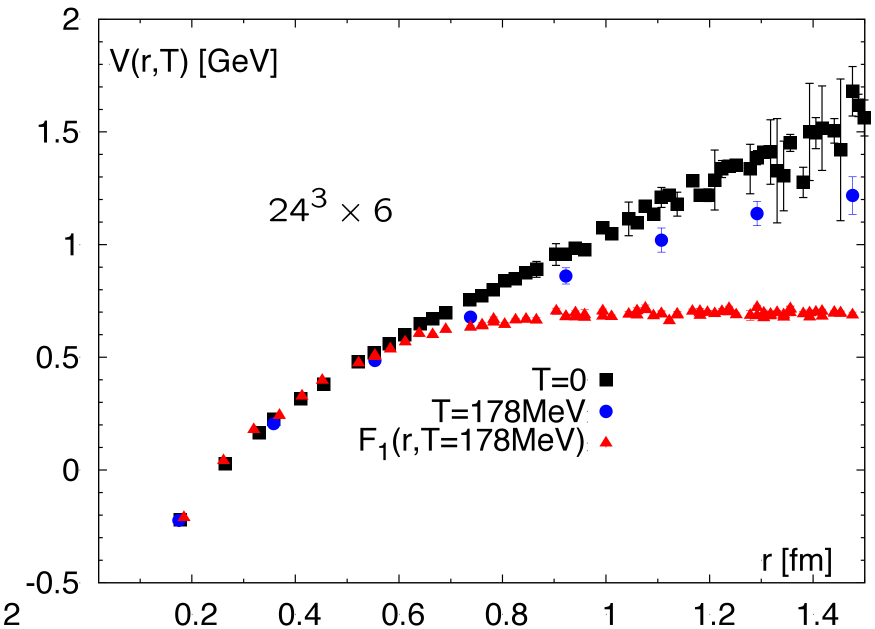
Ground state and first excited states are from lattice QCD
Michael, Shindler, Wagner, arXiv1004.4235
Wagner, Wiese, JHEP 1107 016,2011

Higher excited state energies are estimated from potential model

Gas of static-light mesons only works for T < *145* MeV

Extracting the potential at T>0



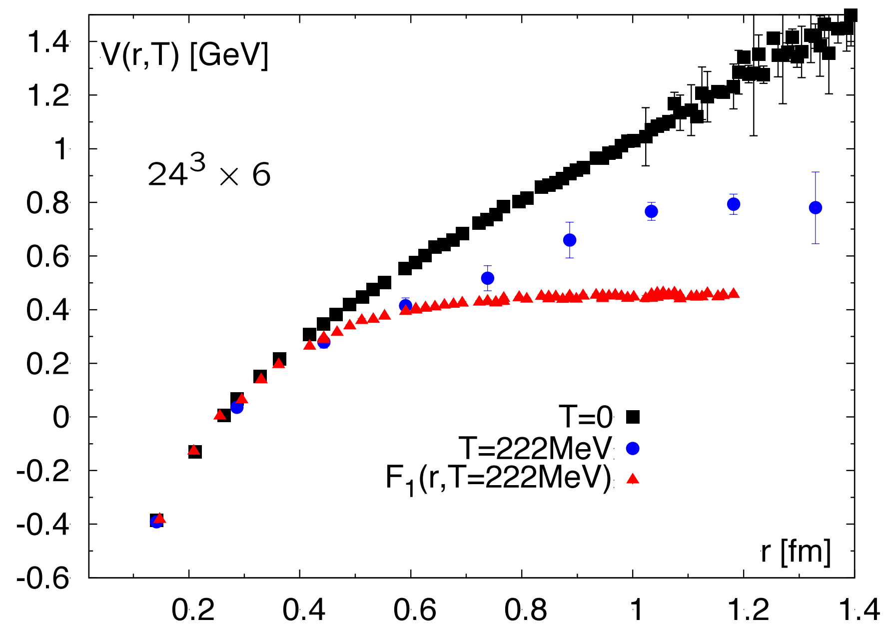


Bazavov, PP, arXiv:1210.6314

•Calculation of the Wilson loops at *T>0* + single state dominance => static quark potential *V(r,T)*

• for T=147 MeV the potential is the same as at *T=0* and agrees with the singlet free energy

• for *150* MeV < *T* < *200* MeV the potential only slightly differs from the *T=0* potential and much larger than the singlet free energy, only for *T>200* MeV it is screened



QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_Q}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

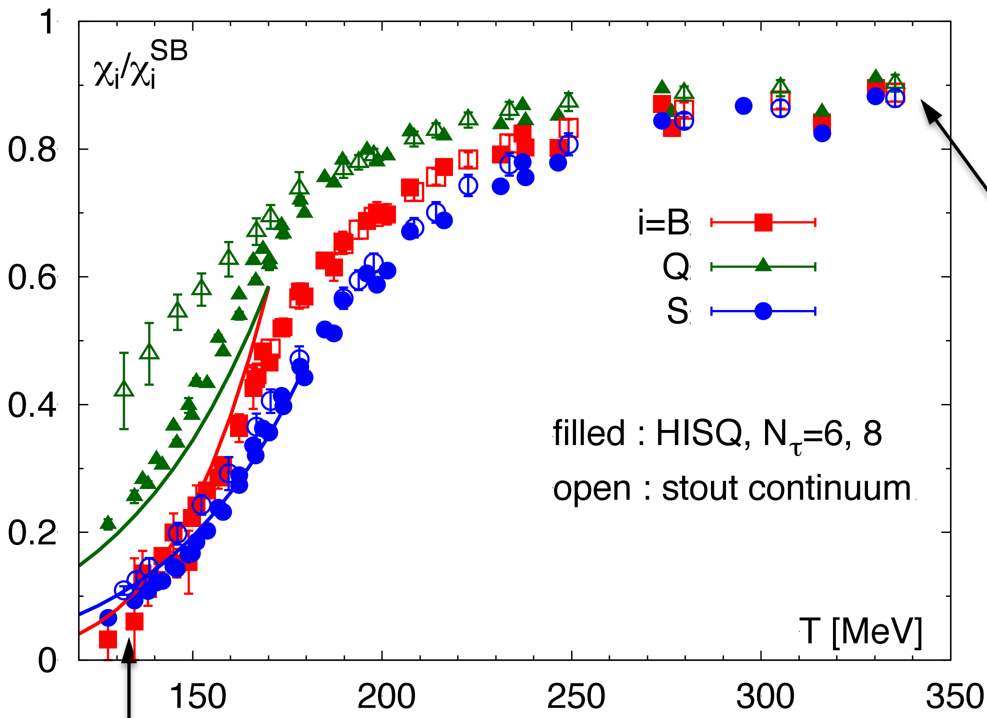
Computation of Taylor expansion coefficients reduces to calculating the product of inverse fermion matrix with different source vectors **=> can be done effectively on GPUs**

Deconfinement : fluctuations of conserved charges

$$\chi_B^{SB} = \frac{1}{VT^3}(\langle B^2\rangle - \langle B\rangle^2)$$ baryon number

$$\chi_Q^{SB} = \frac{1}{VT^3}(\langle Q^2\rangle - \langle Q\rangle^2)$$ electric charge

$$\chi_S^{SB} = \frac{1}{VT^3}(\langle S^2\rangle - \langle S\rangle^2)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_B^{SB} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

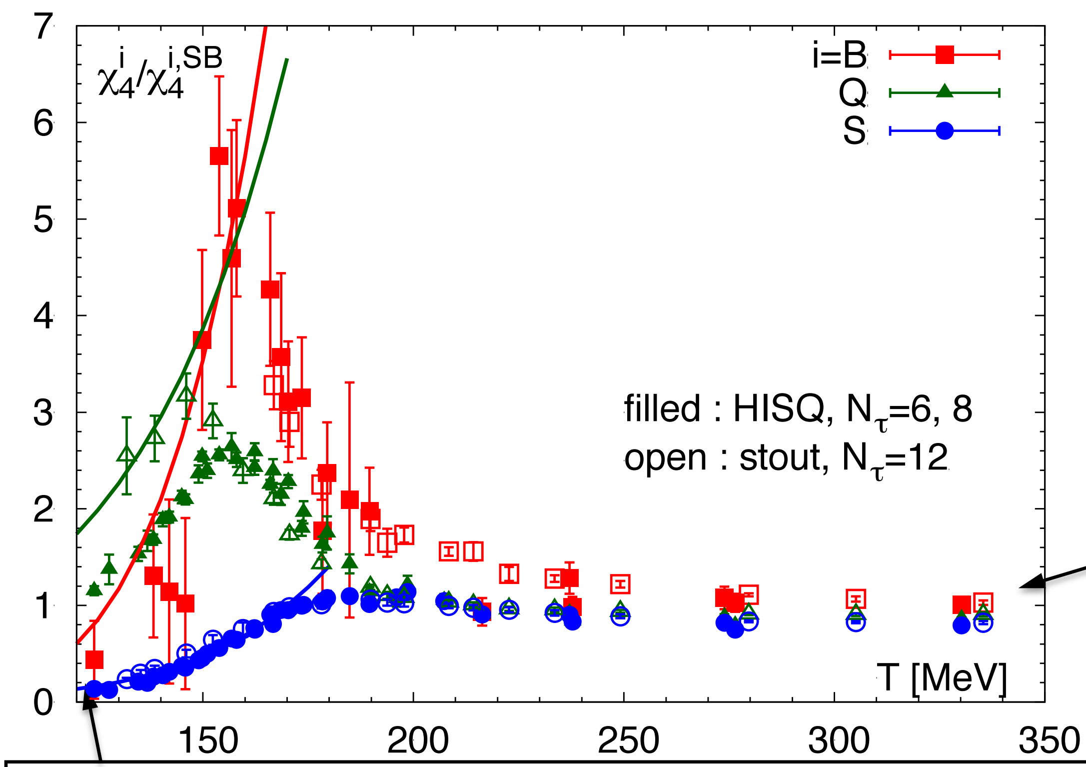
BW: JHEP 1201 (2012) 138,

Deconfinement : fluctuations of conserved charges

$$\chi_4^B = \frac{1}{VT^3}(\langle B^4\rangle - 3\langle B^2\rangle^2)$$ baryon number

$$\chi_4^Q = \frac{1}{VT^3}(\langle Q^4\rangle - 3\langle Q^2\rangle^2)$$ electric charge

$$\chi_4^S = \frac{1}{VT^3}(\langle S^4\rangle - 3\langle S^2\rangle^2)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

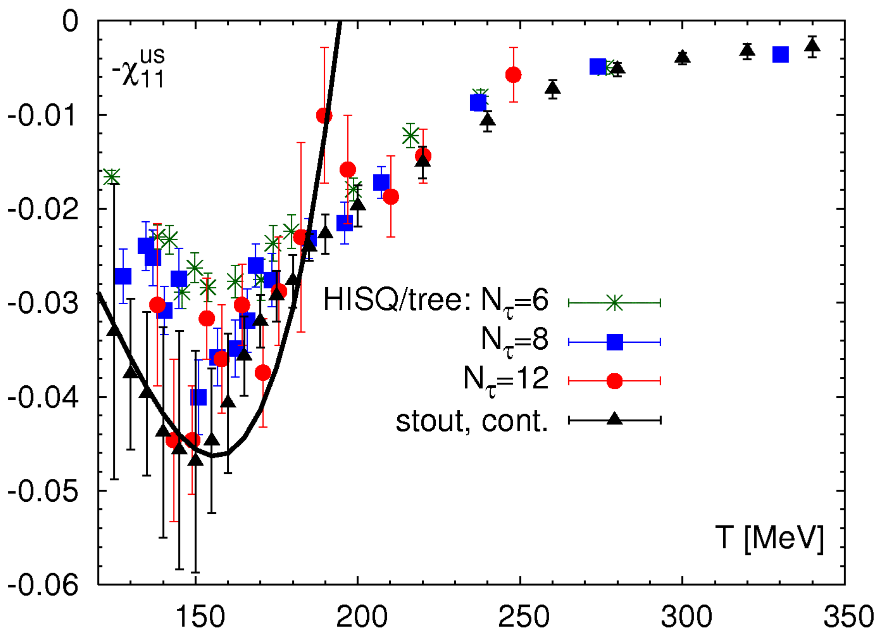
$$\chi_4^B{}_{SB} = \frac{2}{9\pi^2} \qquad \chi_4^Q{}_{SB} = \frac{4}{3\pi^2}$$

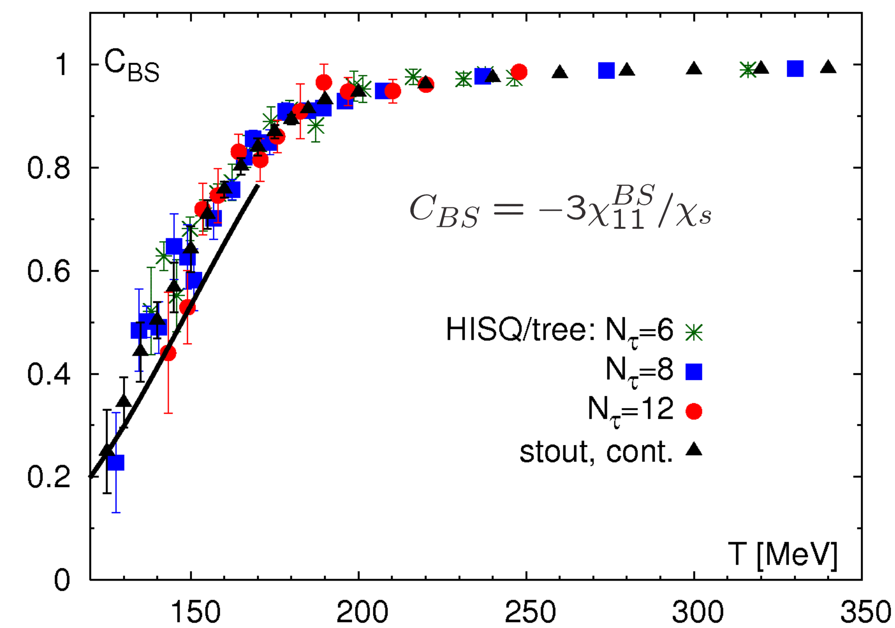
$$\chi_4^S{}_{SB} = \frac{6}{\pi^2}$$

conserved charges carried by light quarks

BNL-Bielefeld : talk by C. Schmidt
BW: talk by Borsanyi
@ Confinement X conference

Correlations of conserved charges





P.P. J.Phys. G39 (2012) 093002

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)* => weakly interacting quark gas

- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ (100-150) MeV

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

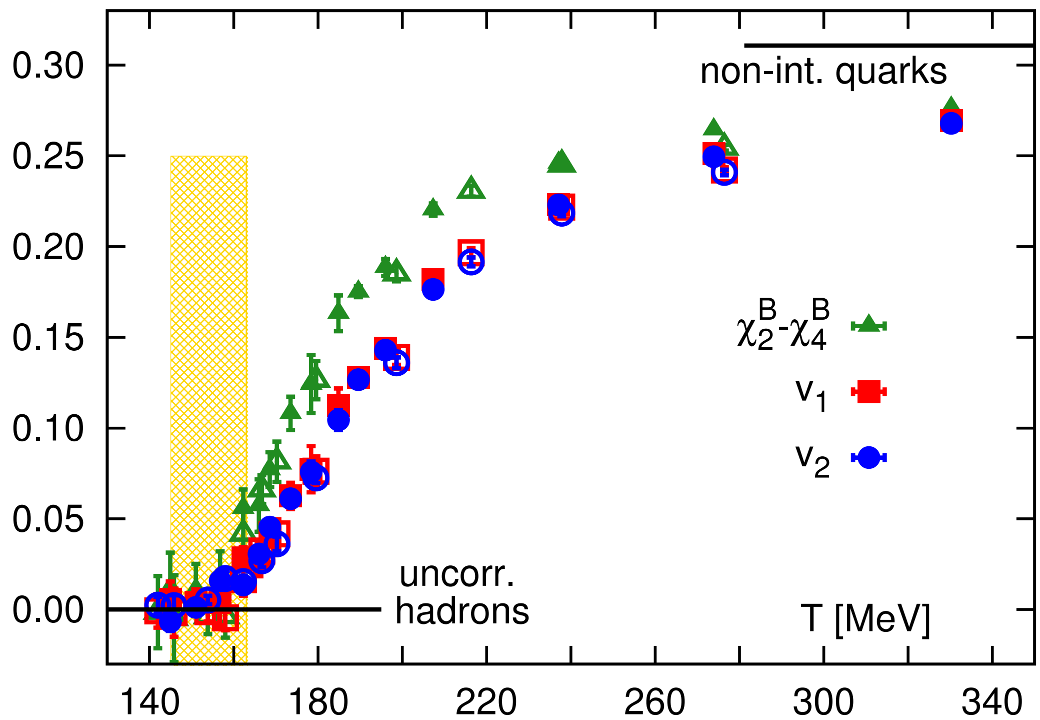
$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} M(T) \cosh\left(\frac{\mu_S}{T}\right) +$$
$$B_{S=1}(T) \cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T) \cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T) \cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$
$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- **v_1** and **v_2** do vanish within errors at low *T*
- **v_1** and **v_2** rapidly increase above the transition region, eventually reaching non-interacting quark gas values

BNL-Bielefeld, arXiv:1304.7220



Deconfinement of strangeness (cont'd)

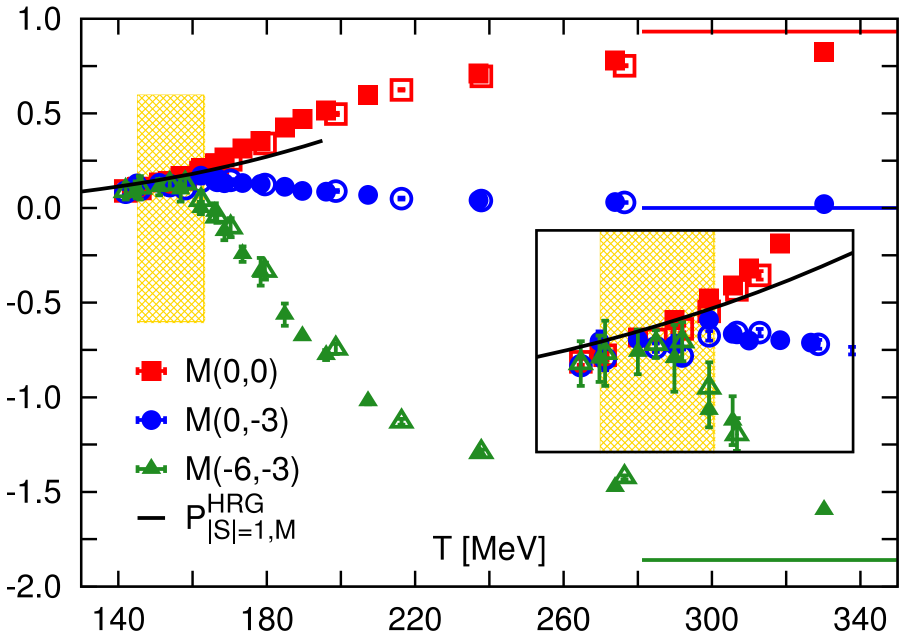
Using the six Taylor expansion coefficients related to strangeness

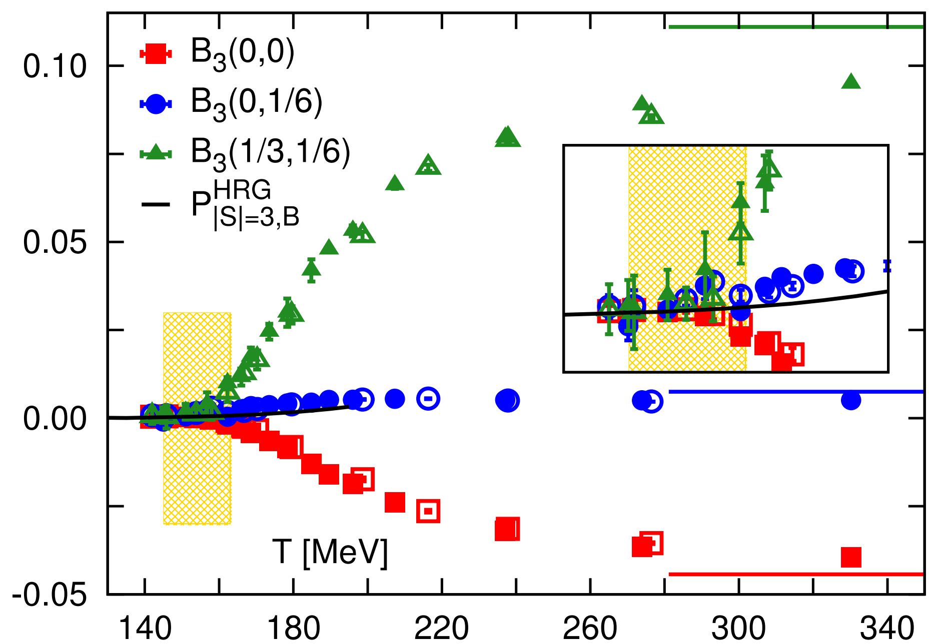
$$\chi_2^S,\ \chi_4^S,\ \chi_{13}^{BS},\ \chi_{22}^{BS},\ \chi_{31}^{BS}$$

it is possible to construct combinations that give

$$M(T),\ B_{S=1}(T),\ \ B_{S=2}(T),\ \ B_{S=3}(T)$$

up to terms $c_1 v_1 + c_2 v_2$





Hadron resonance gas descriptions breaks down for all strangeness sectors above T_c

BNL-Bielefeld, arXiv:1304.7220

Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

2nd order quark number fluctuations

4th order quark number fluctuations

Andersen, Mogliacci, Su, Vuorinen, PRD87 (2013) 074003

- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd order quark number fluctuations
- For 4th order the weak coupling results are in reasonable agreement with lattice

Summary

• Lattice QCD show that at high temperatures strongly interacting matter undergoes a transition to a new state QGP characterized by deconfinement and chiral symmetry restoration

• We see evidence that provide evidence that the relevant degrees of freedom are quarks and gluons; lattice results agree well with perturbative calculations, while at low *T* thermodynamics can be understood in terms of hadron resonance gas The deconfinement transition can understood as transition from hadron resonance gas to quark gluon gas, it starts at around the chiral crossover but it is very gradual, including for strangeness

• The chiral aspects of the transition are very similar to the transition in spin system in external magnetic fields: it is governed by universal scaling. Different calculations with improved staggered actions agree in the continuum limit resulting in a chiral transition temperature *(154 ± 9)* MeV

•The effective restoration of $U_A(1)$ symmetry happens for $T>200$MeV and thus does not effect the chiral transition

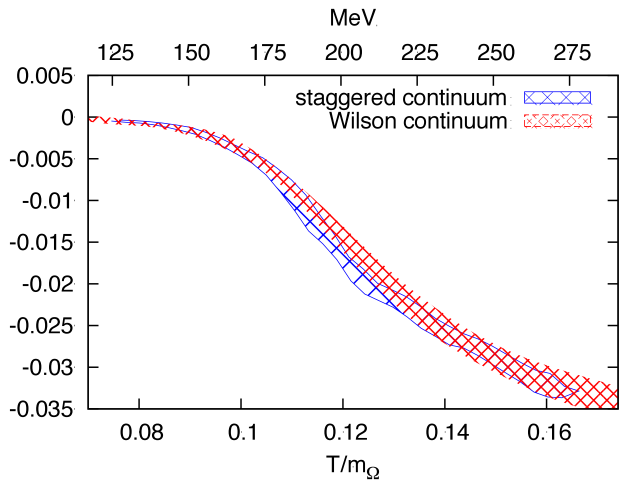
•Color screening can be seen at temperatures $T>200$ MeV

Staggered versus Wilson and Overlap Fermioms

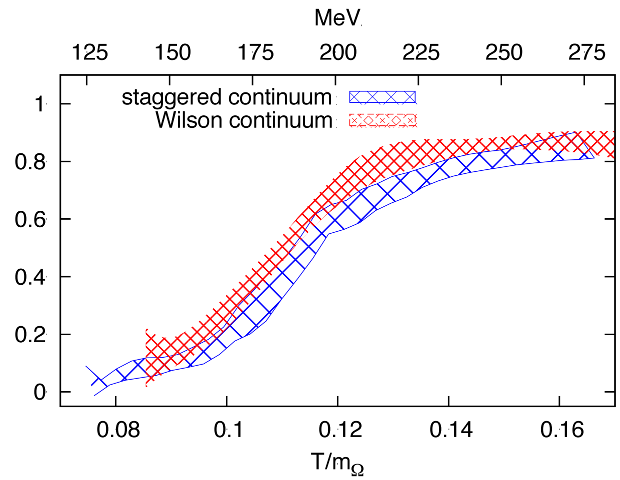
Comparison with Wilson Fermion calculations, $m_\pi \approx 500$ MeV,

Borsányi et al, arXiv:1205.0440

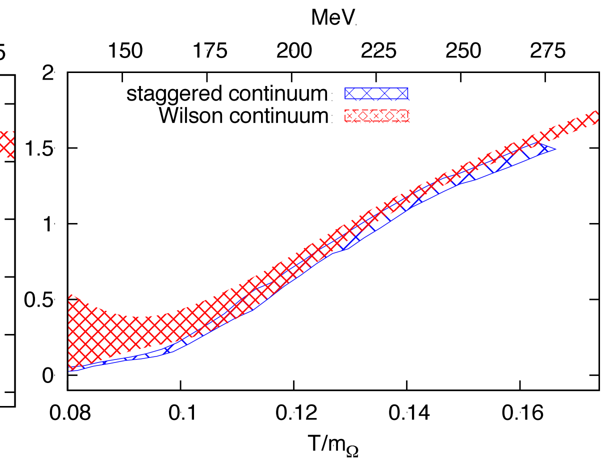
Chiral condensate



Strangeness susceptibility

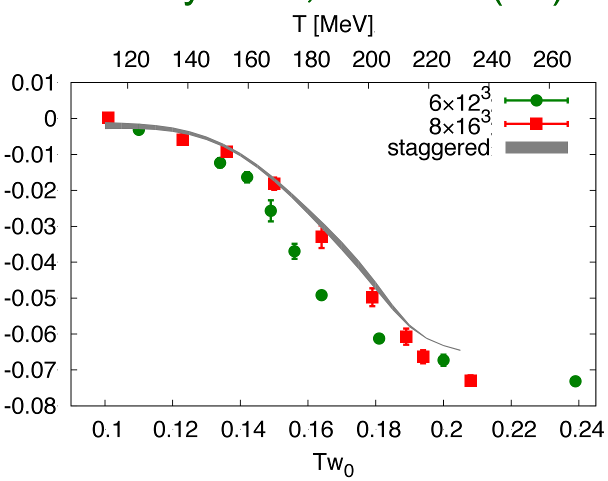


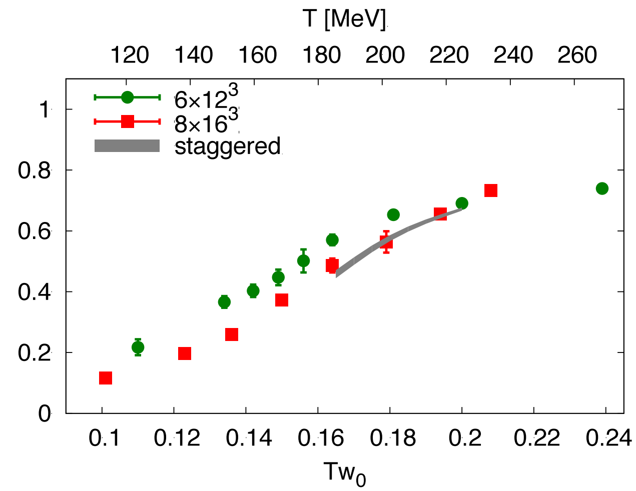
Polyakov loop

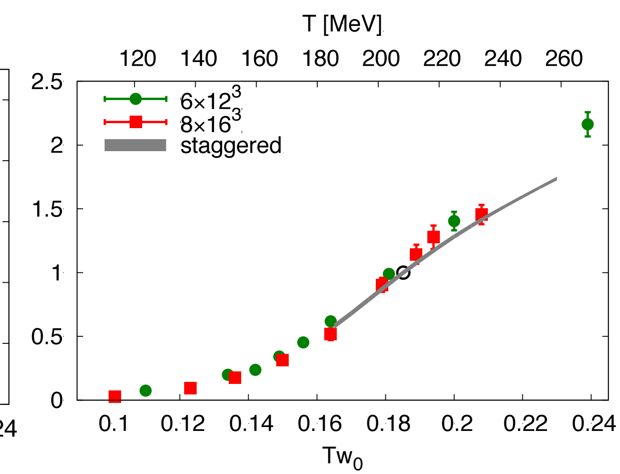


Comparison with overlap Fermion calculations, $m_\pi \approx 350$ MeV

Borsányi et al, PLB713 (12) 342







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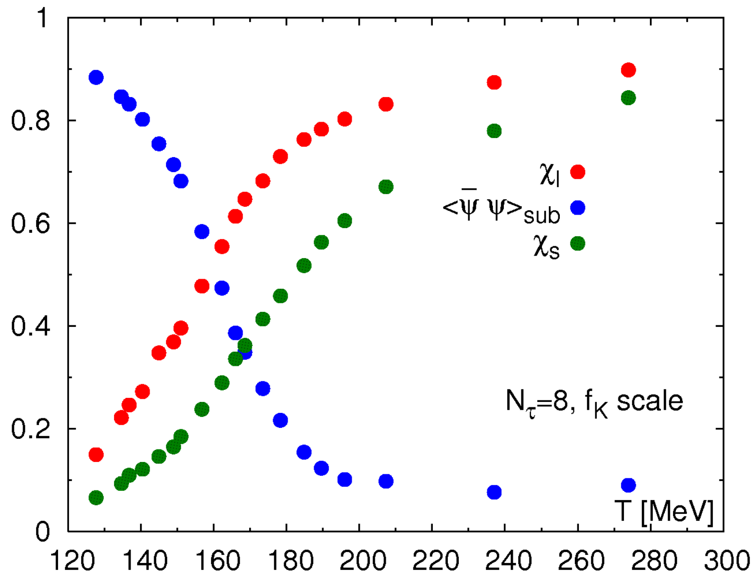
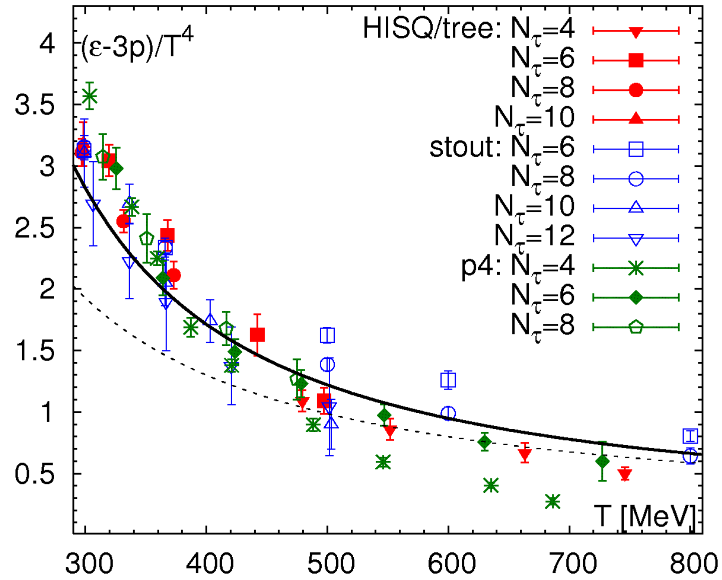
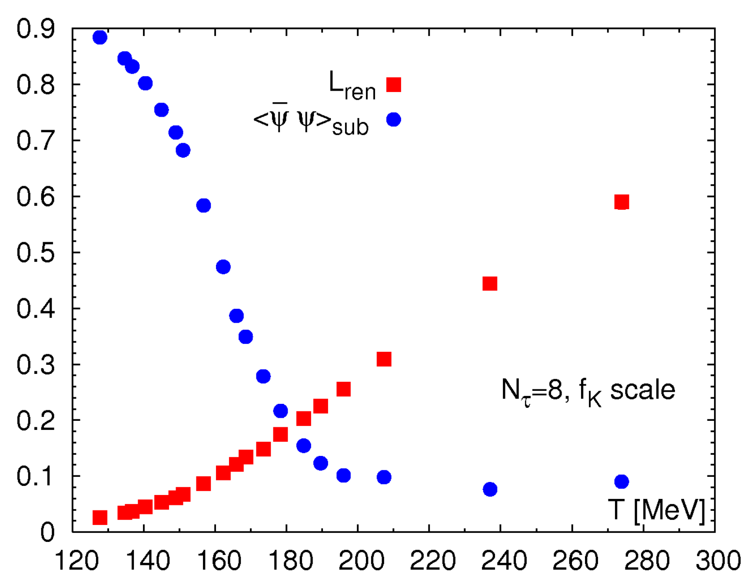


χl
<ψ̄ ψ>sub
χs
Nτ=8, fK scale
T [MeV]



(ε-3p)/T4
HISQ/tree: Nτ=4
Nτ=6
Nτ=8
Nτ=10
stout: Nτ=6
Nτ=8
Nτ=10
Nτ=12
p4: Nτ=4
Nτ=6
Nτ=8
T [MeV]



Lren
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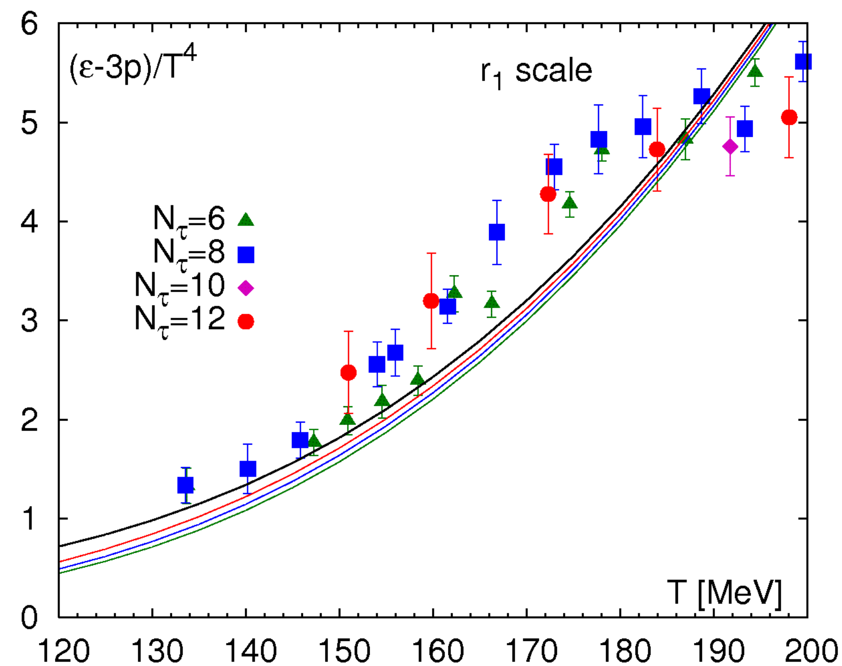
Nτ=8, fK scale
T [MeV]



(ε-3p)/T4
r1 scale
Nτ=6
Nτ=8
Nτ=10
Nτ=12
T [MeV]

Improved staggered calculations at finite temperature

cutoff effects are different in :

$a = 1/(TN_\tau)$

low T region

$T<200$ MeV

$\mathcal{O}(\alpha_s^n (a\Lambda_{QCD})^2)$ errors

a>0.125fm

hadronic degrees of freedom

improvement of the flavor symmetry is important $\longrightarrow$ fat links

$N_\tau = 8$

for #flavors < 4 rooting trick

$detD \rightarrow (detD)^{\frac{n_f}{4}}$

high-T region

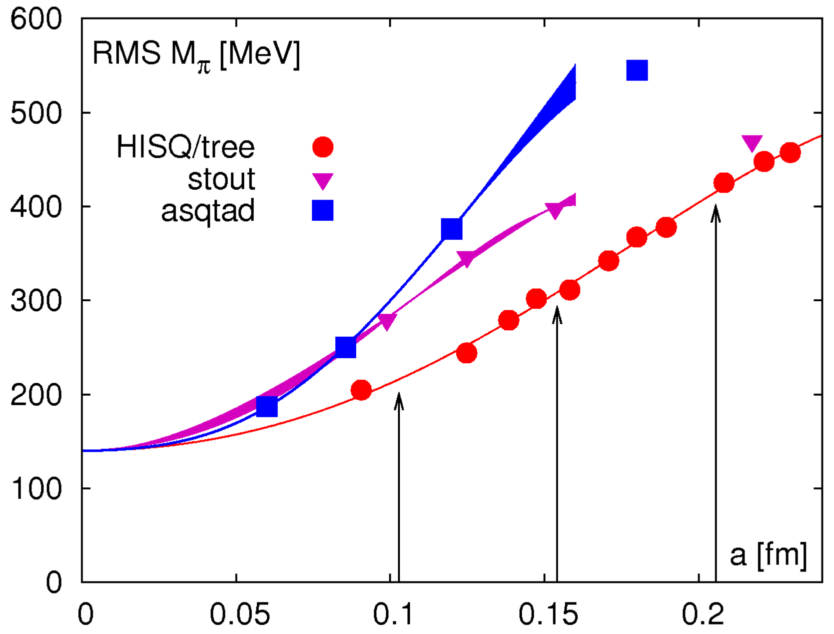
$T>200$MeV

$\mathcal{O}((aT)^2)$ errors

a<0.125fm

quark degrees of freedom

quark dispersion relation



p4, asqtad, HISQ, stout